Relativistic simulation in nanotube-graphene devices

Lorentz invariance and atomic collapse

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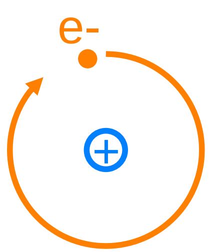
Atomic collapse in a nutshell



momentum **p**
radius **r**

Atomic number Z

$$E = \frac{\boldsymbol{p}^2}{2m} - \frac{Ze^2}{4\pi\epsilon_0 \boldsymbol{r}}$$

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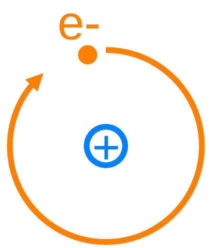

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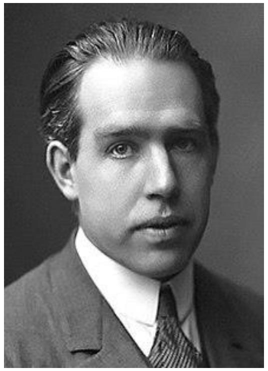
Niels Bohr



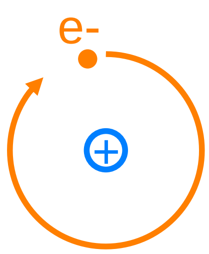
$$2\pi\boldsymbol{r} = n\lambda$$

Louis de Broglie

$$\boldsymbol{p} = h/\lambda$$

$$\boldsymbol{p} = \hbar/\boldsymbol{r}$$

Atomic collapse in a nutshell



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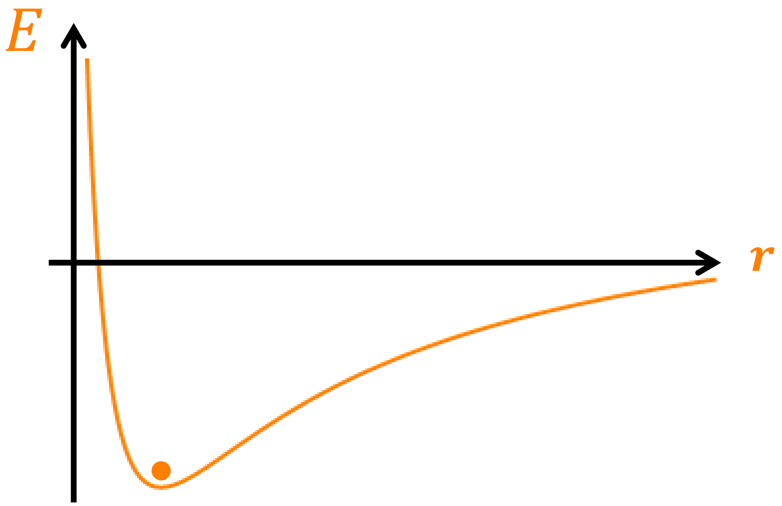

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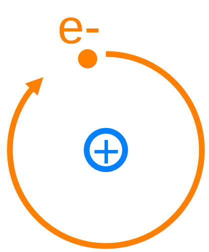

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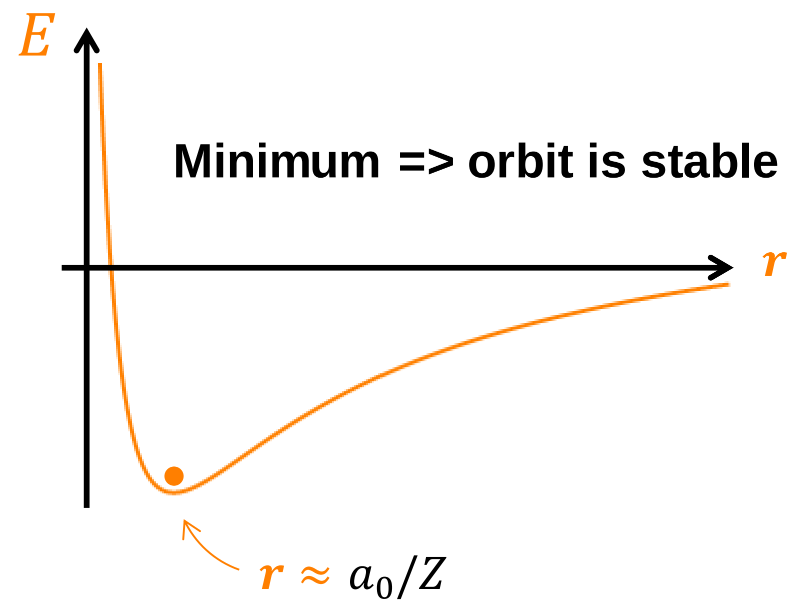

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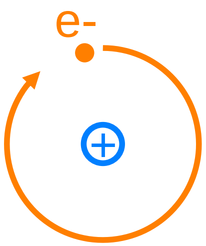
Atomic collapse in a nutshell

What about atoms with large Z ?



Atomic number Z

momentum **p**
radius **r**

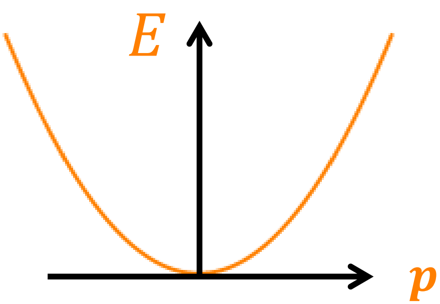
$\boldsymbol{r} \approx \frac{a_0}{Z} \rightarrow 0$ and $\boldsymbol{p} \rightarrow \infty$

=> requires relativistic correction

Atomic collapse in a nutshell

What about atoms with large Z ?

Classical particle

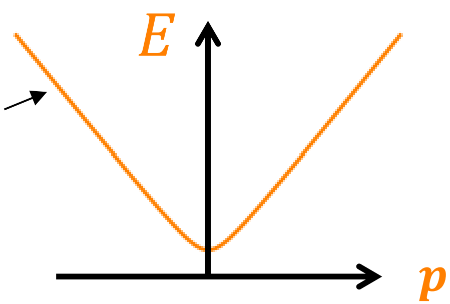


$$E = \frac{\boldsymbol{p}^2}{2m}$$

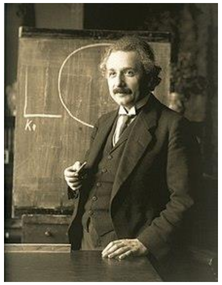
Relativistic particle

slope=velocity

$v \sim c$



$$E = \sqrt{m^2c^4 + \boldsymbol{p}^2c^2}$$



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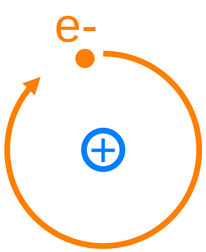

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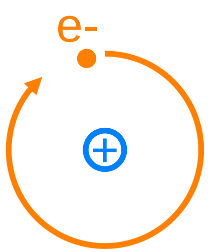
Kinetic energy for relativistic particle

Atomic collapse in a nutshell

What about atoms with large Z ?



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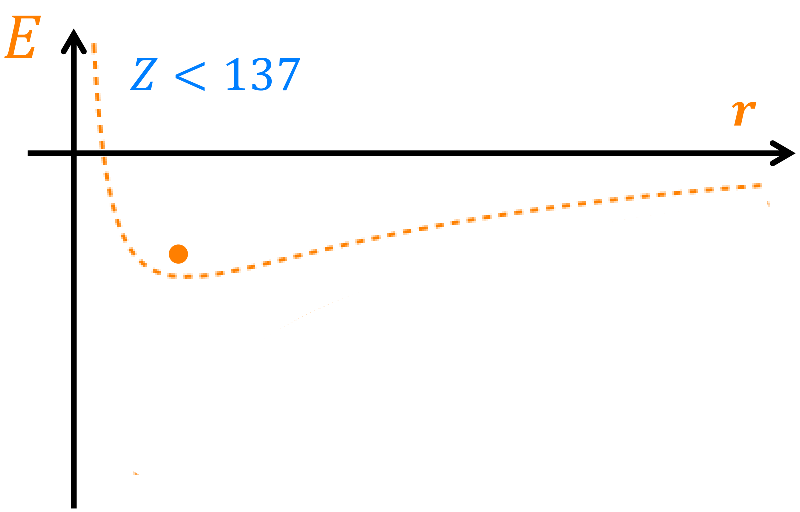

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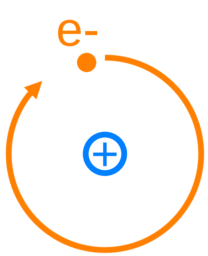

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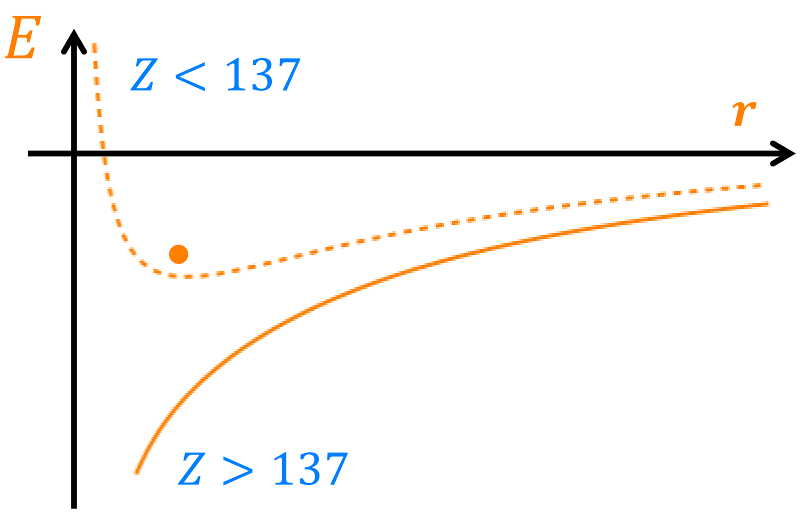

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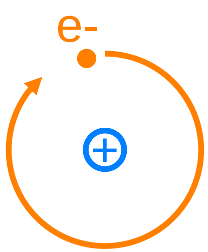

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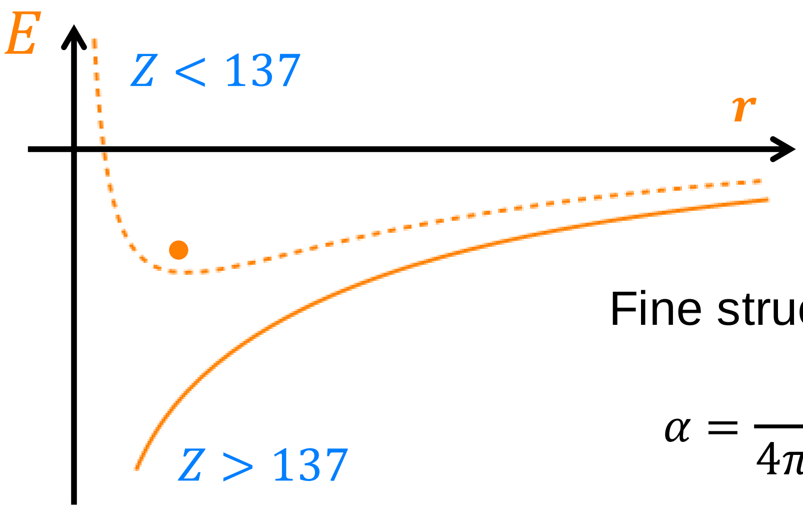

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Fine structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$$

Atomic collapse in a nutshell

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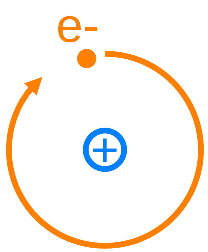

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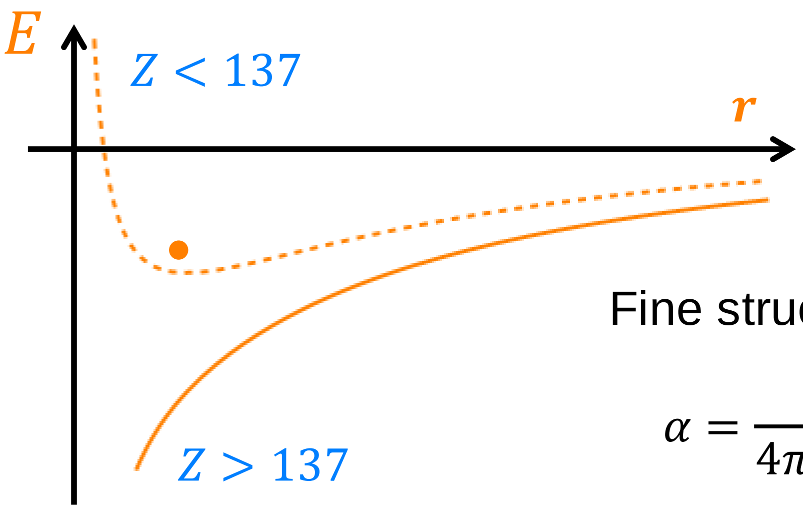

No minimum for $Z > 1/\alpha$
=> heavy atoms should collapse

Pomeranchuk et al.,
J. of Phys. 1945

Fine structure constant

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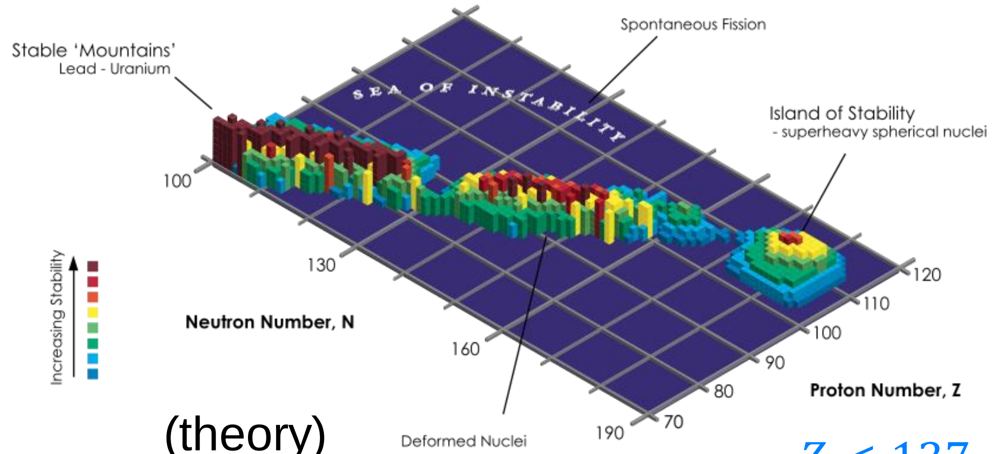
Atomic collapse in a nutshell

Can we observe atomic collapse ? Not really…



(theory)

$Z < 137$

We don't have nuclei with large enough atomic number

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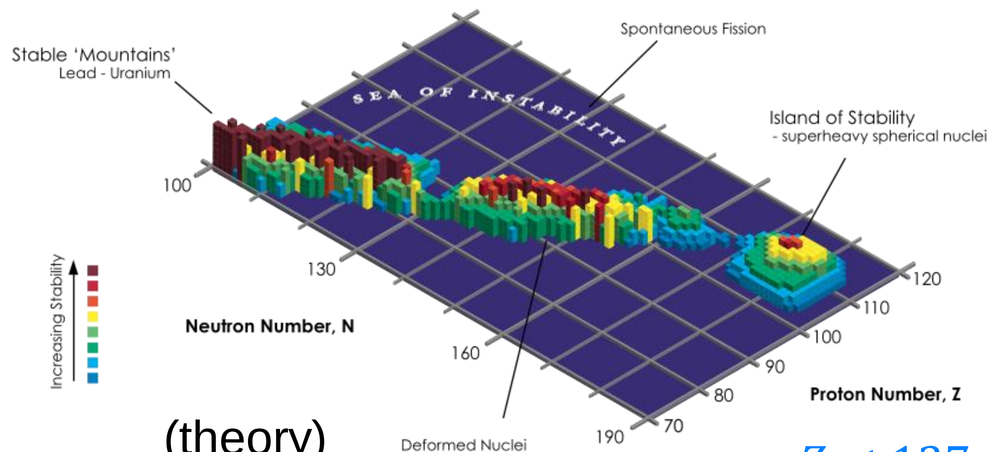

(theory)

$Z < 137$

We don't have nuclei with large enough atomic number

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137}$$

=> Only solution: decrease the speed of light… or use graphene

Shytov et al., PRL 2007

Outline

1) Why is graphene a solution to simulate relativistic effects?
2) Our strategy: a hybrid nanotube-graphene circuit
3) Signature of quasi-relativistic effects in graphene
4) Driving the circuit towards atomic collapse

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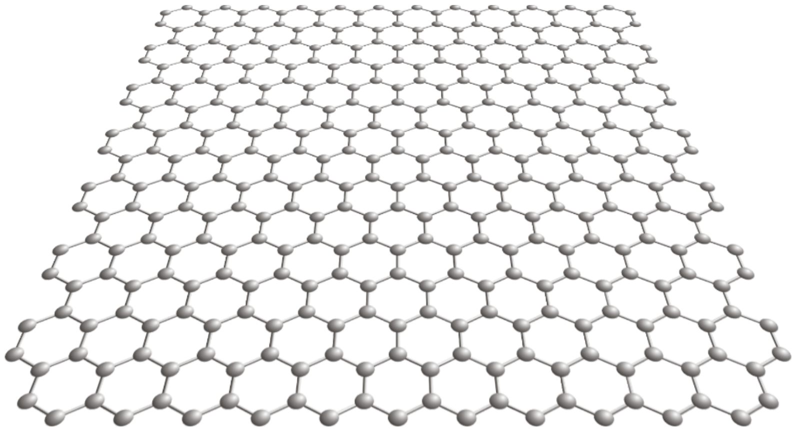
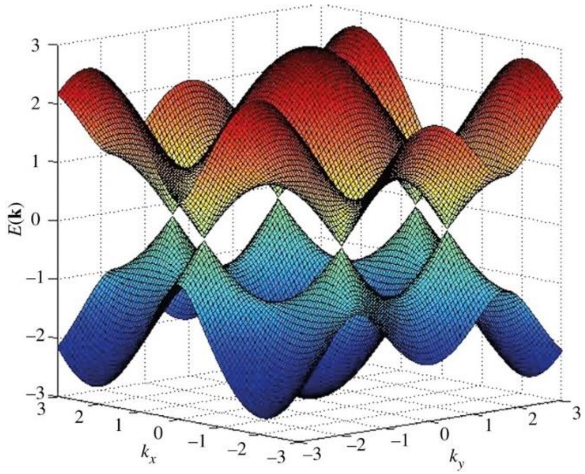
Graphene

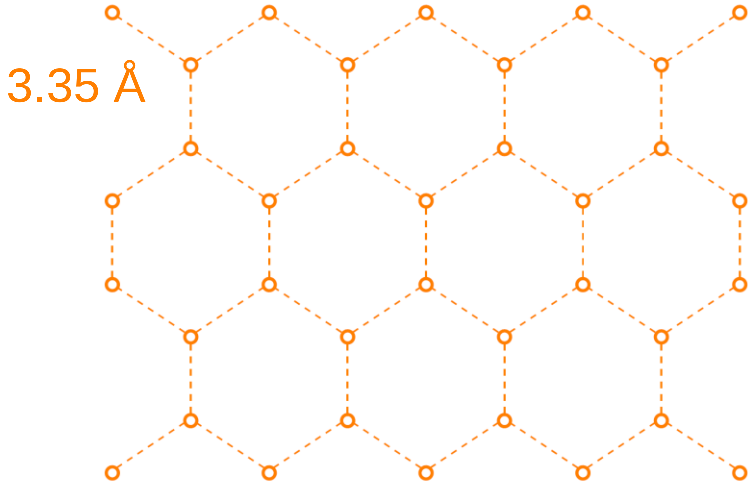
Crystalline structure of carbon atoms



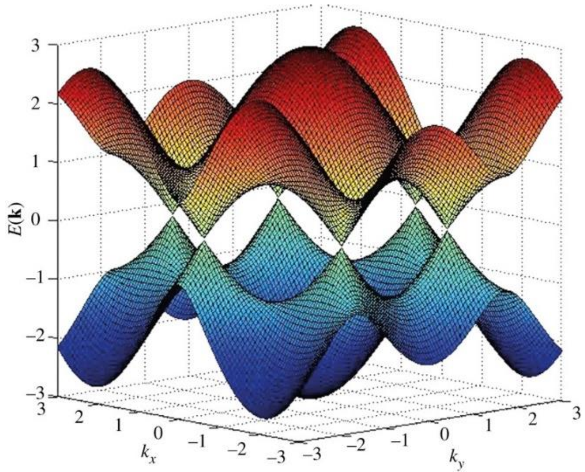
Graphene

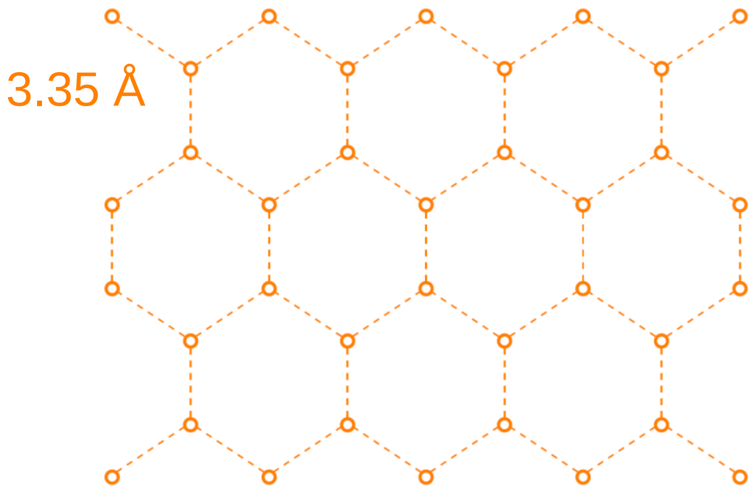
Graphene



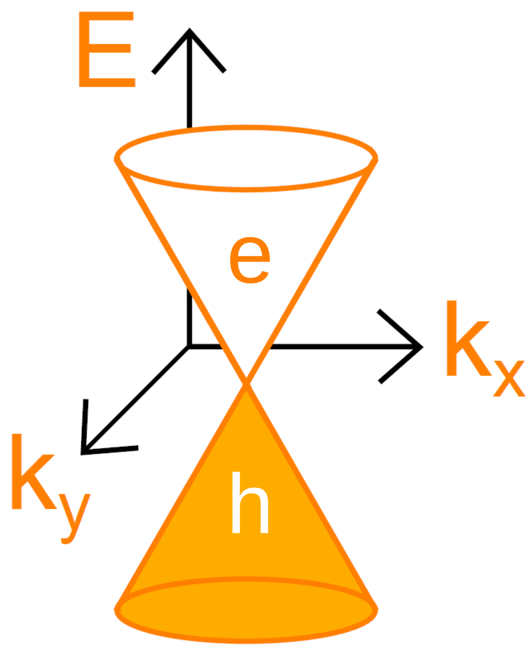


Graphene: a test-bed for relativity





Dirac cone in the band structure



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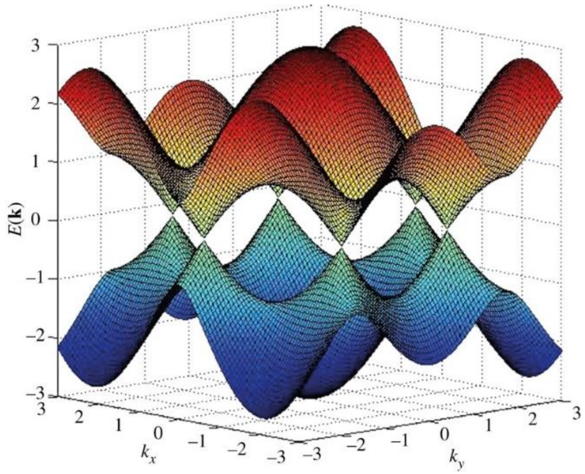

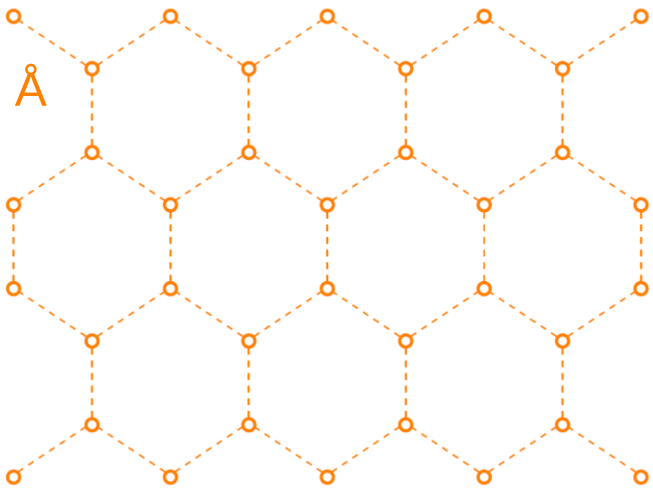

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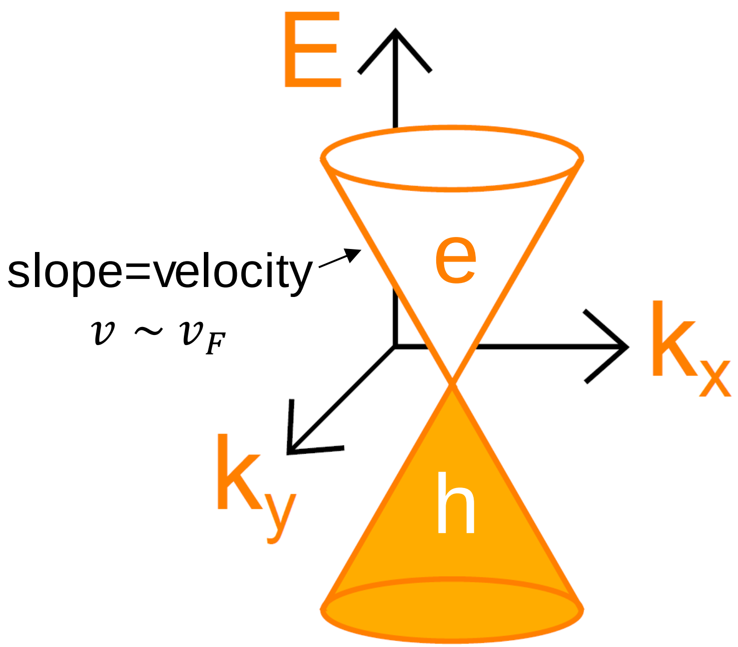
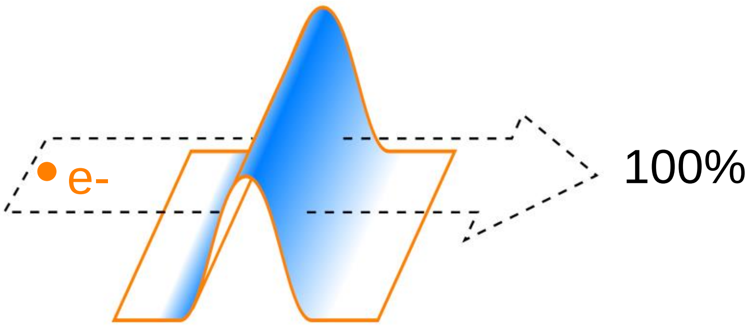


$$v_F \approx c/300$$

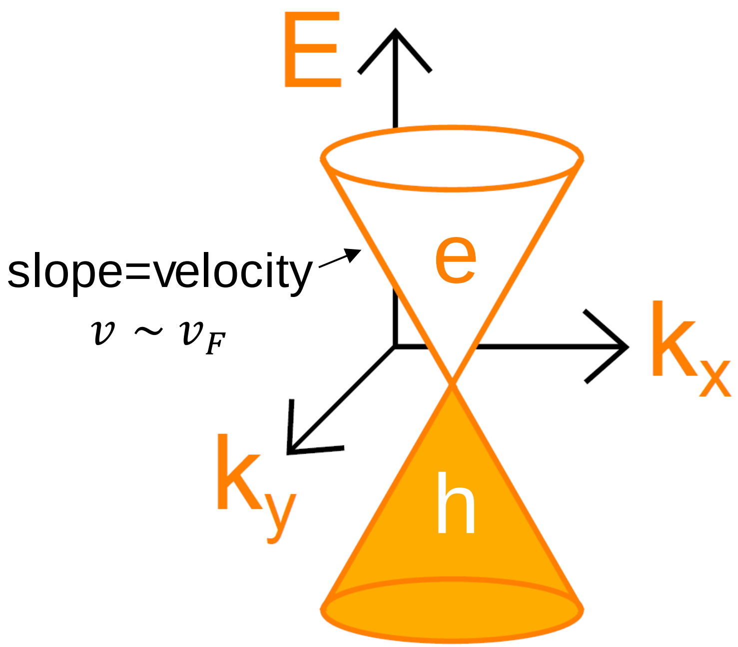
Graphene: a test-bed for relativity

Klein tunneling

Perfect transmission through a barrier



Standter et al., PRL 2009
Young et al., Nat. Phys. 2009

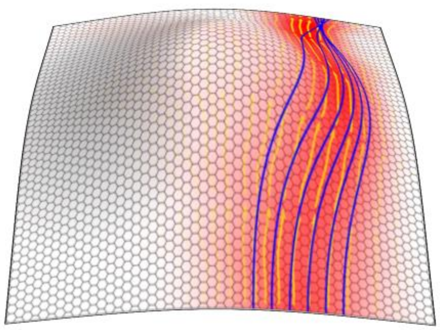


$v_F \approx c/300$

Manipulate relativistic particles

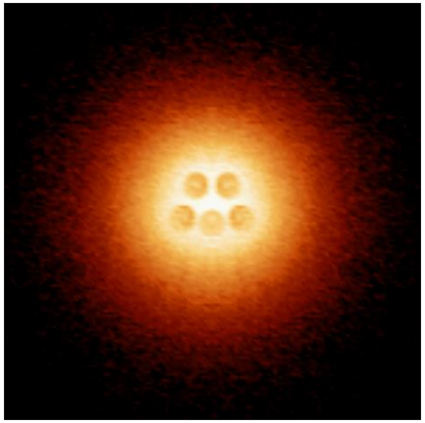
Requires atomic scale defects

Curvature

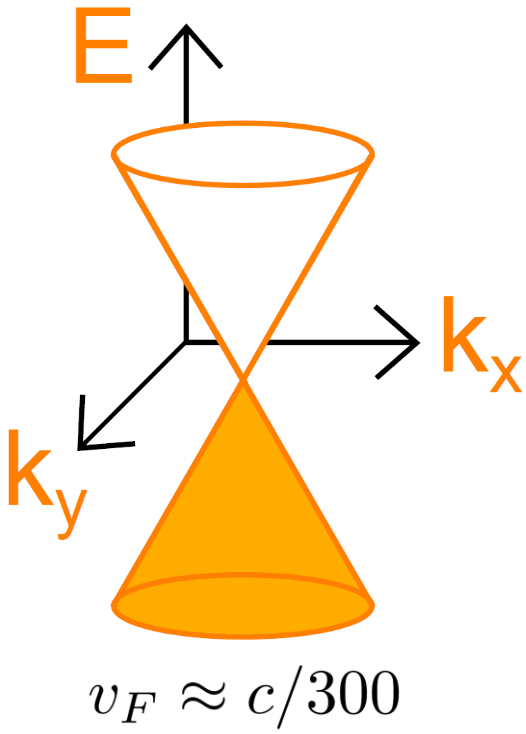


Stegmann et al., New J of Phys. 2016

Localized charges



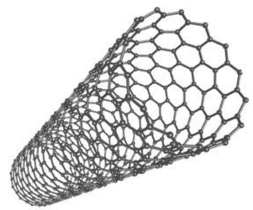
Wang et al., Science 2013
(atomic collapse)

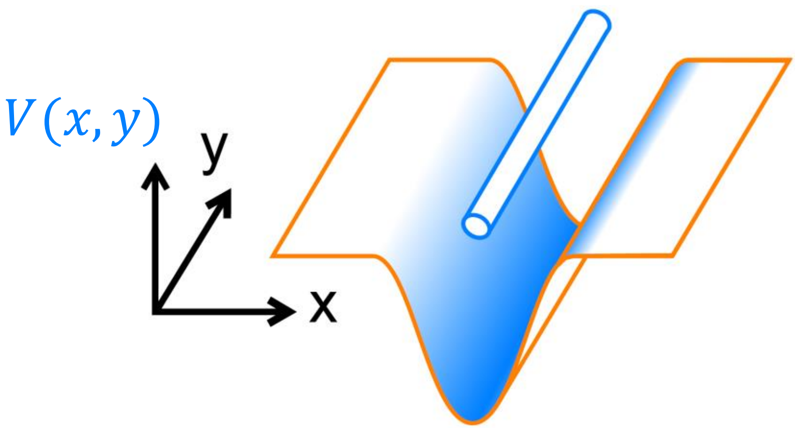


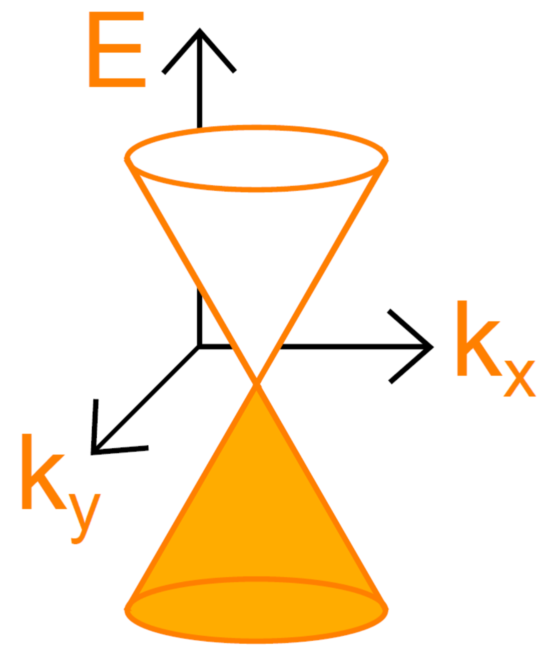
Manipulate relativistic particles

Our solution

A carbon nanotube







$$v_F \approx c/300$$

<u>2 functions</u>: 1) an artificial/tunable **nucleus**

2) a **probe**

Outline

1) Why is graphene a solution to simulate relativistic effects?

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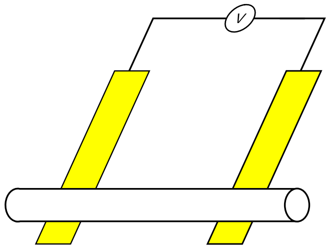
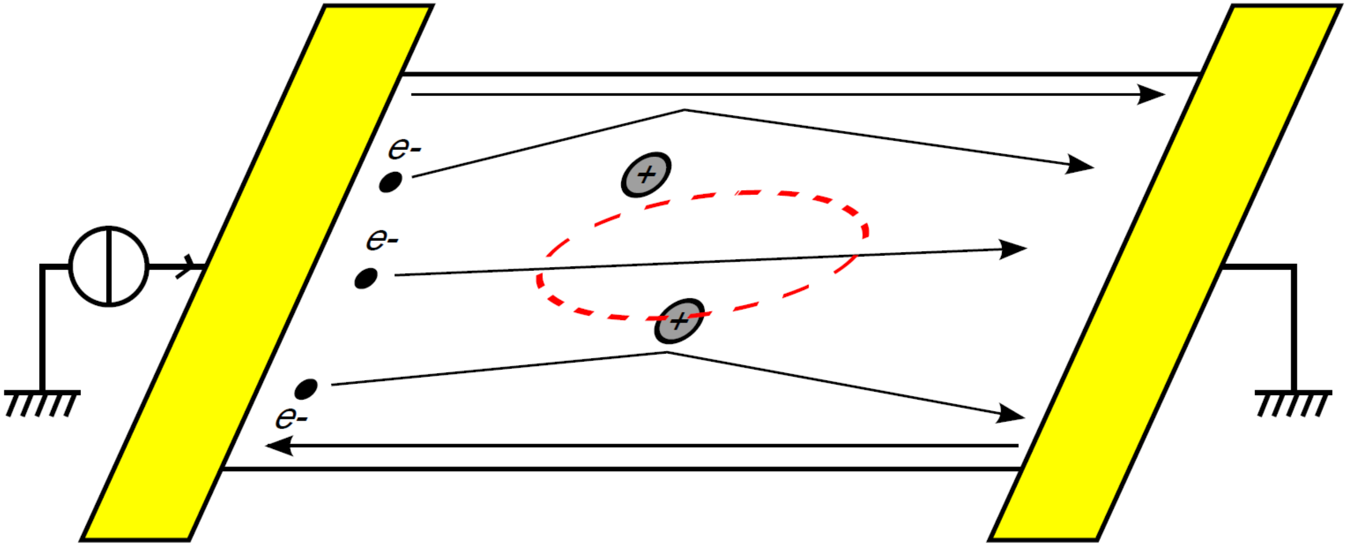
   Structure and functioning

3) Signature of quasi-relativistic effects in graphene

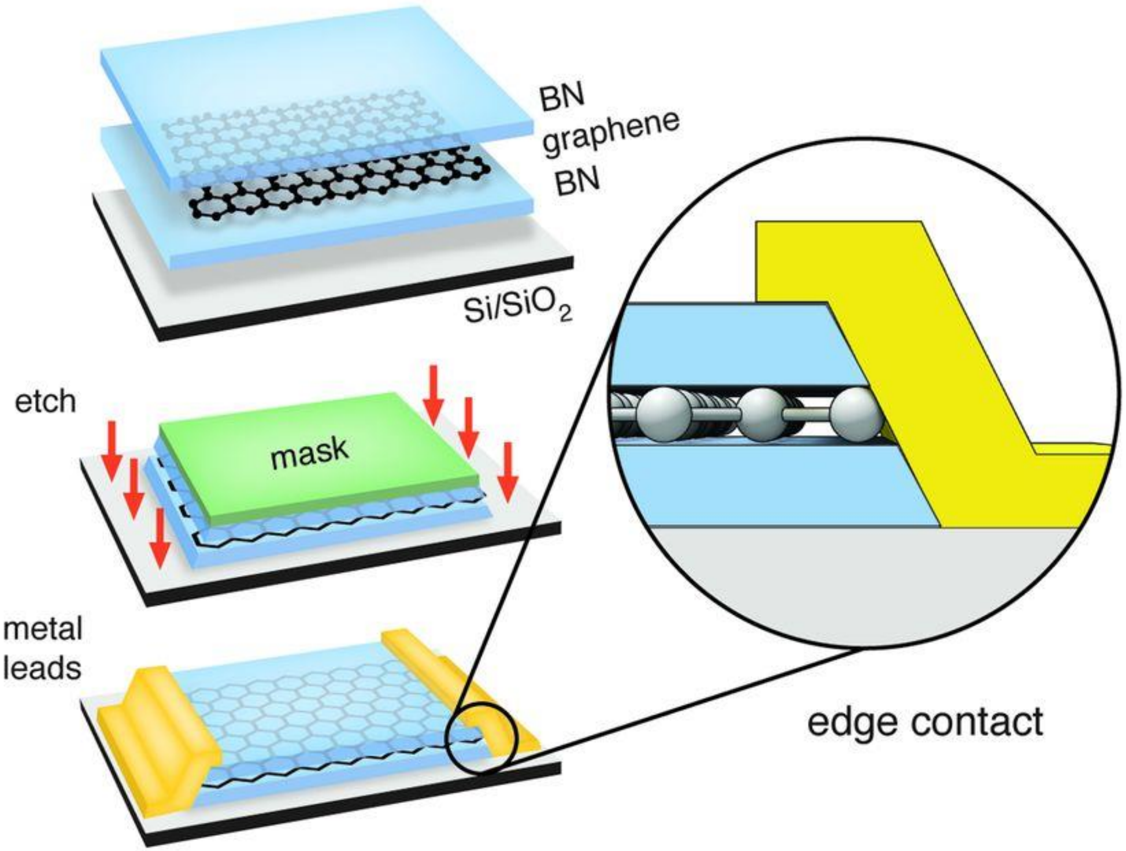
4) Driving the circuit towards atomic collapse

1D defect: a carbon nanotube



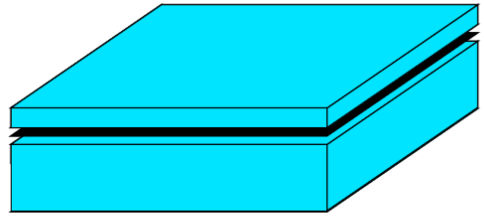


Graphene-based electrical device



Wang et al., Science 2013

CNT on h-BN encapsulated graphene



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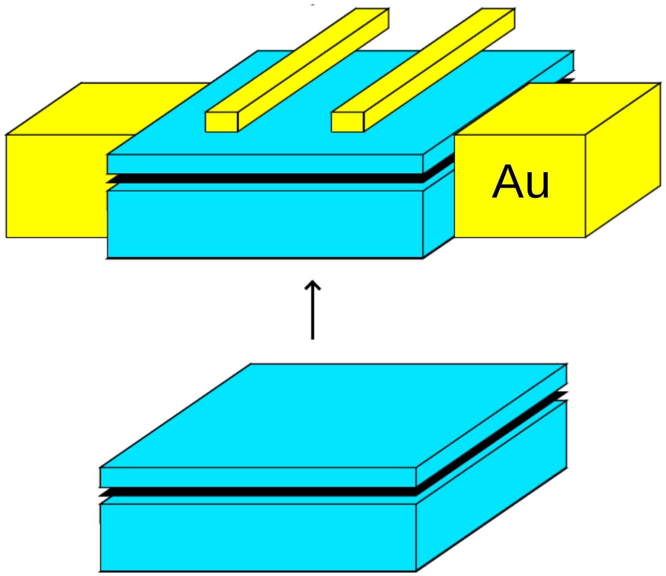

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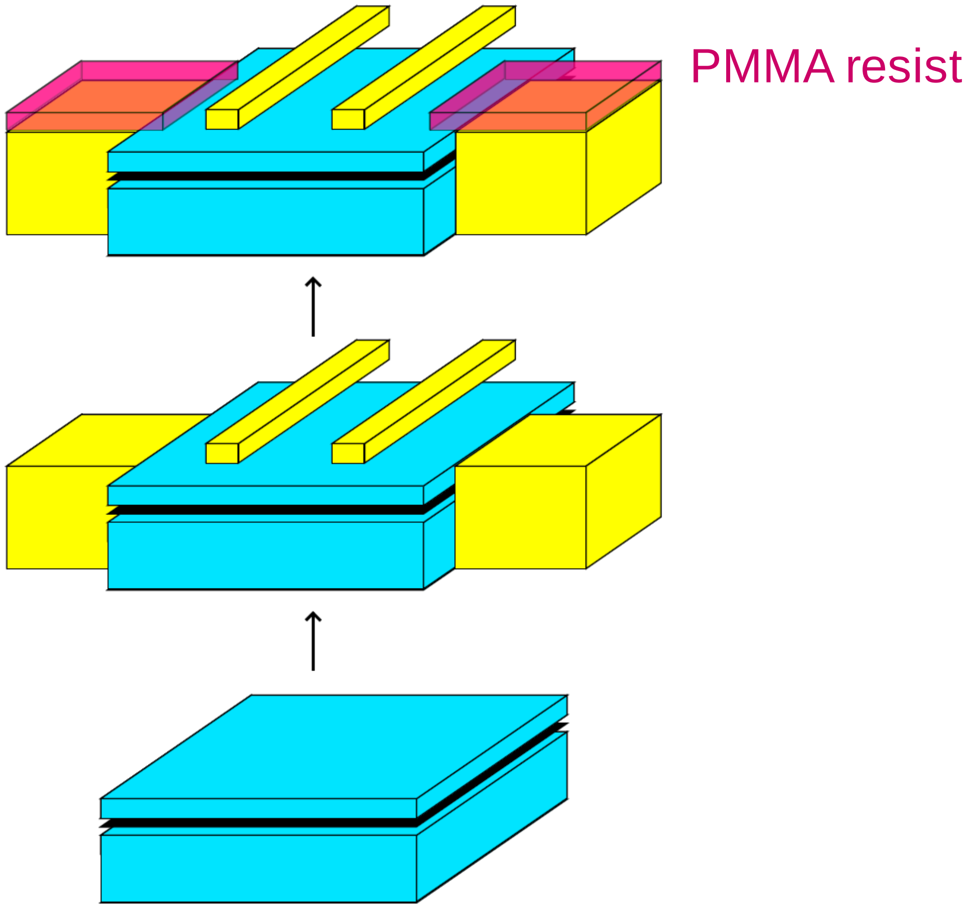
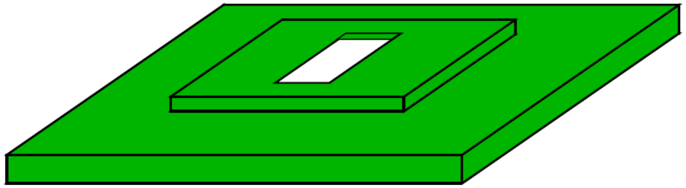
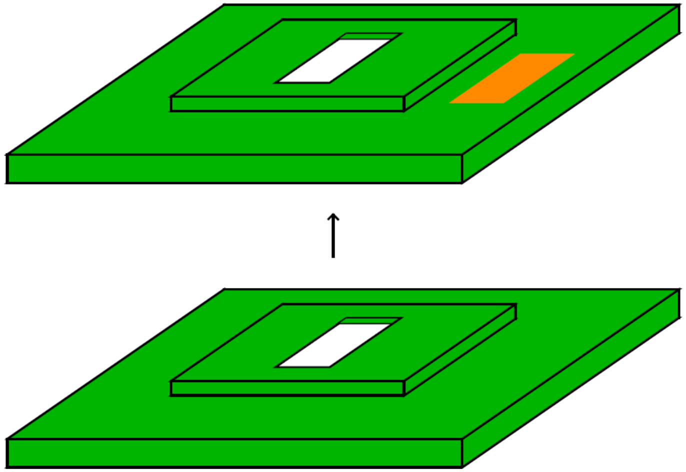


Carbon Nanotube growth and characterization



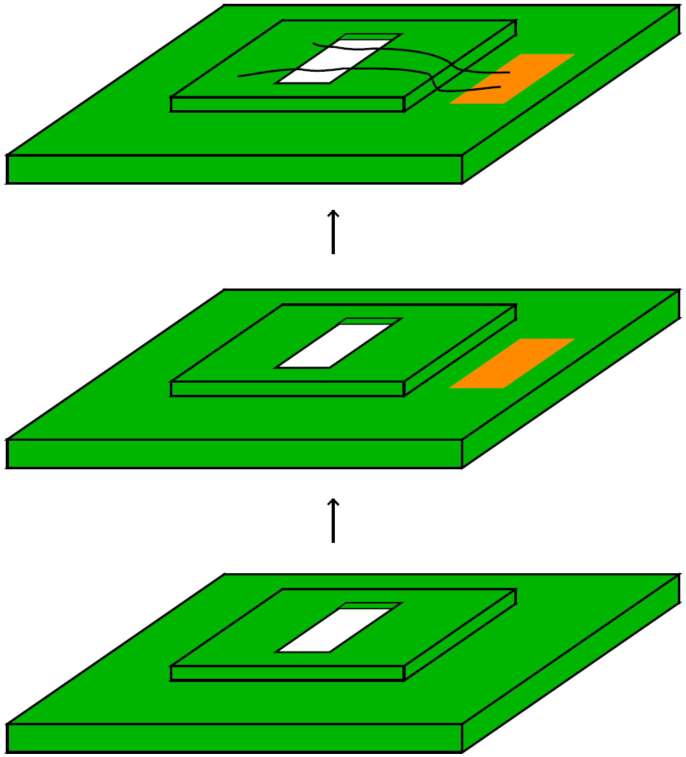
Silicon chip with a slit

Carbon Nanotube growth and characterization



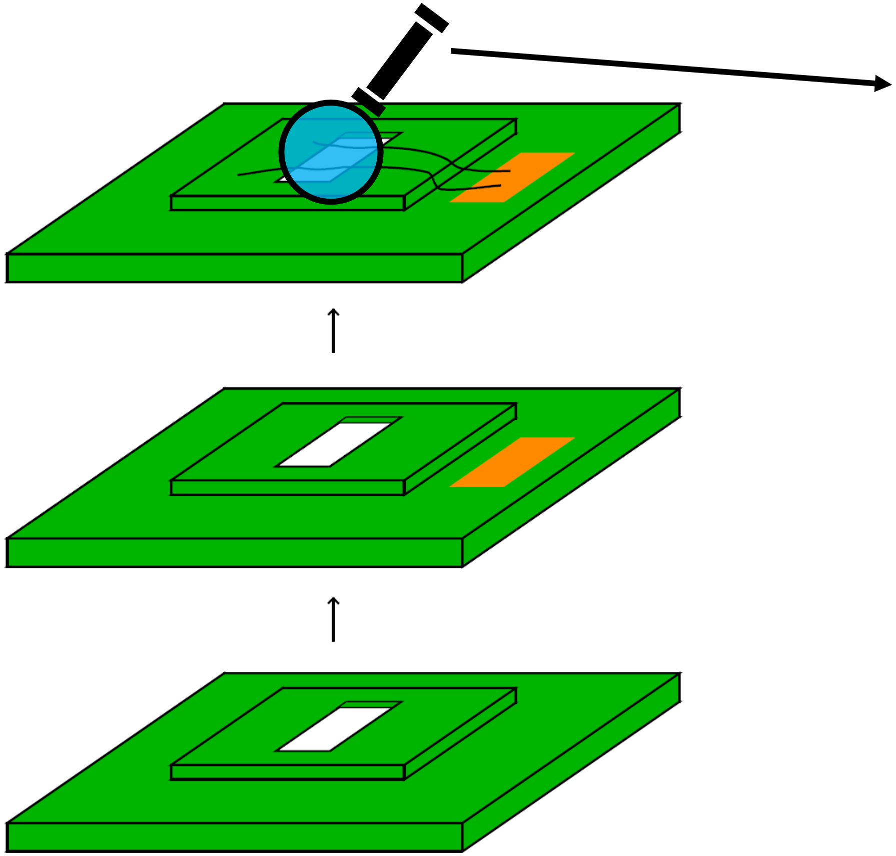
Deposition of catalyst

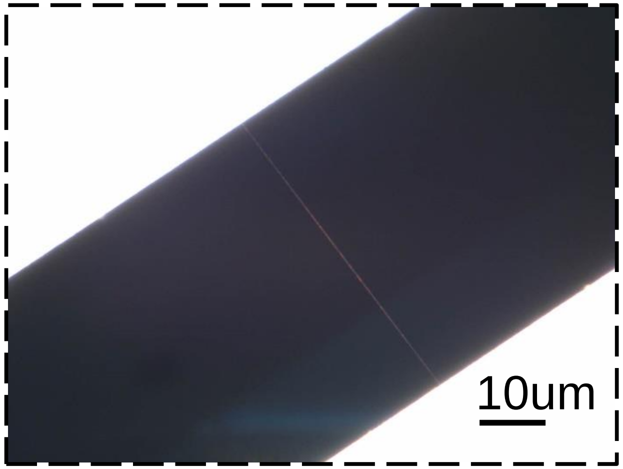
Carbon Nanotube growth and characterization



CVD growth

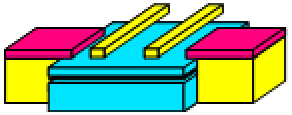
Carbon Nanotube growth and characterization





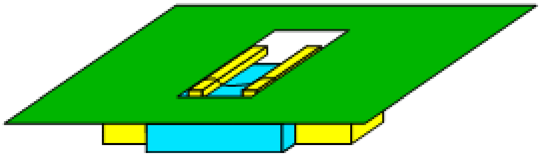
Optical image of CNT
Covered with 30nm Au

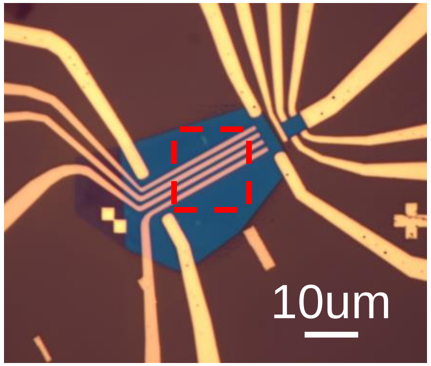
Deposition of CNT on h-BN encapsulated graphene



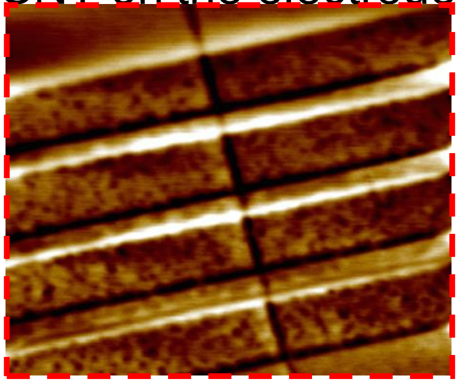
Melt resist by warming up to 180°C

Deposition of CNT on h-BN encapsulated graphene



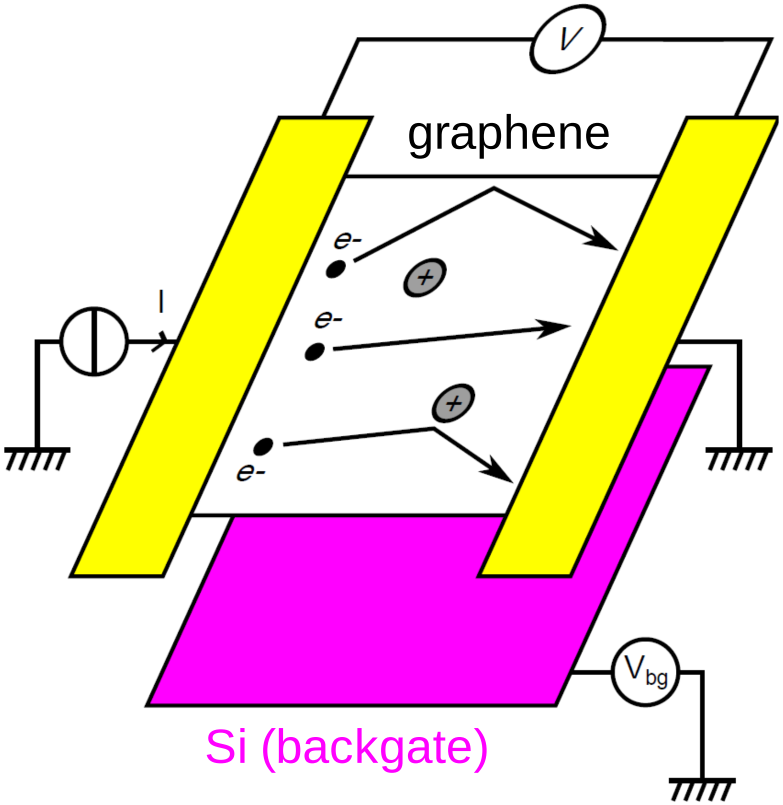


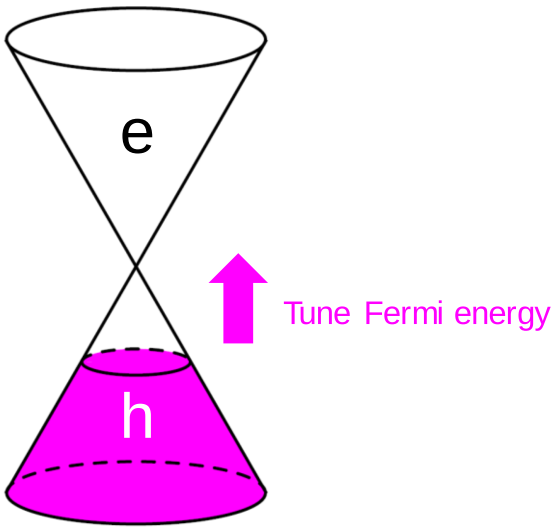
Leave the CNT on the electrodes



EFM image of CNT

Characterization of graphene





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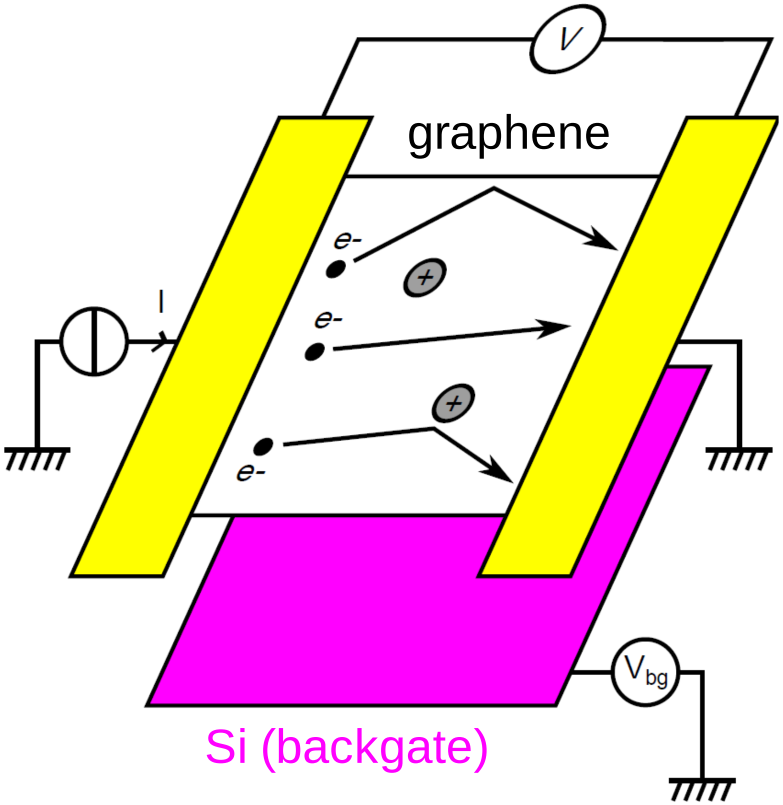
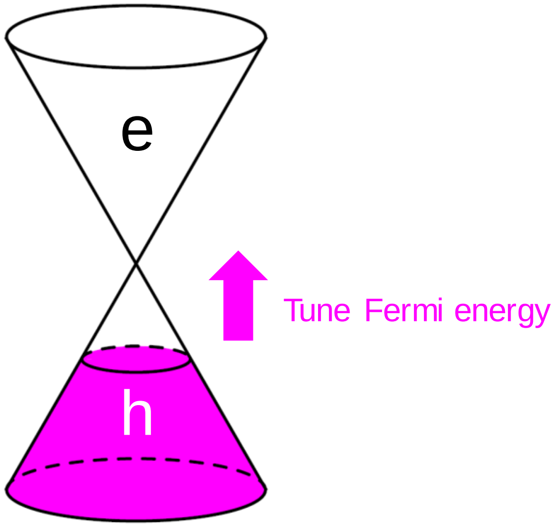
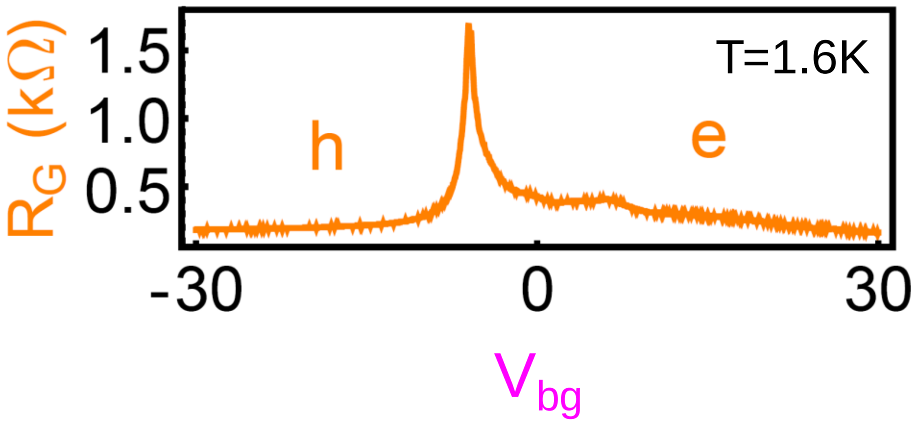
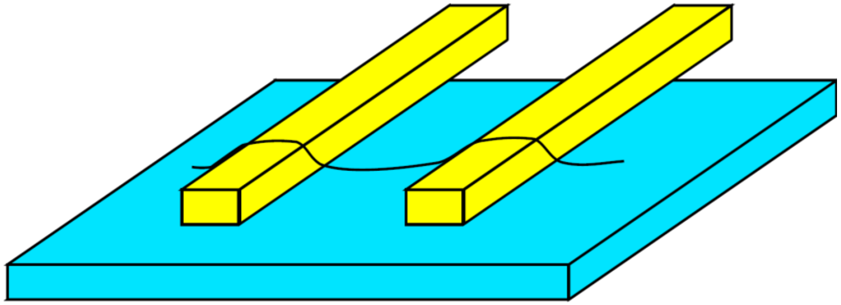


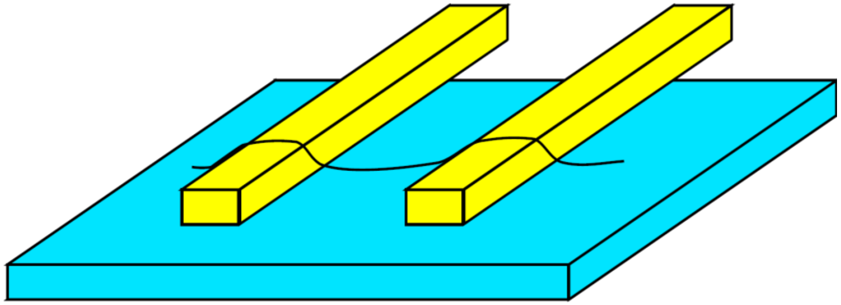




CNT electronic behavior

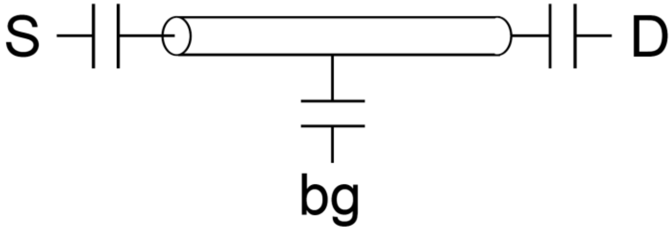


CNT electronic behavior



Coulomb repulsion
=> Charging energy

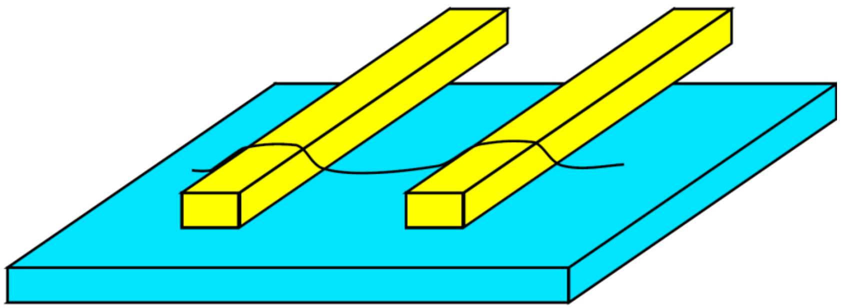
$$U = \frac{e^2}{2C}$$

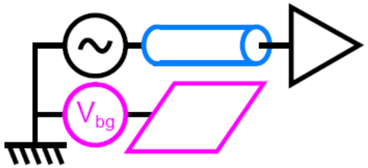


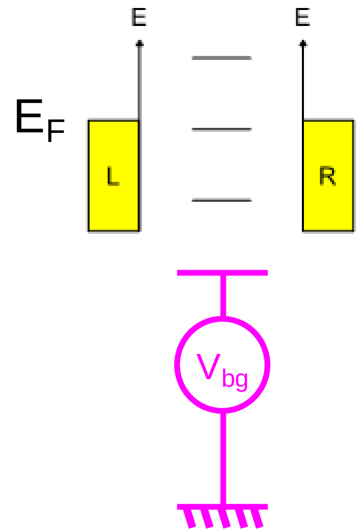
$$U \approx \mathrm{meV} \gg kT$$

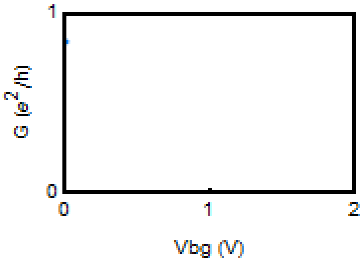
(T=1.5K)

CNT behaves as a Quantum Dot

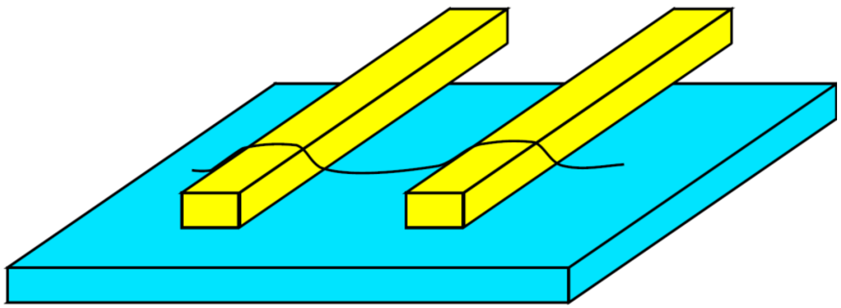


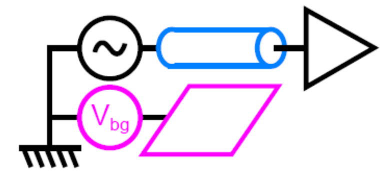


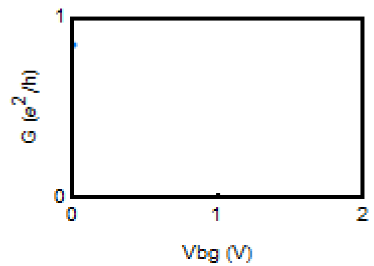


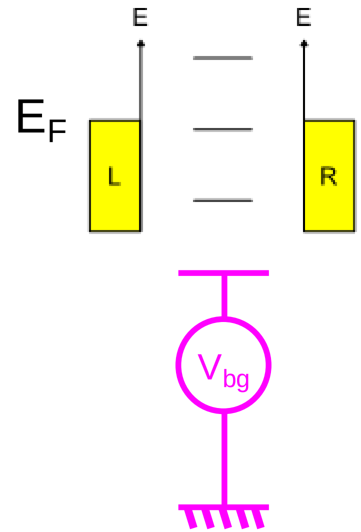


CNT behaves as a Quantum Dot

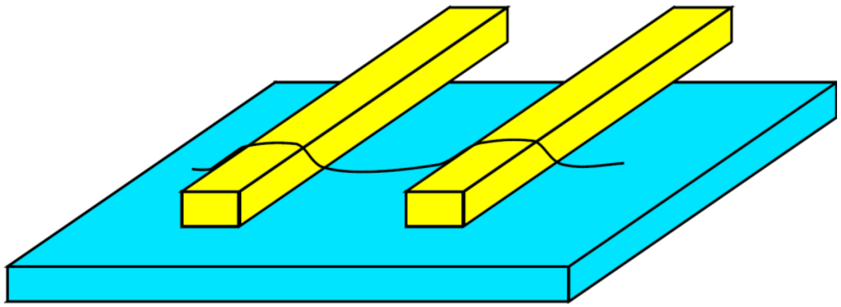


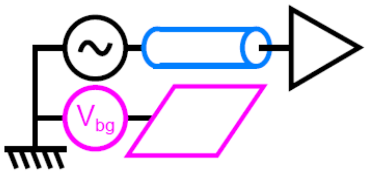


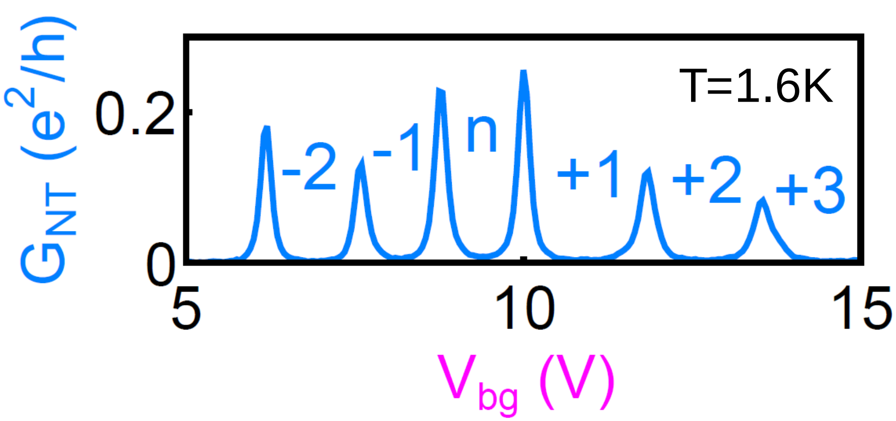


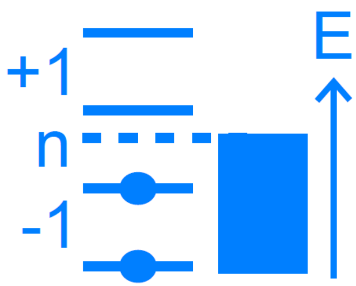


CNT behaves as a Quantum Dot



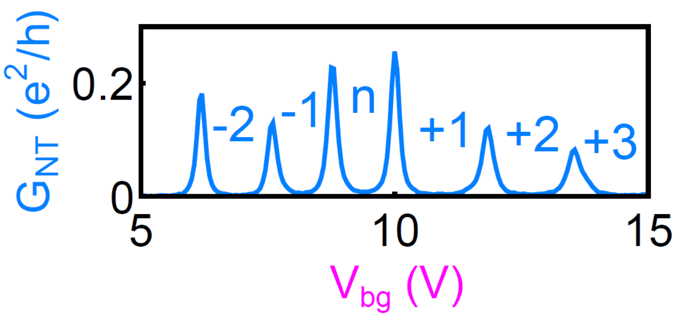






Nanotube as an artificial nucleus

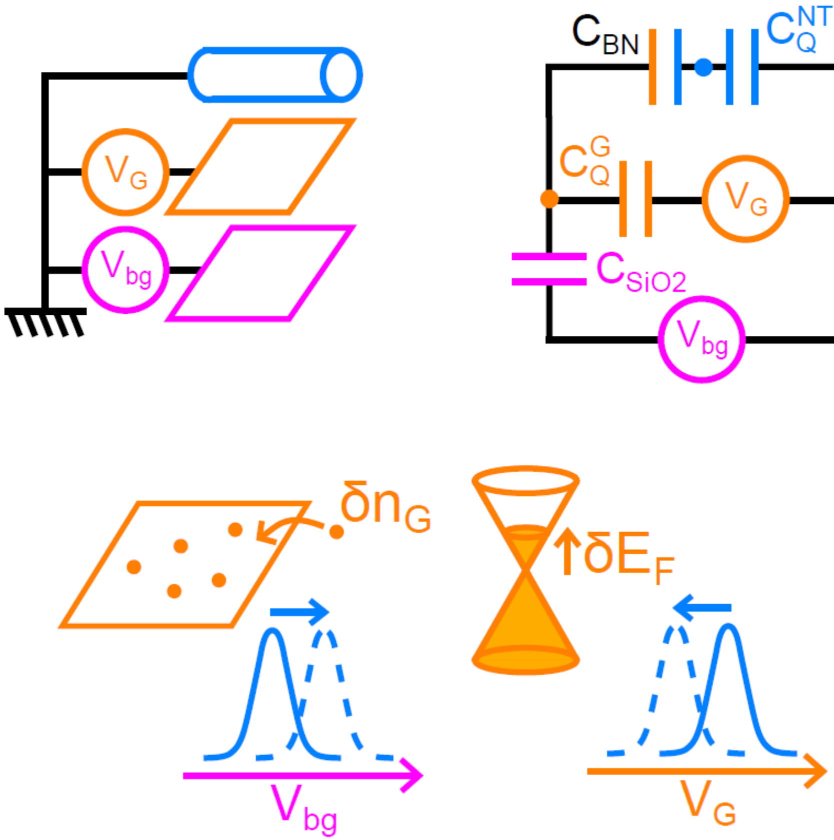
Nanotube can be used as a <u>tunable</u> nucleus…



$$Z \propto n \pm 1,2,3\ldots$$

Nanotube as a detector

…and a charge <u>detector</u>



Outline

1) Why is graphene a solution to simulate relativistic effects?

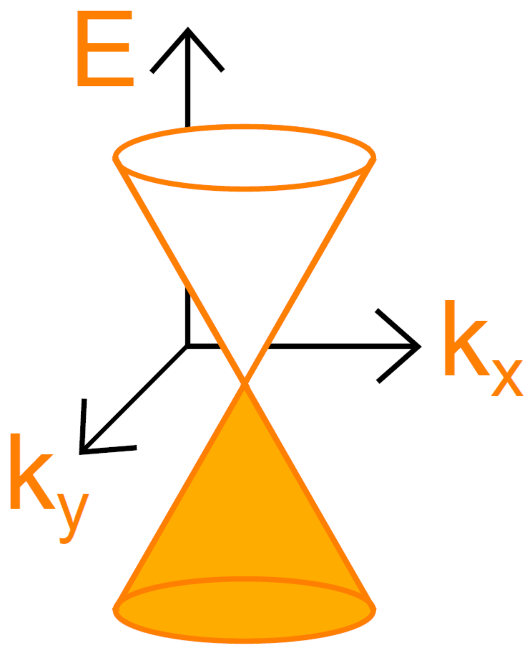
2) Our strategy: a hybrid nanotube-graphene circuit

3) Signature of quasi-relativistic effects in graphene
   Linear dispersion relation and Lorentz invariance

4) Driving the circuit towards atomic collapse

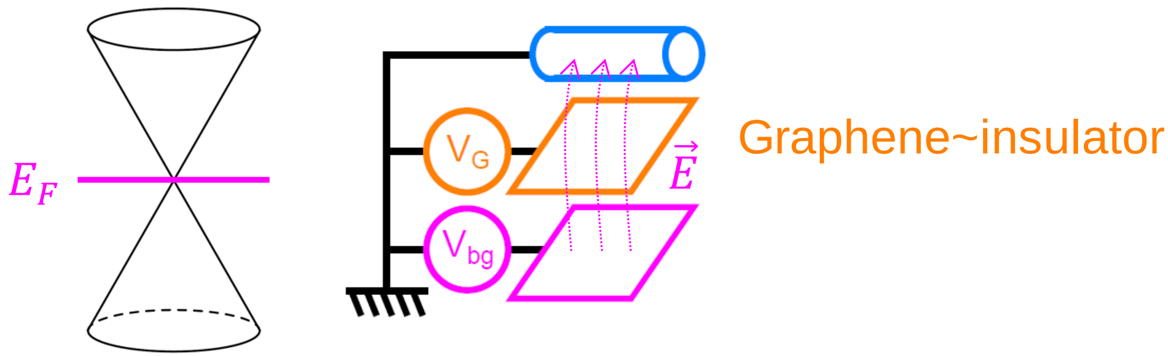
Test #1: Linear dispersion relation

Is the Fermi velocity v_F a constant?

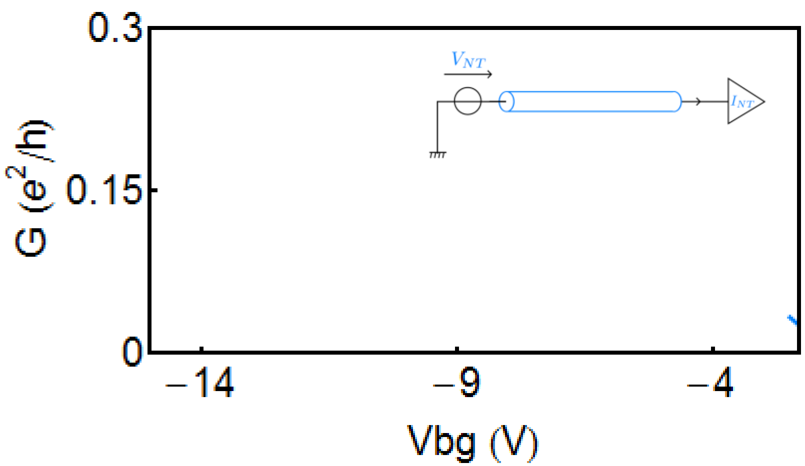


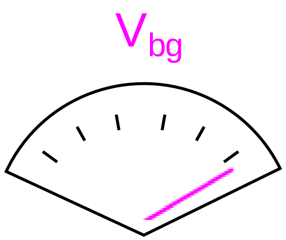
$$v_F \approx c/300$$

Charging up graphene



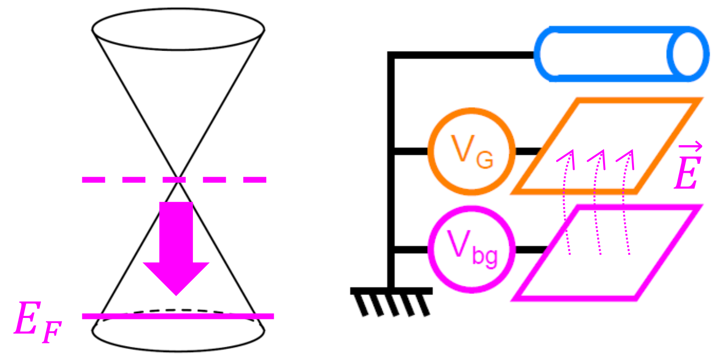
Vbg=0 low density of states in graphene





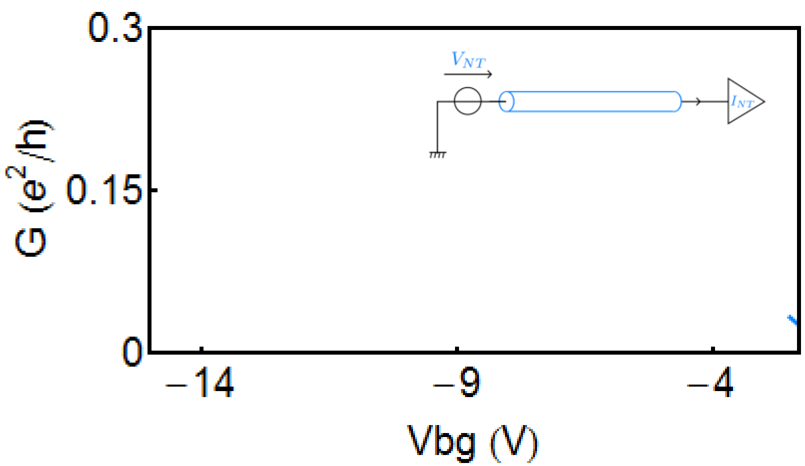
(T=1.5K)

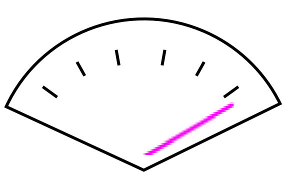
Charging up graphene



Graphene~metal
=>screening

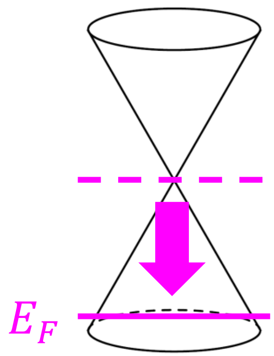
Vbg<<0 large density of states in graphene

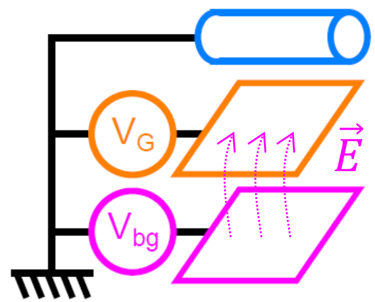




(T=1.5K)

Charging up graphene





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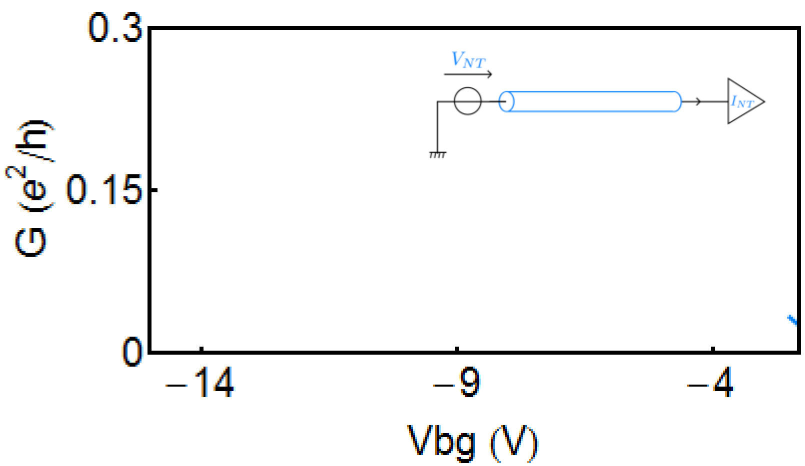
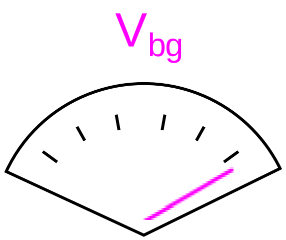
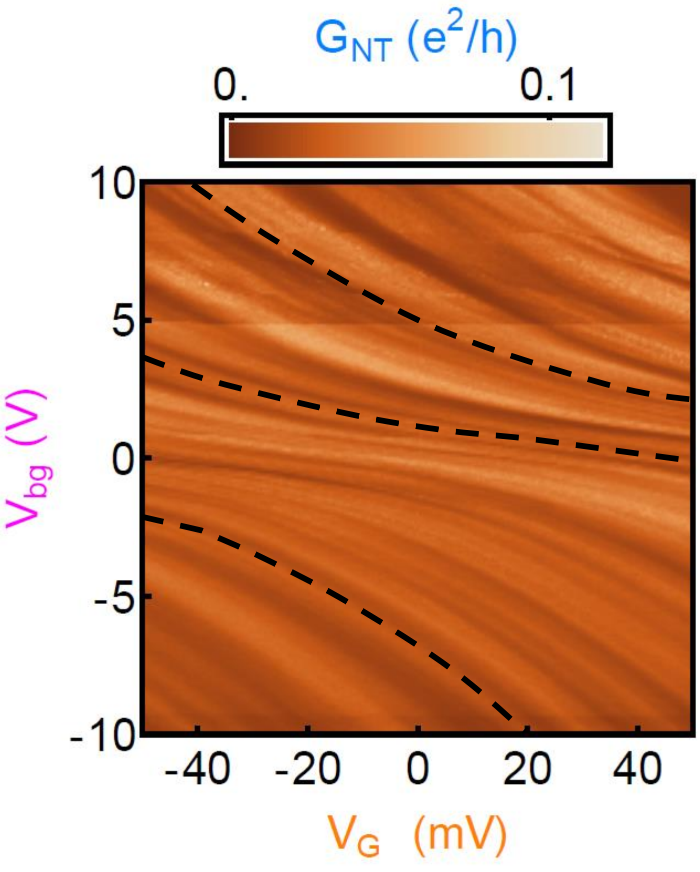
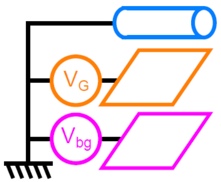


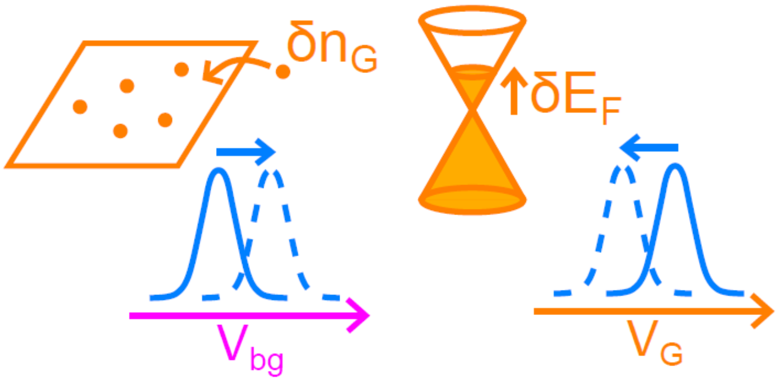


Nanotube less and less sensitive
to the back gate because of screening
=>slower oscillations

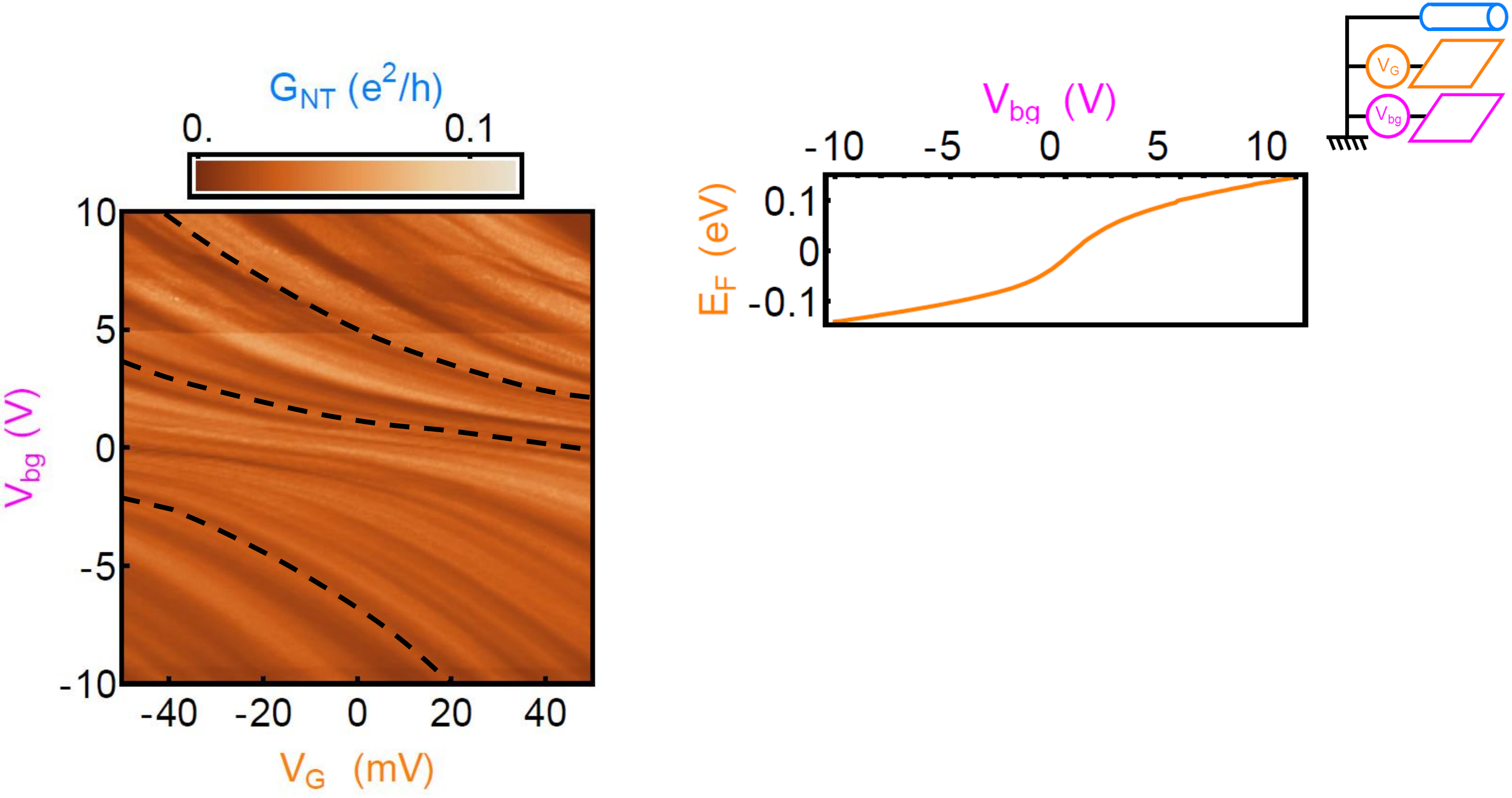
Measurement of Fermi energy and Fermi velocity



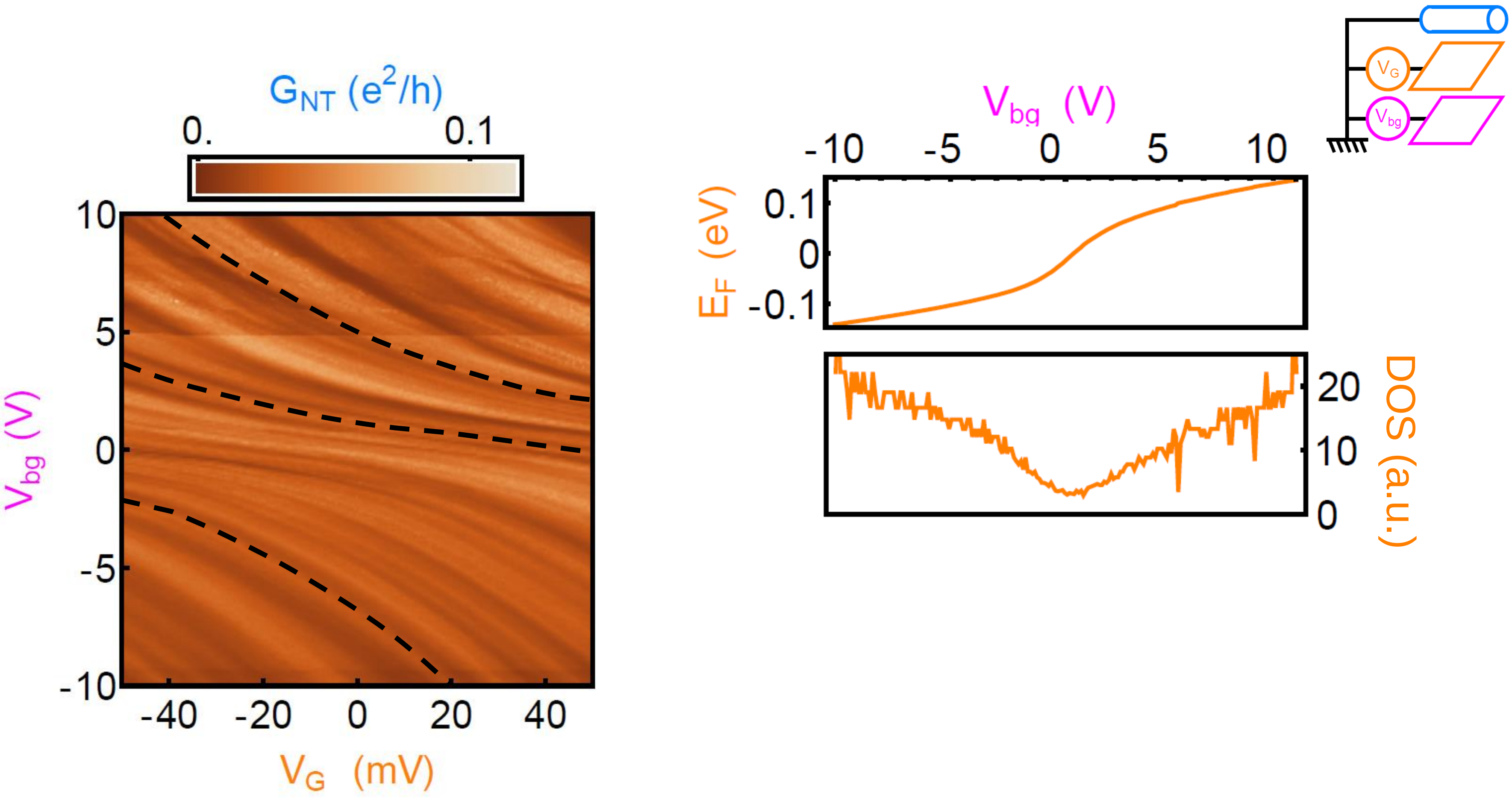




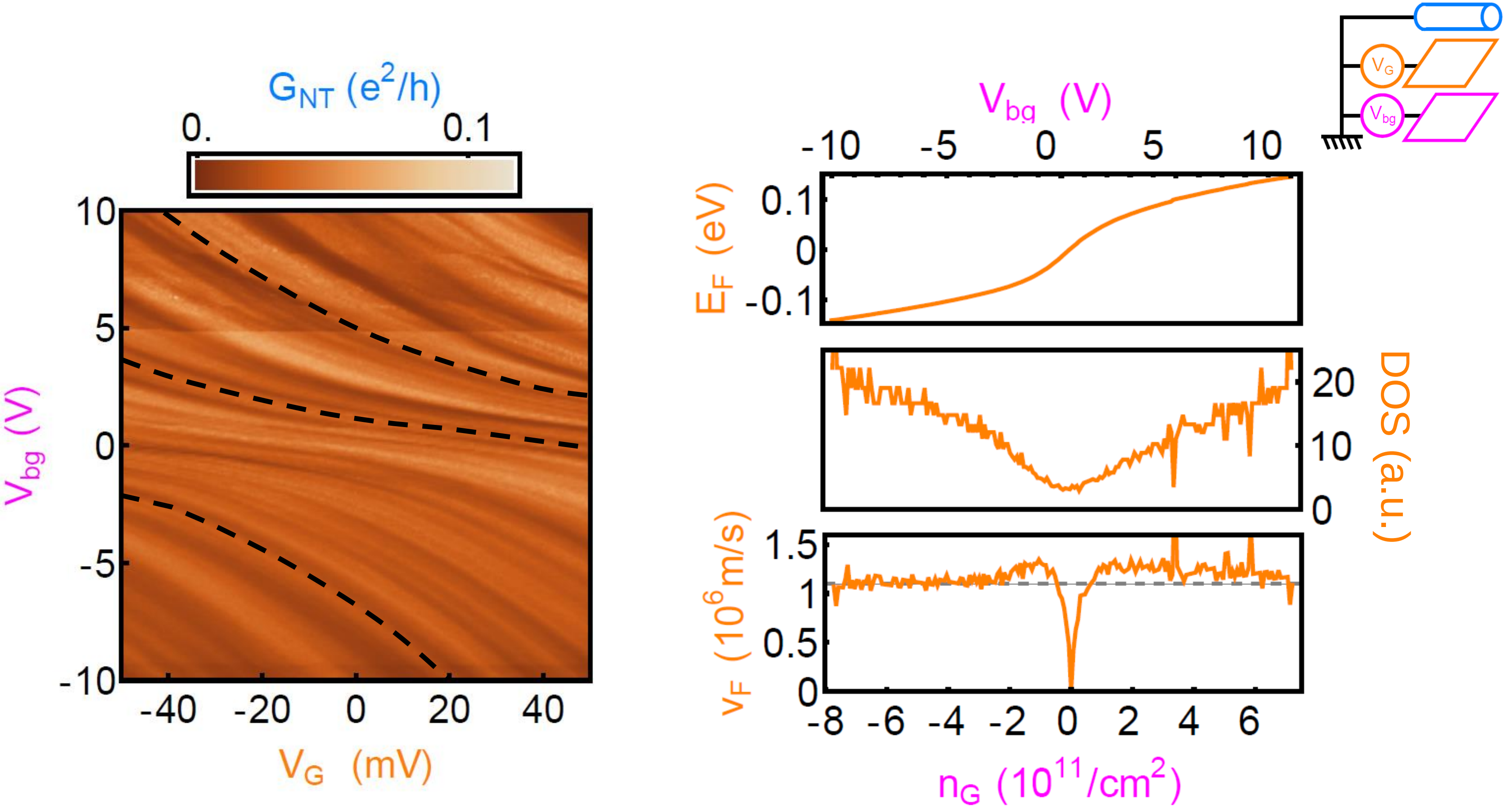
Measurement of Fermi energy and Fermi velocity



Measurement of Fermi energy and Fermi velocity

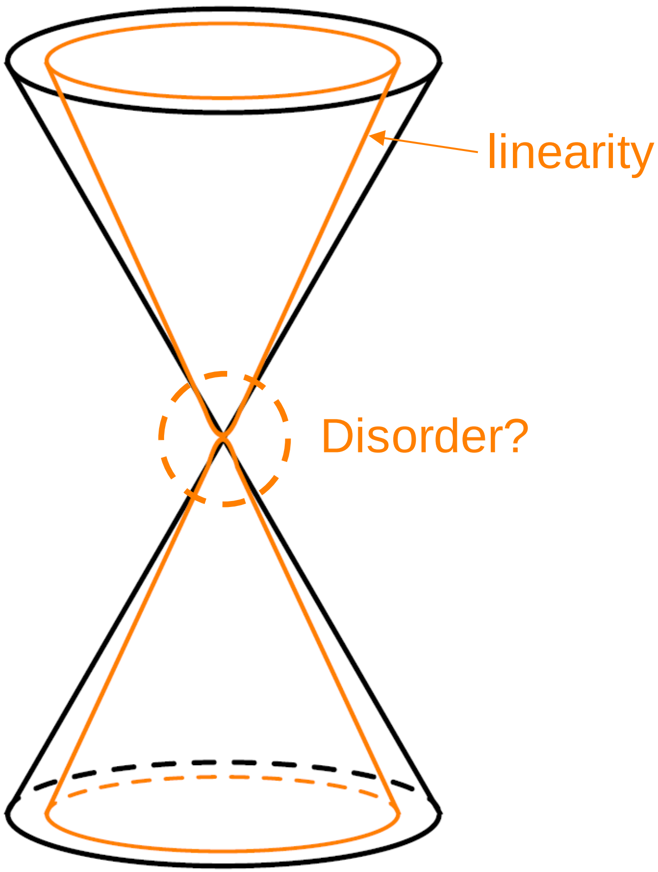


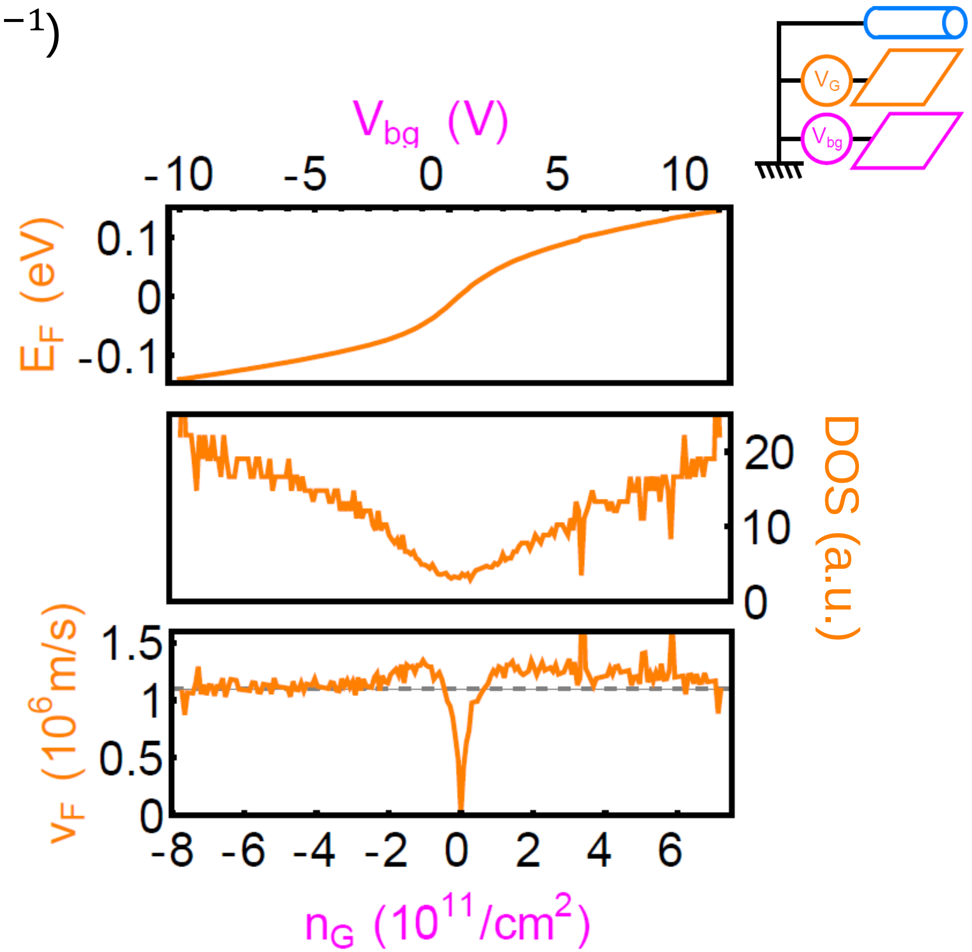
Measurement of Fermi energy and Fermi velocity



Measurement of Fermi energy and Fermi velocity

$v_F > v_F$ from tight-binding ($10^6\ m.s^{-1}$)

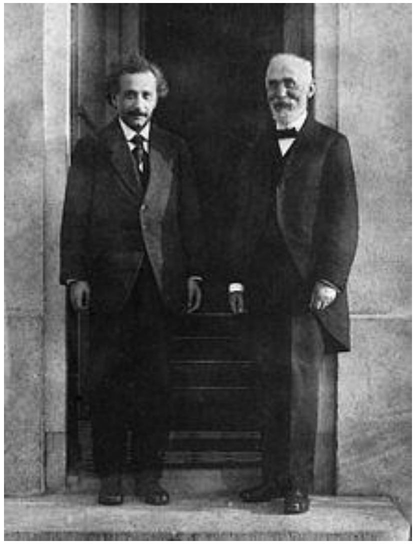




Linear over a large energy range

Test #2: Lorentz invariance

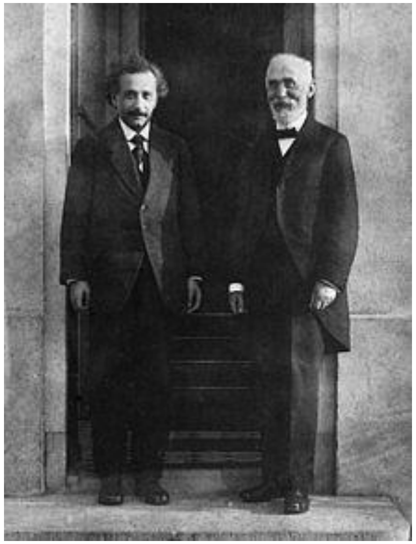
Are laws of Physics **Lorentz invariant** ?



Test #2: Lorentz invariance

Are laws of Physics **Lorentz invariant** ?

"Electricity and magnetism are not independent of one another, but <u>are intimately related</u>, so that both sets of phenomena should be regarded as parts of one vast system, embracing all Nature."



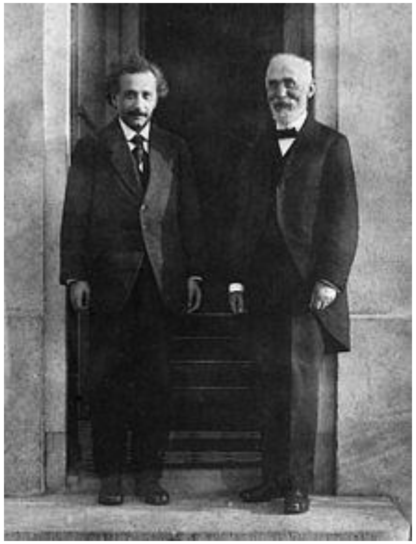
Hendrik Antoon Lorentz
The Einstein Theory of Relativity

Test #2: Lorentz invariance

Are laws of Physics **Lorentz invariant** ?

"Electricity and magnetism are not independent of one another, but <u>are intimately related</u>, so that both sets of phenomena should be regarded as parts of one vast system, embracing all Nature."

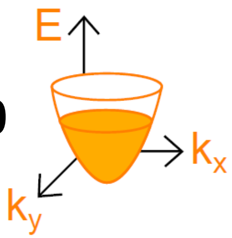
"The relation of the two is, however, of such a character that it is <u>perceptible only in a very few instances</u>, and then only to refined observations."

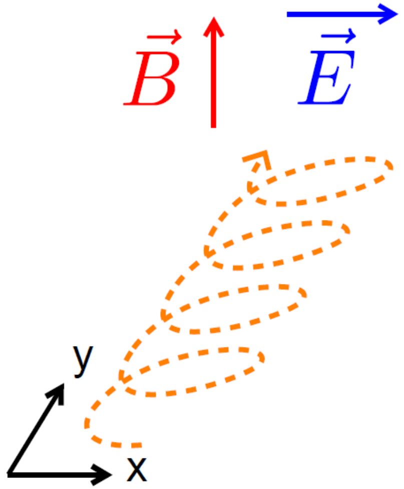


Hendrik Antoon Lorentz
The Einstein Theory of Relativity

Non-relativistic electron

Massive electrons m ≠ 0

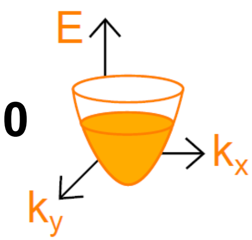


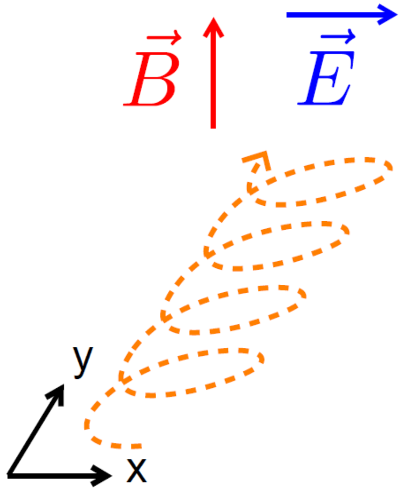


$$\vec{v}_d = \frac{\vec{E} \times \vec{B}}{B^2}$$

Non-relativistic electron

Massive electrons m ≠ 0





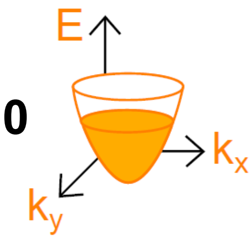
$$\vec{v}_d = \frac{\vec{E}\times\vec{B}}{B^2}$$

Galilean transformation

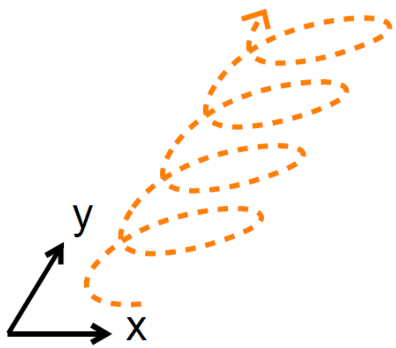
$$x' = x$$
$$y' = y - v_d t$$
$$t' = t$$

Non-relativistic electron

Massive electrons m ≠ 0



$\vec{B}$ $\vec{E}$

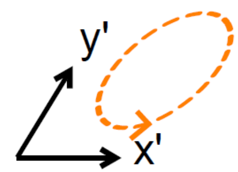


$$\vec{v}_d = \frac{\vec{E}\times\vec{B}}{B^2}$$

Galilean transformation

$$x' = x$$
$$y' = y - v_d t$$
$$t' = t$$

$\vec{B}$ $\vec{E}' = 0$

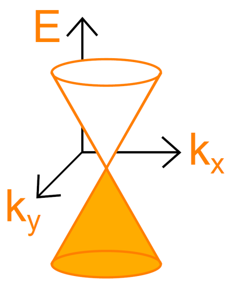


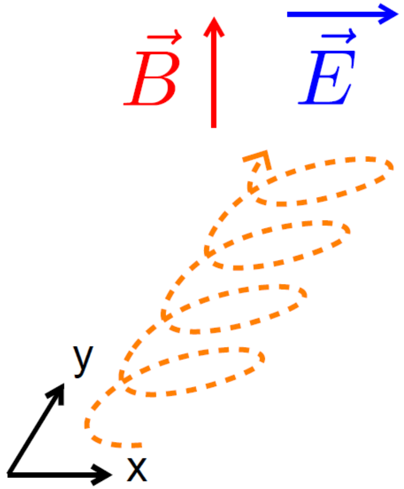
Electric field cancels in moving frame but magnetic field remains identical

=> For slow particles, E and B look independent

What about relativistic electrons?

Massless electrons m = 0





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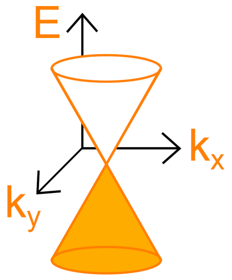
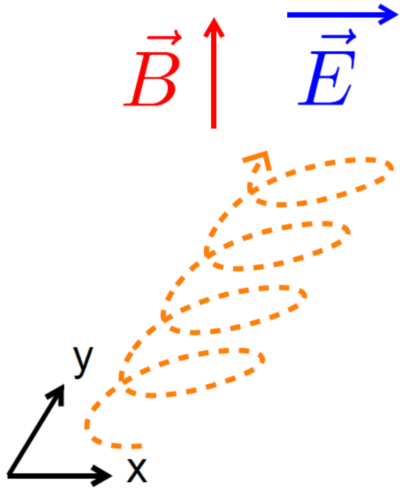




Dirac equation

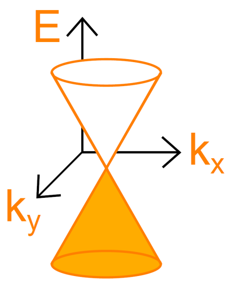
$$\gamma^\mu \left(\hbar \partial_\mu - e A_\mu\right) \psi = 0$$

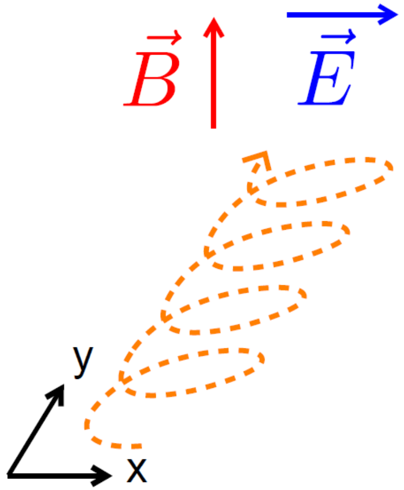
Electromagnetic tensor

$$A_\mu = \left(v_F^{-1} E x, 0, -B x\right)$$

What about relativistic electrons?

Massless electrons m = 0





$$\gamma = \frac{1}{\sqrt{1-v_d^2/v_F^2}} < 1$$

Lorentz transformation

$$x' = x$$
$$y' = \gamma(y - v_d t)$$
$$t' = \gamma(t - v_d y/v_F^2)$$

?

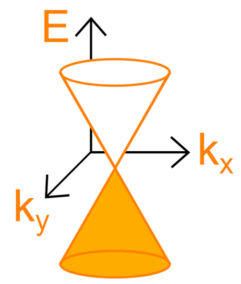
Dirac equation

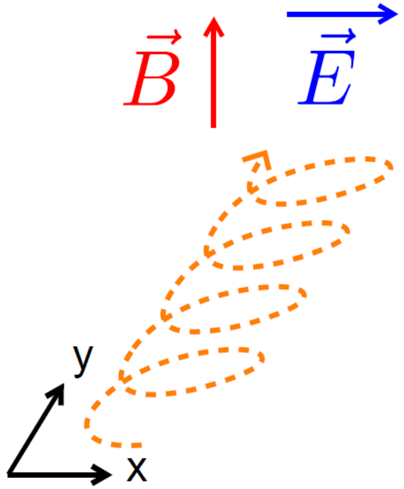
$$\gamma^\mu \left(\hbar\partial_\mu - eA_\mu\right)\psi = 0$$

$$A_\mu = \left(v_F^{-1}Ex, 0, -Bx\right)$$

What about relativistic electrons?

Massless electrons m = 0





$$\gamma = \frac{1}{\sqrt{1-v_d^2/v_F^2}} < 1$$

Lorentz transformation

$$E' = \gamma\left(E - v_d B\right)$$
$$B' = \gamma\left(B - v_d/v_F^2 \times E\right)$$

$$x' = x$$
$$y' = \gamma(y - v_d t)$$
$$t' = \gamma(t - v_d y/v_F^2)$$

Dirac equation

$$\gamma^\mu \left(\hbar\partial_\mu - eA_\mu\right)\psi = 0 \qquad A_\mu = \left(v_F^{-1}Ex, 0, -Bx\right)$$

Shytov et al., PRL 2007
Goerbig et al., EPL 2009

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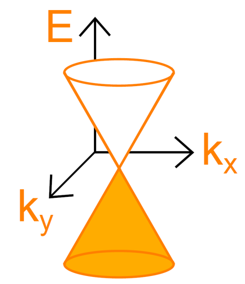

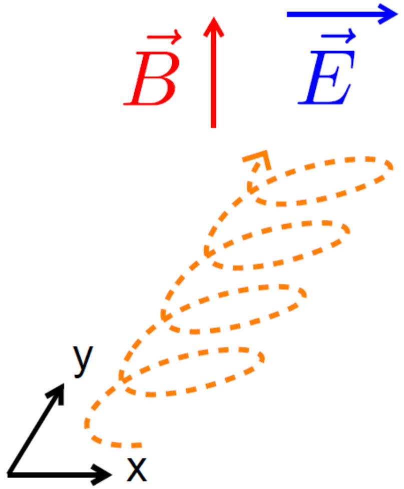

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Lorentz transformation

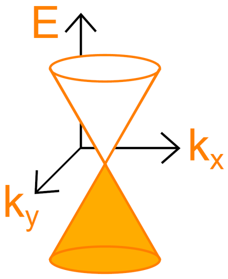
$$E' = \gamma\left(E - v_d B\right)$$
$$B' = \gamma\left(B - v_d/v_F^2 \times E\right)$$

Electric and magnetic field are different aspect of the same force.

$$A_\mu = \left(v_F^{-1} Ex, 0, -Bx\right)$$

Shytov et al., PRL 2007
Goerbig et al., EPL 2009

What about relativistic electrons?



m = 0

What are the implications?

$$E' = \gamma \left(E - v_d B \right)$$
$$B' = \gamma \left(B - v_d / v_F^2 \times E \right)$$

Shytov et al., PRL 2007
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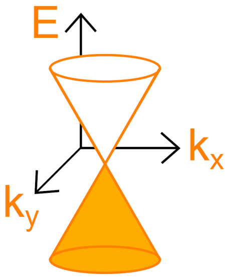
What about relativistic electrons?



m = 0

What are the implications?

$$E' = \gamma\left(E - v_d B\right)$$
$$B' = \gamma\left(B - v_d/v_F^2 \times E\right)$$

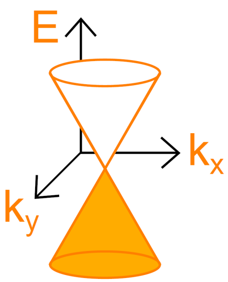
For $B < E/v_F$

There exists a frame of velocity
$v_d = v_F^2 \times B/E$ where $B' = 0$

Electrons should be insensitive to B in every frame

Shytov et al., PRL 2007
Georbig et al., EPL 2009

What about relativistic electrons?



m = 0

What are the implications?

$$E' = \gamma\left(E - v_d B\right)$$
$$B' = \gamma\left(B - v_d / v_F^2 \times E\right)$$

For $B < E/v_F$

There exists a frame of velocity $v_d = v_F^2 \times B/E$ where $B' = 0$

Electrons should be insensitive to B in every frame

For $B > E/v_F$

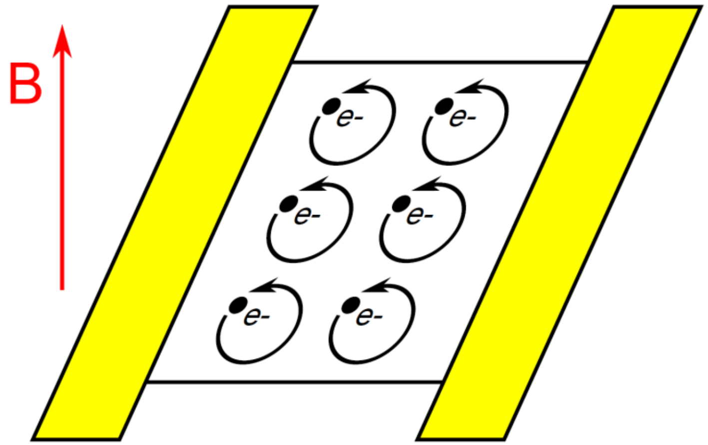
There exists a frame of velocity $v_d = E/B$ where $E' = 0$

$B' = B\sqrt{1-(E/v_F B)^2}$

Electrons should orbit as if there was no E

Shytov et al., PRL 2007
Georbig et al., EPL 2009

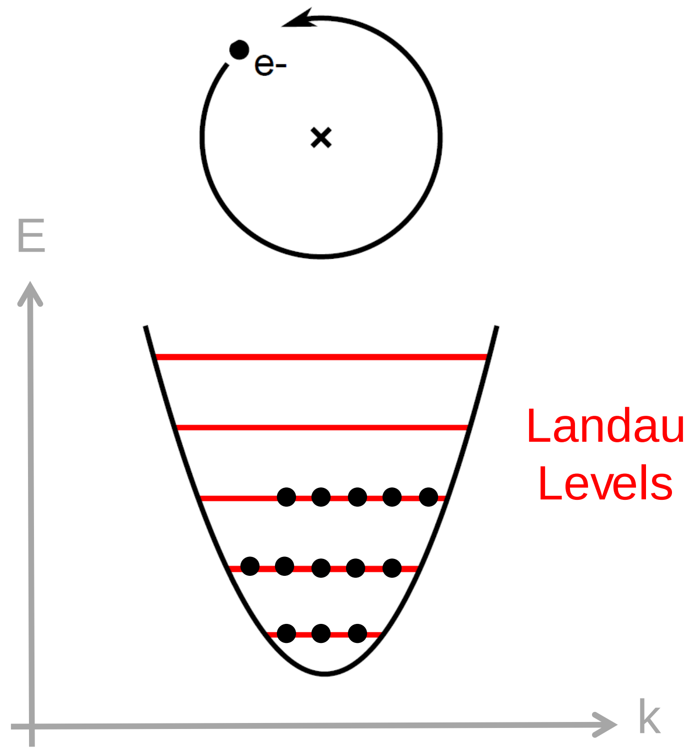
Quantum Hall effect in graphene



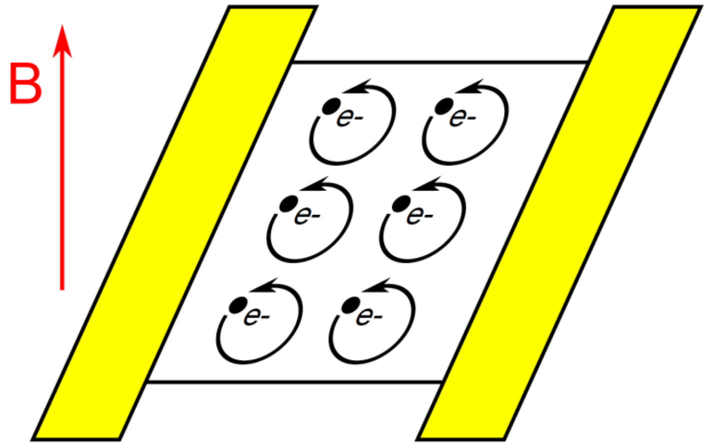
$$E_n = \hbar\omega_c(n + \frac{1}{2})$$

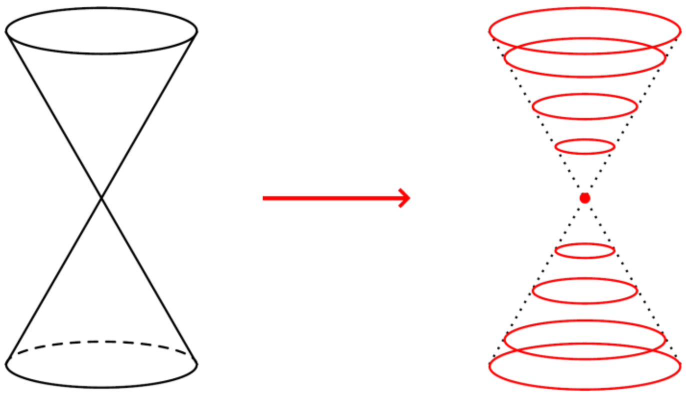
$$\omega_c = \frac{eB}{m}$$

Free electron in magnetic field
= harmonic oscillator
=> Quantization into discrete levels



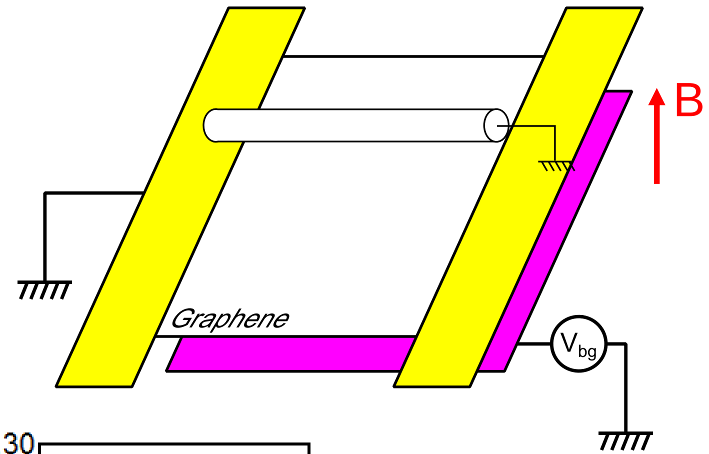
Quantum Hall effect in graphene

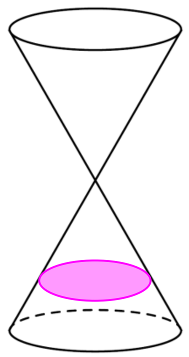


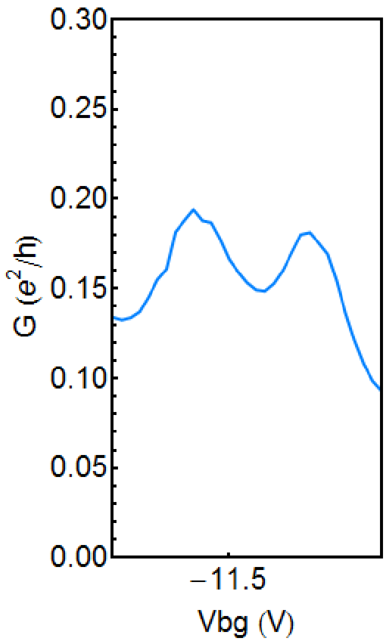


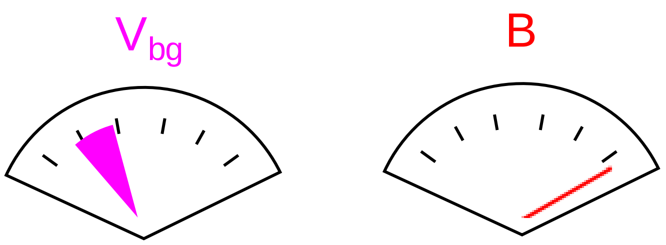
$$E_n = sign(n)\sqrt{2\hbar Ben}$$

Sensing Landau levels

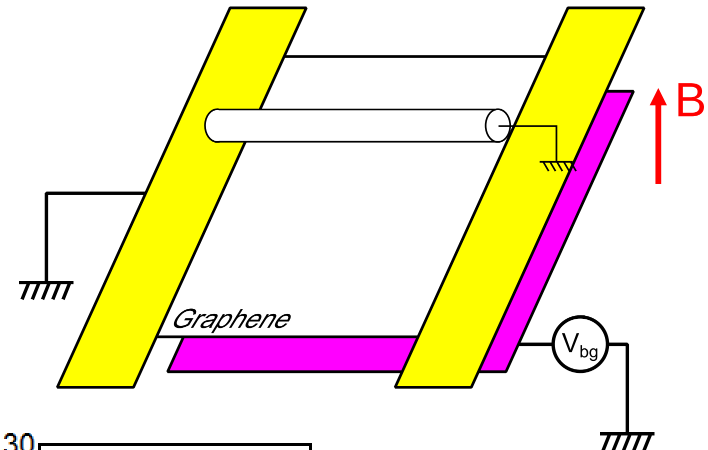


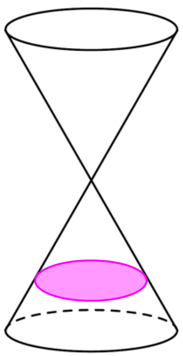


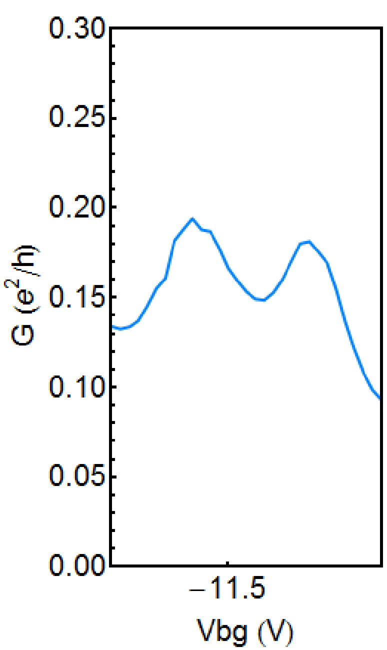


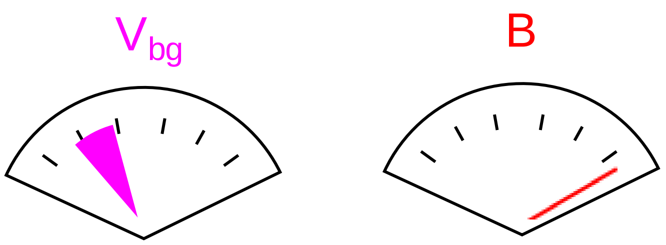


Sensing Landau levels

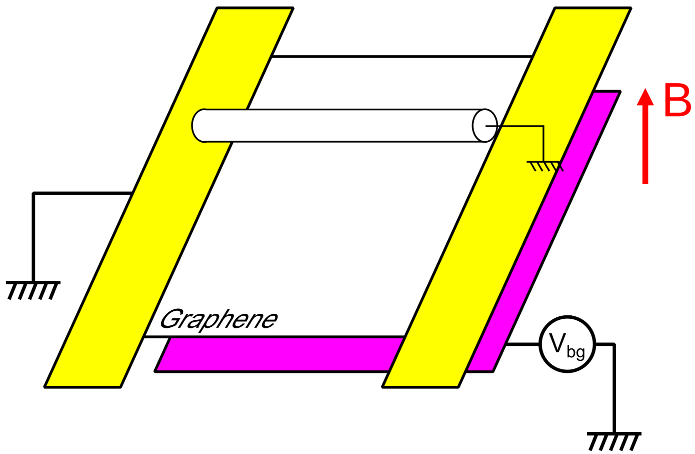


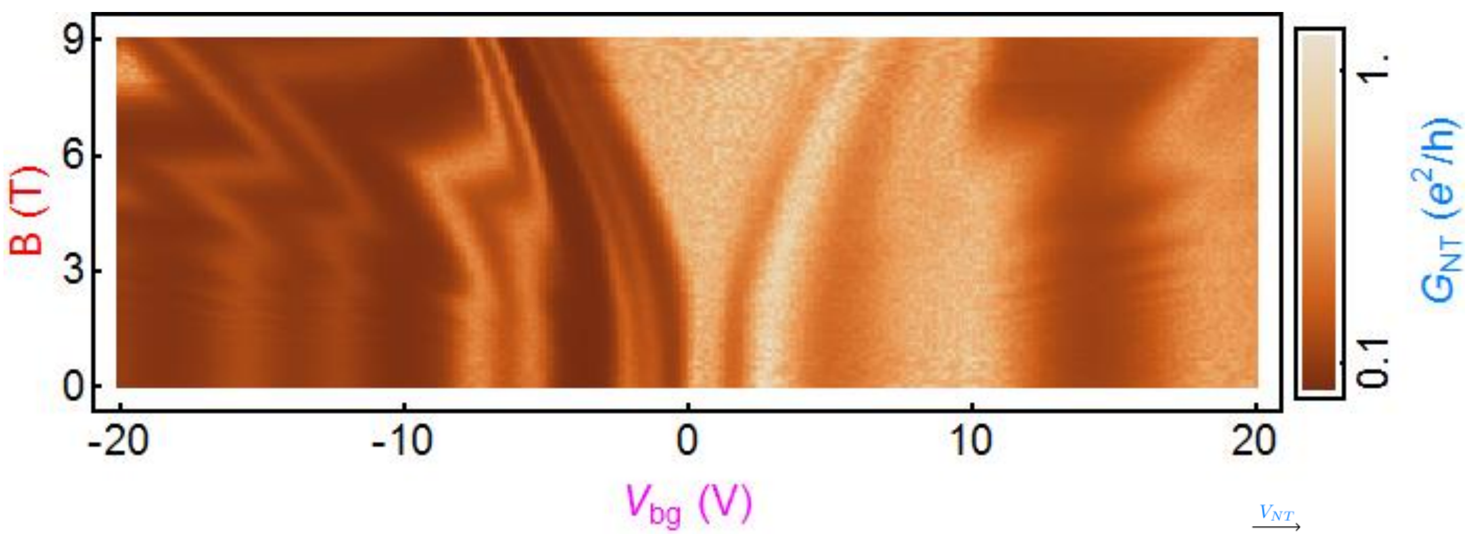


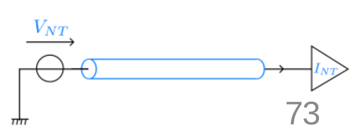




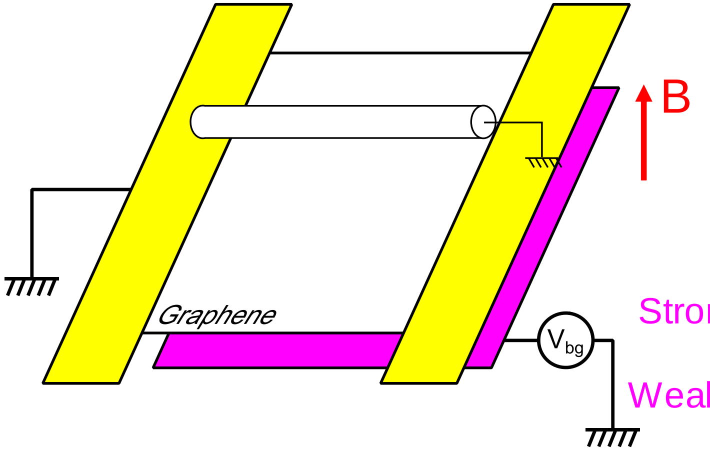
Sensing Landau levels

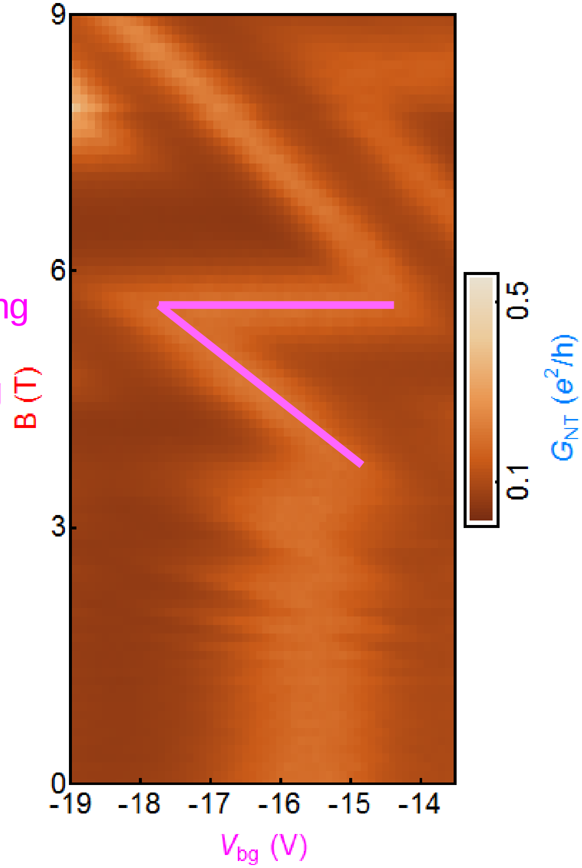


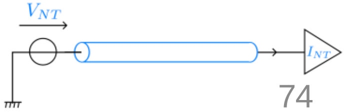




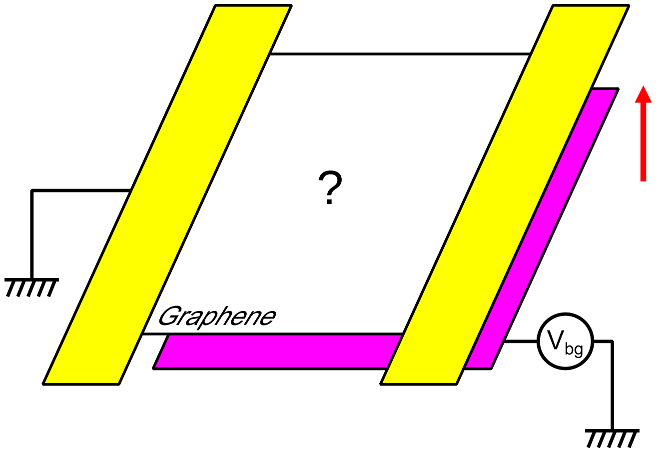
Screening of individual Landau levels

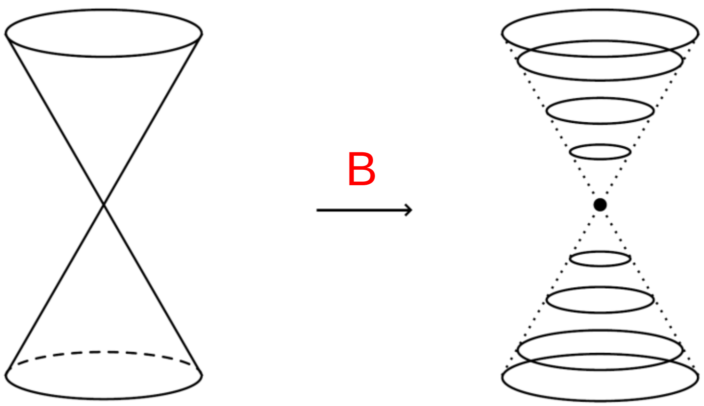


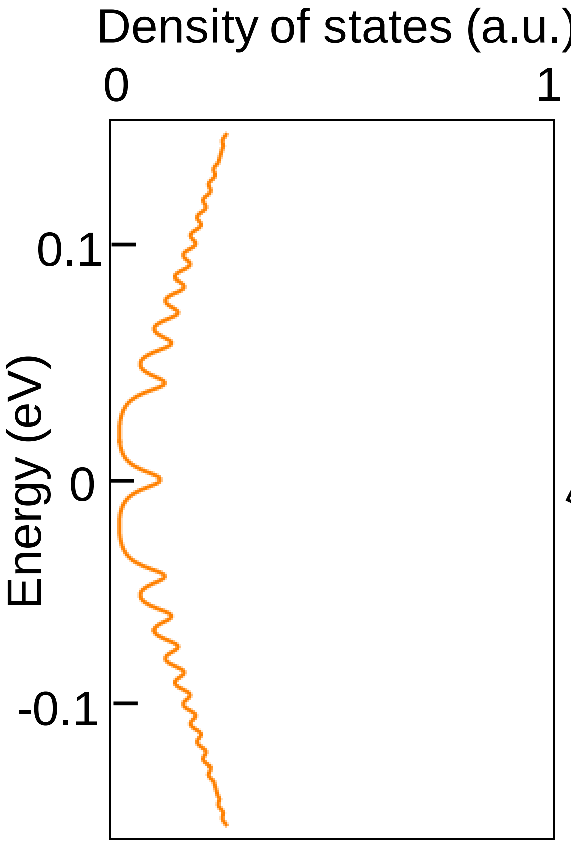


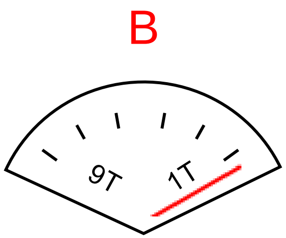


Screening of individual Landau levels

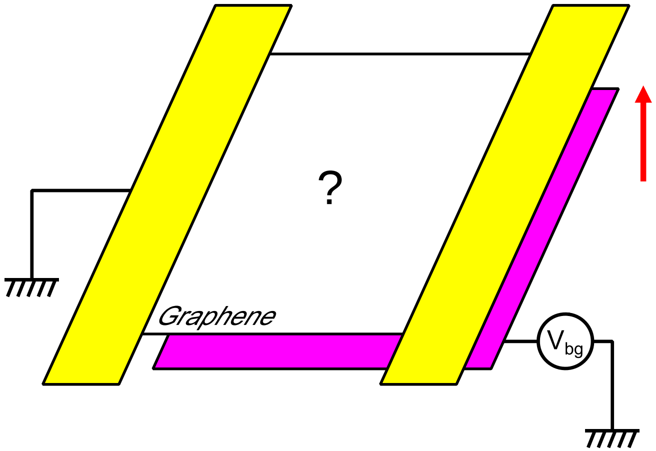


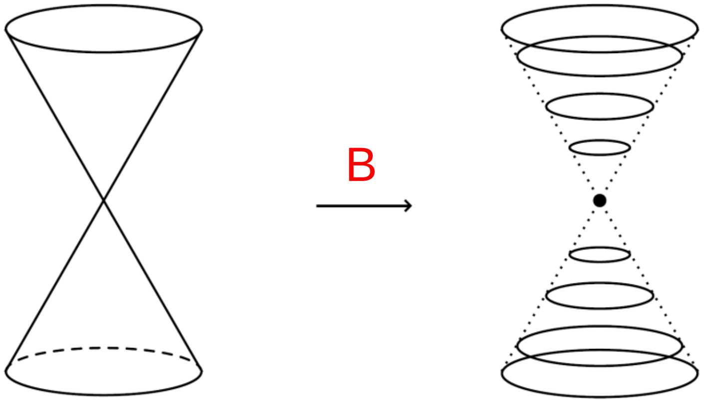


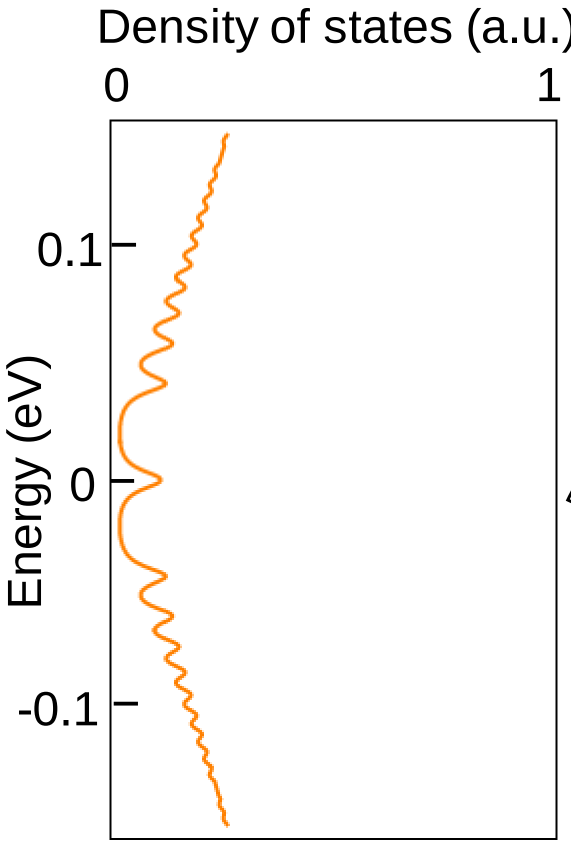


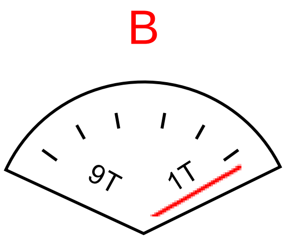


Screening of individual Landau levels

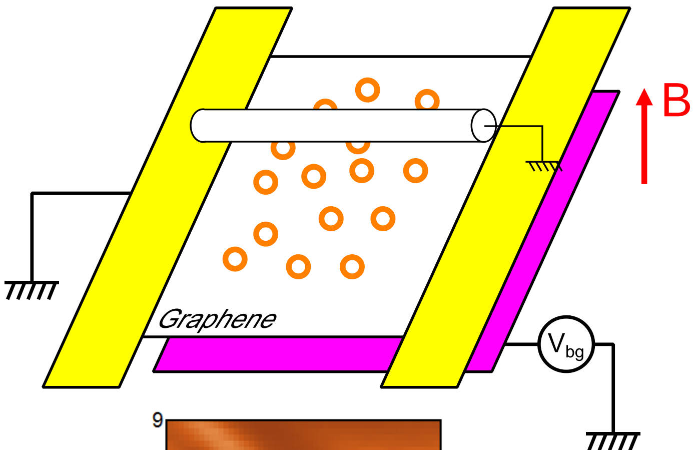




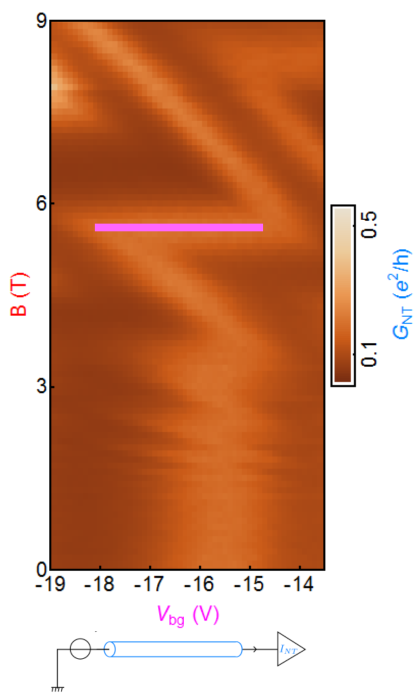


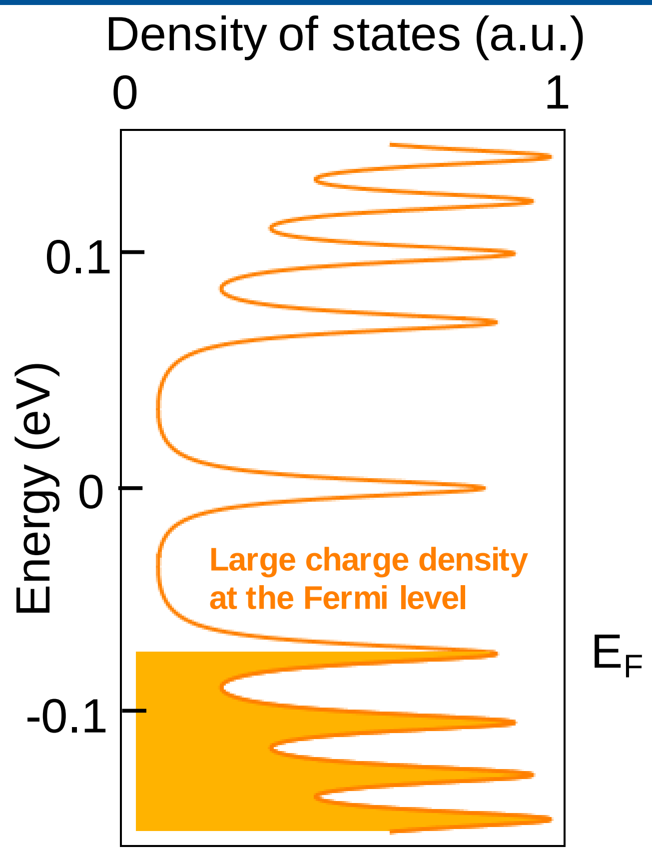


Screening of individual Landau levels

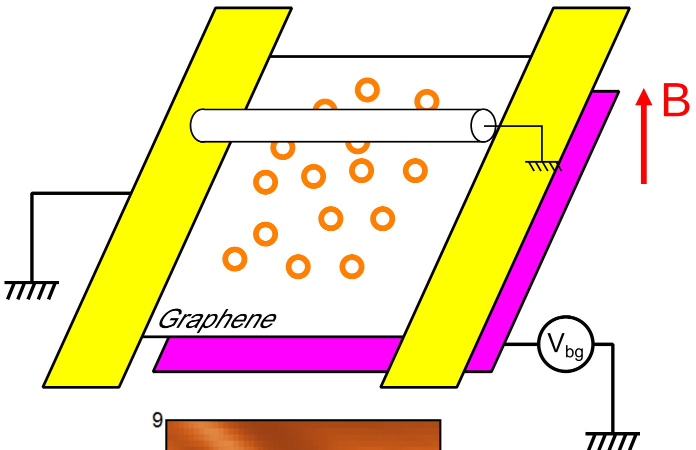


Strong screening

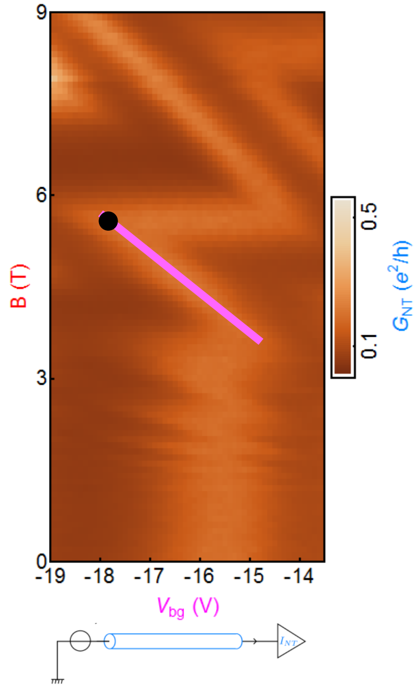


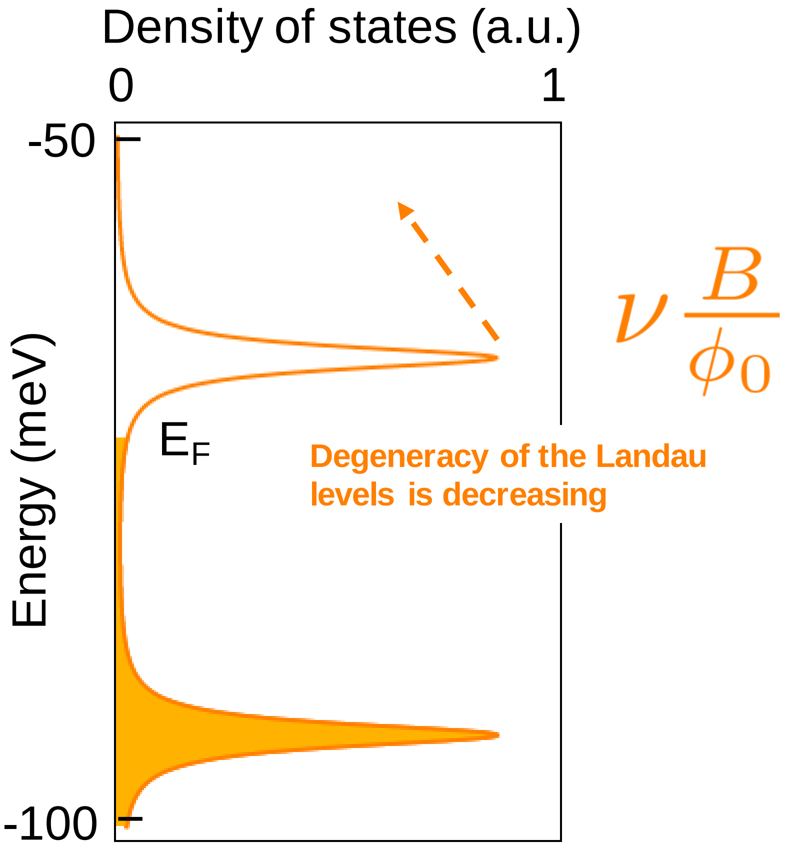


Screening of individual Landau levels

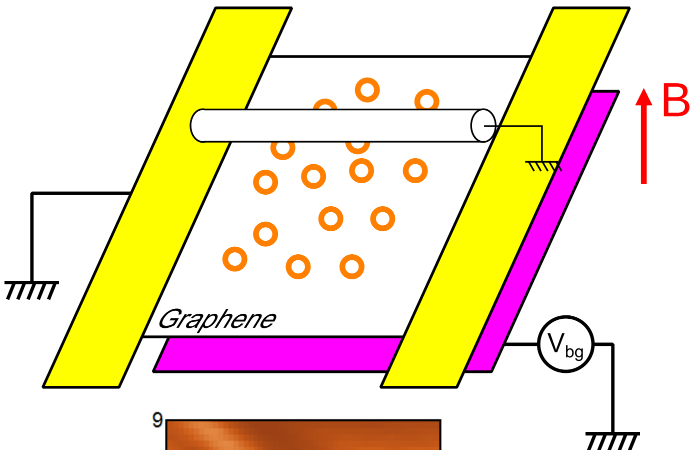


Weak screening

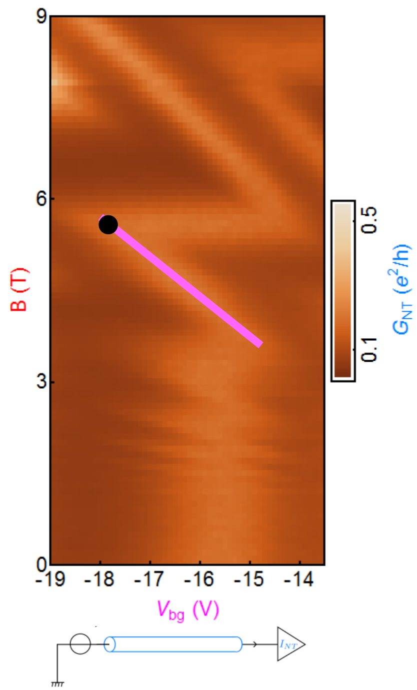


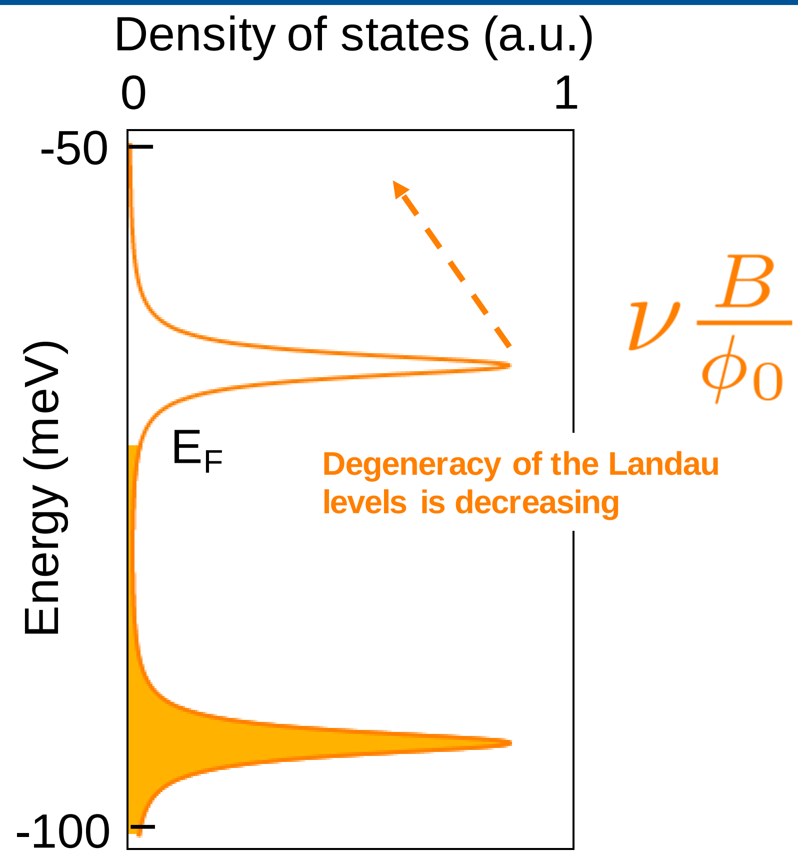


Screening of individual Landau levels

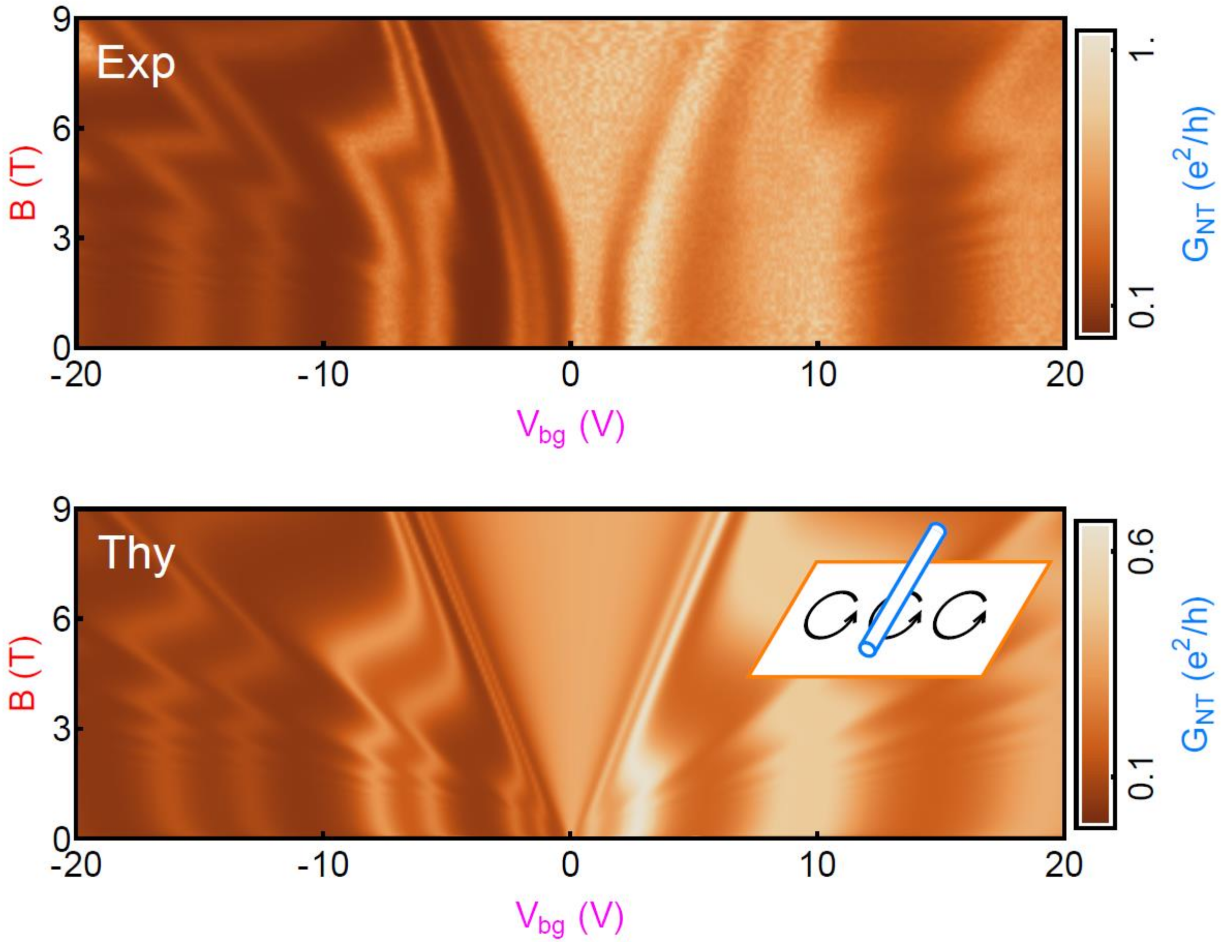


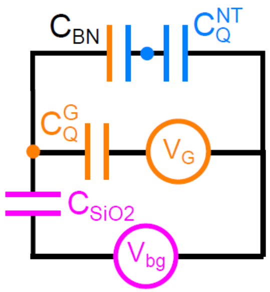
Weak screening



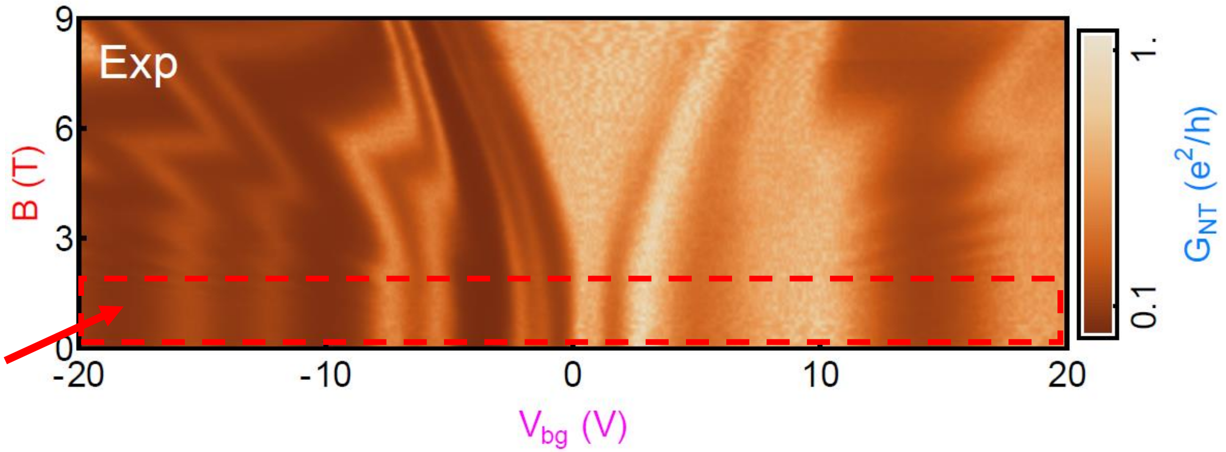


Electrostatic description

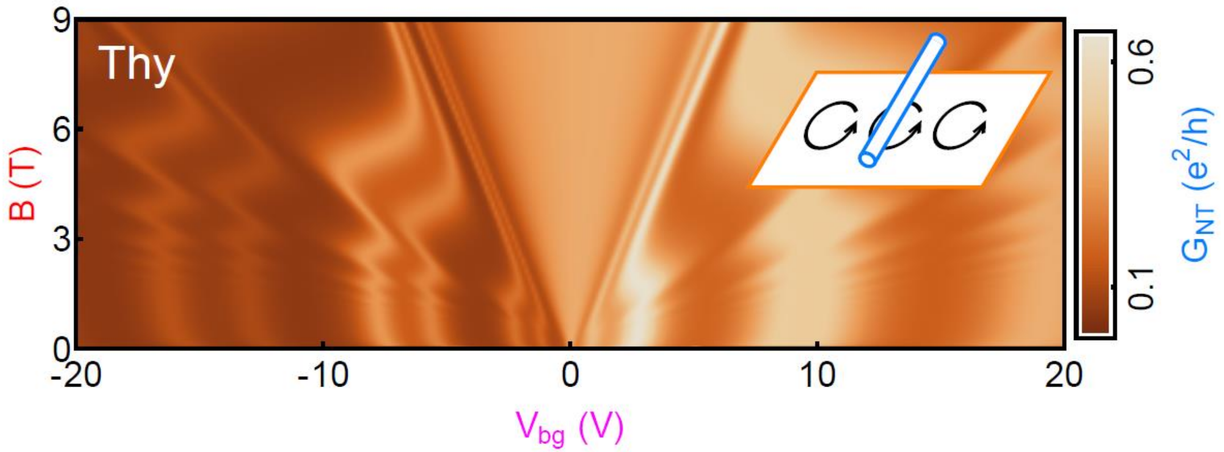




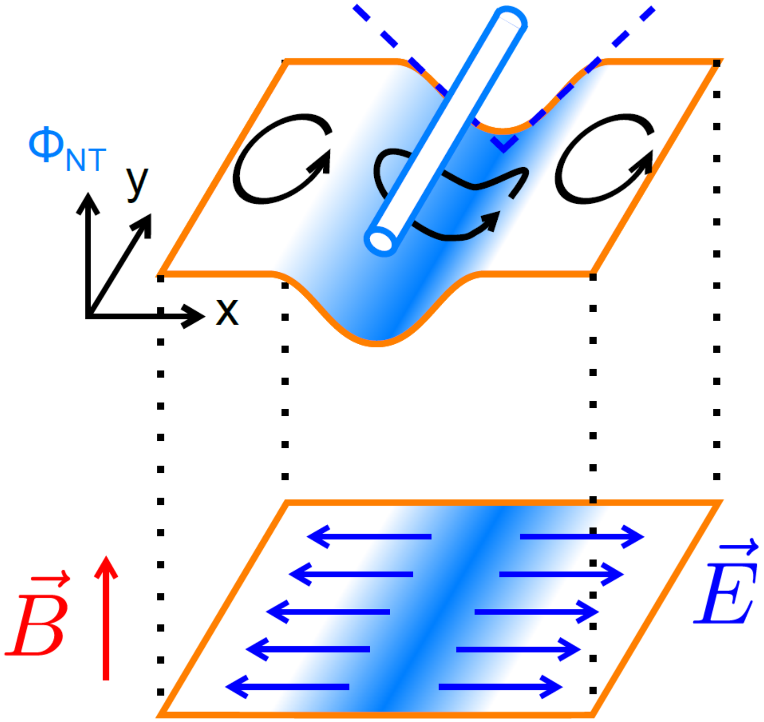
Electrostatic description



Poor description at low magnetic field

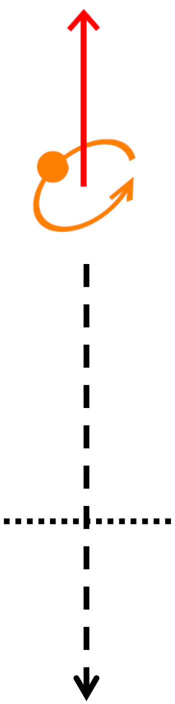


Electric field generated by nanotube

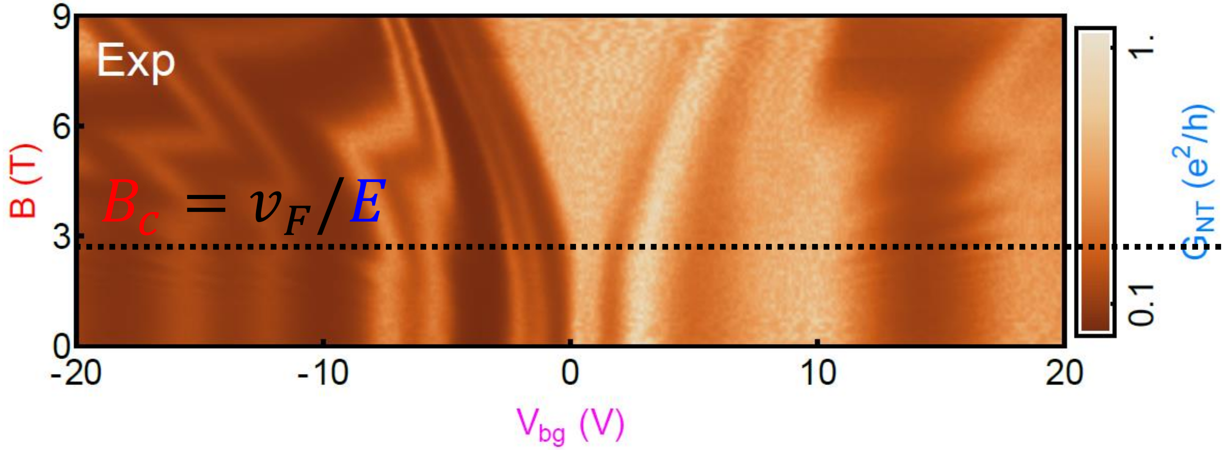


Collapse of Landau Levels

$$E' = 0$$
$$B' = B\sqrt{1 - (E/v_F B)^2}$$



$$l_c = \sqrt{\frac{\hbar}{eB'}}$$



$$B_c = v_F/E$$

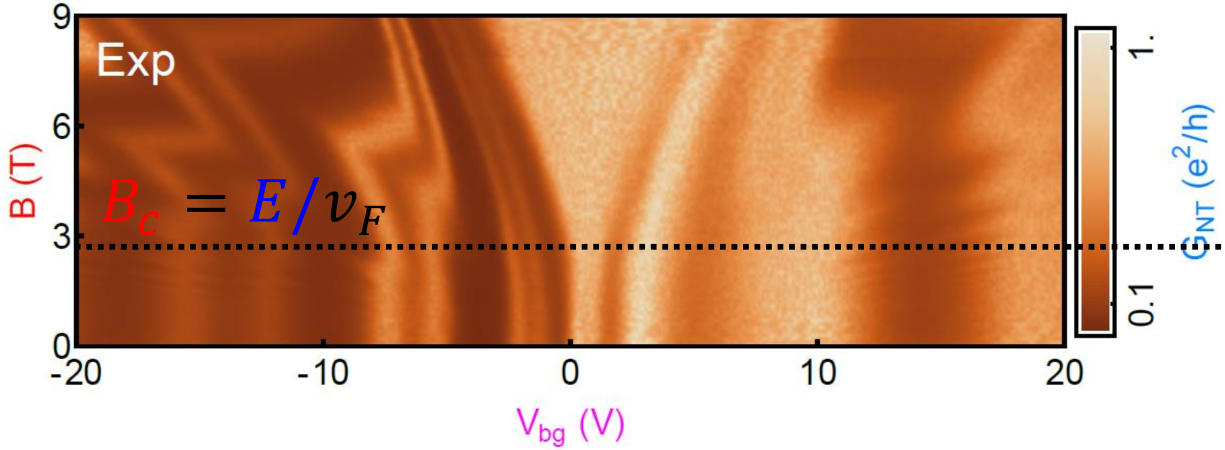
$$l_c = \infty$$

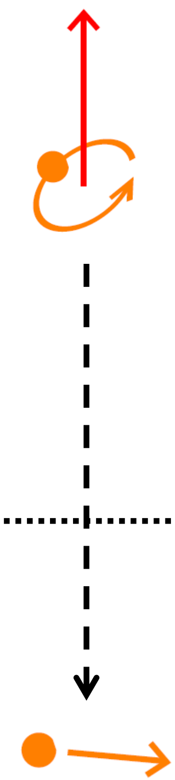
Lukose et al, PRL 2007
Gu et al., PRL 2010

Collapse of Landau Levels

$$E' = 0$$

$$B' = B\sqrt{1 - (E/v_F B)^2}$$





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Collapse of Landau Levels

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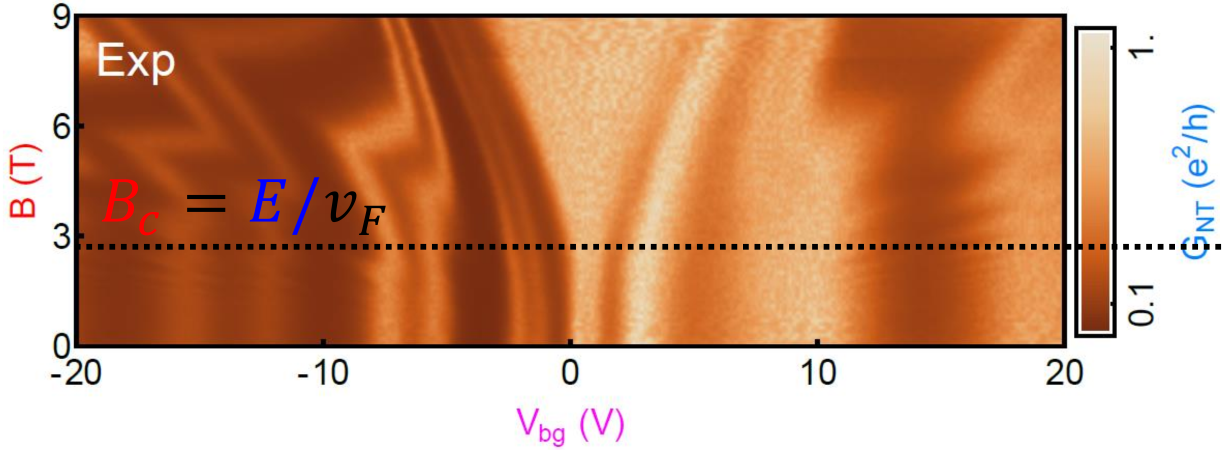
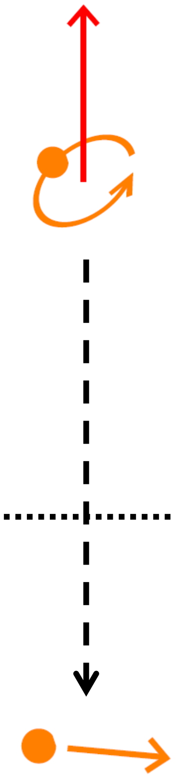




$$l_c = \sqrt{\frac{\hbar}{eB'}}$$

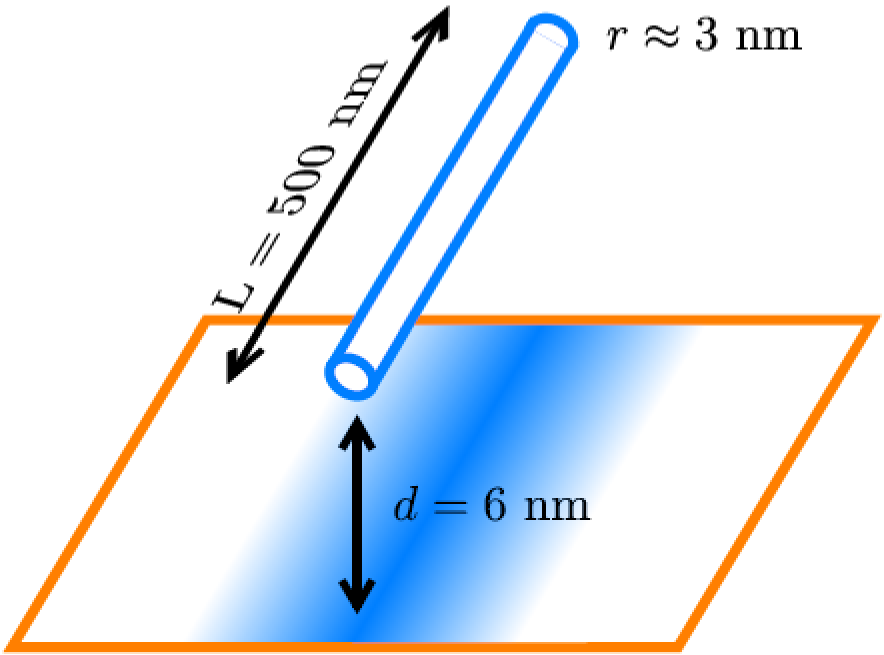
$$l_c = \infty$$

$$B' = 0$$

This insentivity of magnetic field for $B < B_c$ is a signature of the Lorentz invariant Physics of quasi-relativitic electrons.

Lukose et al, PRL 2007
Gu et al., PRL 2010

Critical magnetic field



$$C = \log\left(2d/r\right)/\pi\epsilon L$$

Critical magnetic field

$$B_c = \boxed{n_{NT}/dC} v_F$$

Electric field

Dielectric response of graphene

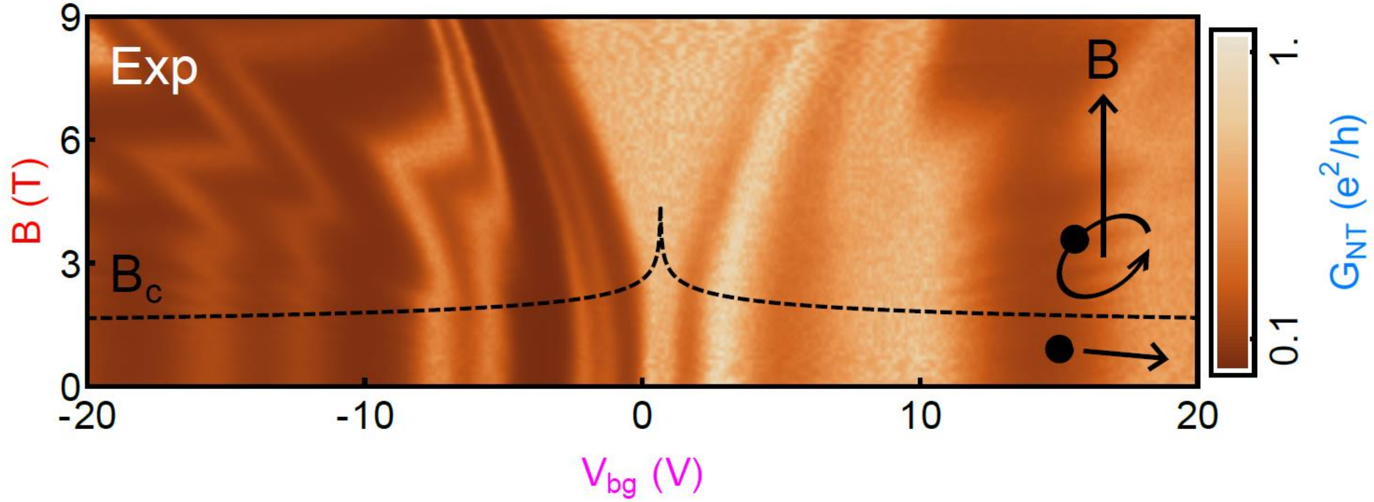
$$d_{eff} = d + d_{TF}$$

Thomas Fermi screening length

$$d_{TF} = 2/\sqrt{\pi n_G}$$

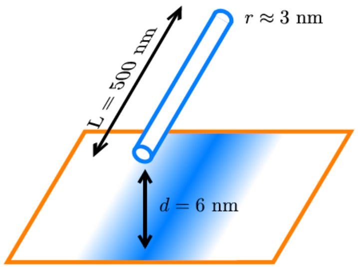
Jiang PRB 2015

Critical magnetic field



$$n_{NT} = 2$$ Chemical doping

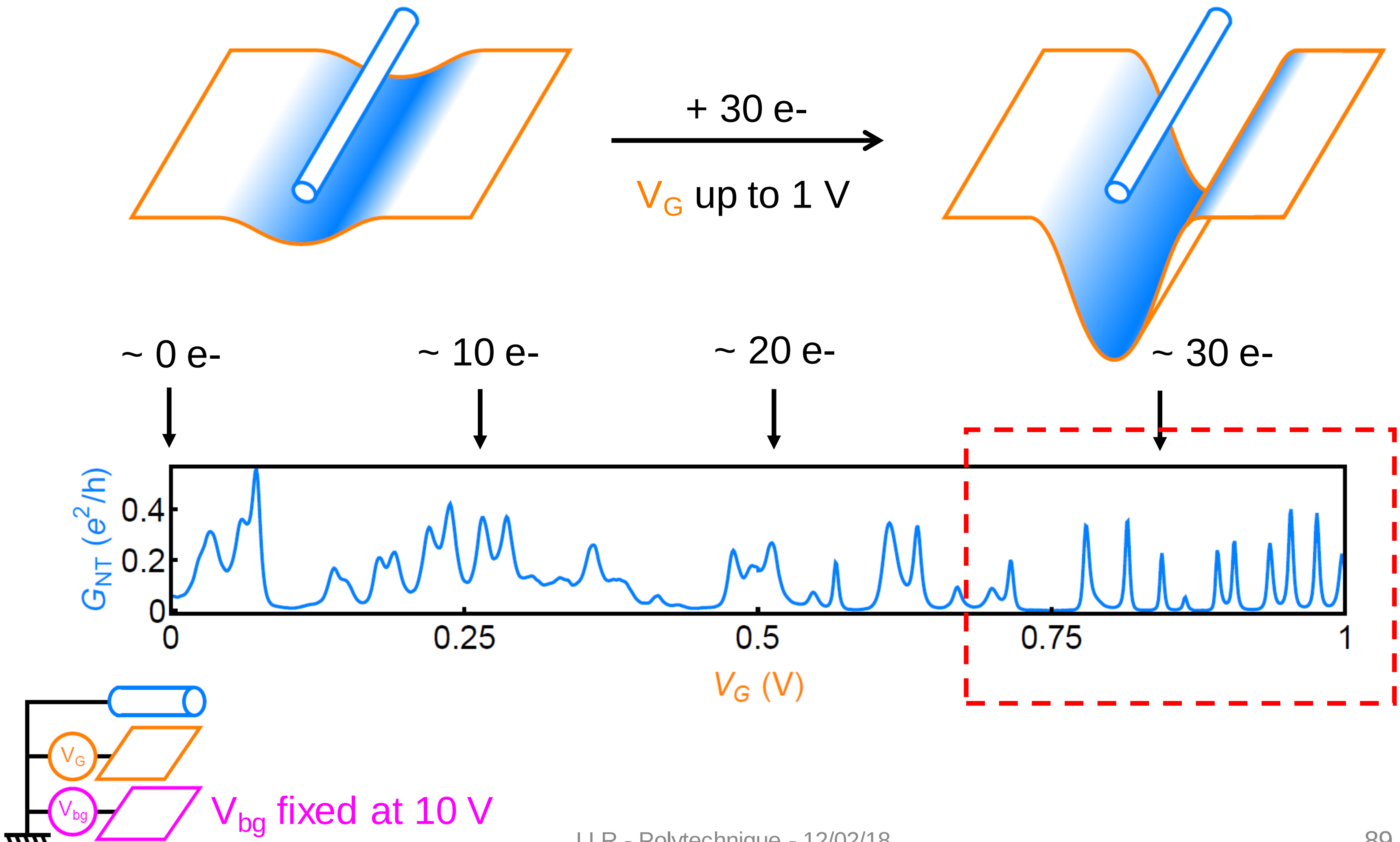
$$|E| = 1.25 \times 10^6 \ \mathrm{V.m}^{-1}$$



Outline

1) Why is graphene a solution to simulate relativistic effects?
2) Our strategy: a hybrid nanotube-graphene circuit
3) Signature of quasi-relativistic effects in graphene
4) Driving the circuit towards atomic collapse
   Heavily charging the artificial nucleus (nanotube)

What happens for large number of charge n_{NT} in the nanotube?

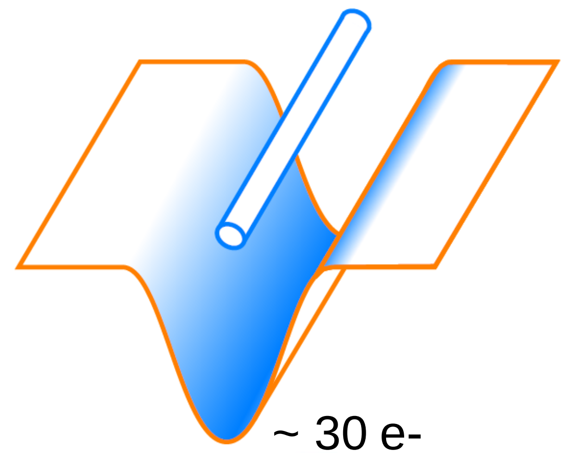


V_{bg} fixed at 10 V

What happens for large number of charge n_{NT} in the nanotube?

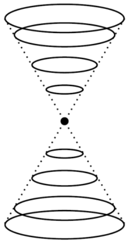
+ 35 e-

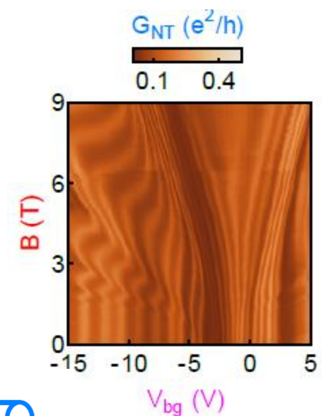
V_G up to 1 V

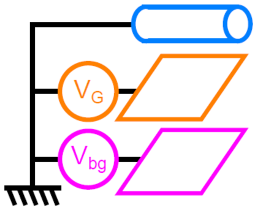


~ 0 e-

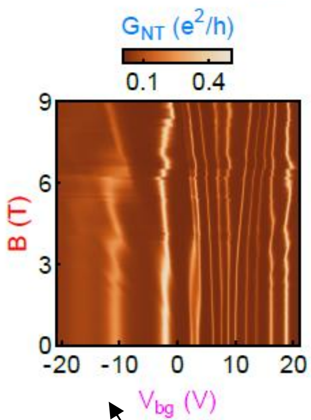
~ 30 e-







Landau levels's manifestation much weaker



?

Irregular spacing of NT e- levels

Analogy with atomic collapse

Atomic number Z

Analogy with atomic collapse

Atomic number Z

Real atom

The atom is stable only if

$$Z < \frac{1}{\alpha} \approx 137$$

Analogy with atomic collapse

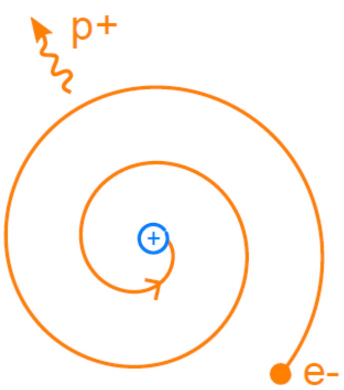
Atomic number Z

Real atom

For a heavy nucleus

$$Z > \frac{\hbar c}{e^2} \approx 137$$

$\Rightarrow$ Atomic collapse



Signature: emission of an escaping positron

Analogy with atomic collapse

In graphene

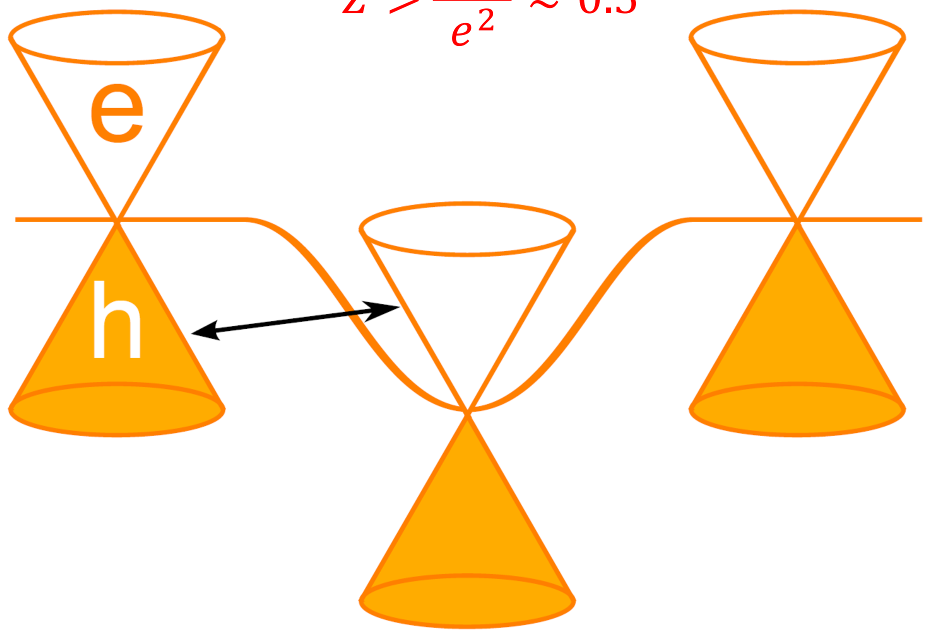
$$Z > \frac{\hbar v_F}{e^2} \approx 0.5$$

Analogy with atomic collapse

Atomic number Z

In graphene

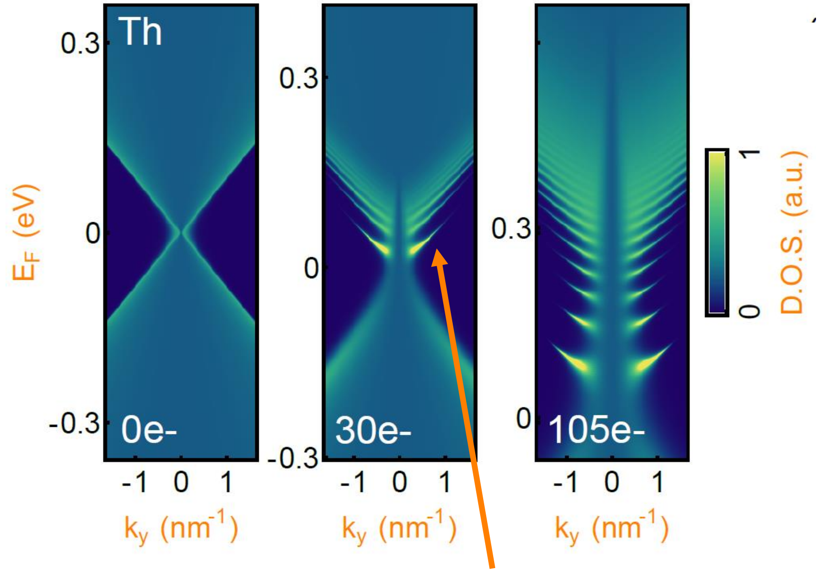
$$Z > \frac{\hbar v_F}{e^2} \approx 0.5$$



Tunneling between electrons and holes bands
=> e-h quasi-bound states

Analogy with atomic collapse

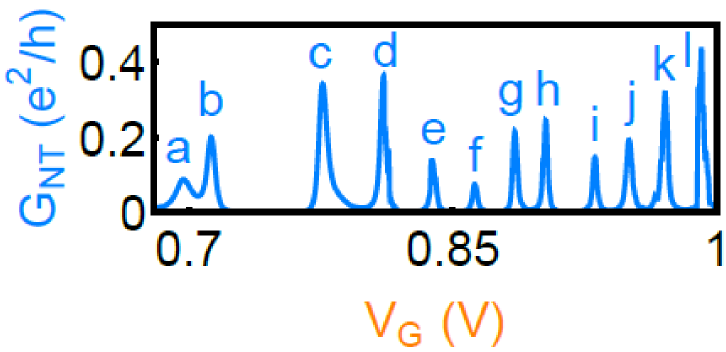
$$\begin{pmatrix} \Phi_{NT}(x) & p_x - ip_y \\ p_x + ip_y & \Phi_{NT}(x) \end{pmatrix} \psi = E\psi$$

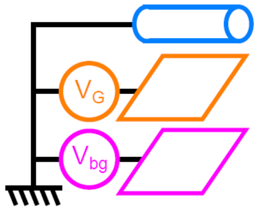


$$\Phi_{NT}(x) = \frac{e^2 n_{NT}}{2\pi L\epsilon} \log \frac{L}{\sqrt{x^2+d^2}}$$

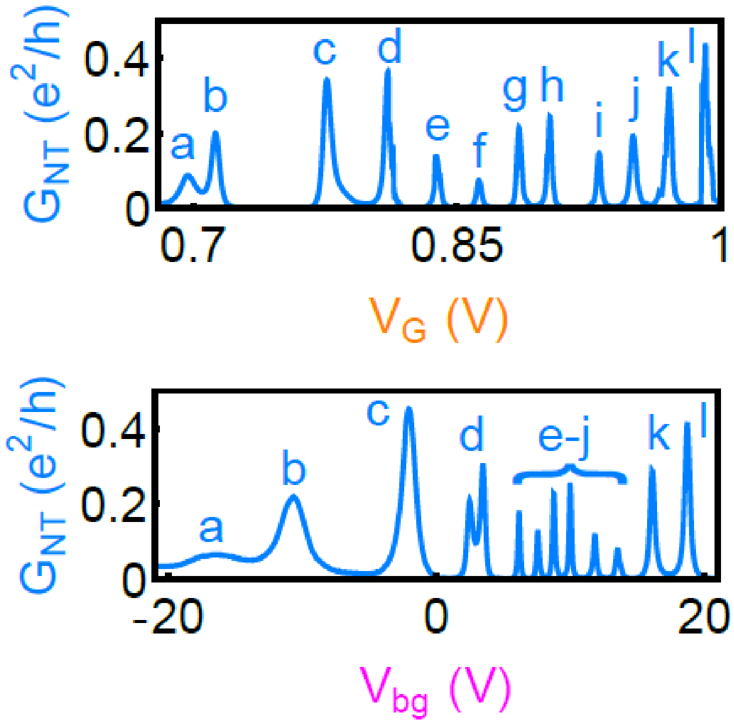
Atomic collapse quasi-bound states
e-h plasmonic mode

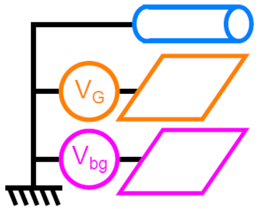
Extracting density of states



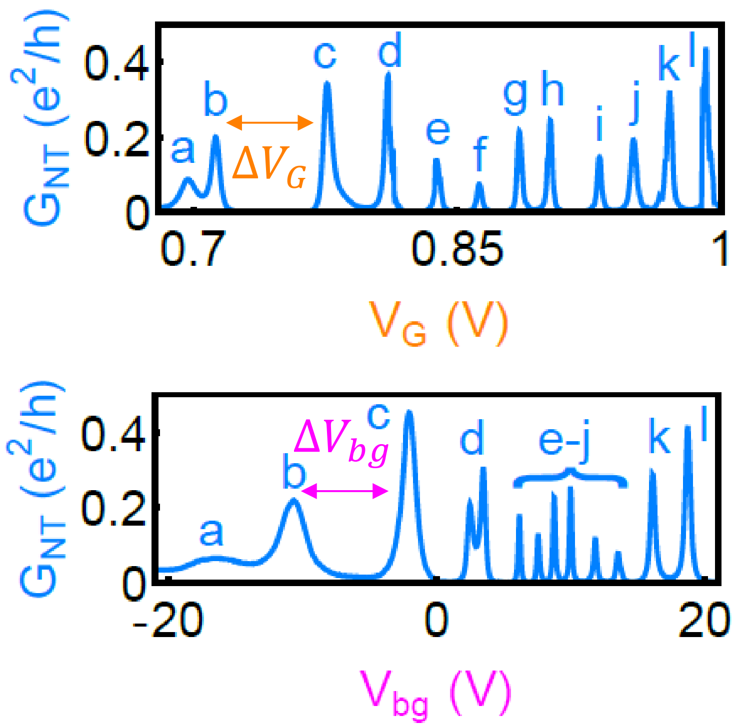


Extracting density of states

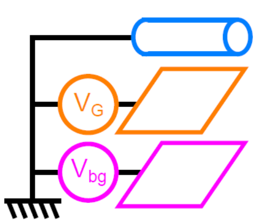




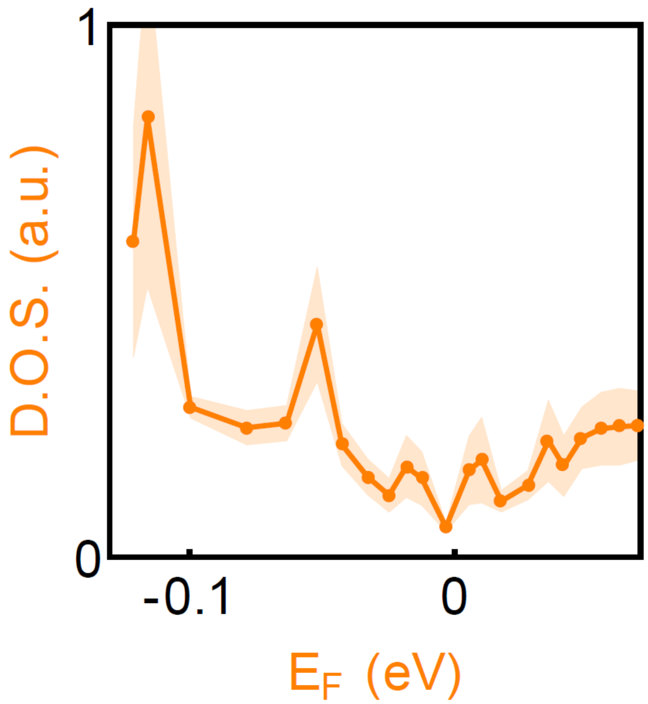
Extracting density of states



$$\text{D.O.S.} = \frac{dn_G}{dE_F} \propto \frac{\Delta V_{bg}}{\Delta V_G}$$



Extracting density of states

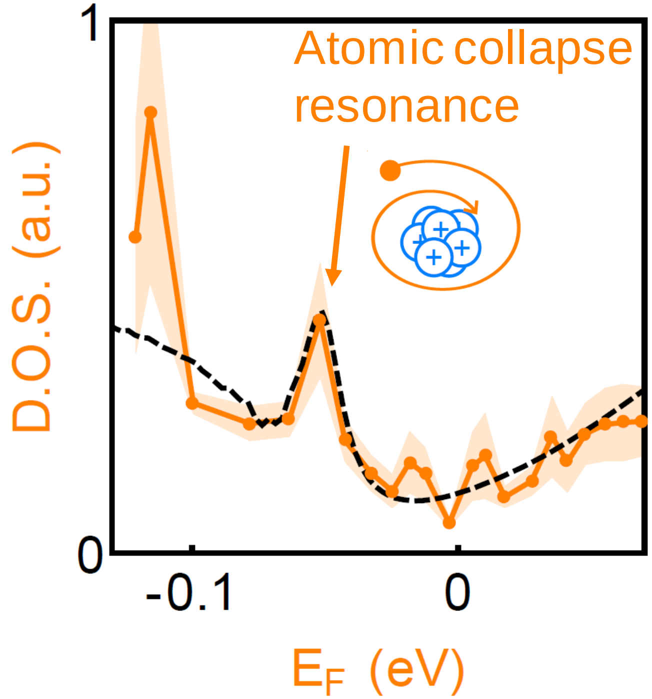


$$\text{D.O.S.} = \frac{dn_G}{dE_F} \propto \frac{\Delta V_{bg}}{\Delta V_G}$$

Extracting density of states

No fit parameter

$n_{NT} \approx 30, L \approx 500\ nm, d \approx 6\ nm, \epsilon \approx 4,\ v_F \approx 1.15\ m.s^{-1}$

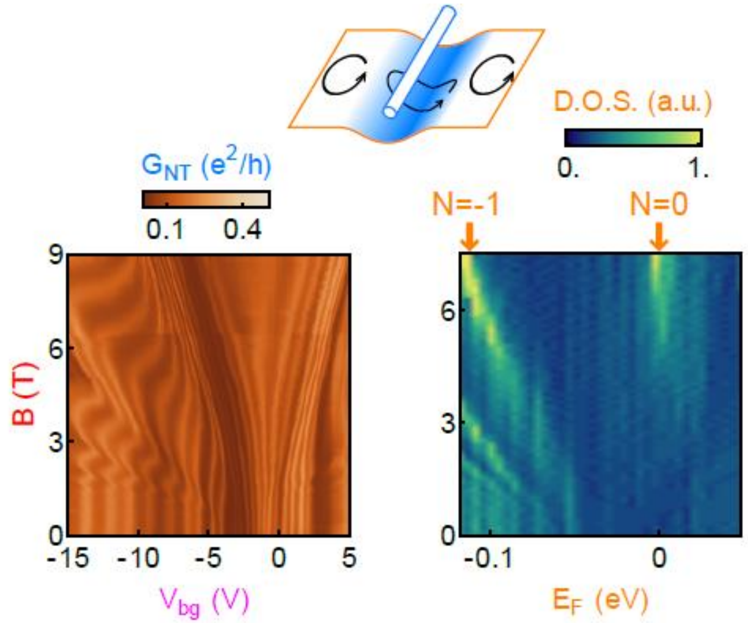


$$\begin{pmatrix} \Phi_{NT}(x) & p_x - ip_y \\ p_x + ip_y & \Phi_{NT}(x) \end{pmatrix} \psi = E\psi$$

$$\Phi_{NT}(x) = \frac{e^2 n_{NT}}{2\pi L \epsilon} \log \frac{L}{\sqrt{x^2 + d^2}}$$

Evolution with magnetic field

$n_{NT} \approx 0$



Evolution with magnetic field

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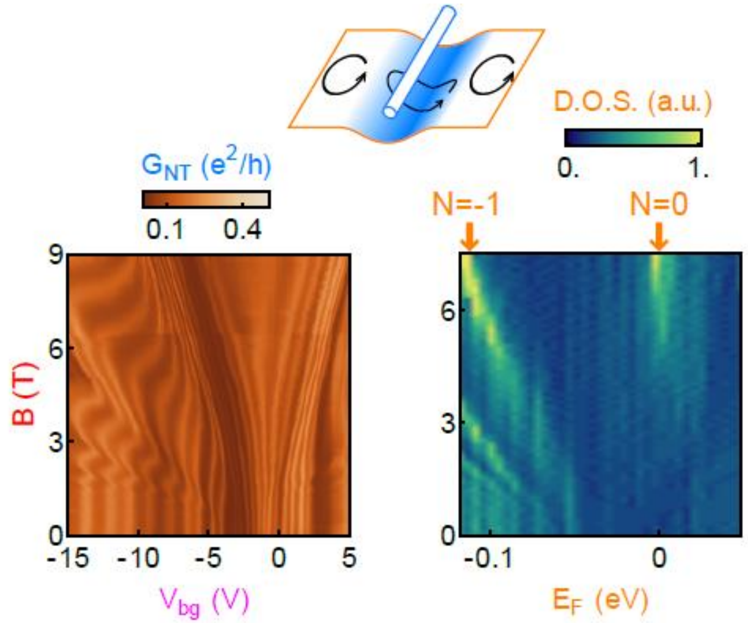
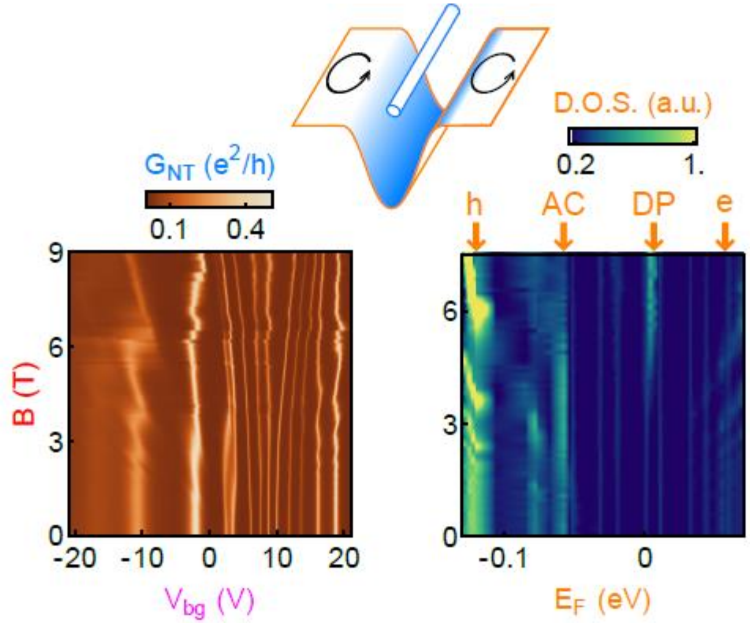
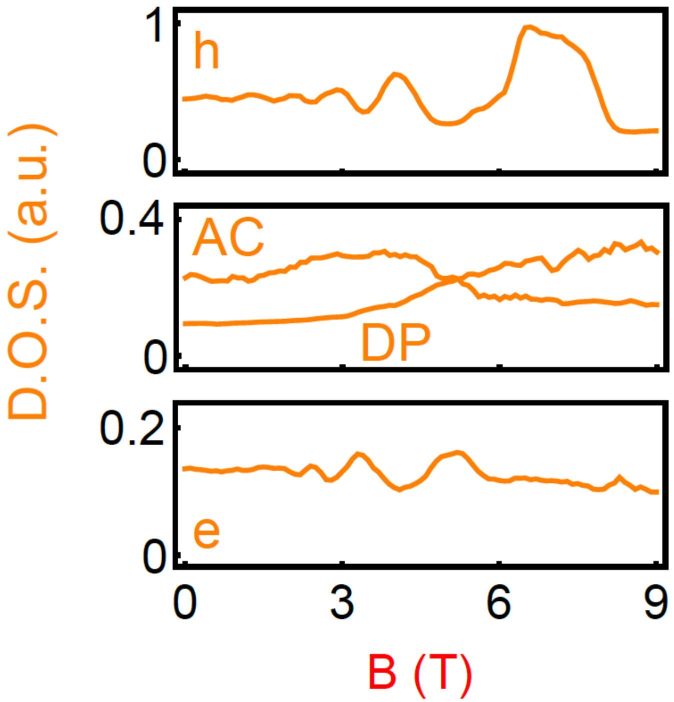


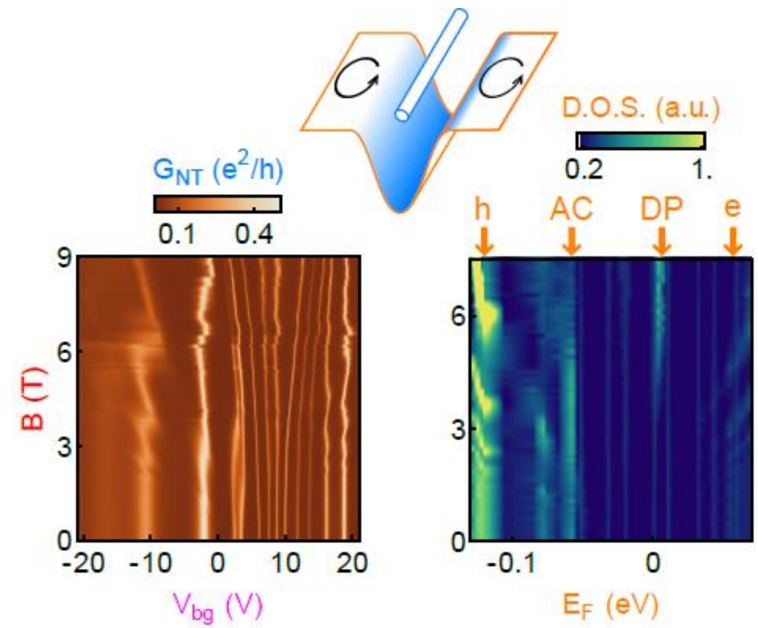
$n_{NT} \approx 30$



Evolution with magnetic field



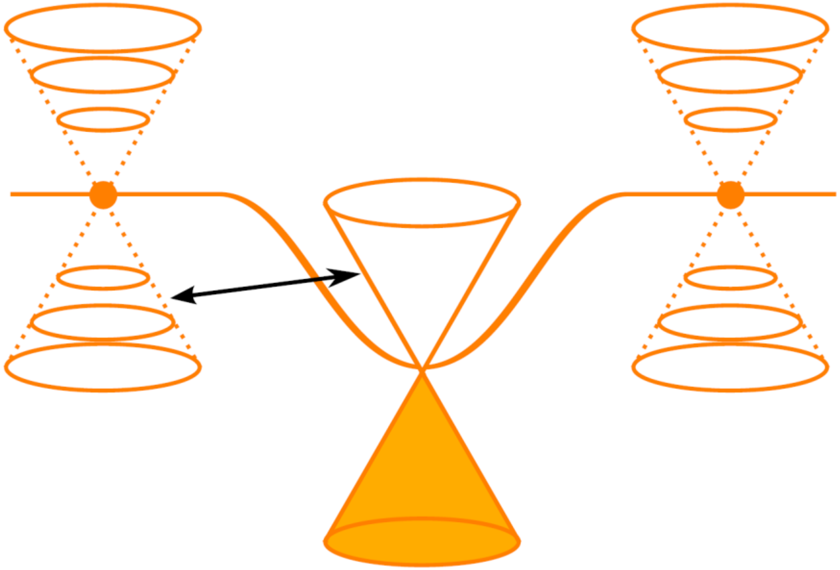
$$n_{NT} \approx 30$$

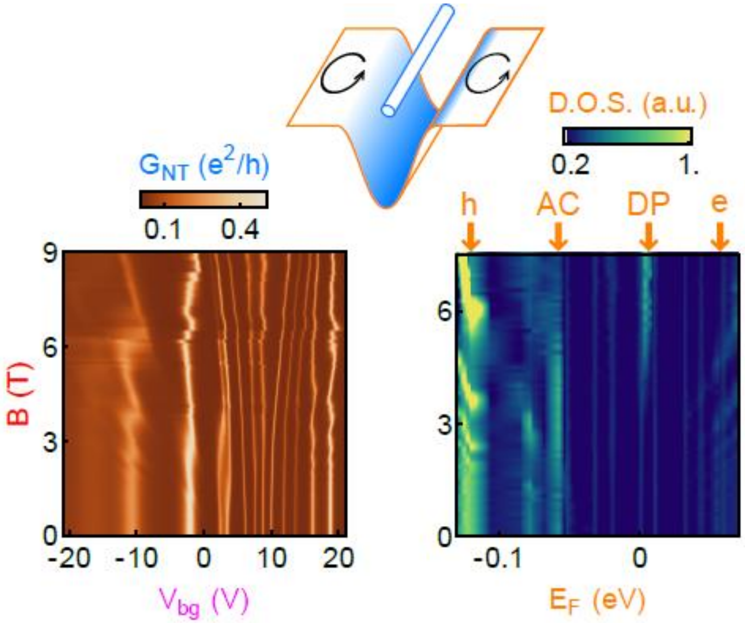


Evolution with magnetic field

Hybridization of hole Landau levels with electrons

$n_{NT} \approx 30$

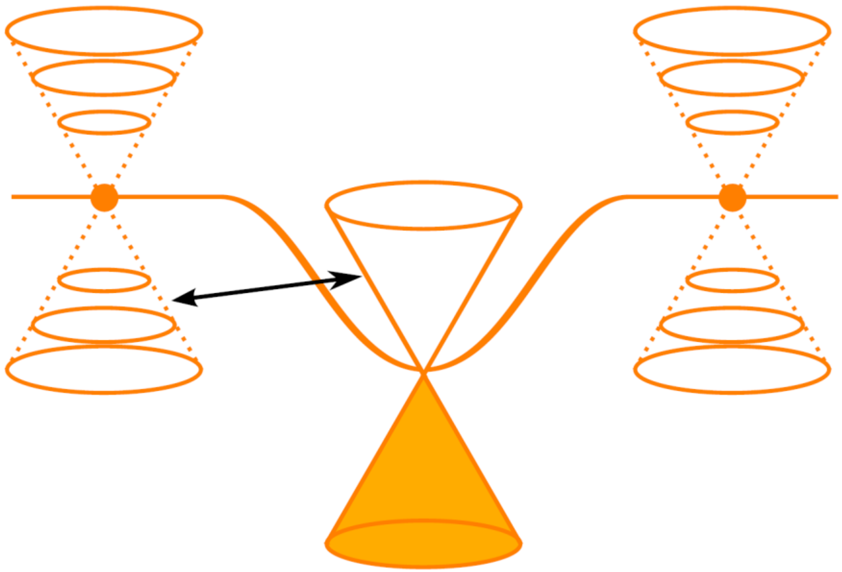


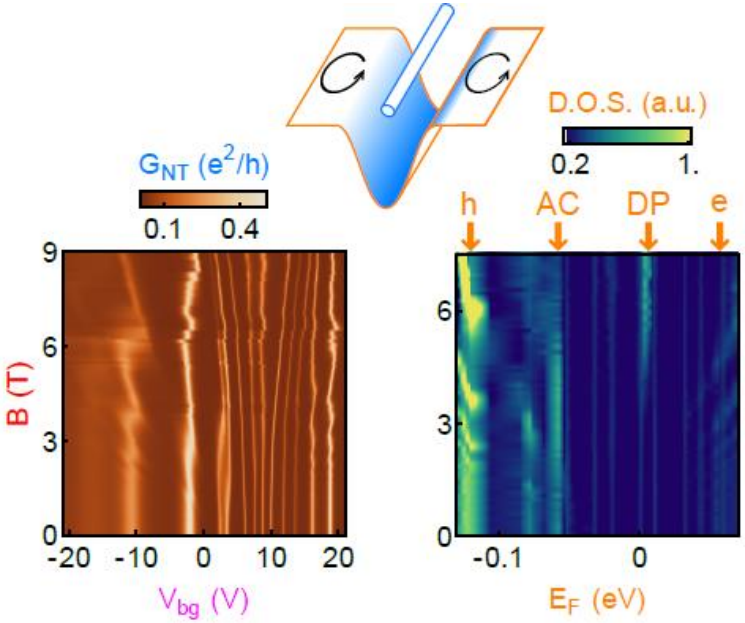


Evolution with magnetic field

Hybridization of hole Landau levels with electrons

$n_{NT} \approx 30$



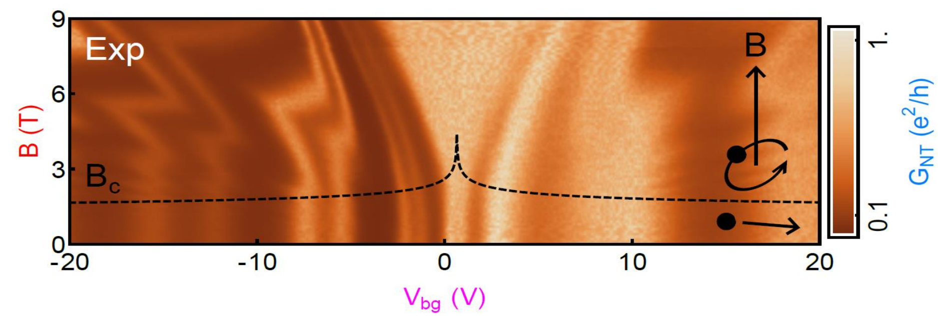


We can turn ON and OFF atomic collapse with magnetic field

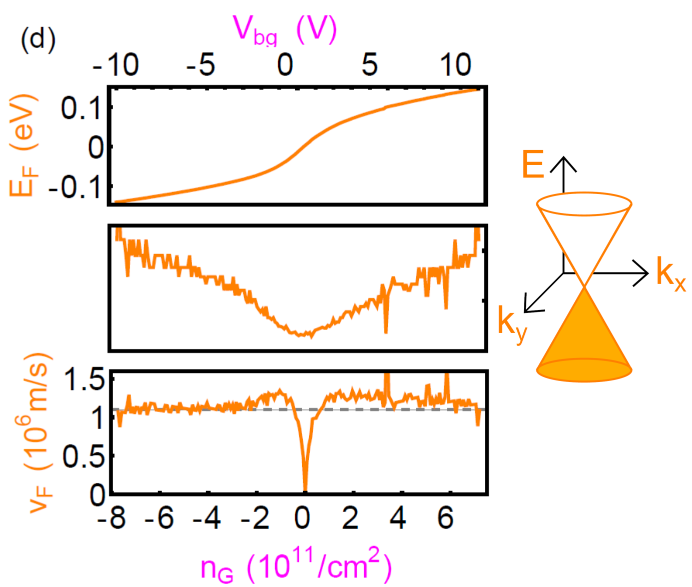
Conclusions

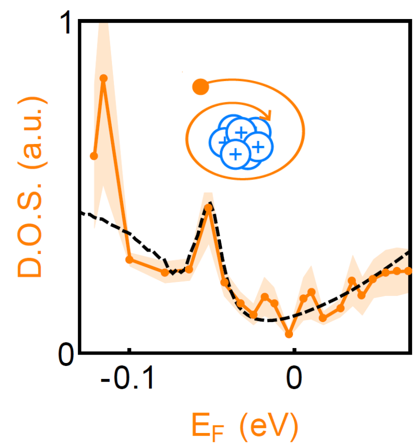
Analogue relativity in nanotube-graphene devices

Lorentz invariance



Constant velocity

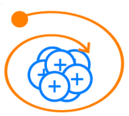




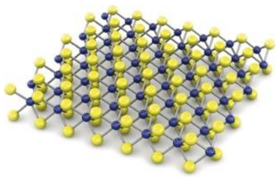
Atomic collapse

Perspectives

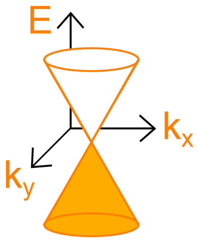
Continuous monitoring of atomic collapse as a function of electric field intensity, plasmonic effect Akkaravarawong et al. arXiv 2015



Use NT single electron transistor on other two-dimensional materials (MoS_2, WSe_2…)
Novoselov et al., Science 2016



Probe relativistic-like effects in other "Dirac" materials (Weyl semi-metal, etc…)
Tchoumakov et al. PRL 2016



Acknowledgments

Philip Kim
Harvard University

Austin Cheng
Harvard University

Cory Dean
Columbia University

Relativistic simulation in nanotube-graphene devices

Lorentz invariance and **atomic collapse**

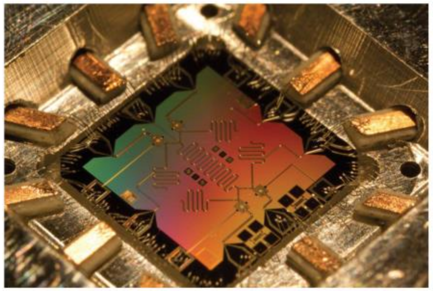
Jean-Damien Pillet

Quantum Circuit and Matter in Polytechnique (QCMX)

Laboratoire des Solides Irradiés (LSI)

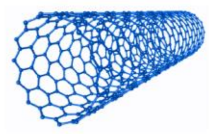
Quantum Circuit and Matter X

Quantum Technologies

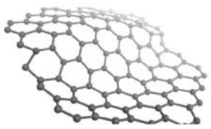

UCSB

Superconducting circuits

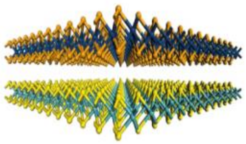
Low dimensional materials


Stanford

Carbon nanotube


NRNU

Graphene


Nature

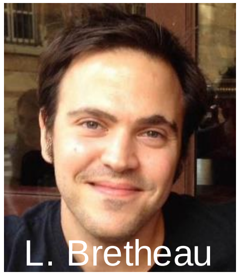
2D materials

Probe Quantum properties of matter

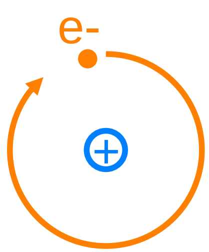
Develop new quantum technologies

Simulate complex many-body problem

With



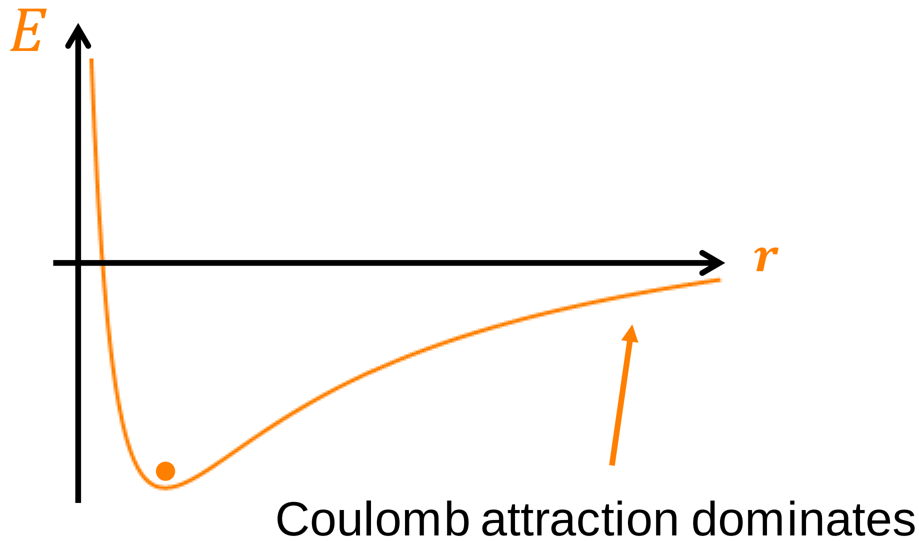
Atomic collapse in a nutshell



momentum **p**
radius **r**

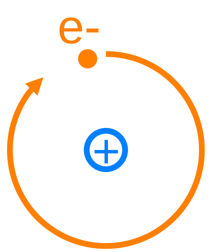
Atomic number Z

$$E = \frac{\boldsymbol{p}^2}{2m} - \frac{Ze^2}{4\pi\epsilon_0 \boldsymbol{r}}$$



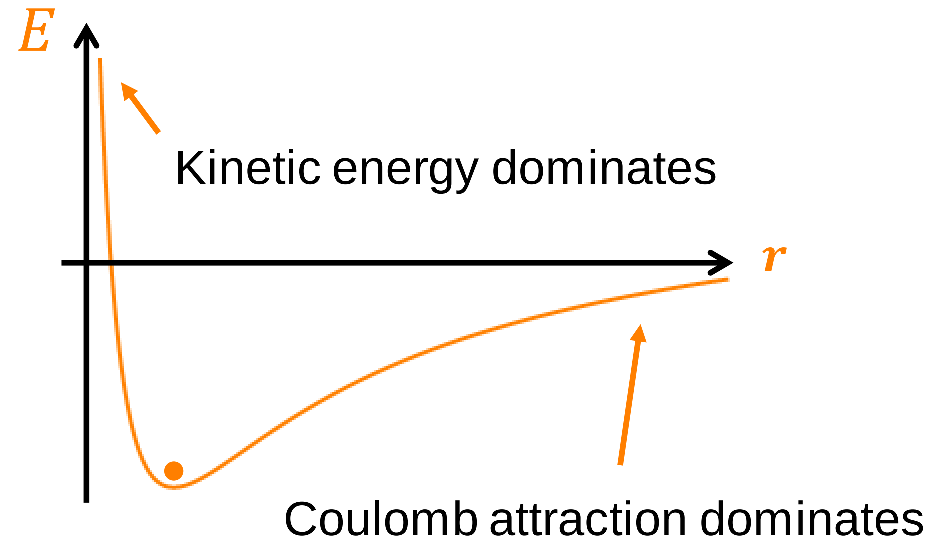
$$\boldsymbol{p} = \hbar/\boldsymbol{r}$$

Atomic collapse in a nutshell



momentum **p**
radius **r**

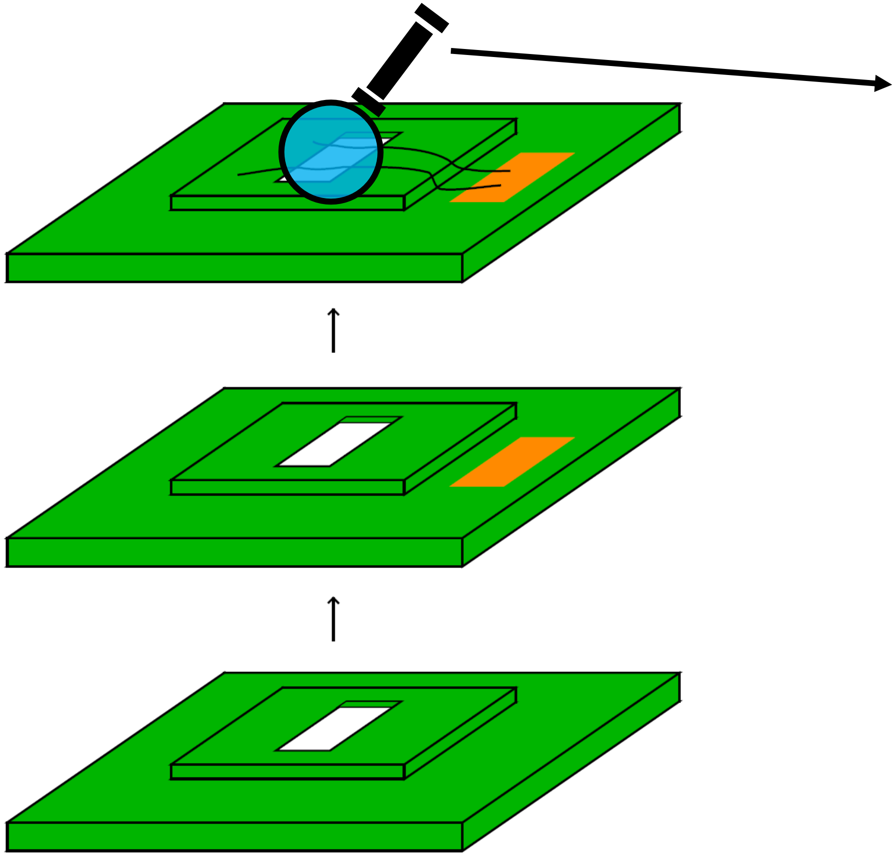
Atomic number Z

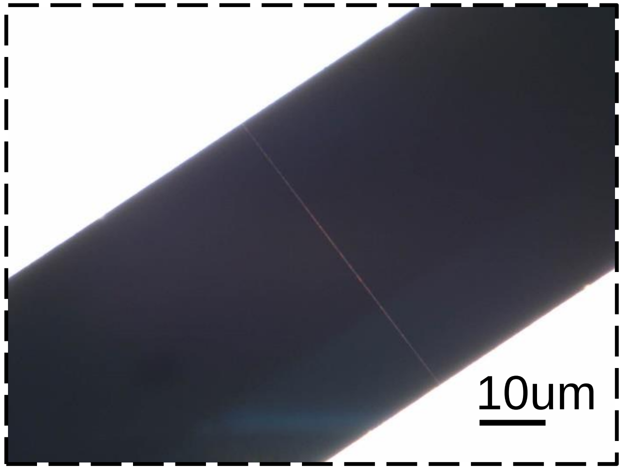


$$E = \frac{\boldsymbol{p}^2}{2m} - \frac{Ze^2}{4\pi\epsilon_0 \boldsymbol{r}}$$

$$\boldsymbol{p} = \hbar/\boldsymbol{r}$$

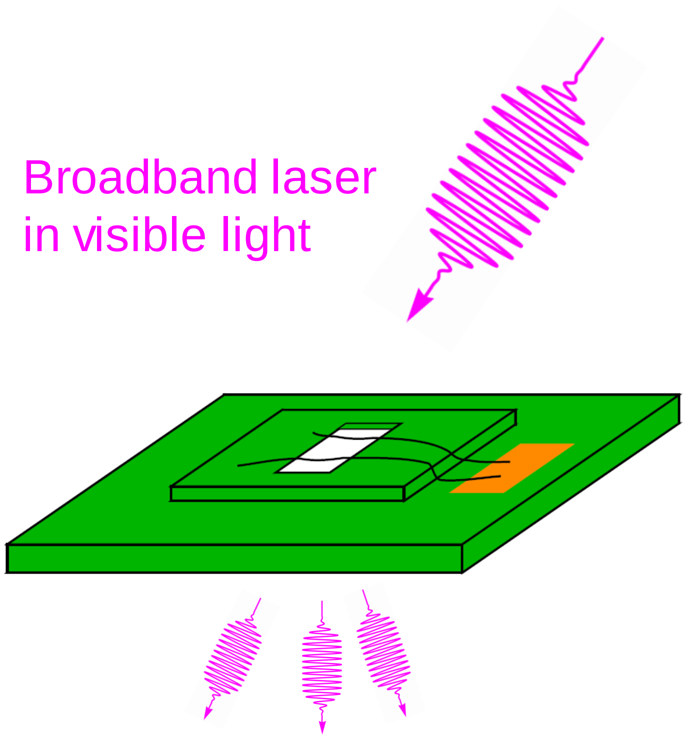
Carbon Nanotube growth and characterization

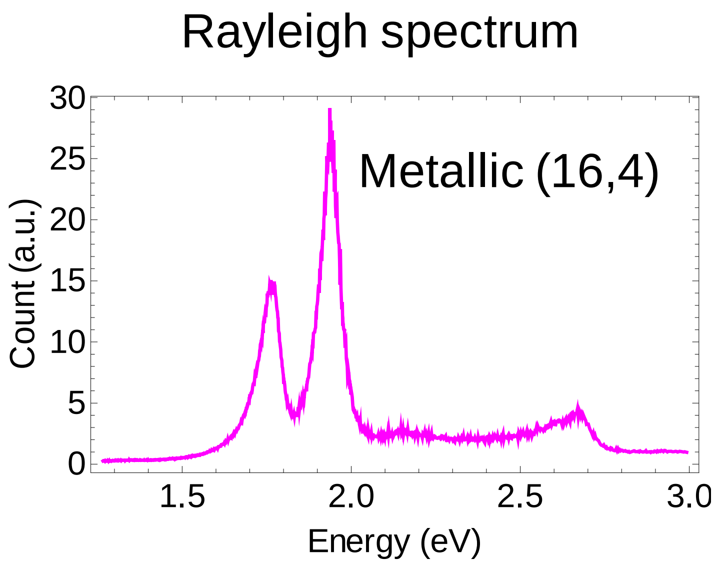




Optical image of CNT
Covered with 30nm Au

Carbon Nanotube growth and characterization

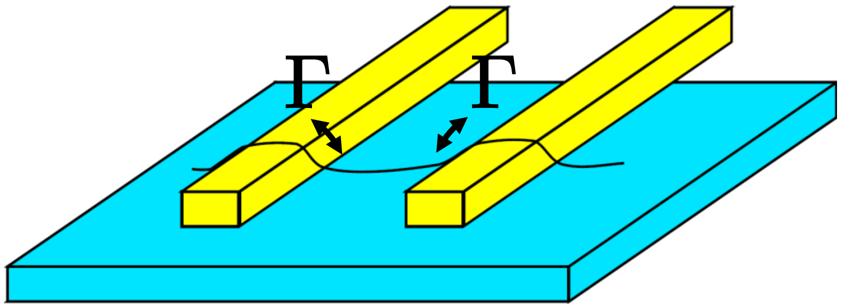




Chirality and nature (semiconducting or metallic)

Sfeir et al., Science 2004

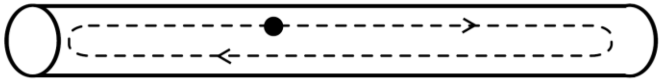
CNT behaves as a Quantum Dot



$$\Delta E \approx U \approx \mathrm{meV} \gg kT$$

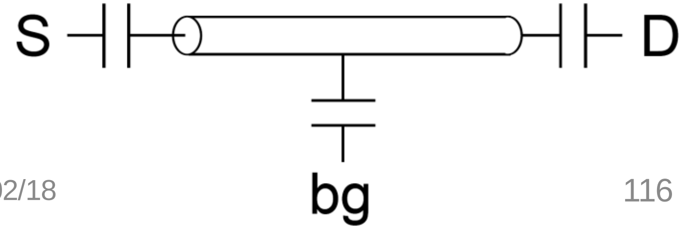
Quantization due to spatial confinement

$$\Delta E = \frac{h v_F}{2L}$$

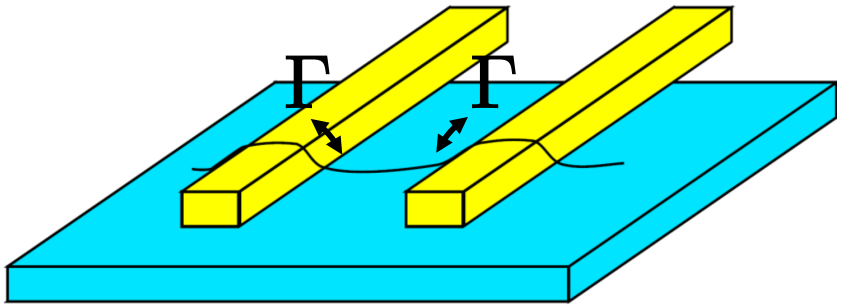


Coulomb repulsion
=> Charging energy

$$U = \frac{e^2}{2C}$$



CNT behaves as a Quantum Dot



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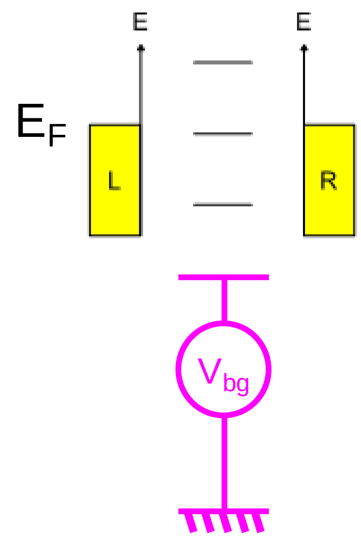

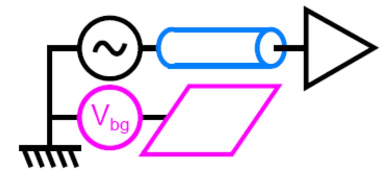

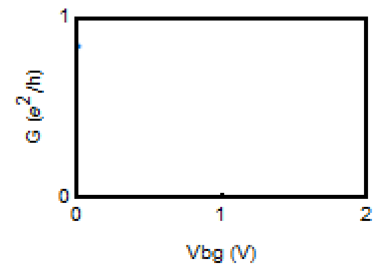

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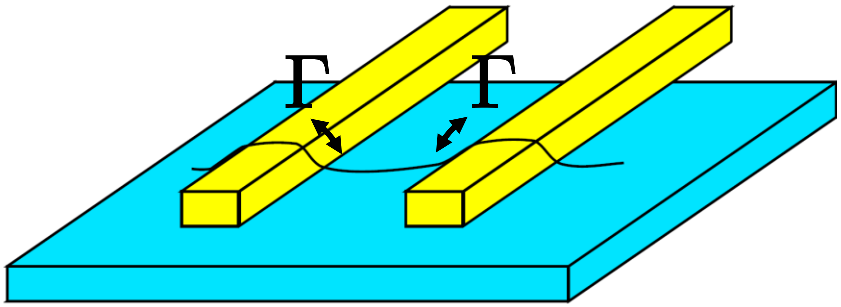

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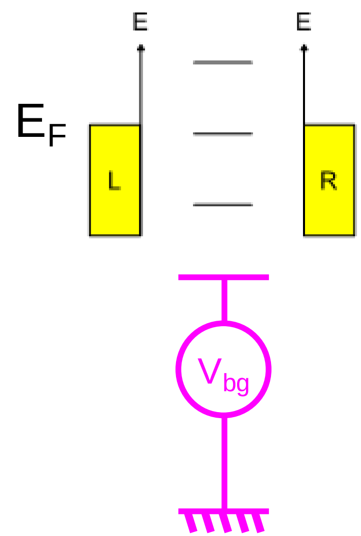
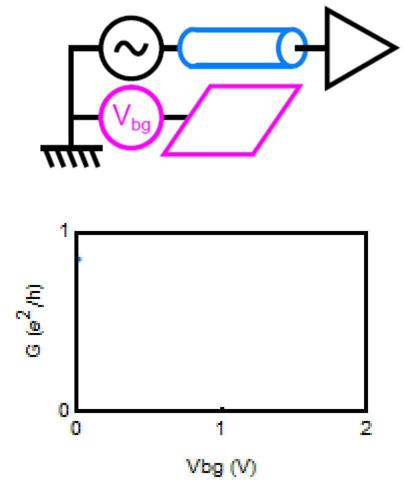
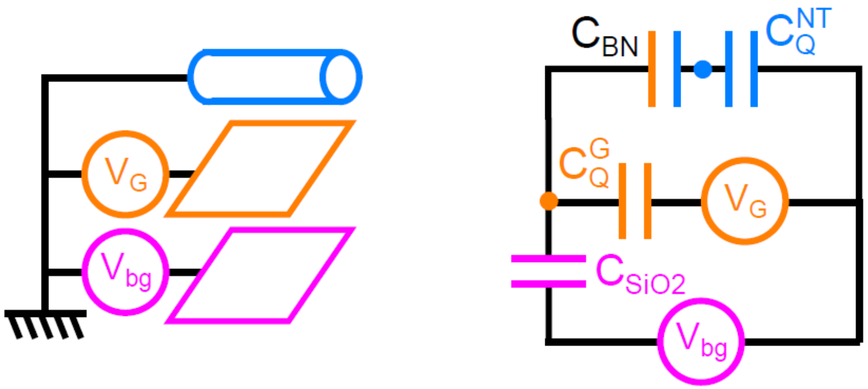




A network of capacitances

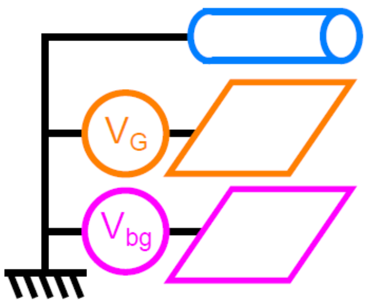
Independent control of charges in nanotube and graphene

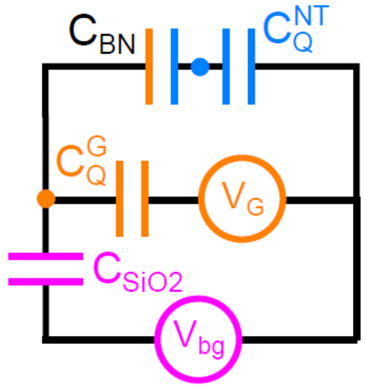


Two voltages: V_{bg} and V_G

Principle of nanotube detector

Kim et al., PRL 2012



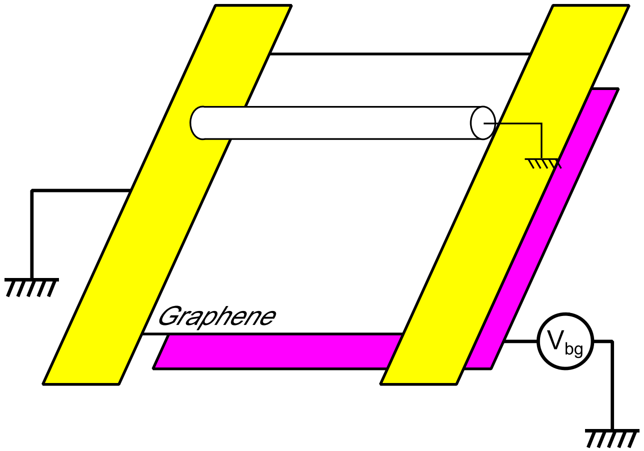


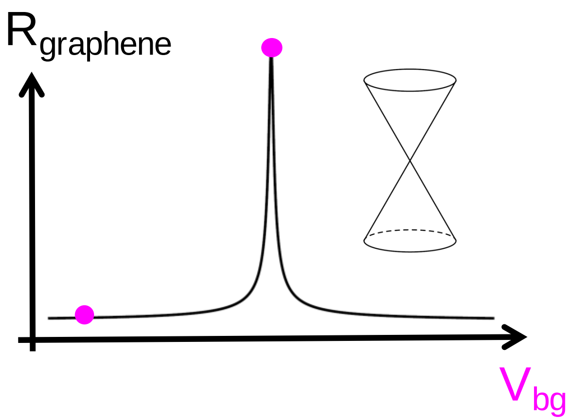
$$H = e^2 \frac{(n_G + n_{NT})^2}{2C_{SiO_2}} + e^2 \frac{n_{NT}^2}{2C_{BN}}$$ Charging energy

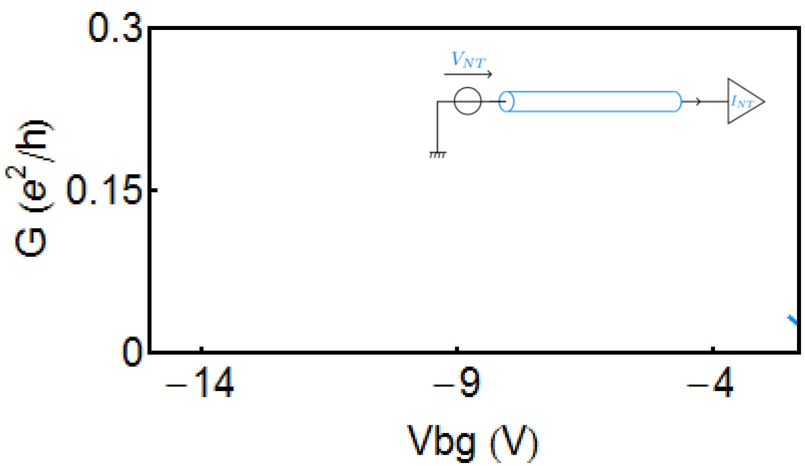
$$+ \int_{-\infty}^{E_F^G} D_G(E)\, E\, dE + \int_{-\infty}^{E_F^{NT}} D_{NT}(E)\, E\, dE$$ Quantum capacitance

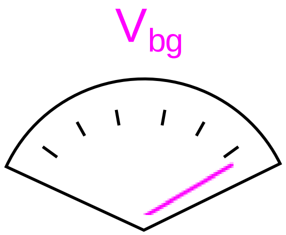
$$+ eV_{bg}(n_G + n_{NT}) - eV_G n_G$$ Work of voltage sources

Charging up graphene



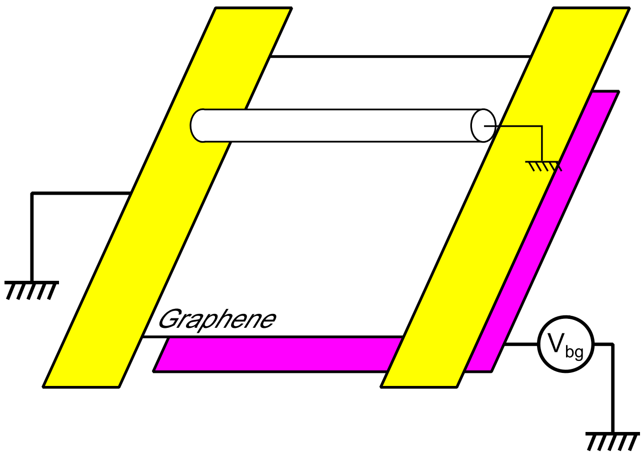


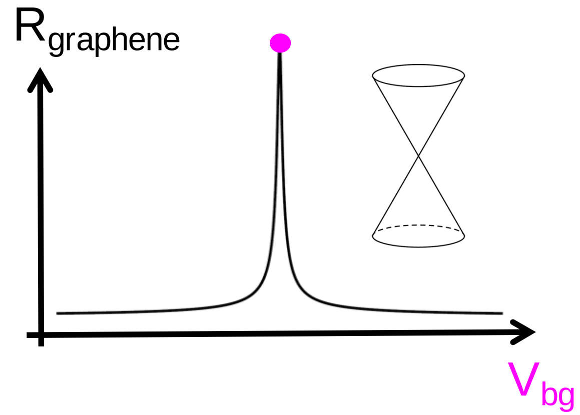


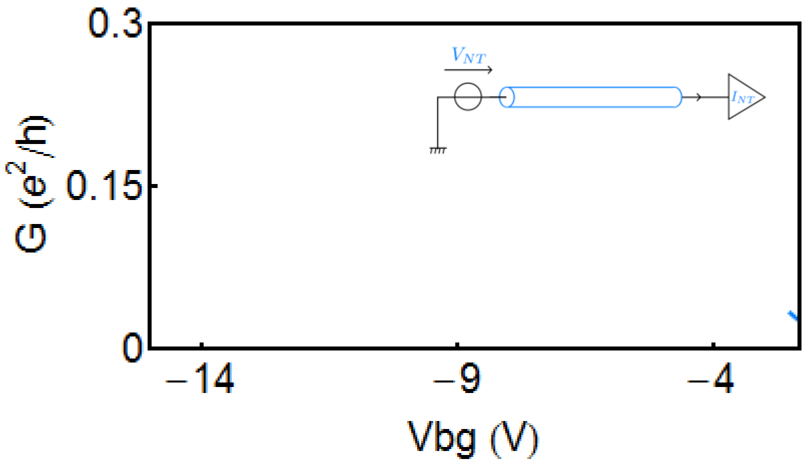


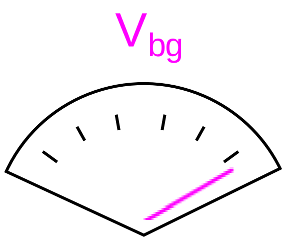
(T=1.5K)

Charging up graphene

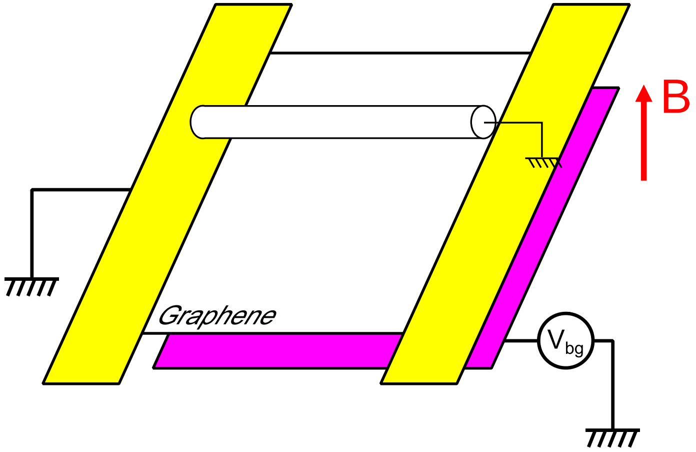




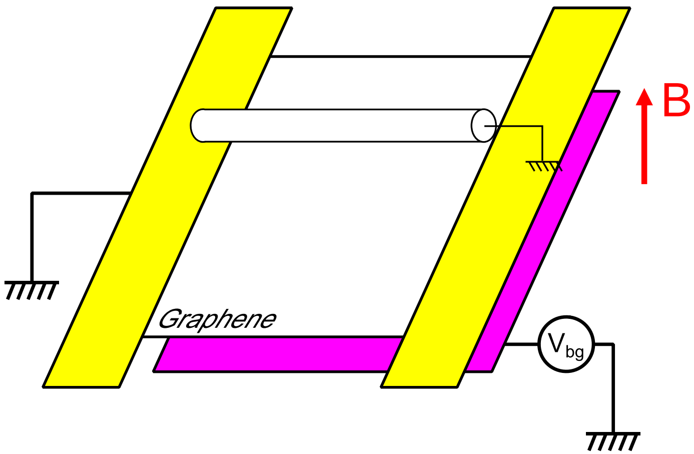


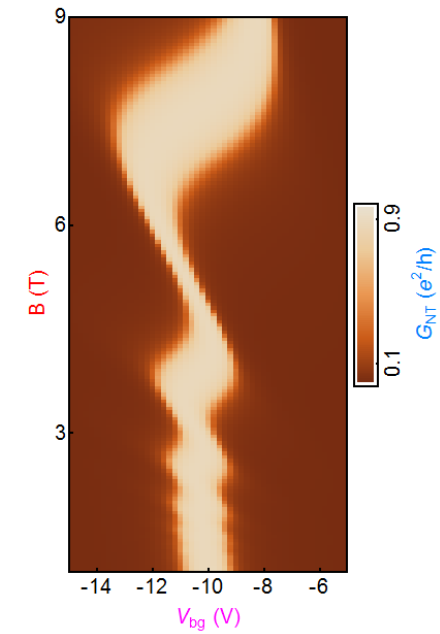


Electrostatic description



Electrostatic description

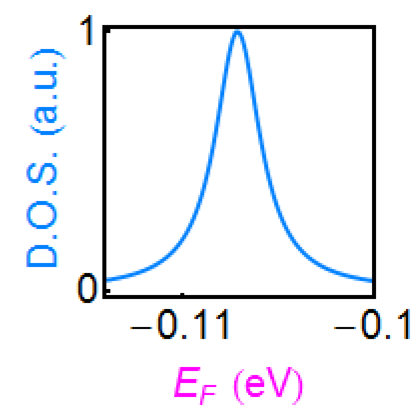




$$H = e^2 \frac{(n_G + n_{NT})^2}{2C_{SiO_2}} + e^2 \frac{n_{NT}^2}{2C_{BN}}$$

$$+ \int_{-\infty}^{E_F^G} D_G(E)\, E\, dE + \int_{-\infty}^{E_F^{NT}} D_{NT}(E)\, E\, dE$$

$$+ eV_{bg}(n_G + n_{NT}) - eV_G n_G$$

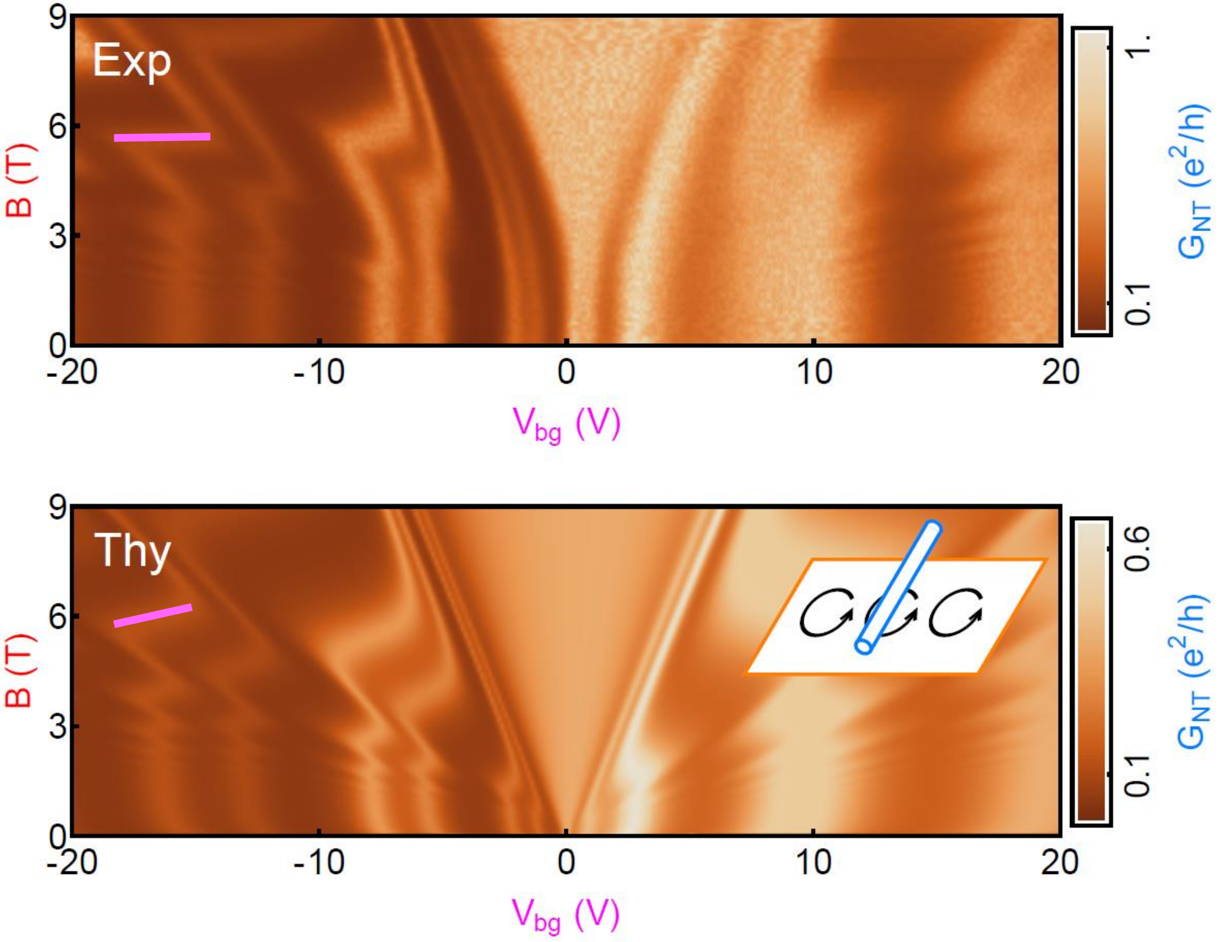


NT probe is non-invasive

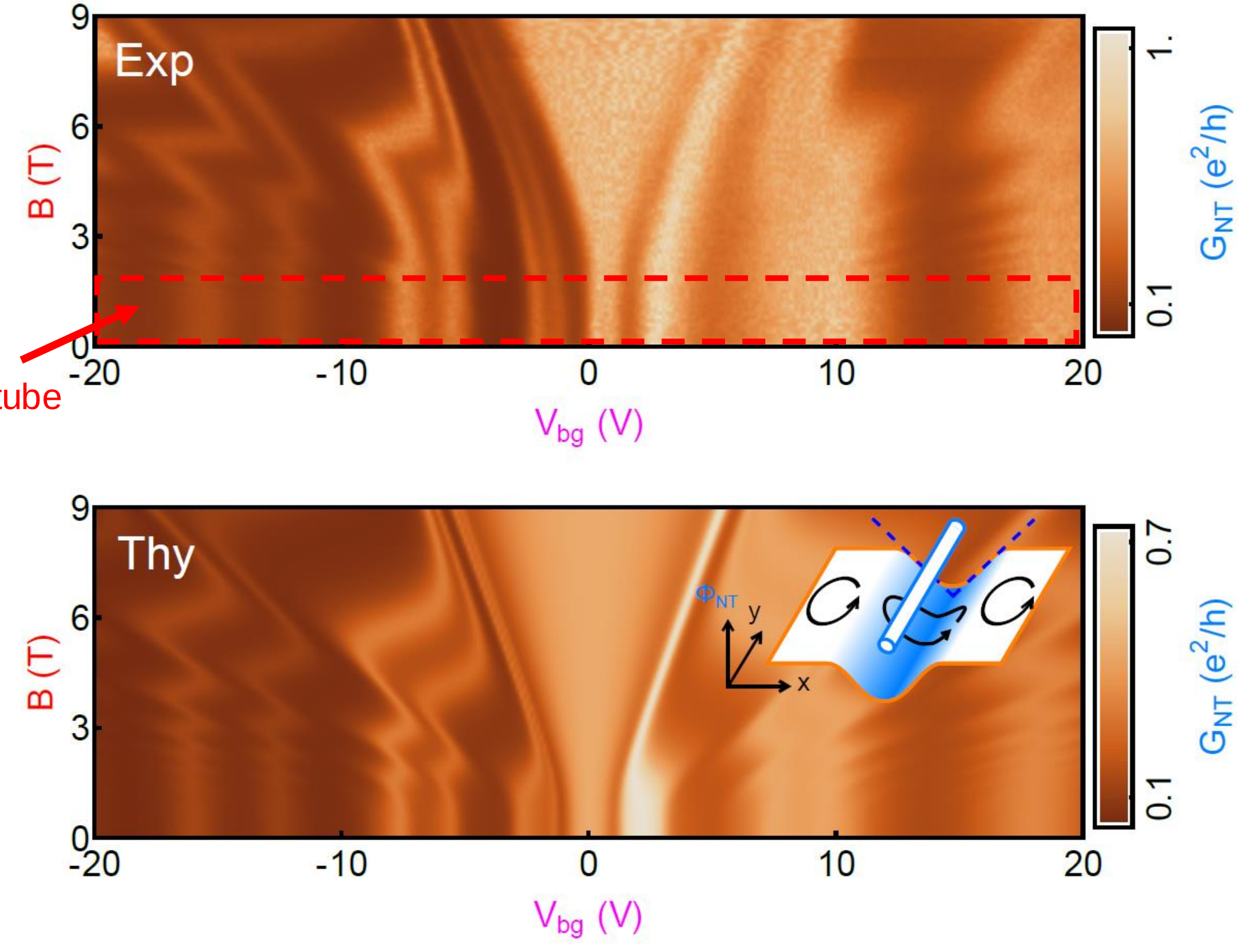
Slope given by
Landau Levels
lifetime γ

γ<<1 meV
Nanotube is an
weakly
invasive probe

γ=5 meV

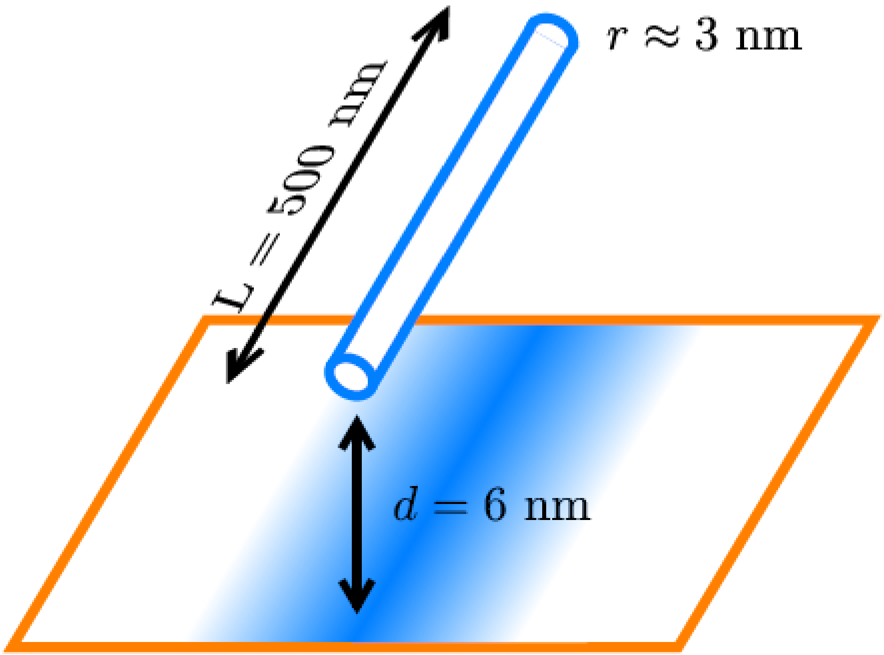


Deviation from electrostatic description



Effect of the nanotube on Landau levels

Critical magnetic field



$$C = \log\left(2d/r\right)/\pi\epsilon L$$

Critical magnetic field

$$B_c = \underbrace{\boxed{n_{NT}/dC}}_{\text{Electric field}} v_F$$

Dielectric response of graphene

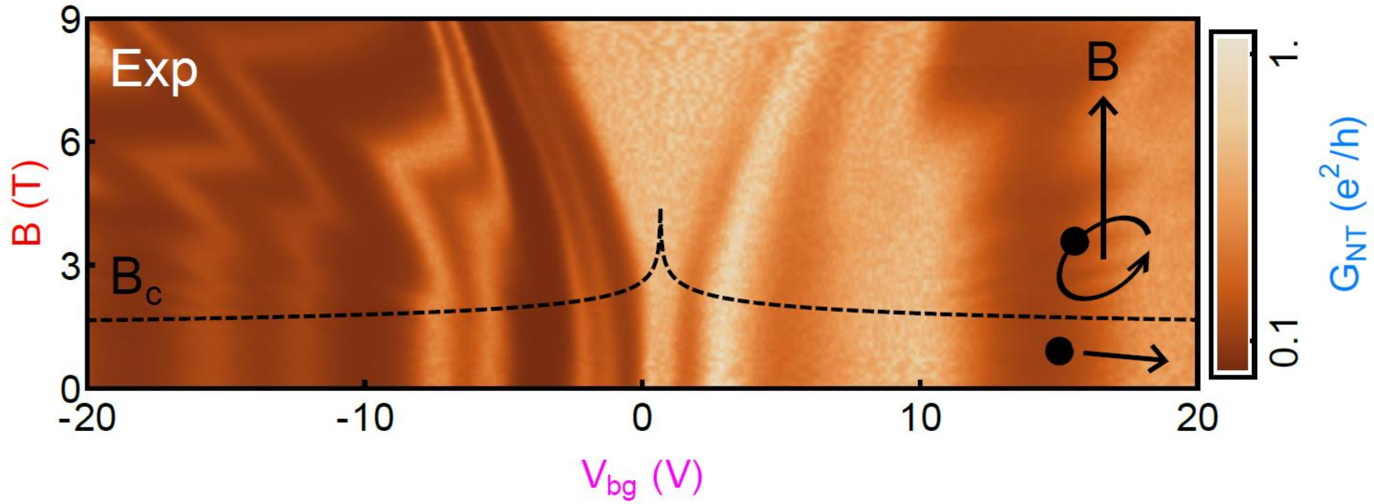
$$d_{eff} = d + d_{TF}$$

Thomas Fermi screening length

$$d_{TF} = 2/\sqrt{\pi n_G}$$

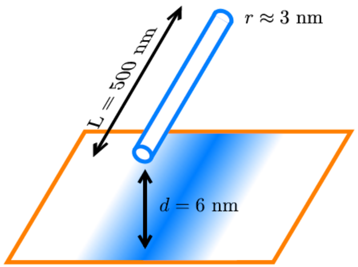
Jiang PRB 2015

Critical magnetic field

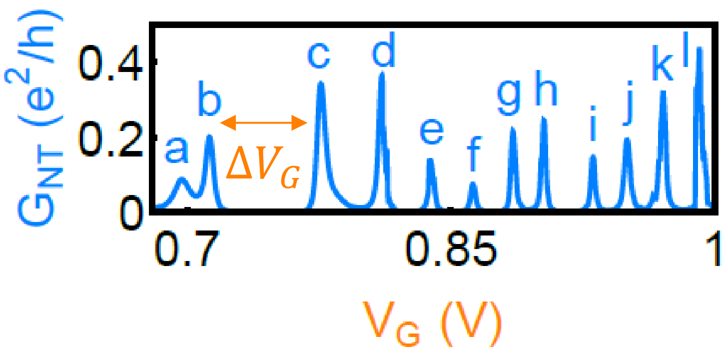


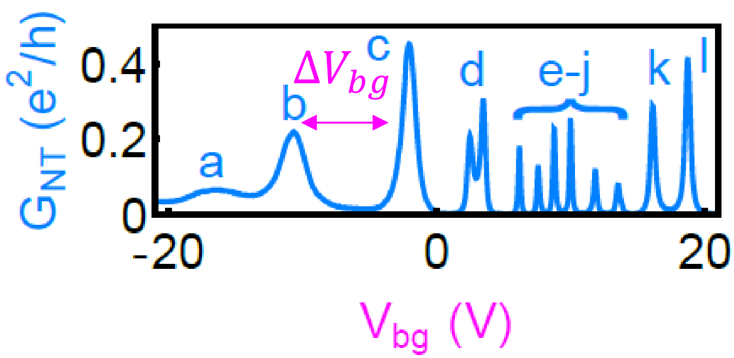
$$n_{NT} = 2$$ Chemical doping

$$|E| = 1.25 \times 10^6 \text{ V.m}^{-1}$$

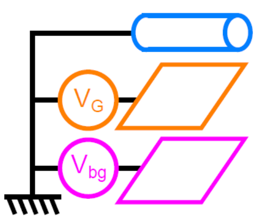


Extracting density of states





$$\text{D.O.S.} = \frac{dn_G}{dE_F} \propto \frac{\Delta V_{bg}}{\Delta V_G}$$



Coulomb spectroscopy calibration