



ISTerre

Institut des Sciences de la Terre



Influence of rotation on thermal convection. Applications to planetary flows

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OSUG

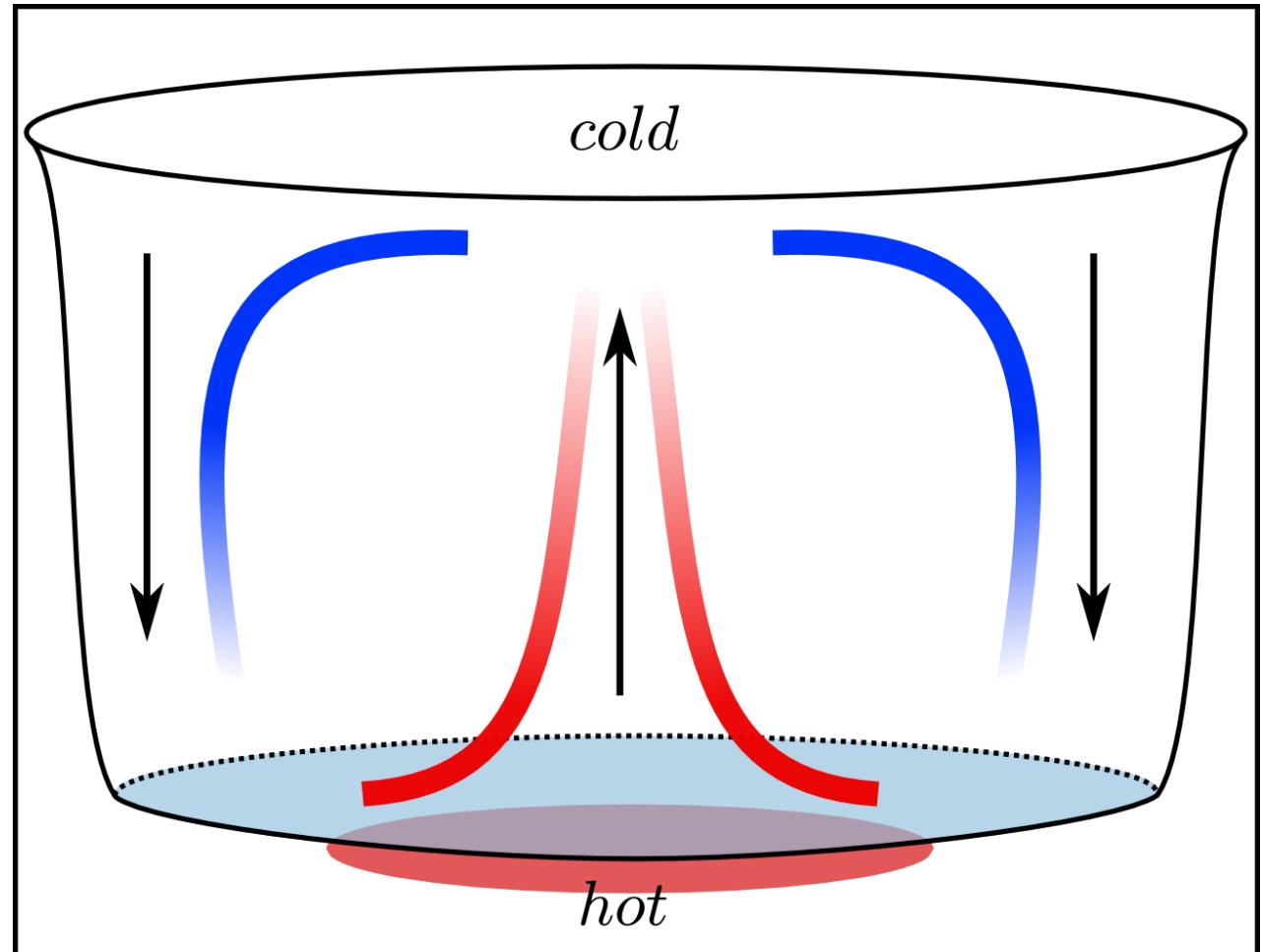


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de Grenoble

LRR, X, 03/18

Thermal convection

Cool, high density fluid sinks

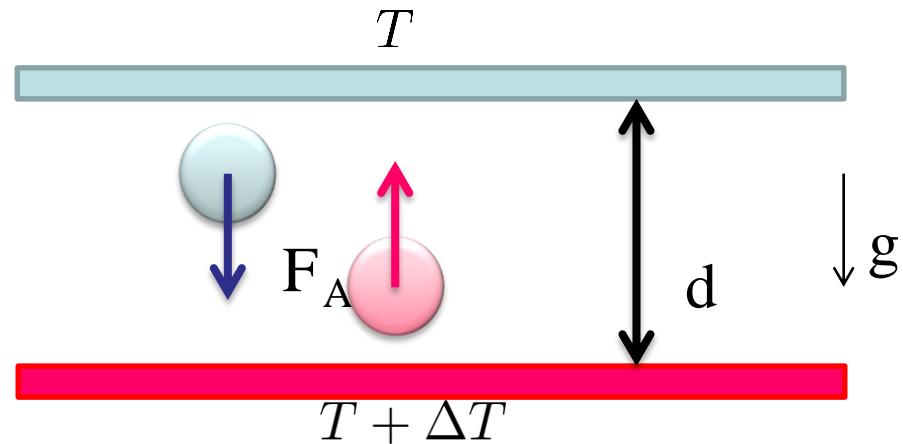


Warm, low density fluid rises

Rayleigh number

Free fall time

$$\tau_{ff} = \sqrt{\frac{d}{\alpha g \Delta T}}$$



Rayleigh number

$$Ra = \frac{\tau_\nu \tau_\kappa}{\tau_{ff}^2} = \frac{\alpha g \Delta T d^3}{\kappa \nu}$$

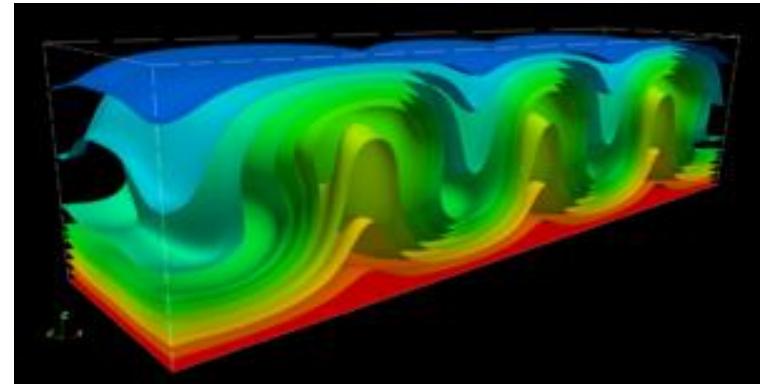
$$\tau_\nu = d^2 / \nu$$

$$\tau_\kappa = d^2 / \kappa$$

critical Rayleigh number

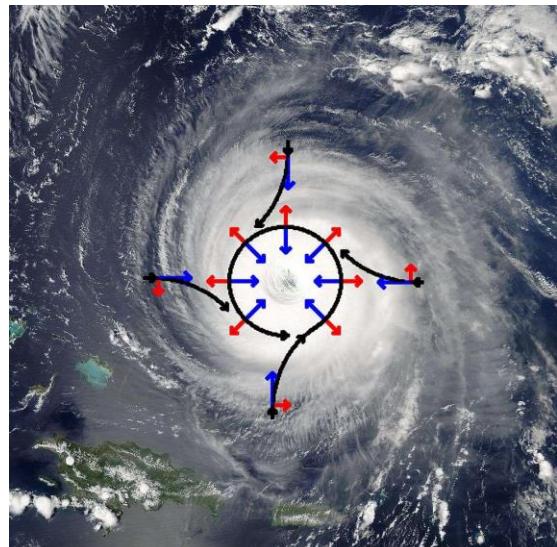
$$Ra_c \approx 1708$$

Thermal convection



Rotation

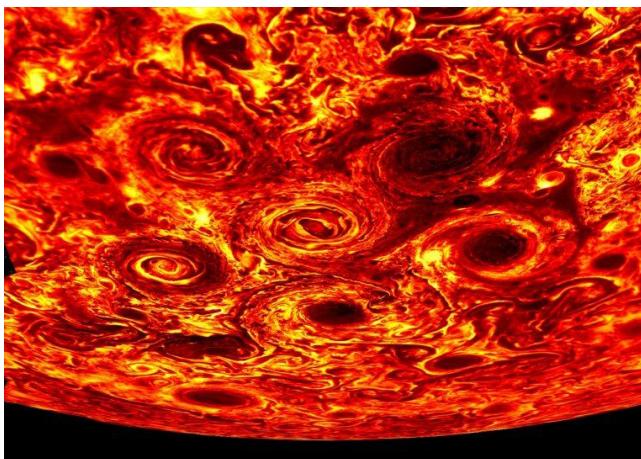
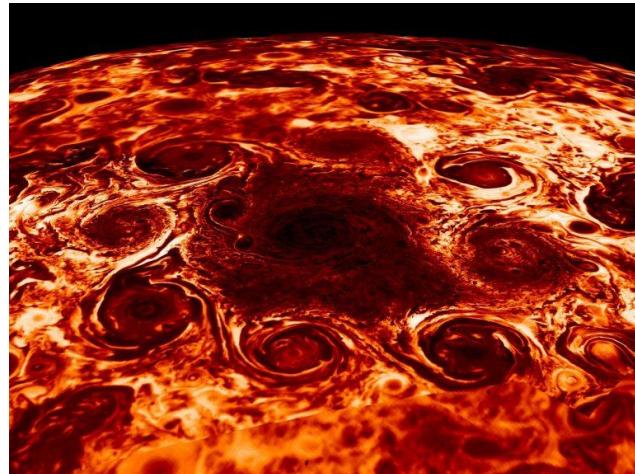
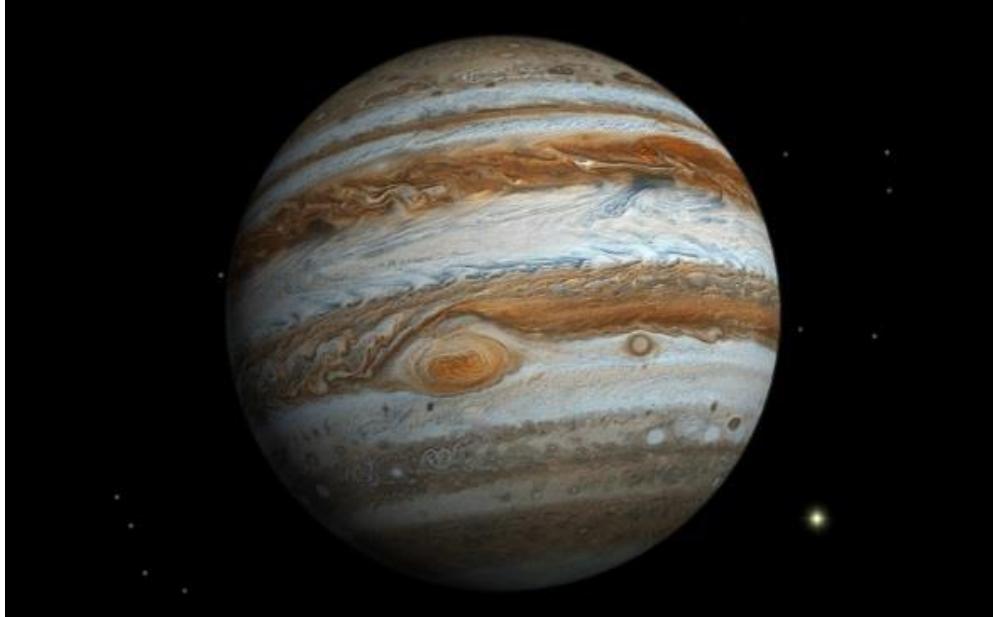
- Référentiel non galiléen.
- Force centrifuge
- Force de Coriolis



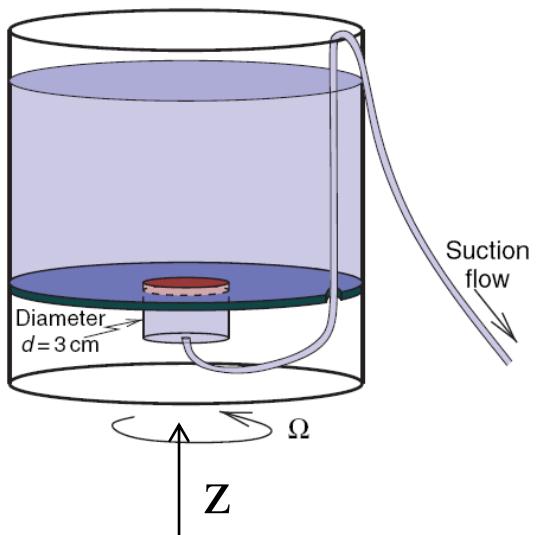
Cyclone hémisphère nord

Équilibre géostrophique
forces de pression (bleu) = forces de coriolis (rouge)

On Jupiter (Juno mission)

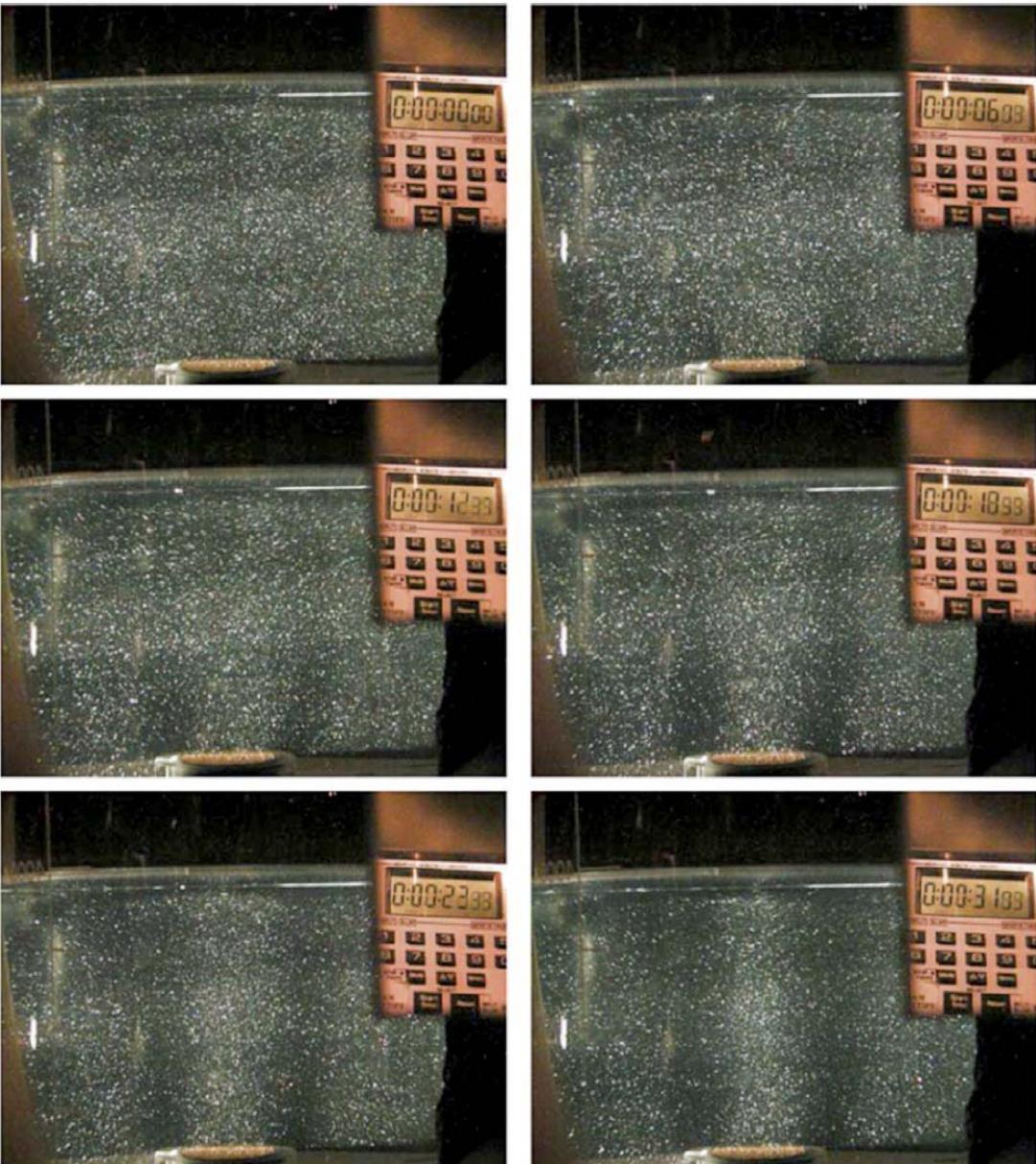


Proudman Taylor theorem



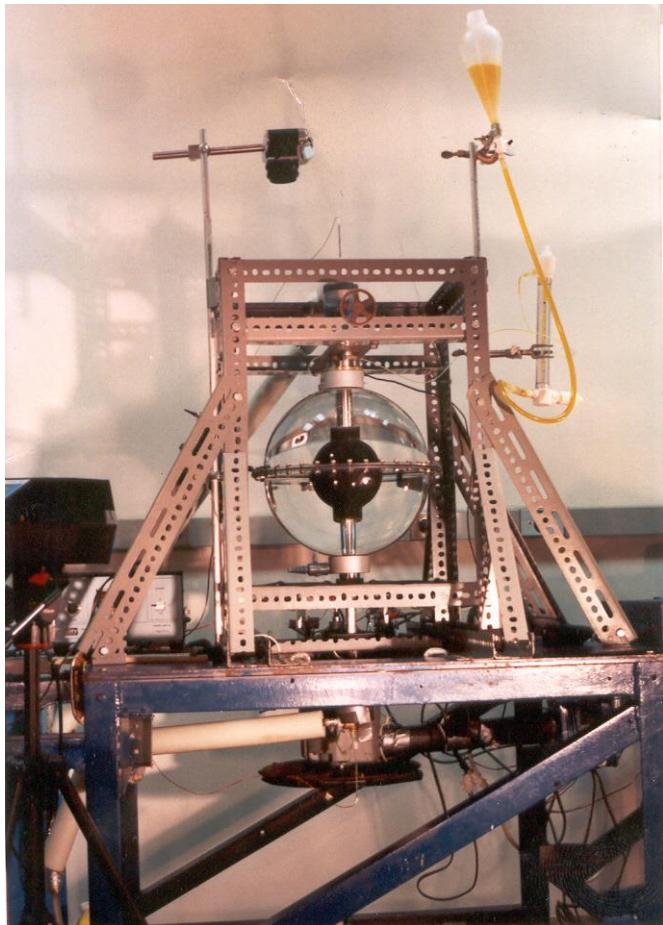
$$\rho\Omega \times \mathbf{u} = -\nabla P$$

$$(\Omega e_z \cdot \nabla) \mathbf{u} = \frac{\partial \mathbf{u}}{\partial z} = 0$$



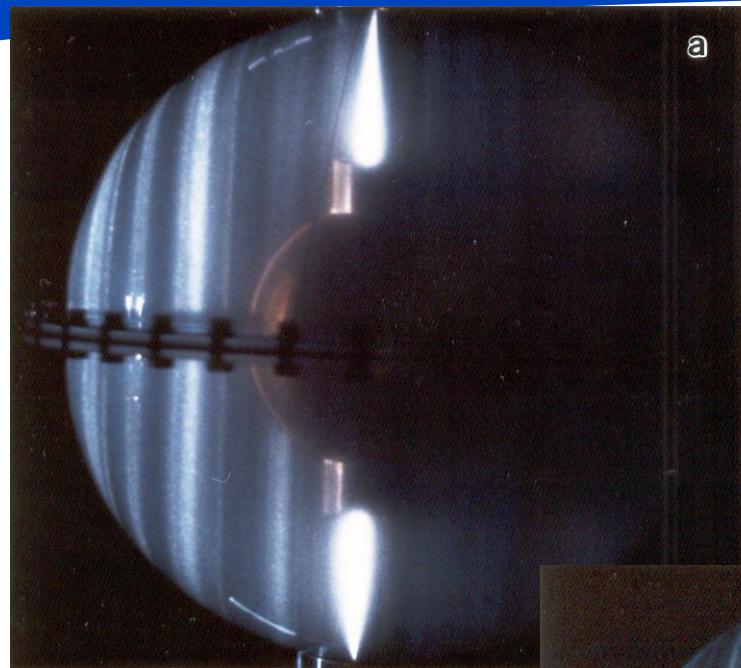
Rotational influence on thermal convection, P. Cardin et al.

Thermal convection in a rotating sphere

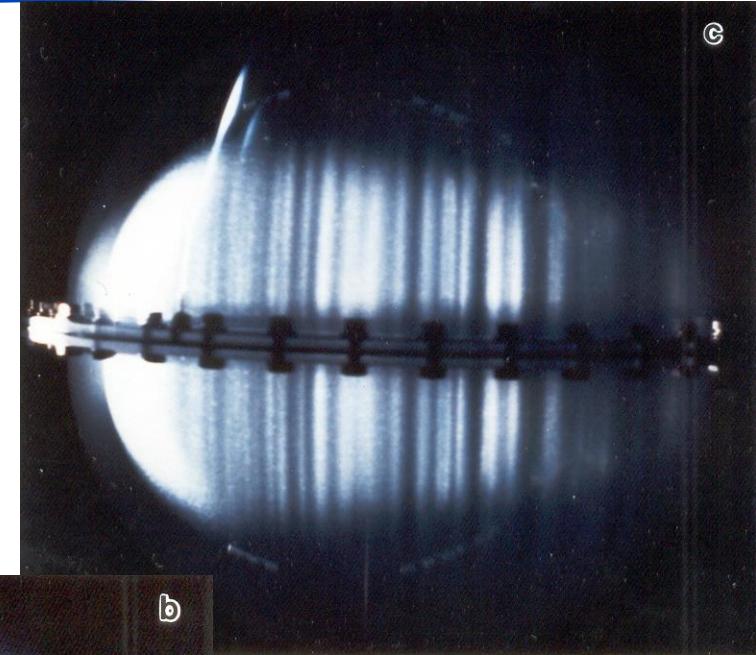


- Working fluid is water
- Rotation is fast >100 rpm
- Gravity is the centrifugal acceleration
- Temperature gradient is reversed : cooling inner sphere
- Flakes and fluoresceine

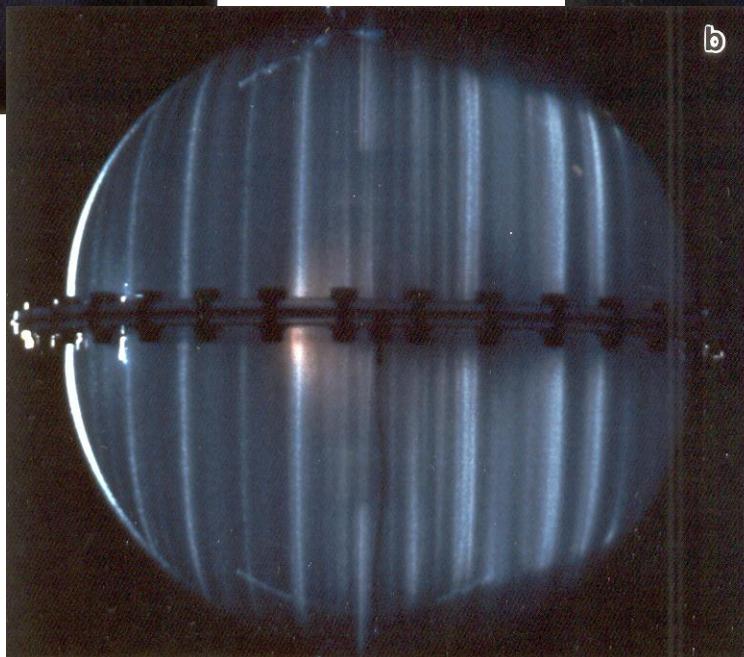
Columnar flow



a



b



Side view
with a
sheet of light

Equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{2}{Ek} \mathbf{e}_z \times \mathbf{u} = -\nabla p + \nabla^2 \mathbf{u} + Ra \Theta \mathbf{r},$$

$$\nabla \cdot \mathbf{u} = 0,$$

$$\frac{\partial \Theta}{\partial t} + \mathbf{u} \cdot \nabla \Theta - \frac{2}{Pr} r u_r = \frac{1}{Pr} \nabla^2 \Theta.$$

$$Pr = \frac{\nu}{\kappa}$$

$$Ra = \frac{\alpha g \Delta T d^3}{\kappa \nu}$$

$$Ek = \frac{\nu}{\Omega R^2}$$

Natural systems

$$Pr = \frac{\nu}{\kappa} \simeq 0.01 - 10$$

Atmosphères
Océans
Noyaux planétaires

$$Ek = \frac{\nu}{\Omega R^2} = 10^{-18} - 10^{-11}$$

Planètes géantes

$$Ra = \frac{\alpha g \Delta T d^3}{\kappa \nu} > 10^{30}$$

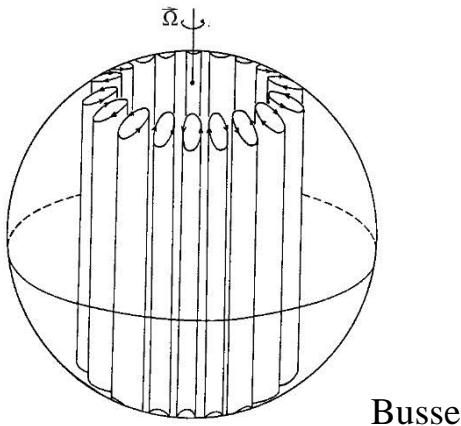
Plasmas
Étoiles
Zones convectives

Disques proto planétaires

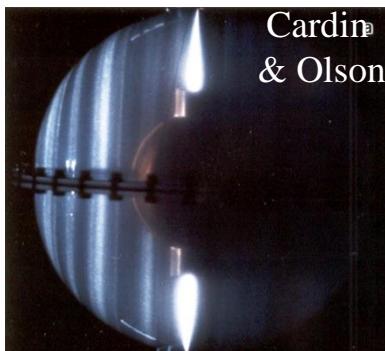
Thermal convection in a rotating sphere

$$Pr = \frac{\nu}{\kappa}$$

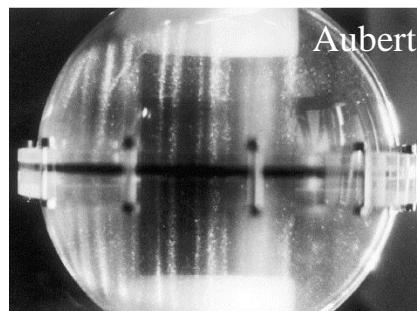
$$Ek = \frac{\nu}{\Omega R^2}$$



*Quasi Geostrophic
Thermal
Rossby Waves*



Cardin
& Olson



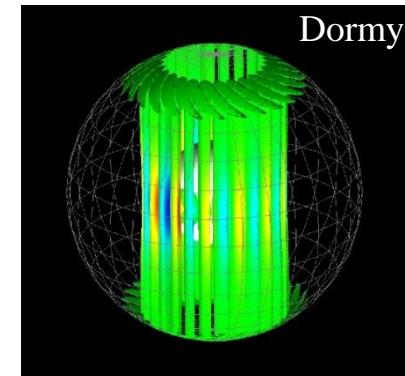
Aubert

$$Ra_c = C_r \left(\frac{Pr}{1 + Pr} \right)^{4/3} Ek^{-4/3}$$

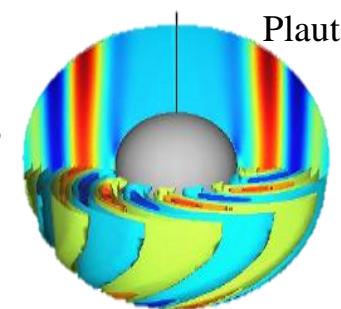
$$m_c = C_m \left(\frac{Pr}{1 + Pr} \right)^{1/3} Ek^{-1/3}$$

$$\omega_c = C_\omega (Pr(1 + Pr)^2)^{-1/3} Ek^{-2/3}$$

Busse, 70
Jones et al, 00

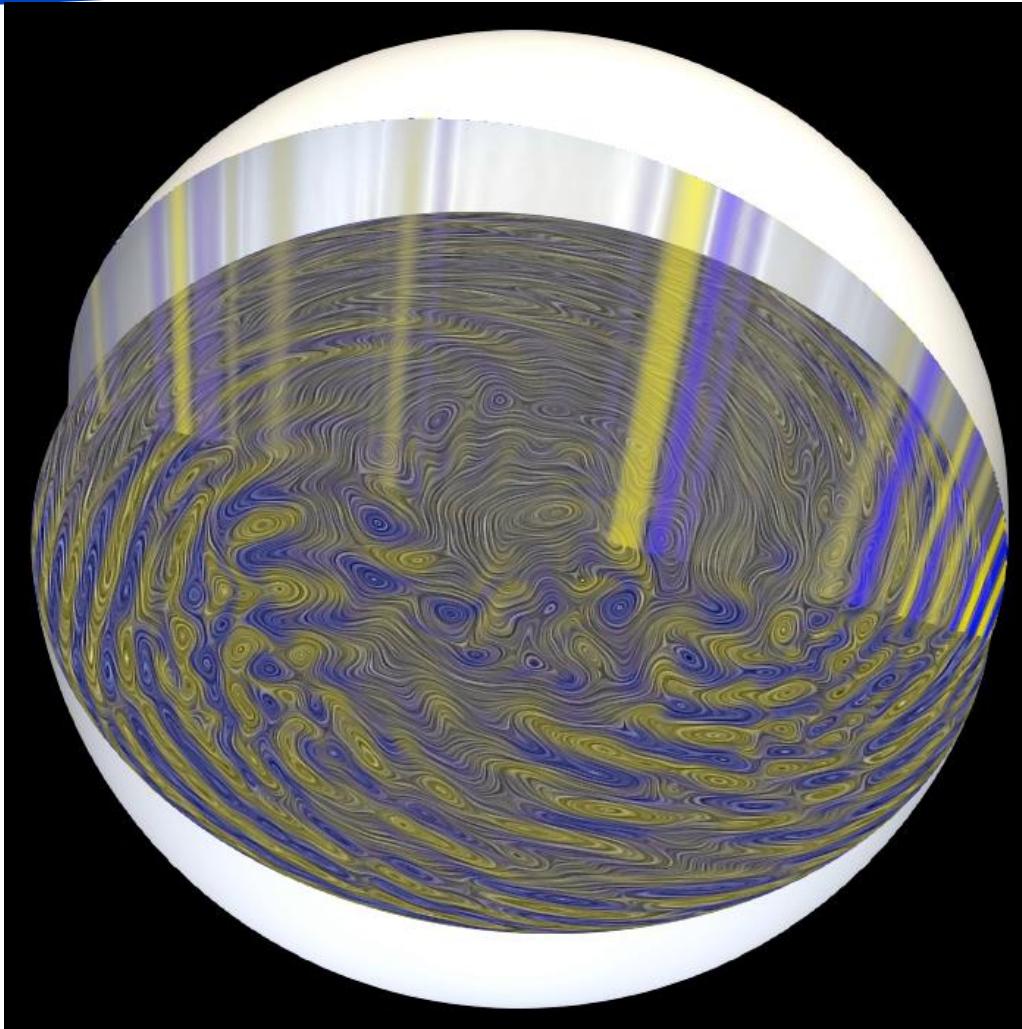


Dormy



Plaut

3D calculation



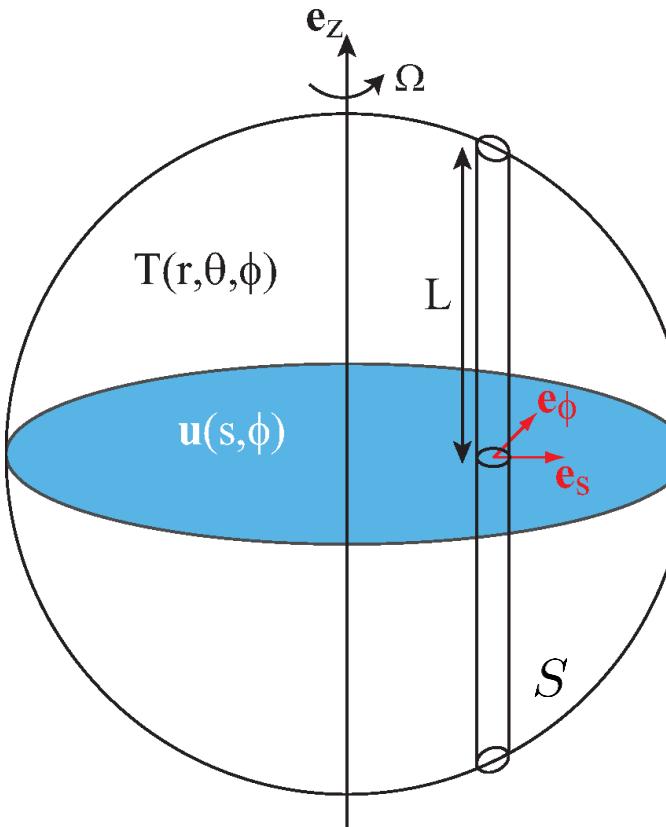
$$Ek = 10^{-7}, Pr = 0.01$$

No inner core – internal heating

$$Ek = \frac{\nu}{\Omega r_o^2}$$

$$Ra = \frac{\alpha g_0 S r_o^6}{6\rho C_p \nu \kappa^2}$$

$$Pr = \frac{\nu}{\kappa}$$



3D

QG-3D

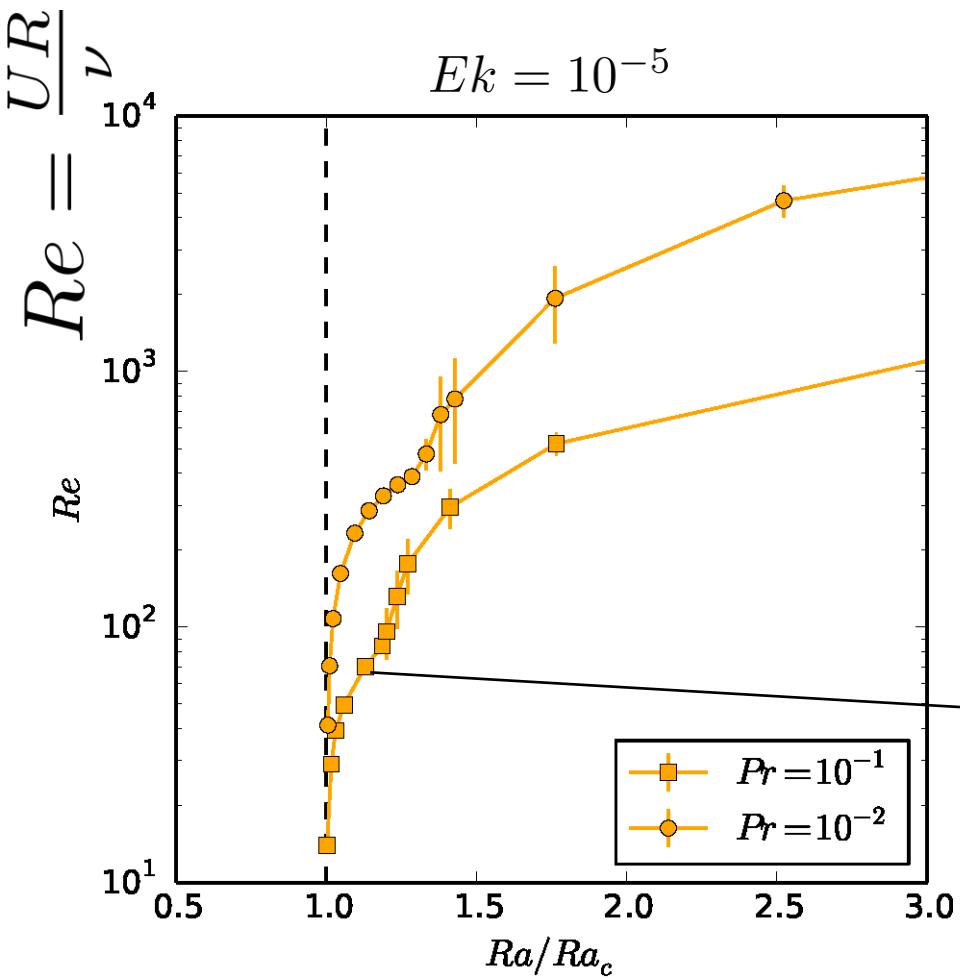
QG-2D

$$\zeta^g = (\nabla \times \mathbf{u}^g) \cdot \mathbf{e}_z,$$

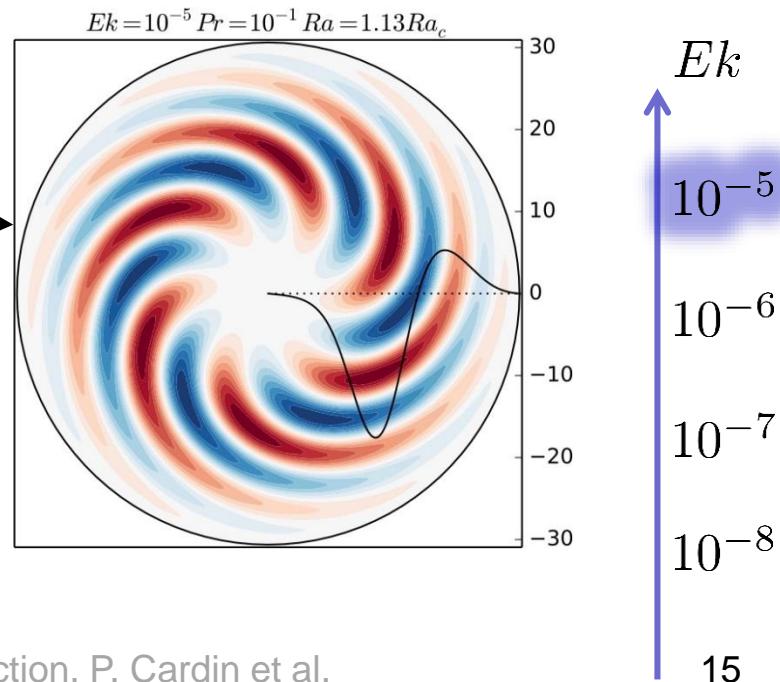
$$\frac{\partial \zeta^g}{\partial t} + (\mathbf{u}^g \cdot \nabla) \zeta^g - \left(\frac{2}{Ek} + \zeta^g \right) \left\langle \frac{\partial u_z}{\partial z} \right\rangle = \nabla_h^2 \zeta^g - Ra \left\langle \frac{\partial \Theta}{\partial \phi} \right\rangle,$$

$$\frac{\partial \Theta}{\partial t} + \mathbf{u} \cdot \nabla \Theta - 2Pr^{-1} r u_r = Pr^{-1} \nabla^2 \Theta.$$

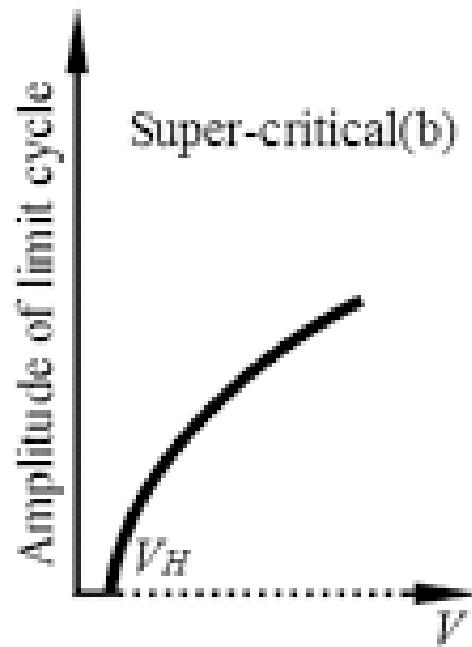
Supercritical thermal convection



- Continuous transition at $\text{Ra} = \text{Ra}_c$
- Thermal Rossby waves at low Pr : tilted/spiralling pattern (Zhang 1992)
- Production of a zonal flow by non-linear interactions of the convective velocities

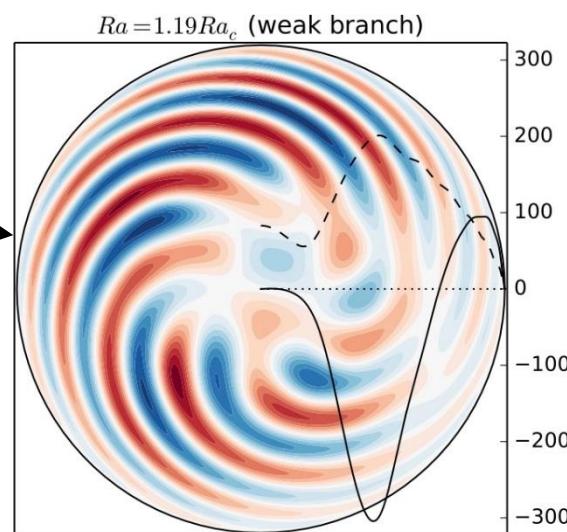
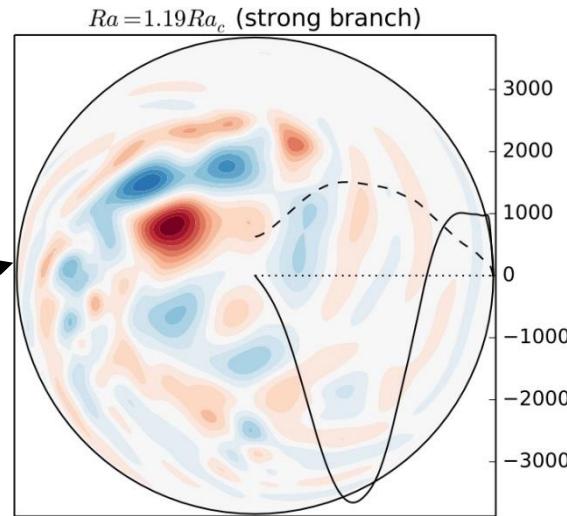
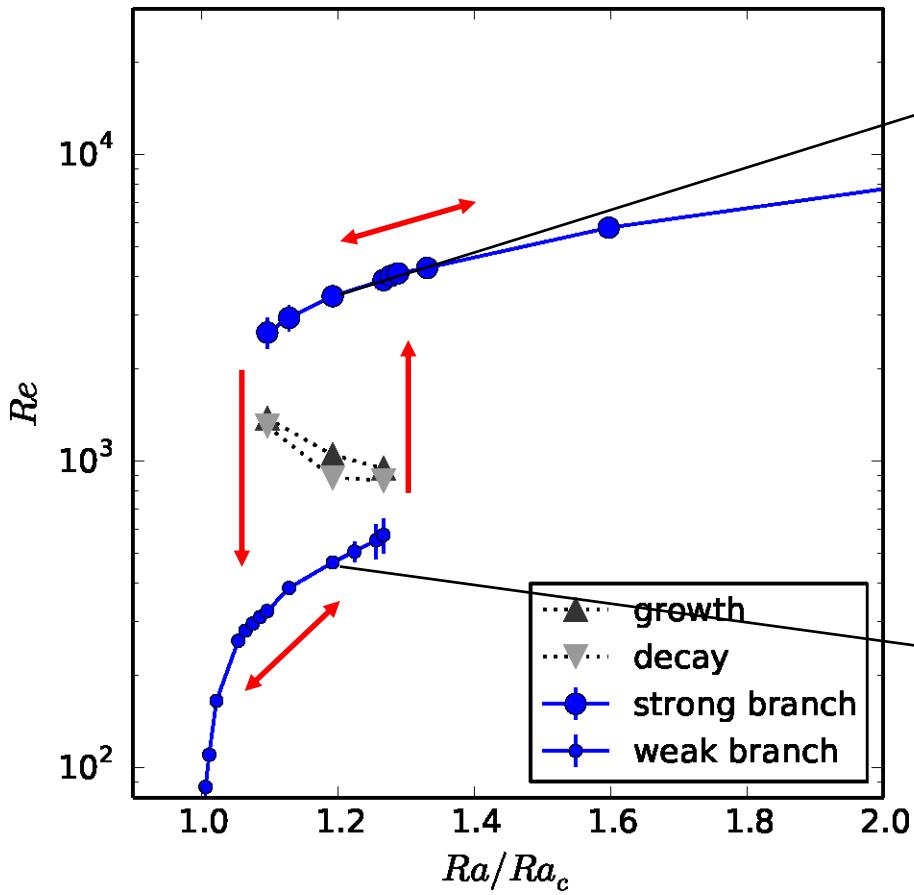


Bifurcation diagrams

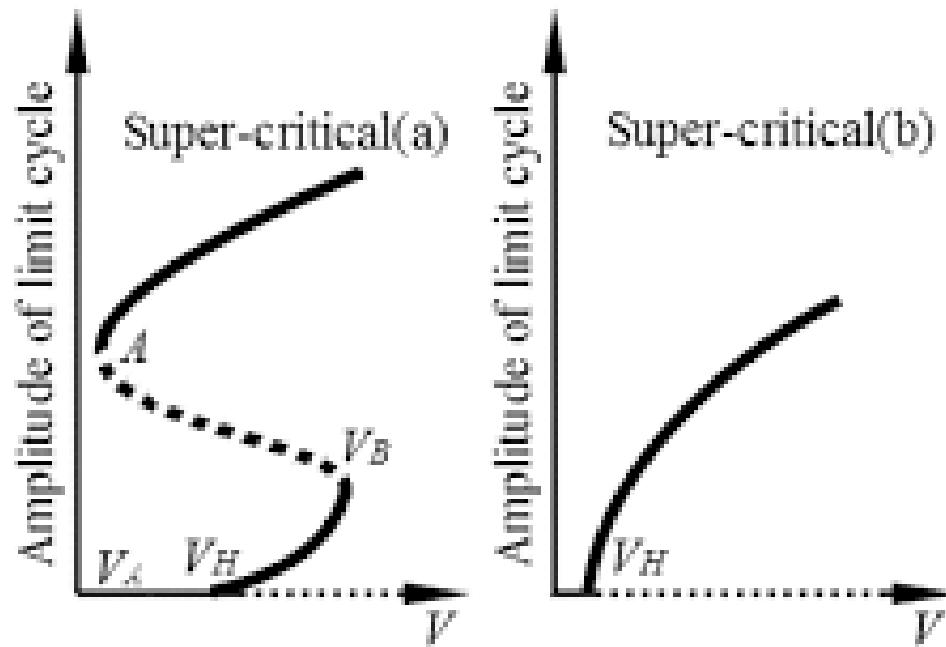


Hysteresis

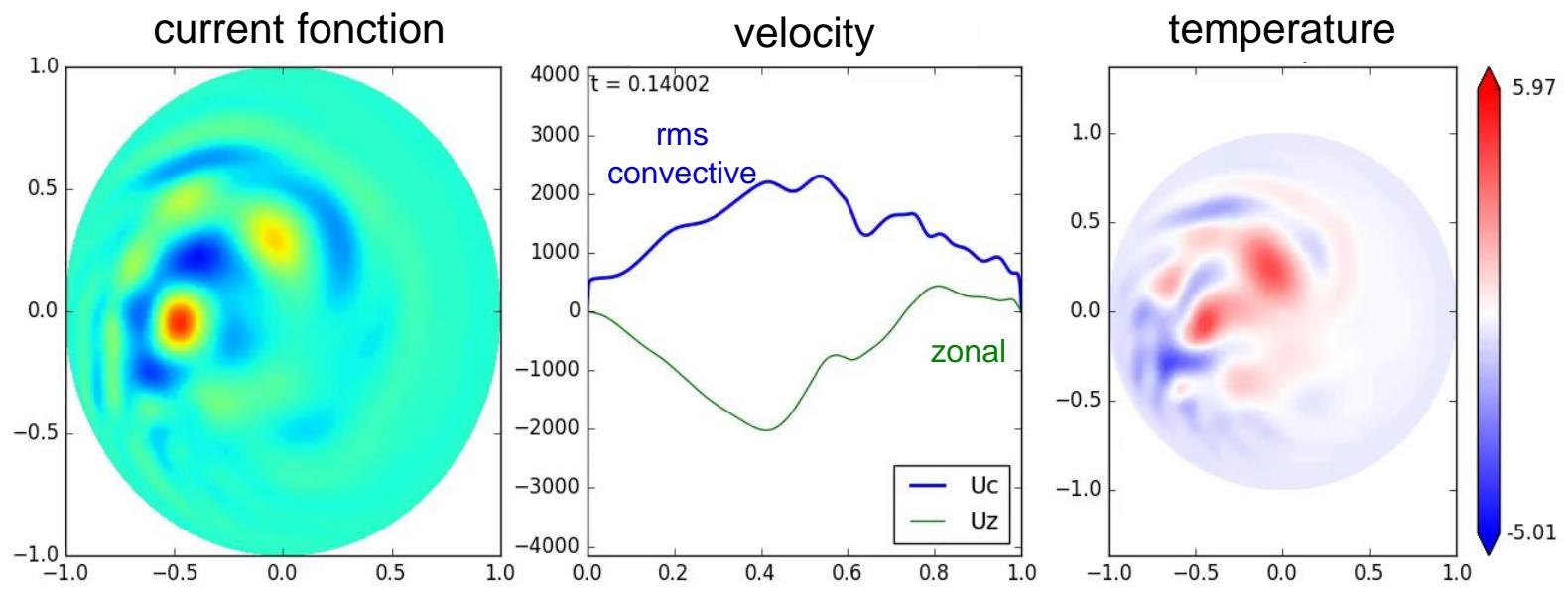
$$Ek = 10^{-6}, Pr = 0.01$$



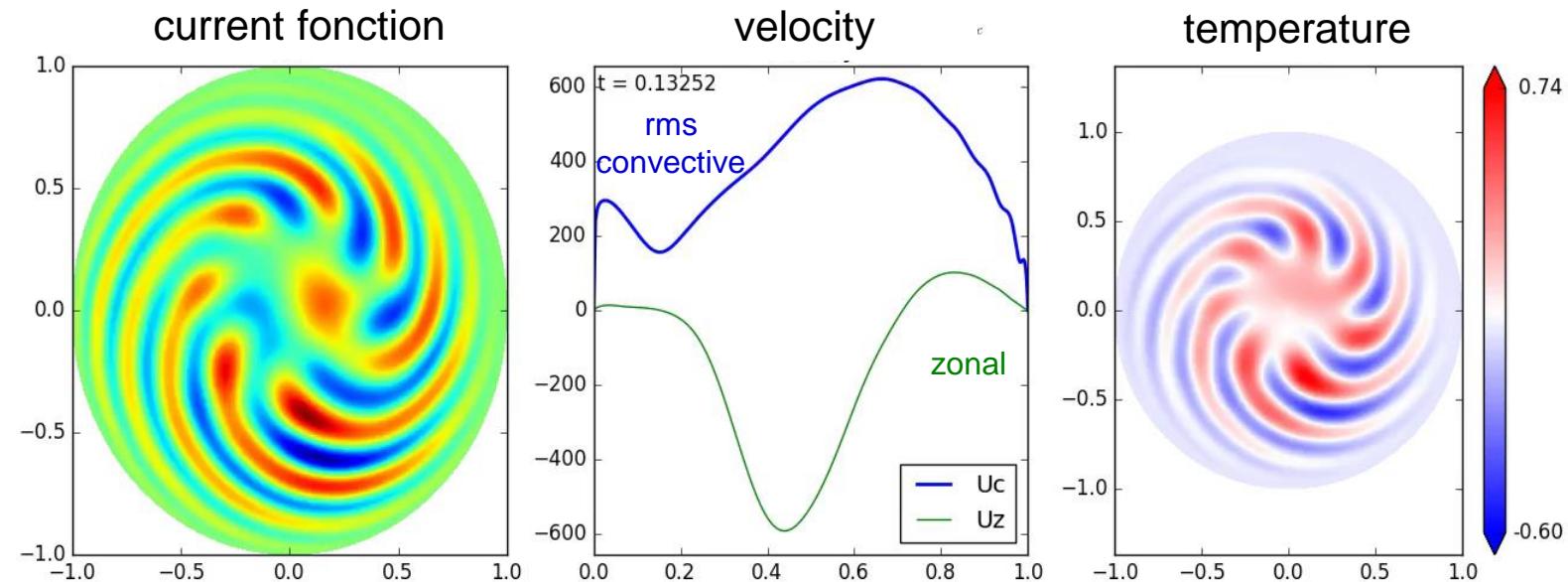
Bifurcation diagrams



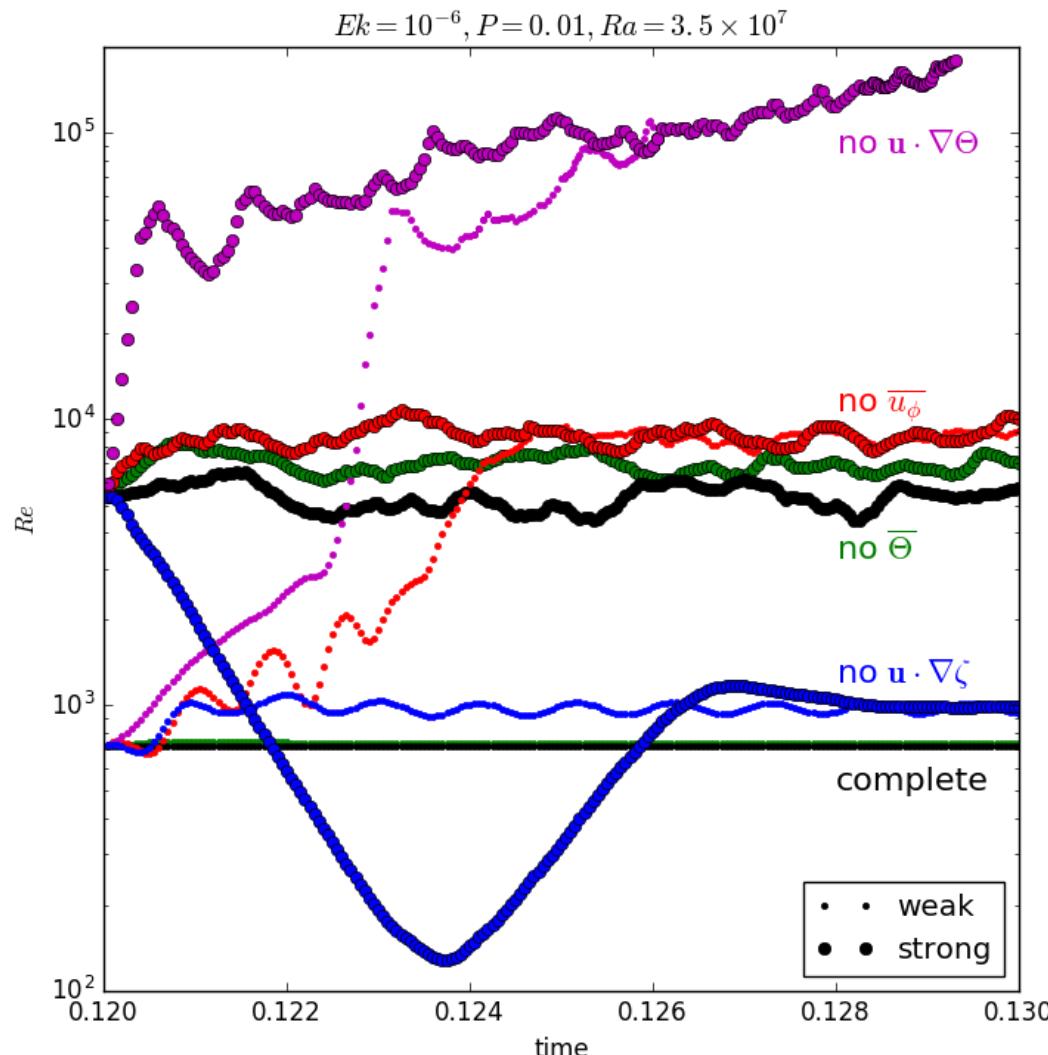
Strong branch



Weak branch



Non linear terms effect



What is responsible for subcriticality?

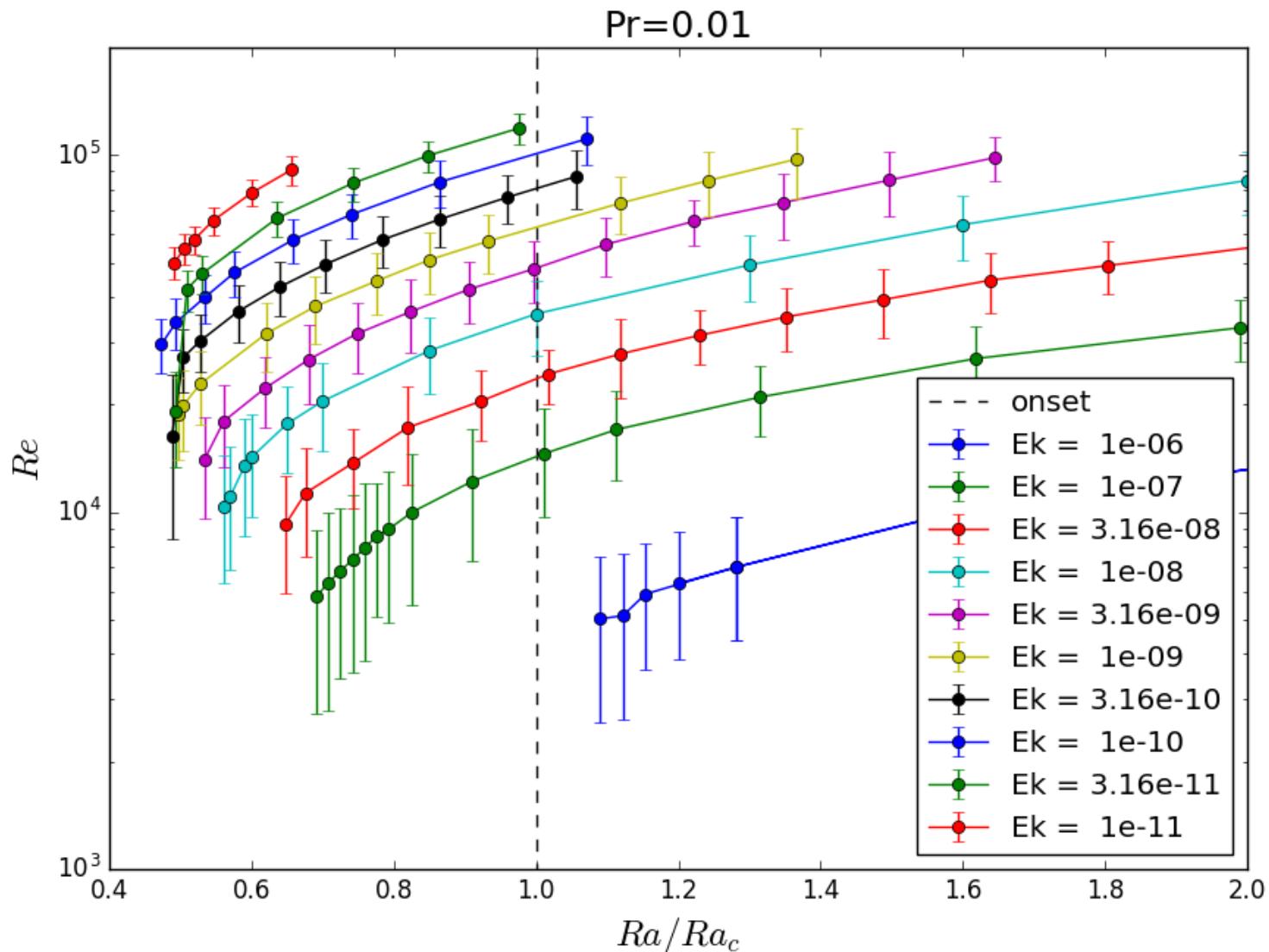
$\mathbf{u} \cdot \nabla \Theta$ is needed in both branches (but not $\overline{\Theta}$)

The suppression of the zonal velocity $\overline{u_\phi}$ leads to the strong branch.

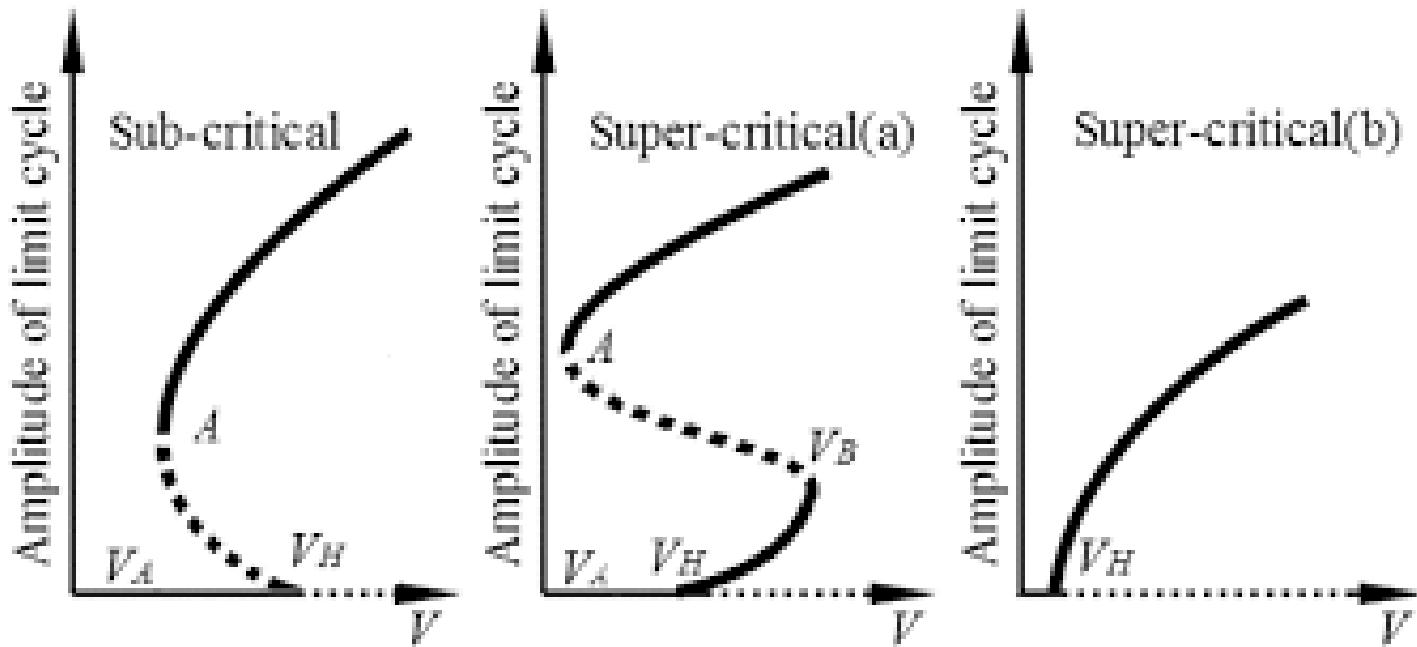
The suppression of Reynolds stress $\mathbf{u} \cdot \nabla \zeta$ leads to the weak branch.

$\overline{u_\phi} \frac{\partial}{\partial \phi} \Theta$ saturates the weak branch (thermal)

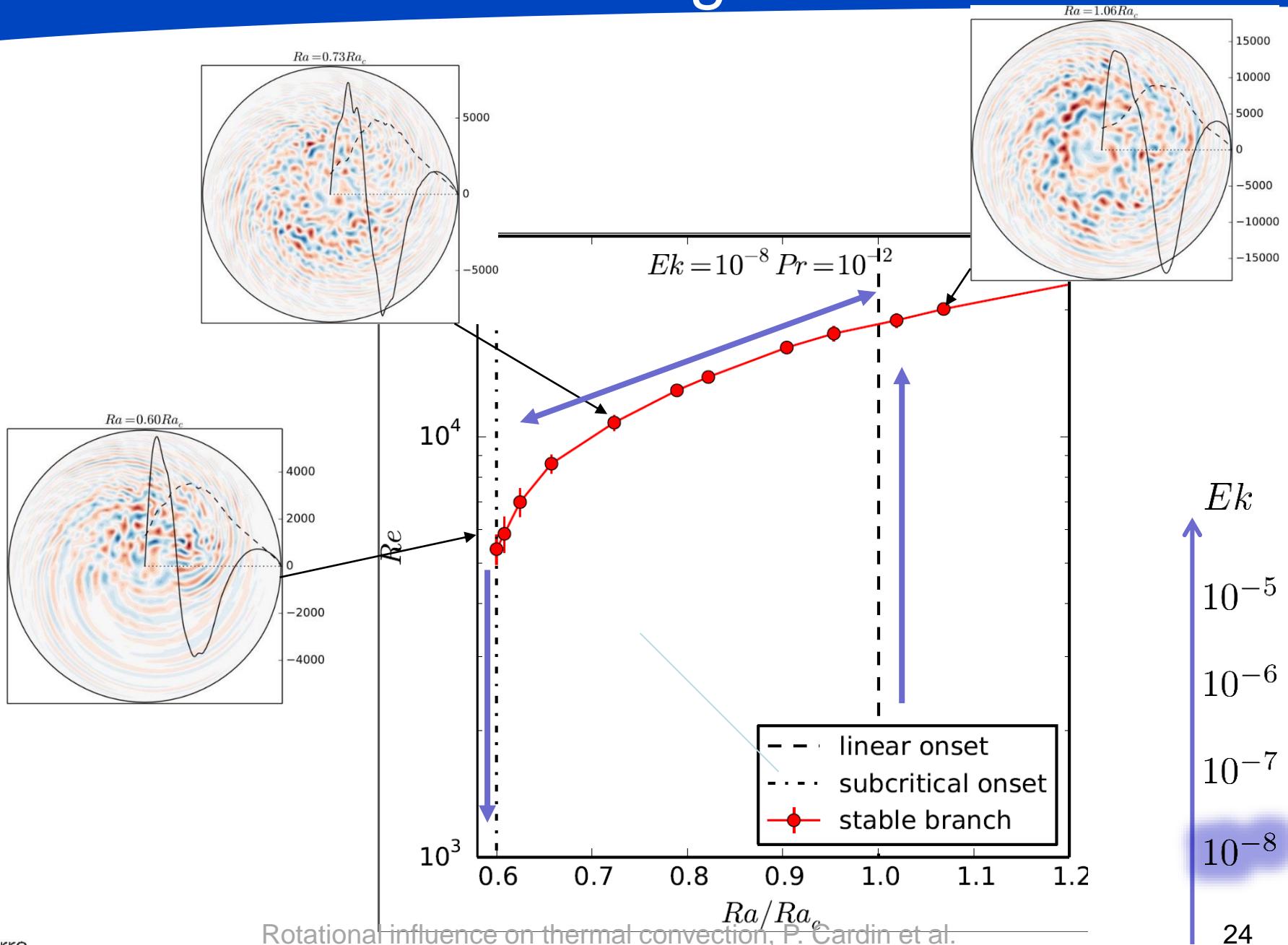
Strong branches



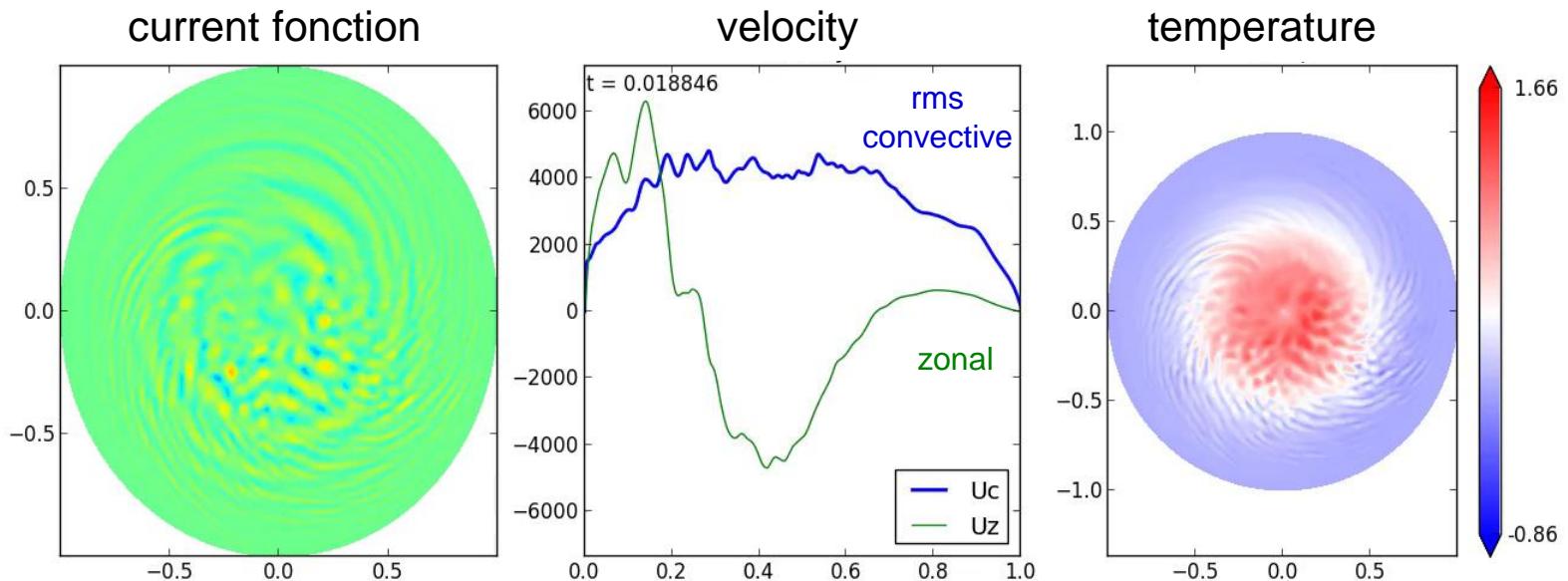
Bifurcation diagrams



Subcritical strong branche

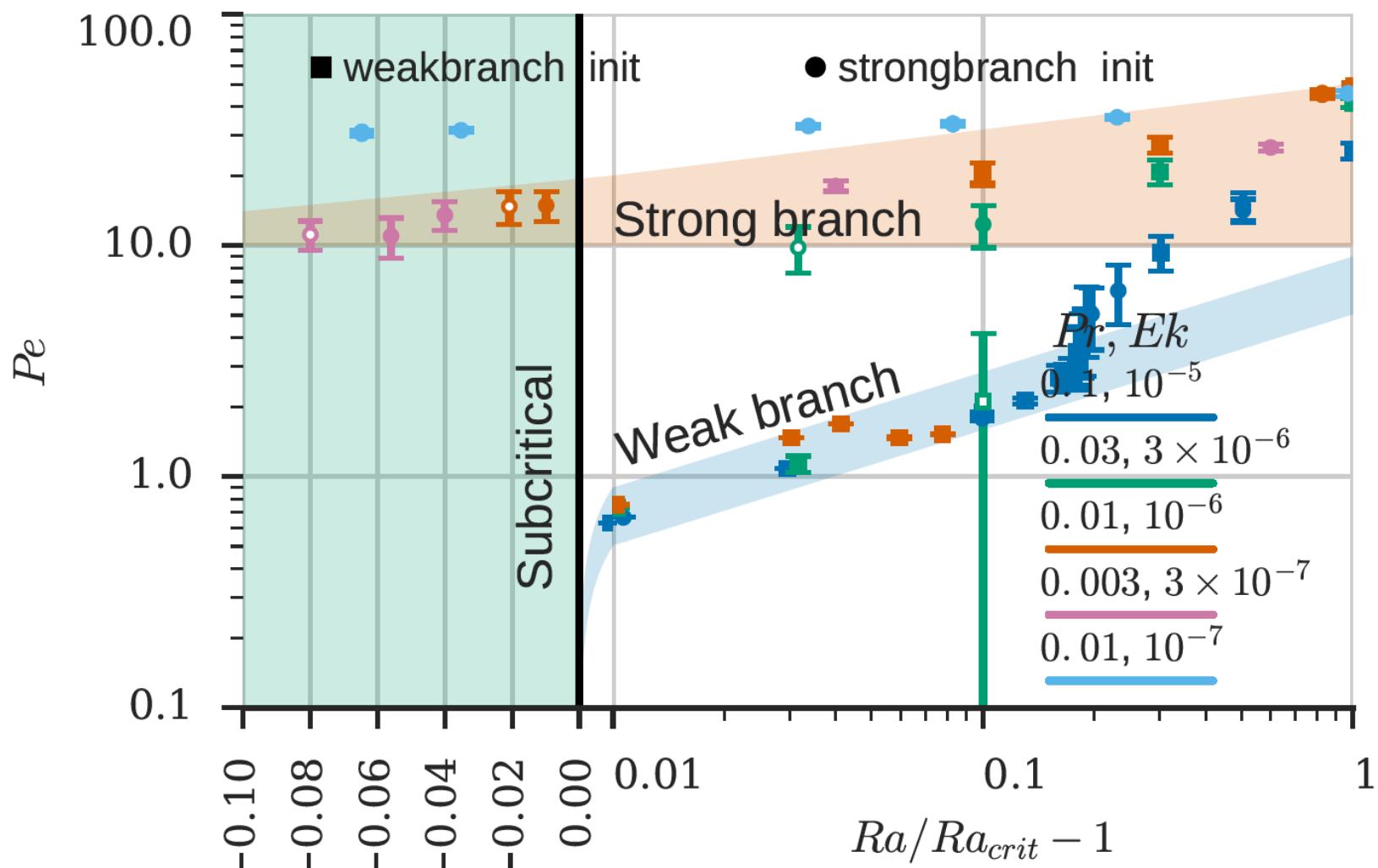


Subcritical convection

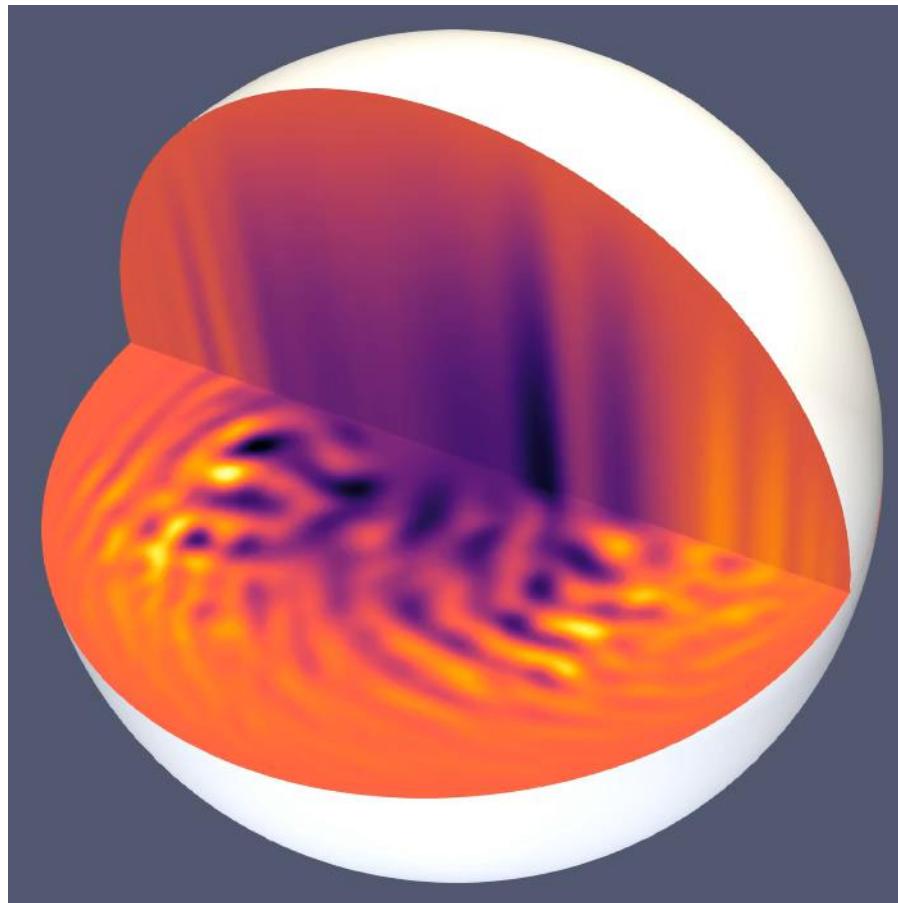


$$Ek = 10^{-8}, Pr = 0.01, Ra = 0.60Ra_c$$

3D calculations

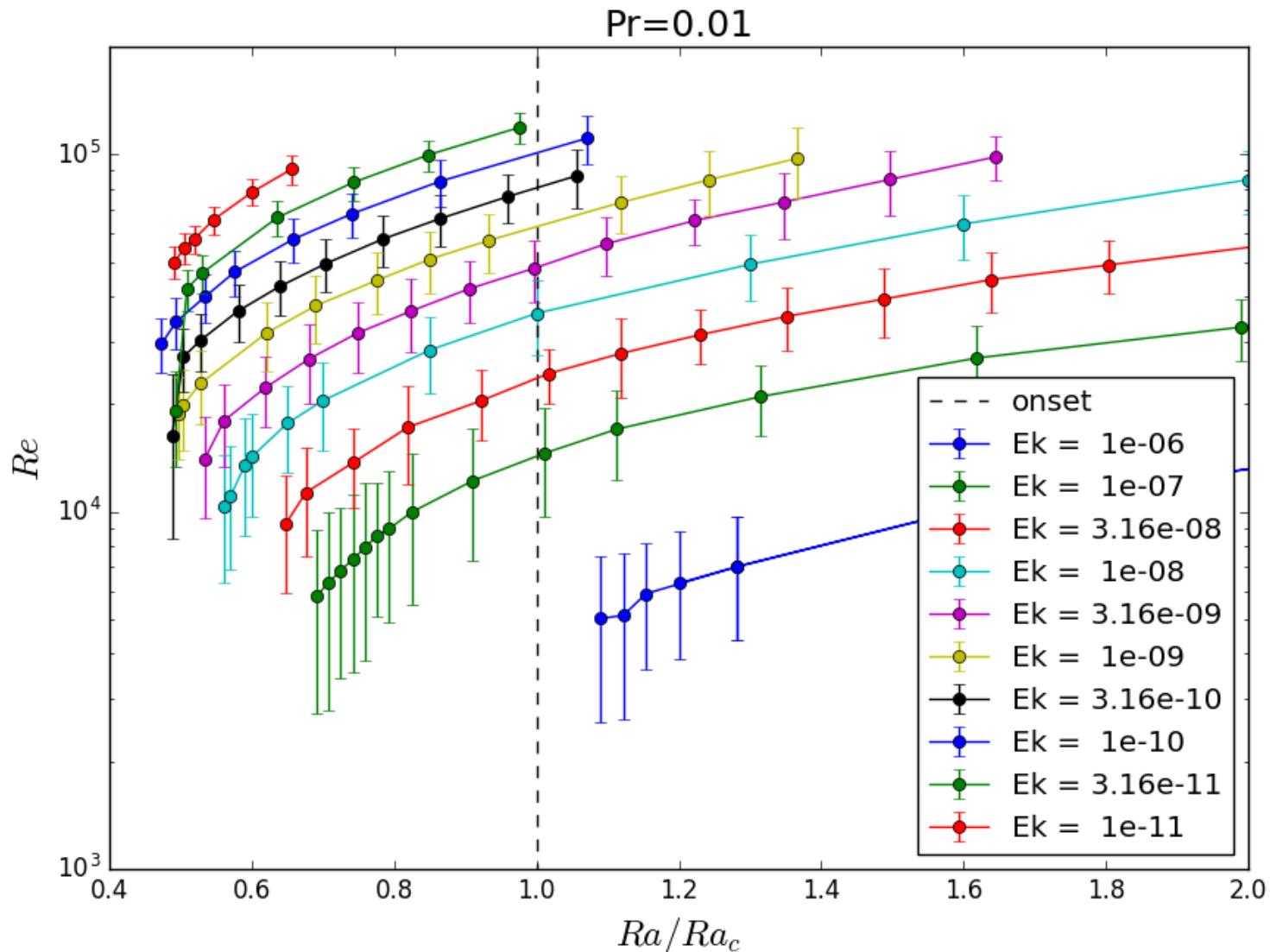


3D calculation



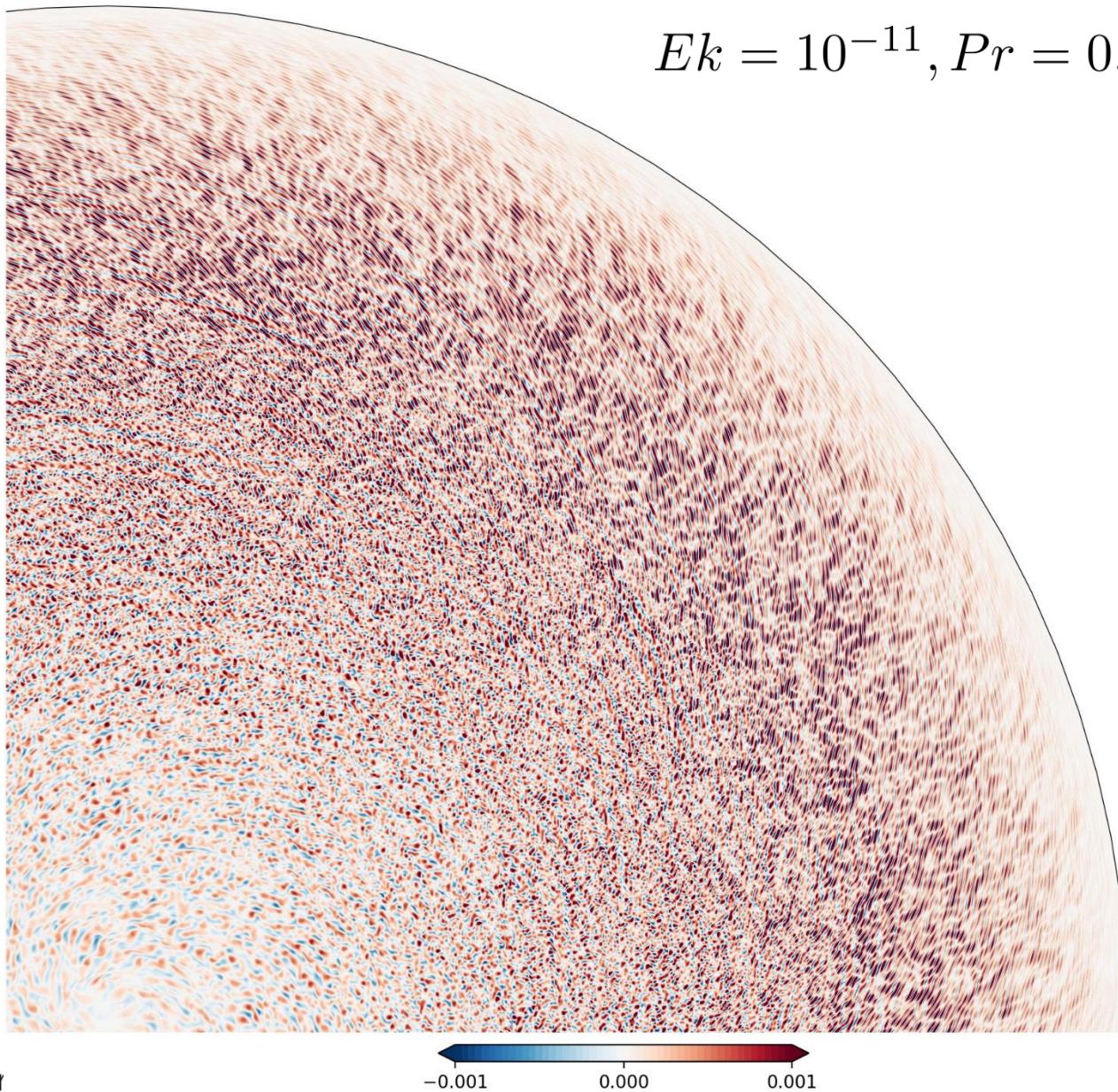
$$Ek = 10^{-7}, Pr = 0.01$$

Asymptotic subcriticality



Vorticity

$$Ek = 10^{-11}, Pr = 0.01, Ra = 2.5e13$$



Ideal balance

$$Bu = \frac{\alpha g_0 S}{6\rho C_p \kappa \Omega^2} = \frac{Ra E k^2}{Pr}.$$

Dimensionless time Ω^{-1}

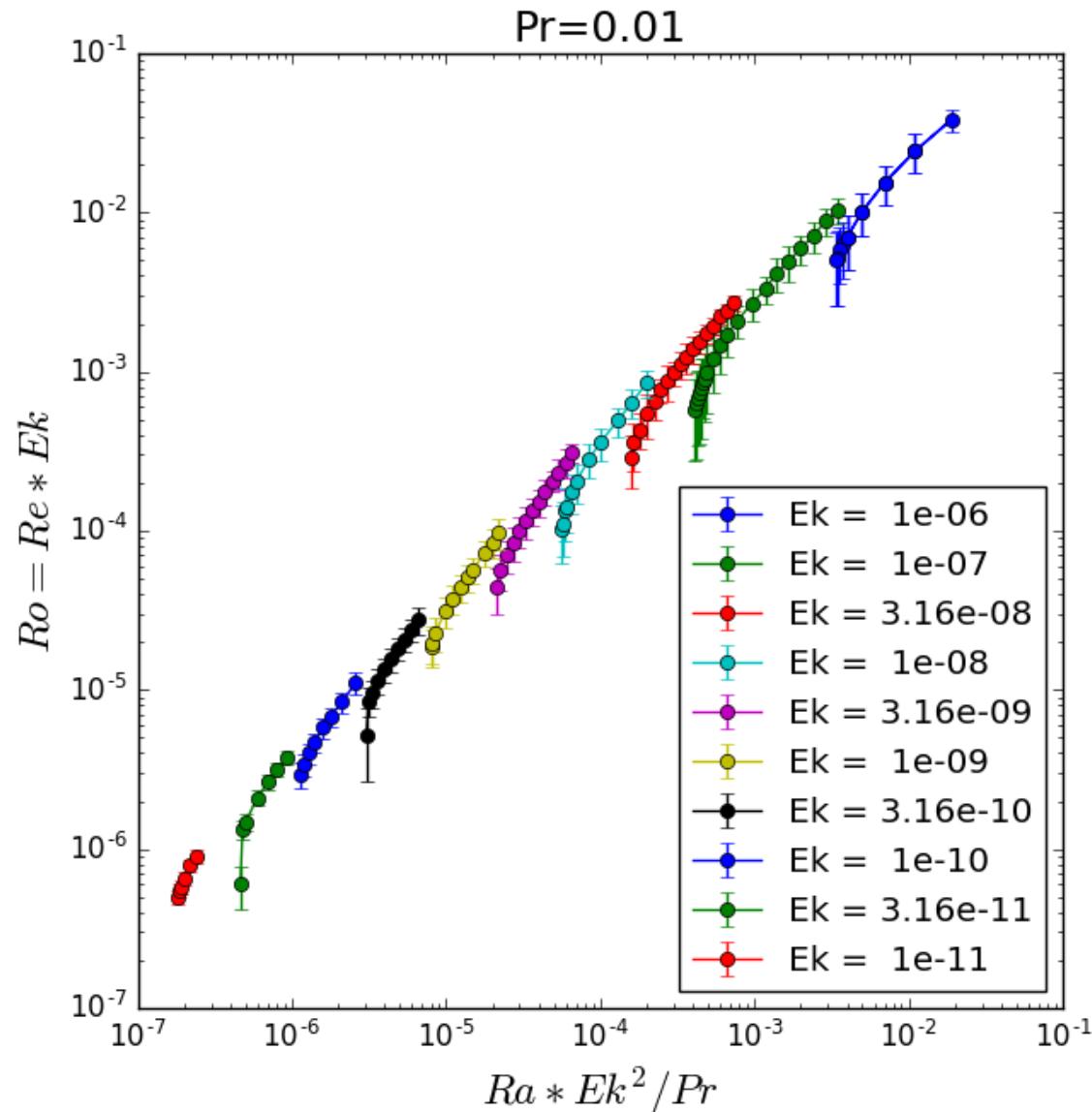
$$\frac{\partial \zeta}{\partial t} + (\mathbf{u} \cdot \nabla) \zeta - 2\beta u_s = -Bu \left\langle \frac{\partial \Theta}{\partial \phi} \right\rangle,$$

$$\frac{\partial \Theta}{\partial t} + \mathbf{u} \cdot \nabla \Theta = 2su_s, \quad \delta \approx \Theta/s$$

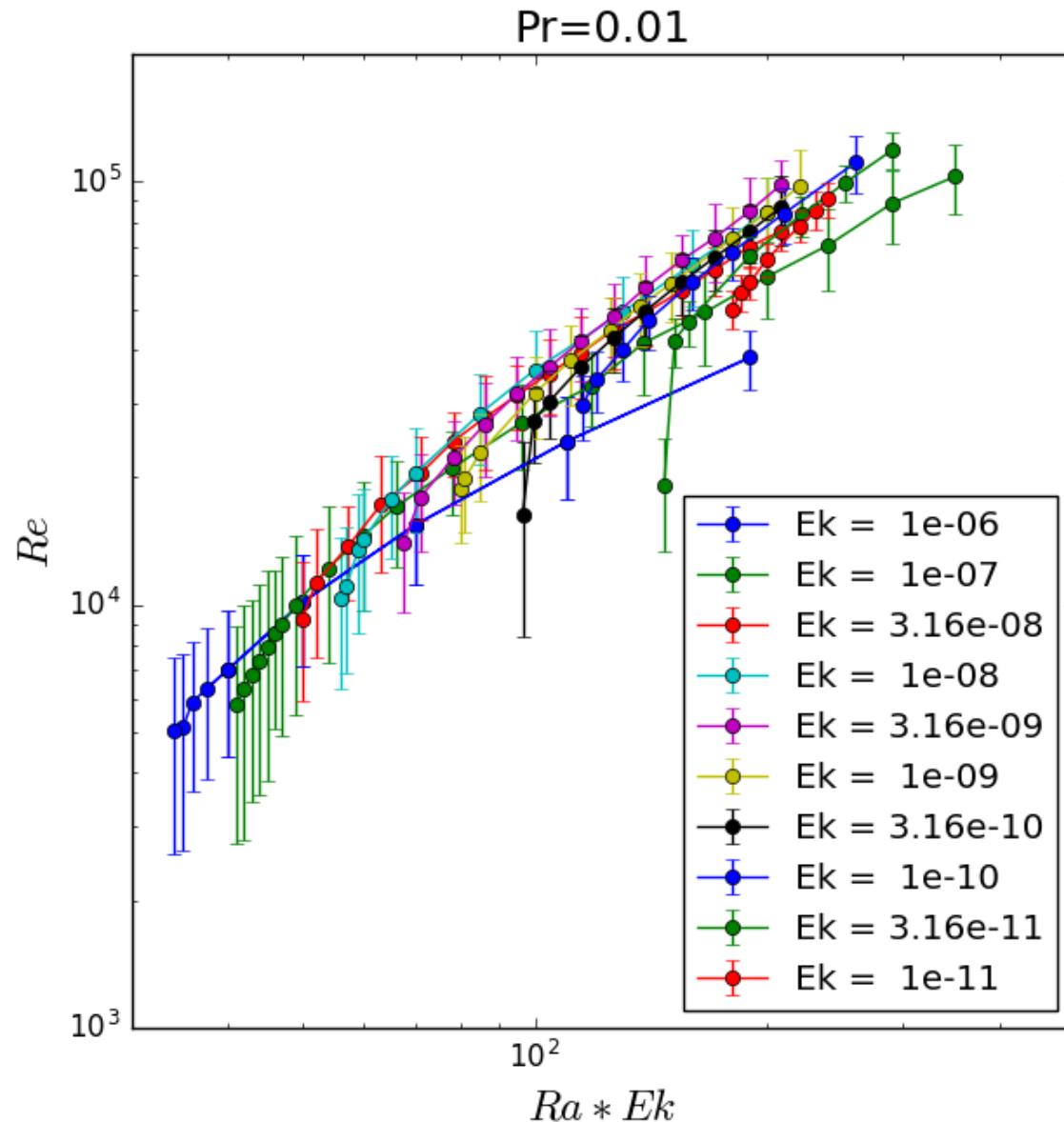
$$u_s \approx u_\phi \approx u = Ro \quad \delta \approx \sqrt{u_s/\beta} \approx \sqrt{Ro(1-s^2)/s}$$

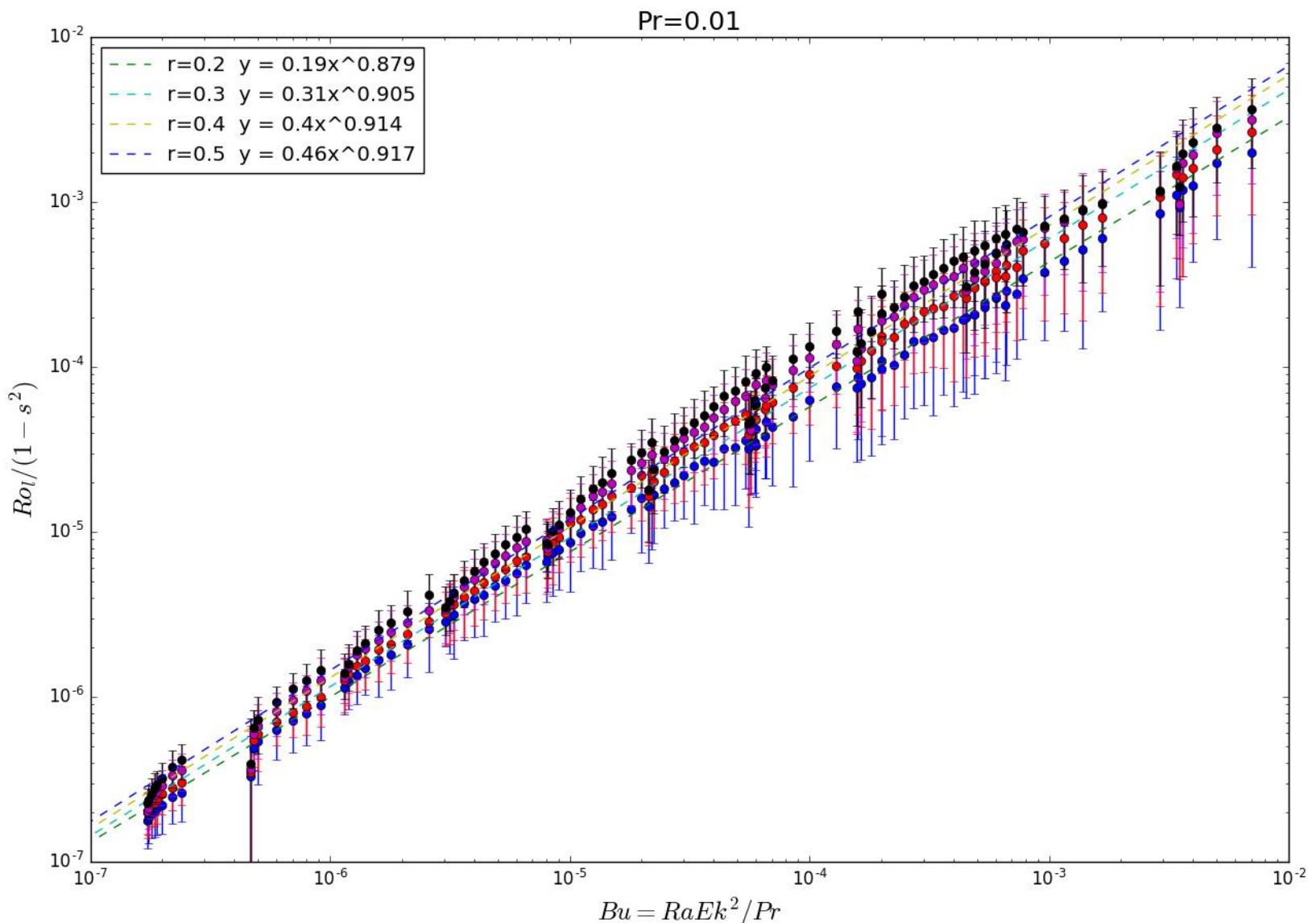
$$Ro \approx Bu \frac{s}{\beta} \approx Bu(1-s^2)$$

Rossby number



$Re = f(Ra^*Ek)$





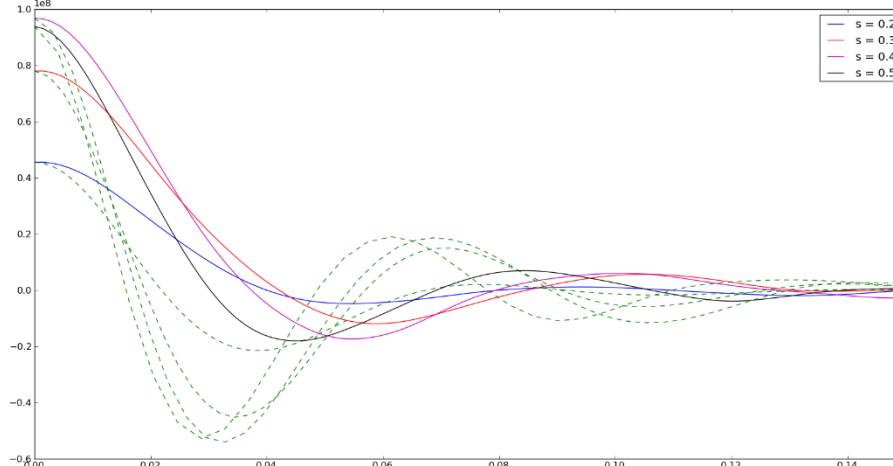
Size of convective cells

$$f_s(dr, t) = \overline{u_s(s, \phi, t) u_s(s + dr, \phi, t)}$$

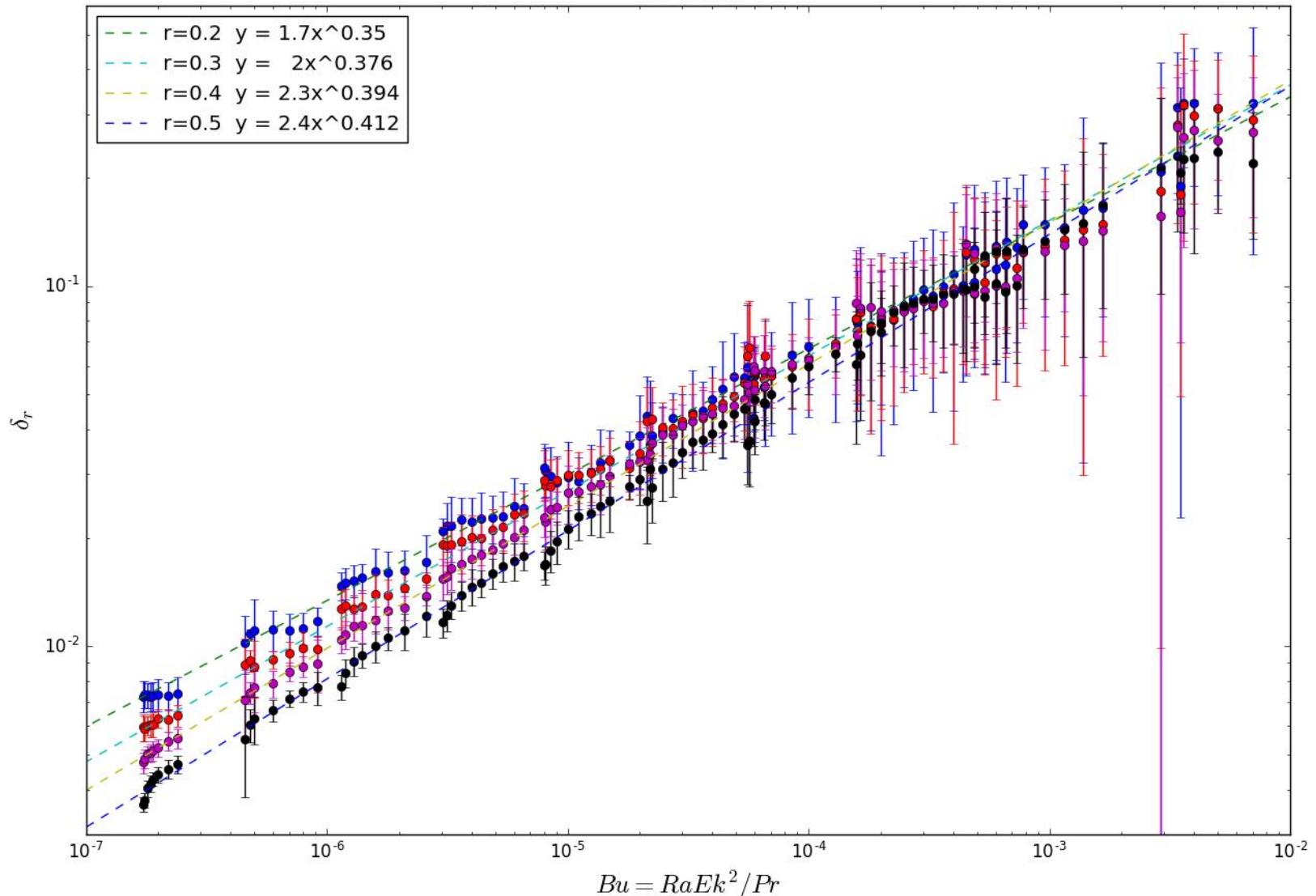
$$\overline{A} \equiv \frac{1}{2\pi} \int_0^{2\pi} A d\phi.$$

$$Ro_l = \sqrt{f_s(dr = 0)}$$

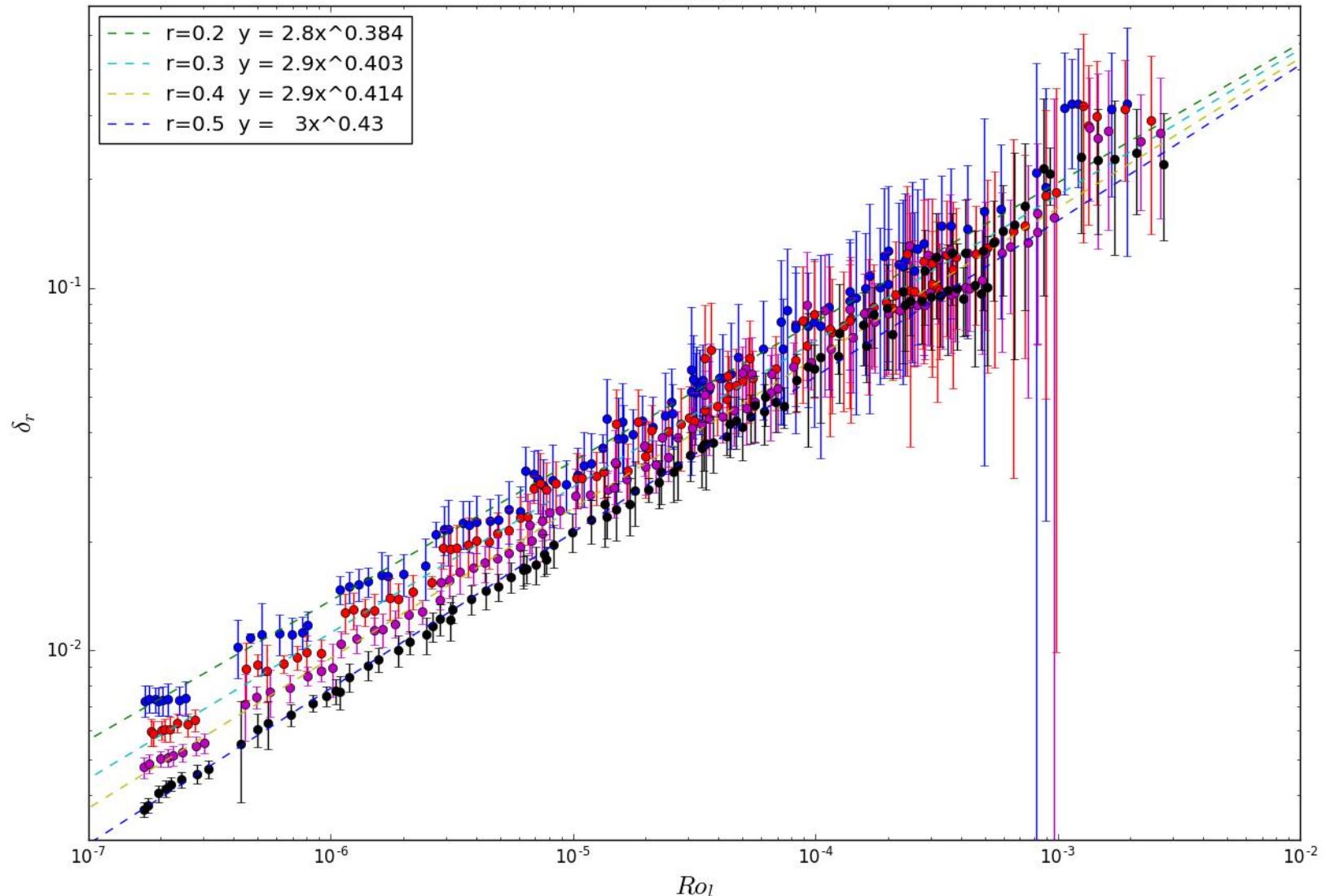
$$f_s(dr = \delta_s) = f_s(dr = 0)/2$$



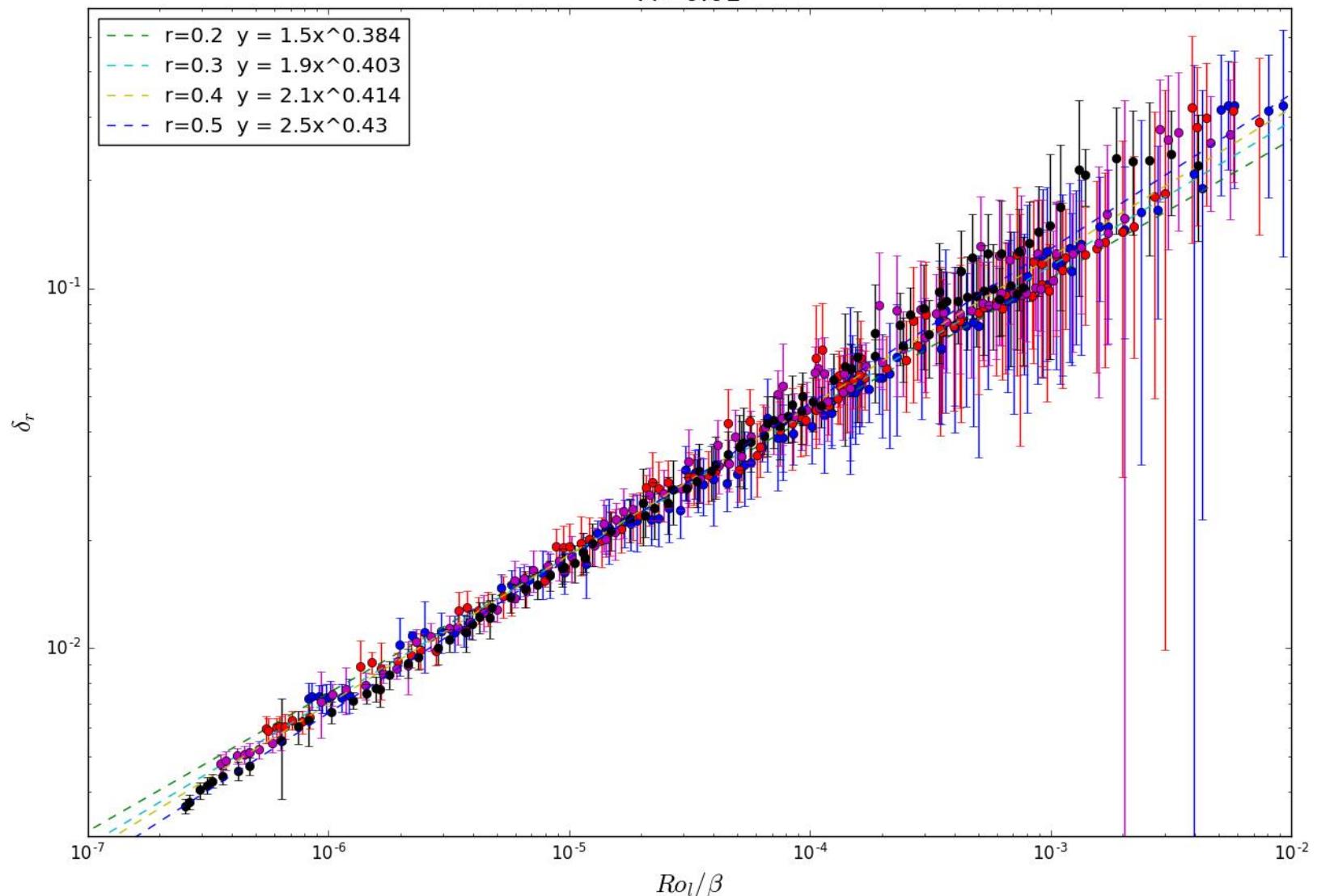
$Pr=0.01$



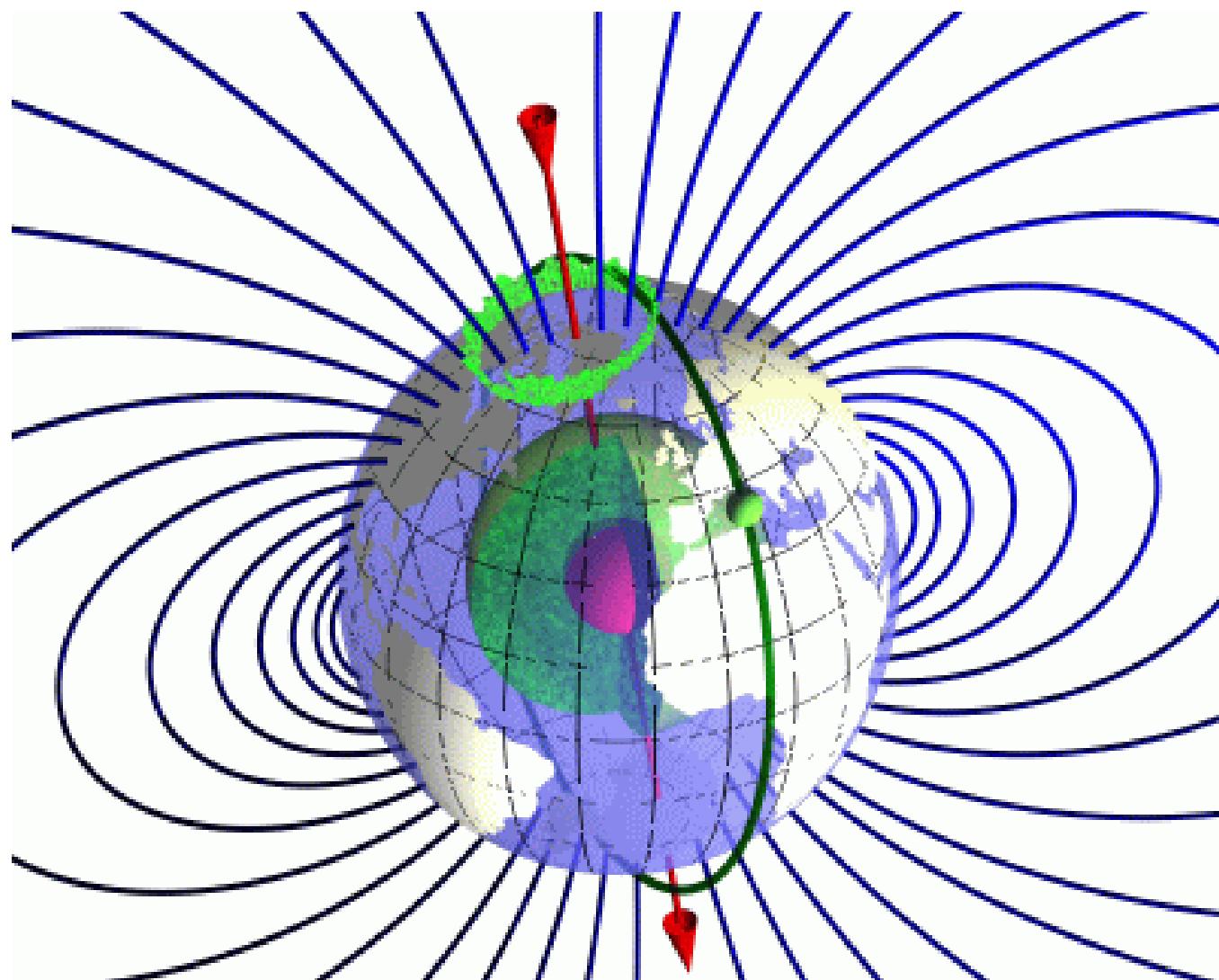
$Pr=0.01$



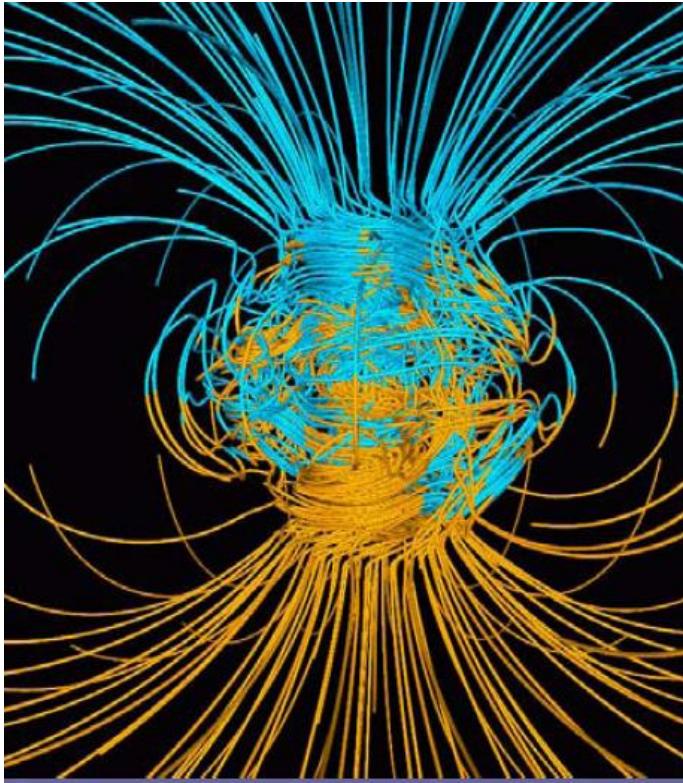
$Pr=0.01$



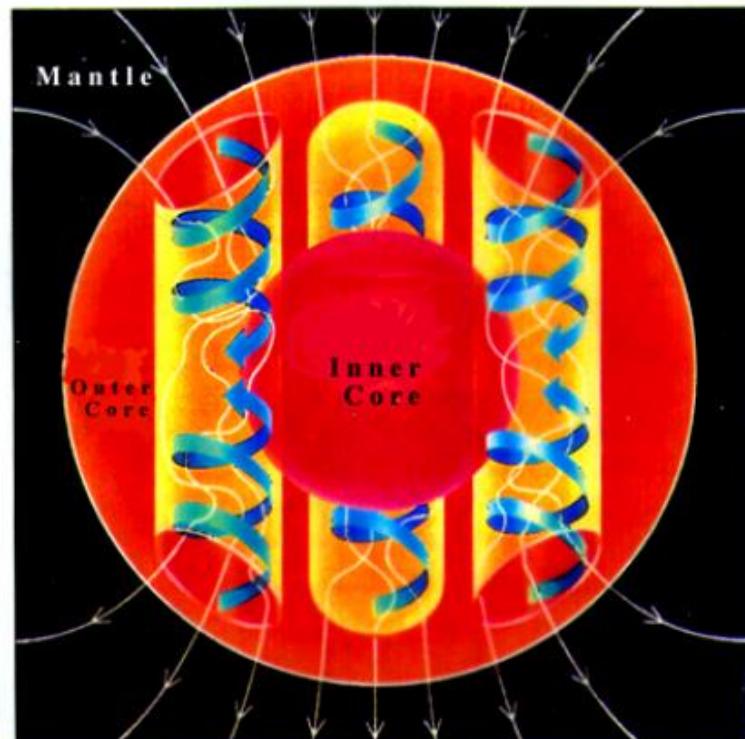
The geodynamo



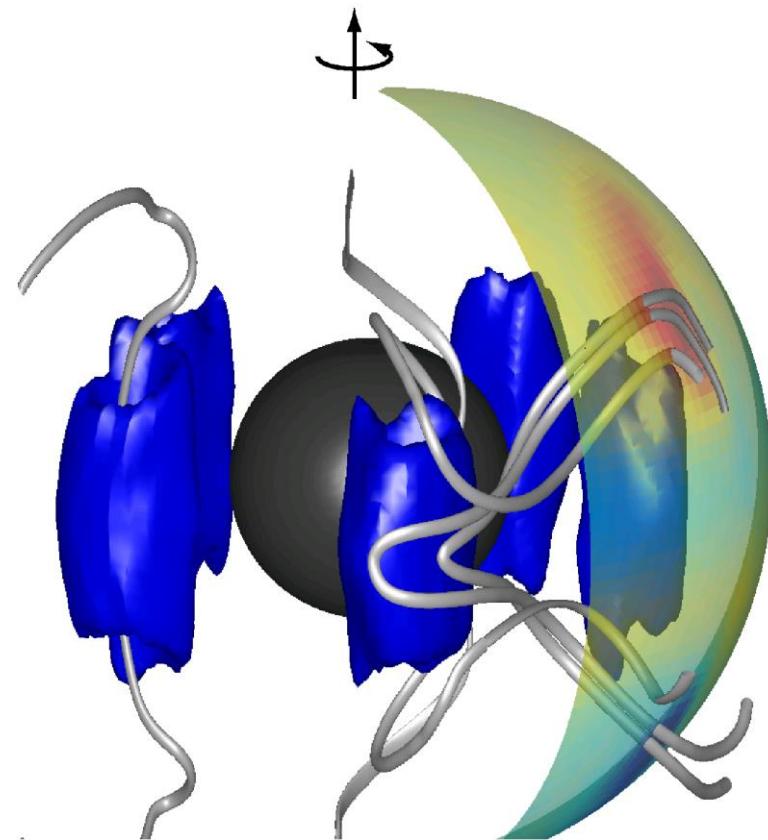
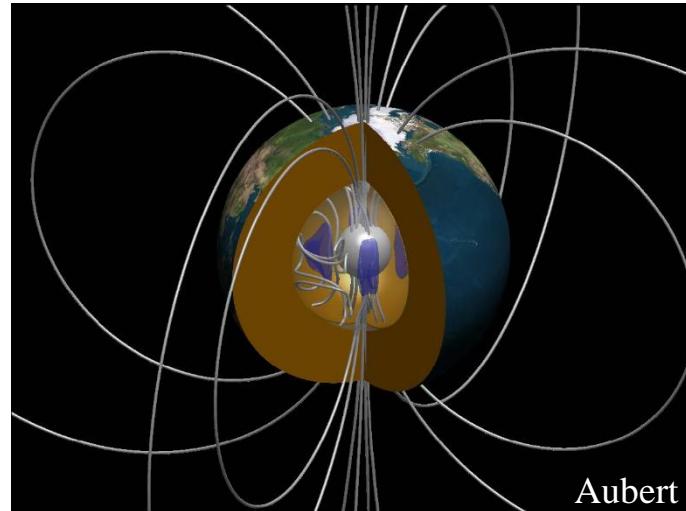
Dynamo calculations



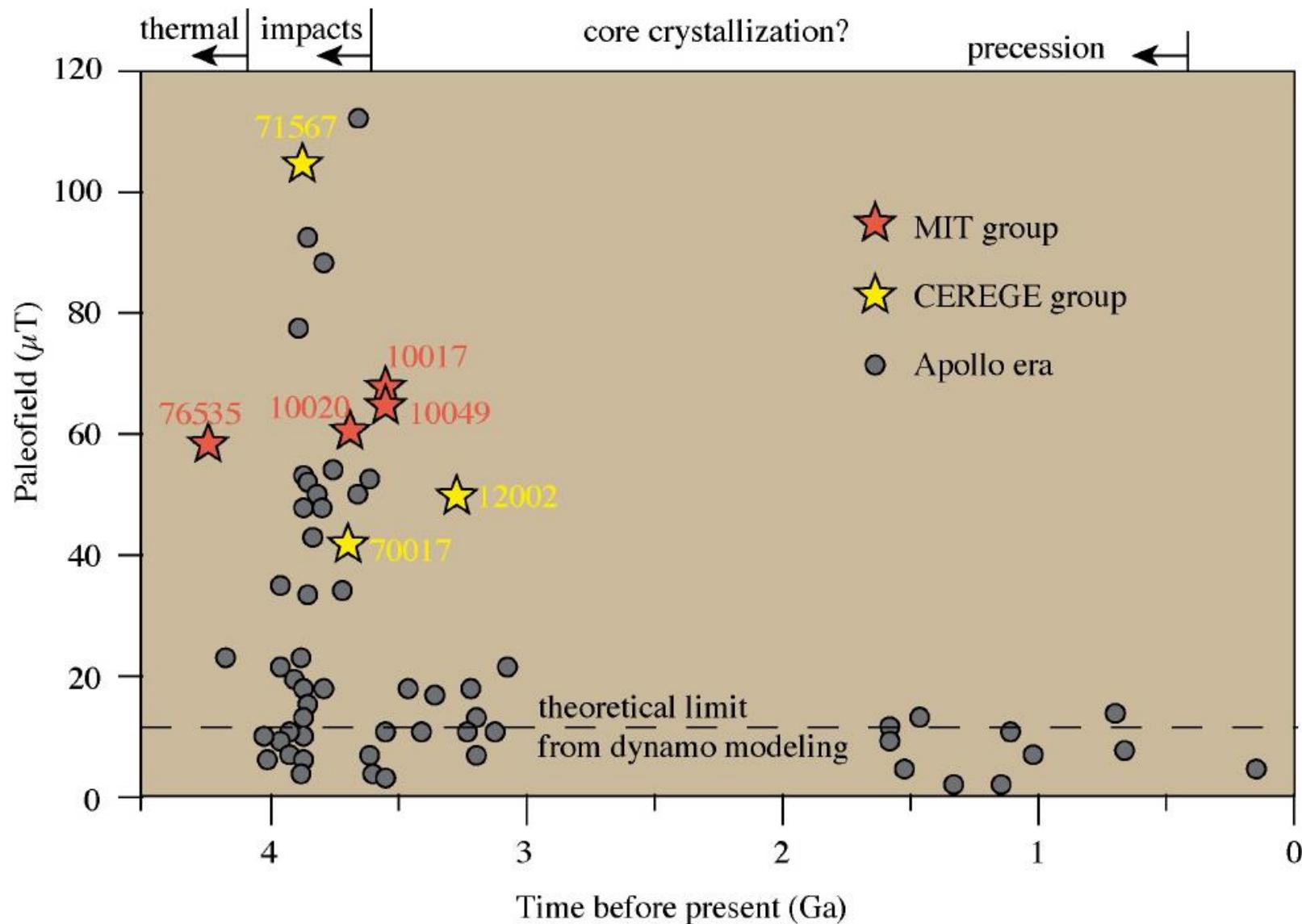
Glatzmaier and Roberts 1995



Numerical geodynamo

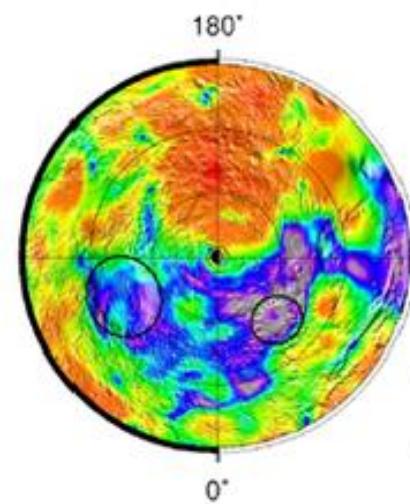


Lunar dynamo

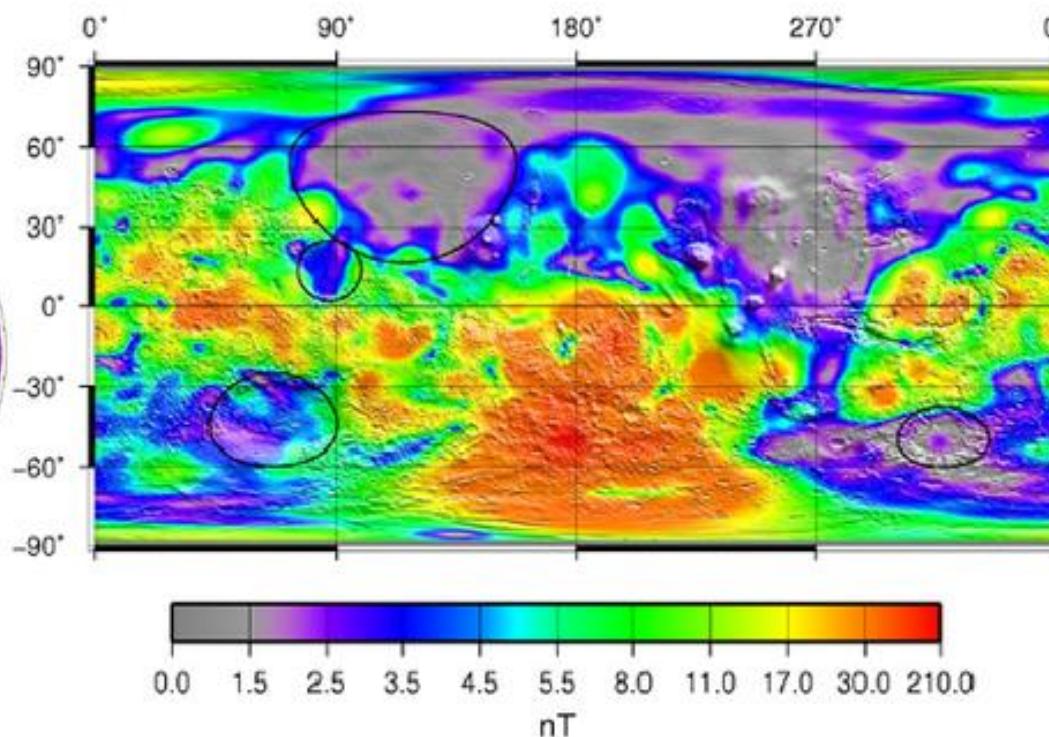


Mars magnetic field

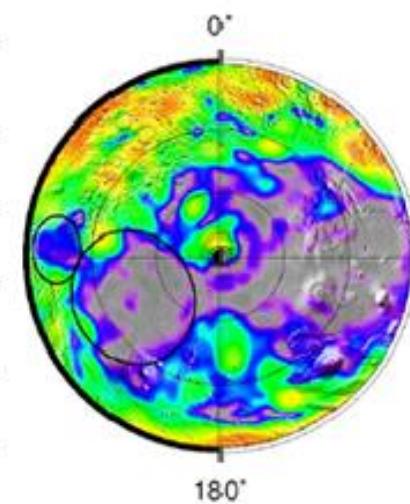
**Magnetic field
magnitude**



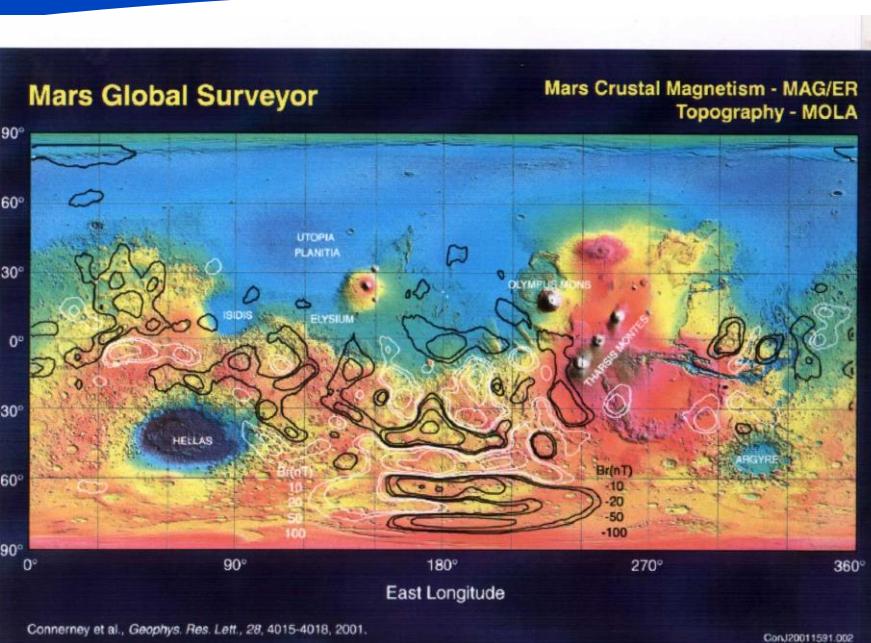
Correlative model



**400 km
altitude**



Mars dichotomy



Correlative model

