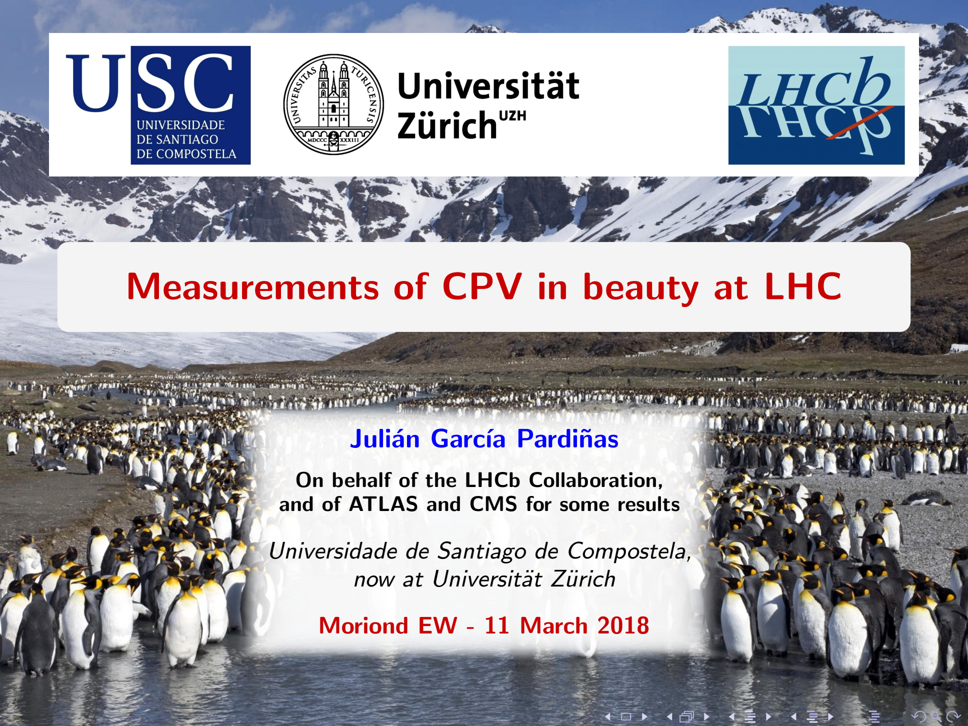

Measurements of CPV in beauty at LHC

Julián García Pardiñas

On behalf of the LHCb Collaboration,
and of ATLAS and CMS for some results

Universidade de Santiago de Compostela,
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Moriond EW - 11 March 2018

CP violation and the CKM matrix

The CP problem

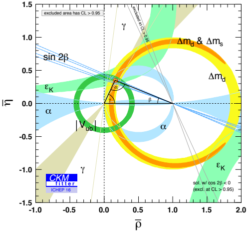
- The huge **matter/anti-matter asymmetry** in the Universe is inconsistent with the small CP in the Standard Model (SM). → **Search for new sources**.
- **Loop-dominated decays** can have significant contributions from new heavy particles that enter in the loops. → Detection via **precision measurements**.

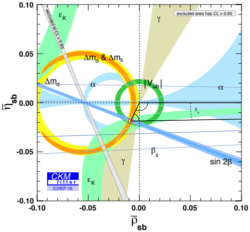
CP violation in the SM

- The only source of CP violation in the SM comes from the **CKM matrix**, governing the quark mixing.
- Unitarity matrix. → **Unitarity triangles**.

$$\begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{ccc} d & s & b \\ \end{array}$$

$$\begin{array}{c|ccc} & d & s & b \\ \hline u & 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ c & -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ t & A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{array}$$



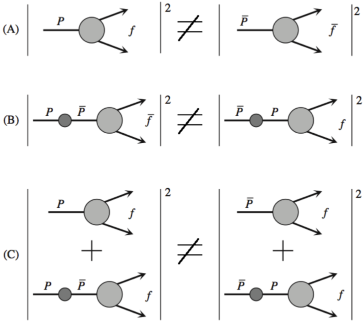


Types of CP violation

The decays of **beauty hadrons (mesons or baryons)** provide excellent scenarios to measure the CKM parameters and look for new sources of CP violation.

$$|P_1\rangle = p\,|P^0\rangle + q\,|\overline{P}^0\rangle$$
$$|P_2\rangle = p\,|P^0\rangle - q\,|\overline{P}^0\rangle$$

$$A_f = \langle f|\,H\,|P\rangle$$
$$\overline{A}_{\overline{f}} = \langle \overline{f}|\,H\,|\overline{P}\rangle$$



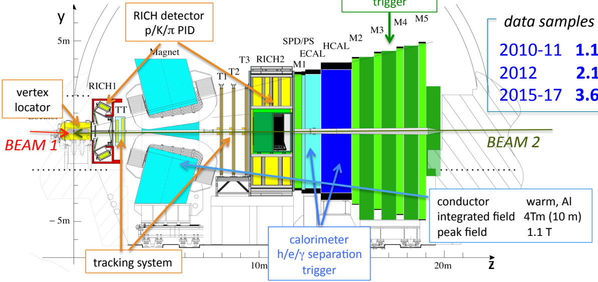
- CP violation in the **decay** (A)
  - $|A_f/\overline{A}_{\overline{f}}| \neq 1$
- CP violation in **mixing** (B)
  - Occurs in neutral mesons
  - $|q/p| \neq 1$
- CP violation in the **interference** between mixing and decay (C)
  - Neutral meson decaying into a non-flavour specific state
  - $\mathrm{Im}\left(\frac{q}{p}\frac{\overline{A}_{\overline{f}}}{A_f}\right) \neq 1$
- Measure CP violating parameters
  - $\frac{\overline{A}_f - A_f}{\overline{A}_f + A_f} = \frac{C_f \cos(\Delta m t) - S_f \sin(\Delta m t)}{\cosh(\frac{\Delta\Gamma t}{2}) + D_f \sinh(\frac{\Delta\Gamma t}{2})}$
  - S_f, D_f: CPV in the interference
  - C_f: CPV in decay

The LHCb experiment

- This talk summarises the **state-of-the-art measurements** of CP violation performed at the **LHC**, mostly focusing on the **LHCb experiment**.

the LHCb detector

LHCb *[Int. J. Mod. Phys. A30, (2015)1530022]*



vertex locator

RICH detector p/K/π PID

muon system
μ/h separation
trigger

tracking system

calorimeter
h/e/γ separation
trigger

conductor	warm, Al
integrated field	4Tm (10 m)
peak field	1.1 T

single-arm forward spectrometer

pseudorapidity range ➔ $2 < \eta < 5$

data samples

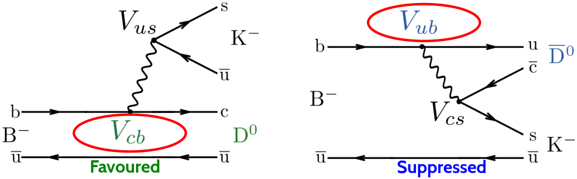
2010-11	**1.1 fb^{-1}**	at 7 TeV
2012	**2.1 fb^{-1}**	at 8 TeV
2015-17	**3.6 fb^{-1}**	at 13 TeV

- Excellent track and vertex reconstruction, good PID separation, flexible trigger.
- Decay-time resolution: $\sim$ 45 fs.
- Flavour tagging power: $4 - 8\%$.

Measurement of the CKM angle γ

The CKM angle γ

- The angle $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$ is the **least known** in the Unitarity Triangle.
- It can be measured from **tree-dominated decays**.
  → Very small theoretical uncertainty, $|\delta_\gamma| \lesssim \mathcal{O}(10^{-7})$ [JHEP 1401 (2014) 051].



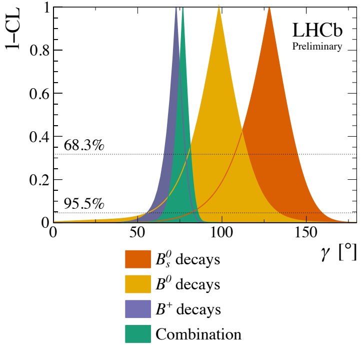
- The suppression of these kind of decays motivates the **combination** of many measurements.
- Two different strategies:
  - **Time-integrated** measurements: e.g. $B \to D^{(*)}K^{(*)}$
  - **Time-dependent** measurements: $B_s \to D_s K$

Measurement of the CKM angle γ

LHCb-CONF-2017-004

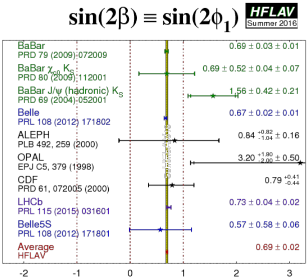
- **New LHCb combination**, following the strategy of the previous LHCb combination [JHEP 12 (2016) 087] but with new analyses and updated measurements.

B decay	D decay	Method	Ref.	Status since last combination [1]
$B^+ \to DK^+$	$D \to h^+h^-$	GLW	[16]	Updated to Run 1 + $2\,\mathrm{fb}^{-1}$ Run 2
$B^+ \to DK^+$	$D \to h^+h^-$	ADS	[17]	As before
$B^+ \to DK^+$	$D \to h^+\pi^-\pi^+\pi^-$	GLW/ADS	[17]	As before
$B^+ \to DK^+$	$D \to h^+h^-\pi^0$	GLW/ADS	[18]	As before
$B^+ \to DK^+$	$D \to K^0_S h^+h^-$	GGSZ	[19]	As before
$B^+ \to DK^+$	$D \to K^0_S K^+\pi^-$	GLS	[20]	As before
$B^+ \to D^*K^+$	$D \to h^+h^-$	GLW	[16]	New
$B^+ \to DK^{*+}$	$D \to h^+h^-$	GLW/ADS	[21]	New
$B^+ \to DK^+\pi^+\pi^-$	$D \to h^+h^-$	GLW/ADS	[22]	As before
$B^0 \to DK^{*0}$	$D \to K^+\pi^-$	ADS	[23]	As before
$B^0 \to DK^+\pi^-$	$D \to h^+h^-$	GLW-Dalitz	[24]	As before
$B^0 \to DK^{*0}$	$D \to K^0_S\pi^+\pi^-$	GGSZ	[25]	As before
$B^0_s \to D^\mp_s K^\pm$	$D^+_s \to h^+h^-\pi^+$	TD	[26]	Updated to $3\,\mathrm{fb}^{-1}$ Run 1



- LHCb combination result: $\gamma = (76.8^{+5.1}_{-5.7})°$
- This is the **most precise determination** of γ from a single experiment to date.
- **Compatible** with the indirect determination: $\gamma = (65.3^{+1.0}_{-2.5})°$ [CKMfitter].

Measurement of the CKM angle β



The CKM angle β

- The angle $\beta = \arg(-V_{cd}V_{cb}^{*}/V_{td}V_{tb}^{*})$ can be determined from the **interference between mixing and decay** in $B^0 \to (c\bar{c})K_s$.
- The B-factories still dominate the world average, but LHCb Run 1 is getting close.

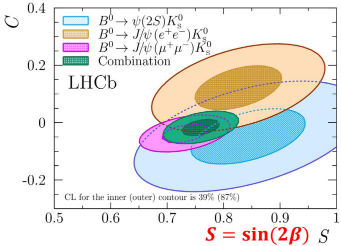
- New Run 1 LHCb analysis of $B^0 \to J/\psi(ee)K_s$ and $B^0 \to \psi(2S)(\mu\mu)K_s$ [JHEP 11 (2017) 170].

  LHCb combination:

$$C = -0.017 \pm 0.029, \qquad S = 0.760 \pm 0.034$$

- Complementary measurement from ATLAS : most precise single determination of the **B^0 mixing width** [JHEP 1606 (2016) 081].

$$\Delta\Gamma_d/\Gamma_d = (-0.1 \pm 1.1(stat.) \pm 0.9(syst.)) \times 10^{-2}$$



$S = \sin(2\beta)$

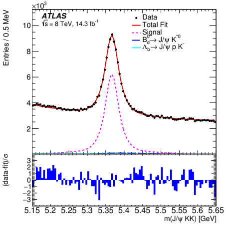
Measurement of the CKM angle β_s

The CKM angle β_s

- The angle $\beta_s = \arg(-V_{cs}V_{cb}^*/V_{ts}V_{tb}^*)$ is determined from the phase $\phi_s^{c\bar{c}s}$ associated to the **interference between mixing and decay** in $b \to c\bar{c}s$.
- $B_s^0 \to J/\psi\phi$ is the golden channel. → **Angular analysis** needed to separate different CP eigenstates.
- The **LHC combined effort** dominates the global picture.

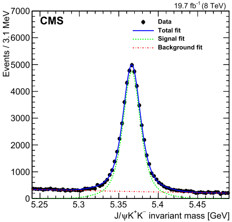
ATLAS: [JHEP 1608 (2016) 147]

75k candidates



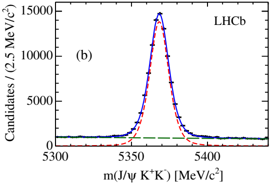
CMS: [PLB 757 (2016) 970]

70.5k candidates

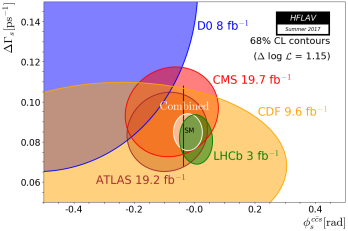


LHCb: [PRL 114, 041801 (2015)]

95k candidates

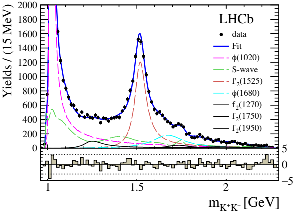


Measurement of the CKM angle β_s



- Global fit: $\phi_s^{c\bar{c}s} = -21 \pm 31$ mrad [HFLAV], **dominated by the LHCb measurement** [PRL 114, 041801 (2015)].
- Focus on **analysing more more data** and studying the **penguin pollution** [JHEP 1503 (2015) 145].
- Stay tuned for the LHCb analysis of Run 2!

- LHCb measurement of $\phi_s^{c\bar{c}s}$ in $B_s^0 \to J/\psi K^+K^-$ above the $\phi(1020)$ region [JHEP 08 (2017) 037].
- First time $\phi_s^{c\bar{c}s}$ is measured in final states **dominated by a tensor**.
- **New LHCb average** (including $J/\psi\phi$ and $J/\psi\pi\pi$): $\phi_s^{c\bar{c}s} = 1 \pm 37$ mrad



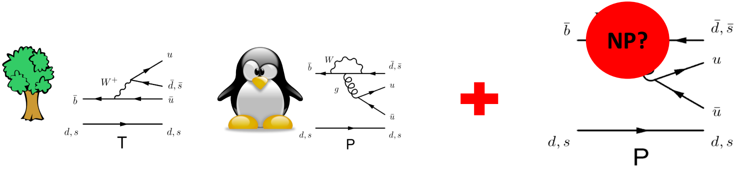
CP violation in $B^0_{(s)} \to hh'$

[New result! Preliminary!]

LHCb-PAPER-2018-006
(in preparation)

Indirect determination of CKM phases

- The study of **time-dependent CP violation** in $\boldsymbol{B^0_{(s)} \to hh'}$ allows the determination of γ and $-2\beta_s$ (also α, when extra input is added) using loop-mediated decays.
- Presence of loop-diagrams $\Longrightarrow$ sensitivity to **New Physics**.



New LHCb measurement using Run 1 data:

- Measure time-dependent asymmetries in $B^0 \to \pi^+\pi^-$ and $B^0_s \to K^+K^-$.
- Measure time-integrated asymmetries in $B^0 \to K^+\pi^-$ and $B^0_s \to \pi^+K^-$.
- Updating previous LHCb measurements: [JHEP 10 (2013) 183,Phys. Rev. Lett. 110 (2013) 221601].

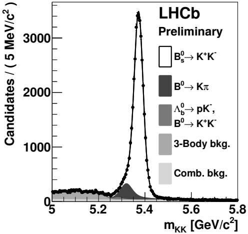
CP violation in $B^0_{(s)} \to hh'$

[**New result!**
Preliminary!]

LHCb-PAPER-2018-006
(in preparation)

Analysis strategy

- Loose pre-selection + PID requirements (creating exclusive $\pi\pi$, KK and $K\pi$ categories) + BDT against combinatorial background.
- Flavour-tagged, decay-time- and hh'-mass-dependent fit to data, simultaneously on the $\pi\pi$, KK and $K\pi$ categories and including signal and background species.
- Production and reconstruction asymmetries (from control samples) and time acceptance and resolution (per event) are included.
- Tagging power of $\sim 4\%$ for the $\pi\pi$ and $\sim 3.7\%$ for the KK.



$$\begin{aligned} C_{\pi^+\pi^-} &= -0.34 \pm 0.06, \\ S_{\pi^+\pi^-} &= -0.63 \pm 0.05, \\ C_{K^+K^-} &= 0.20 \pm 0.06, \\ S_{K^+K^-} &= 0.18 \pm 0.06, \\ A^{\Delta\Gamma}_{K^+K^-} &= -0.79 \pm 0.07, \\ A_{CP}\left(B^0 \to K^+\pi^-\right) &= -0.084 \pm 0.004, \\ A_{CP}\left(B_s^0 \to \pi^+K^-\right) &= 0.213 \pm 0.015, \end{aligned}$$

Most precise measurements from a single experiment.

$(C_{K^+K^-}, S_{K^+K^-}, A^{\Delta\Gamma}_{K^+K^-})$ deviates **4.0 σ** from $(0, 0, -1)$

$\Longrightarrow$ Strongest evidence for time-dependent CP violation in the B_s^0 sector!

Phase $\phi_s^{s\bar{d}d}$ in $B_s^0 \to (K^+\pi^-)(K^-\pi^+)$

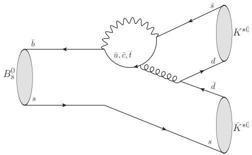
arXiv:1712.08683

$\phi_s^{s\bar{q}q}$ phases

- The $b \to s\bar{q}q$ transitions occur at **loop-level** in the SM. $\Longrightarrow$ **Potential New Physics** entering the decay.
- The phase $b \to s\bar{s}s$ was measured by LHCb using $B_s^0 \to \phi\phi$ decays [Phys. Rev. D 90, 052011 (2014)]. → Compatible with the SM expectation.

First LHCb Run 1 measurement of $\phi_s^{s\bar{d}d}$

The $B_s^0 \to K^{*0}(K^+\pi^-)\overline{K}^{*0}(K^-\pi^+)$ decay proceeds via a gluonic penguin diagram in the SM.



It is sensible to the phase $\phi_s^{s\bar{d}d}$, expected to be ~ 0 in the SM [JHEP 1503 (2015) 145].

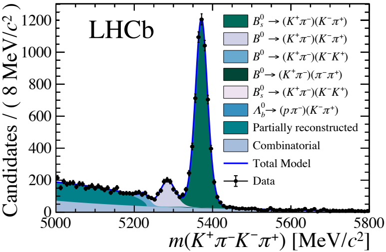
To **increase the statistics**: study $B_s^0 \to (K^+\pi^-)(K^-\pi^+)$ decays with $M(K^\pm\pi^\mp) \in [750, 1600]$ MeV$/c^2$.
Dominant $K\pi$ structures:

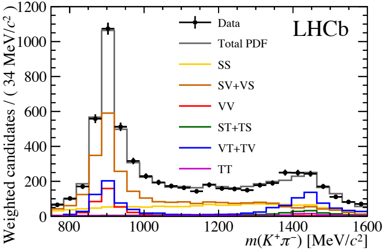
- Scalar ($j = 0$): $K_0^*(800)^0$, $K_0^*(1400)^0$, non-resonant
- Vector ($j = 1$): $K^*(892)^0$
- Tensor ($j = 2$): $K_2^*(1400)^0$

This leads to $3 \times 3 = 9$ channels and 19 polarisation amplitudes in total.
→ Same phase $\phi_s^{s\bar{d}d}$ used for all the amplitudes.

Phase $\phi_s^{s\bar{d}d}$ in $B_s^0 \to (K^+\pi^-)(K^-\pi^+)$

arXiv:1712.08683





- Flavour-tagged, time-dependent, angular and $K\pi$ invariant mass analysis.
- Model with **19 polarisation amplitudes**.
- Fit to **background-subtracted** data performed using **GPUs**.

Results

- CP-violating parameters: $\phi_s^{s\bar{d}d} = -0.10 \pm 0.13 \pm 0.14$ rad, $|\lambda| = 1.035 \pm 0.034 \pm 0.089$. → **Compatible with the SM expectations.**
- Dominant source of systematic uncertainties: size of the simulated samples used to describe the acceptance. → Improvable in the future.

CPV in $B^+ \to D^+_{(s)}\overline{D}^0$

New result! Preliminary!

LHCb-PAPER-2018-007
(in preparation)

Goals

- Measure the direct CP asymmetries in $\boldsymbol{B^+ \to D^+_{(s)}\overline{D}^0}$ decays, where $D^0 \to K^-\pi^+$ or $D^0 \to K^-\pi^+\pi^-\pi^+$, $D^+ \to K^-\pi^+\pi^+$ and $D_s^+ \to K^+K^-\pi^+$, using Run 1 data.
- Interference between Cabbibo-suppressed tree diagrams with loop diagrams predicts CP asymmetries of $\mathcal{O}(10^{-2})$.

Strategy

- Very efficient selection that employs topological and kinematic variables, meson decay times and invariant masses. Divided in pre-selection + BDT.
- Determine raw asymmetries from fits to $M(D^+_{(s)}\overline{D}^0)$ (very clean distributions) and correct them for production and detection asymmetries (from control samples).

$$\mathcal{A}^{CP}(B^+ \to D_s^+\overline{D}^0) = (-0.4 \pm 0.5 \pm 0.5)\%, \qquad \mathcal{A}^{CP}(B^+ \to D^+\overline{D}^0) = (\ \ 2.3 \pm 2.7 \pm 0.4)\%$$

- **No evidence of CP violation** is found.
- **First measurement** in $\boldsymbol{B^+ \to D_s^+\overline{D}^0}$ and **most precise one** in $\boldsymbol{B^+ \to D^+\overline{D}^0}$.

CPV in baryon decays

New result!
Preliminary!

LHCb-PAPER-2018-001
(in preparation)

Goals

- Previously, **first evidence of CPV in a beauty baryon decay**, $\Lambda_b^0 \to p\pi^-\pi^+\pi^-$ [Nature Physics 13, 391-396 (2017)].
- Now, measure CP- and P-violating asymmetries using triple products in $\boldsymbol{\Lambda_b^0 \to pK^+\pi^-\pi^-}$, $\boldsymbol{\Lambda_b^0 \to pK^-K^+K^-}$ and $\boldsymbol{\Xi_b^0 \to pK^-K^-\pi^+}$, with Run 1 data.

$$A_{\widehat{T}} = \frac{N(C_{\widehat{T}} > 0) - N(C_{\widehat{T}} < 0)}{N(C_{\widehat{T}} > 0) + N(C_{\widehat{T}} < 0)}, \quad \bar{A}_{\widehat{T}} = \frac{\overline{N}(-\overline{C}_{\widehat{T}} > 0) - \overline{N}(-\overline{C}_{\widehat{T}} < 0)}{\overline{N}(-\overline{C}_{\widehat{T}} > 0) + \overline{N}(-\overline{C}_{\widehat{T}} < 0)}, \quad a_P^{\widehat{T}\text{-odd}} = \frac{1}{2}\left(A_{\widehat{T}} + \bar{A}_{\widehat{T}}\right), \quad a_{CP}^{\widehat{T}\text{-odd}} = \frac{1}{2}\left(A_{\widehat{T}} - \bar{A}_{\widehat{T}}\right)$$

- Decays mediated by $b \to us\bar{u}$ tree + $b \to su\bar{u}$ penguin transitions. → Interference could lead to CP violation in the decay.

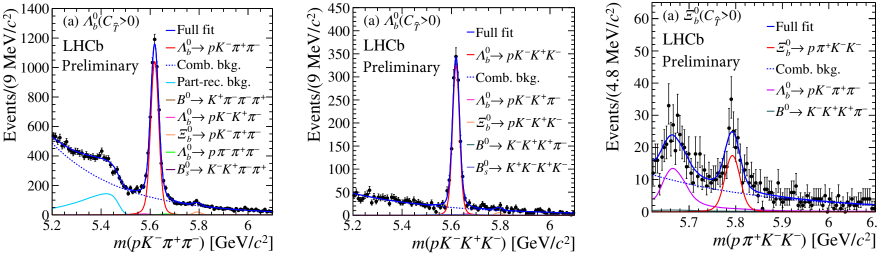
Strategy

- Pre-selection + vetoes against charm resonances + vetoes against mis-ID backgrounds + BDT against combinatorial background + PID requirements.
- **Strategy:** split each sample according to baryon flavour and sign of $C_{\widehat{T}}(\overline{C}_{\widehat{T}})$. Simultaneous fit to $M(pKhh')$ in the four categories to measure the asymmetries.
- **Complementary**, the asymmetries are computed in different bins of the phase-space (two-body-masses and angle $|\Phi|$).

CPV in baryon decays

New result!
Preliminary!

LHCb-PAPER-2018-001
(in preparation)



	$\Lambda_b^0 \to pK^-\pi^+\pi^-$	$\Lambda_b^0 \to pK^-K^+K^-$	$\Xi_b^0 \to pK^-K^-\pi^+$
$a_P^{\hat{T}\text{-odd}}$ (%)	$-0.60 \pm 0.84 \pm 0.31$	$-1.56 \pm 1.51 \pm 0.32$	$-3.04 \pm 5.19 \pm 0.36$
$a_{CP}^{\hat{T}\text{-odd}}$ (%)	$-0.81 \pm 0.84 \pm 0.31$	$1.12 \pm 1.51 \pm 0.32$	$-3.58 \pm 5.19 \pm 0.36$

- Results compatible with neither CP nor P asymmetry.
- **Same conclusion** is reached when looking at **per-bin asymmetries**.

Summary and prospects

- Several **new measurements** of CP violation in beauty decays and more to come in the near future.

  → So far, results are compatible with the SM.

- The role of the LHC experiments is crucial, in particular the one of **LHCb**, leading the World sensitivity in several of these measurements.

- **Precision will improve** in the next years, with more data and upgraded detectors.

Some specific prospects

- Measurement of γ in LHCb: precision of 4° by the end of Run 2 and better than 0.4° with phase-2 upgrade.
- Measurement of β in LHCb: precision of $0.6^\circ(0.2^\circ)$ with Run 2 (phase-1 upgrade).
- Measurement of $\phi_s^{c\bar{c}s}$:
  - ATLAS: increase in sensitivity with a new innermost pixel detector.
  - LHCb: sensitivity with phase-2 upgrade expected to be ≤ 3 mrad.

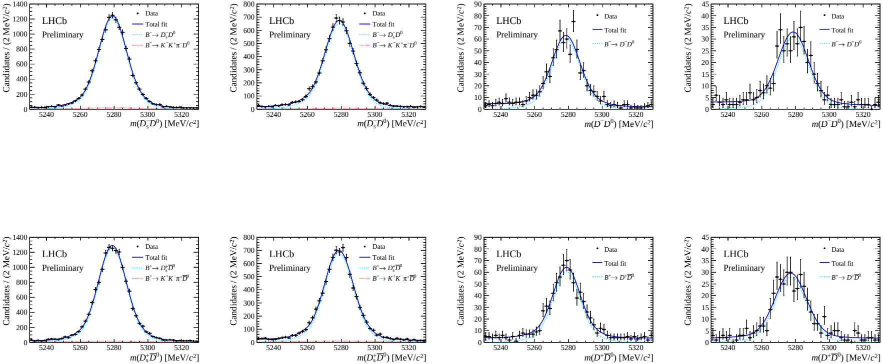
LHCb phase-2 upgrade: [CERN-LHCC-2017-003 (2017)]

Backup slides

CPV in $B^+ \to D^+_{(s)}\overline{D}^0$

[New result! Preliminary!]

LHCb-PAPER-2018-007
(in preparation)



CPV in baryon decays

[New result! Preliminary!]

LHCb-PAPER-2018-001
(in preparation)

The triple products of final state particle momenta in the Λ_b^0 (Ξ_b^0) centre-of-mass frame are defined as $C_{\widehat{T}} = \vec{p}_p \cdot (\vec{p}_{h_1^-} \times \vec{p}_{h_2^+})$ for Λ_b^0 (Ξ_b^0) and $\overline{C}_{\widehat{T}} = \vec{p}_{\overline{p}} \cdot (\vec{p}_{h_1^+} \times \vec{p}_{h_2^-})$ for $\overline{\Lambda}_b^0$ ($\overline{\Xi}_b^0$), where $h_1 = K$, $h_2 = \pi$ for the $\Lambda_b^0 \to pK^-\pi^+\pi^-$ decay, $h_1 = K_{\text{fast}}$, $h_2 = K$ for the $\Lambda_b^0 \to pK^-K^+K^-$ decay and $h_1 = K_{\text{fast}}$, $h_2 = \pi$ for the $\Xi_b^0 \to pK^-K^-\pi^+$ decay, and K_{fast} denotes the K with the highest momentum among those kaons that have the same charge.

CPV in baryon decays

New result!
Preliminary!

LHCb-PAPER-2018-001
(in preparation)

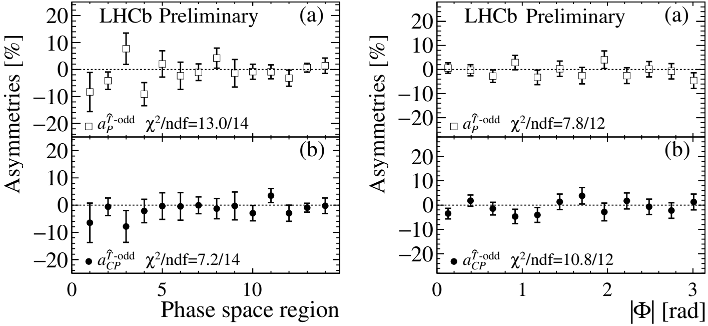


Figure 4: The asymmetries in each region using binning schemes (left) A and (right) B for the $\Lambda_b^0 \to pK^-\pi^+\pi^-$ decay. For $a_P^{\widehat{T}\text{-odd}}$ ($a_{CP}^{\widehat{T}\text{-odd}}$), the values of the χ^2/ndf for the P-symmetry (CP-symmetry) hypothesis, represented by a dashed line, are quoted.

CPV in baryon decays

New result!
Preliminary!

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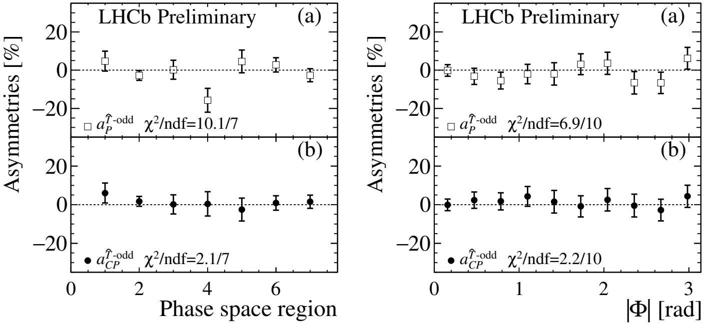

Figure 5: The asymmetries in each region using binning schemes (left) C and (right) D for $\Lambda_b^0 \to pK^-K^+K^-$ decay. For $a_P^{\hat{T}\text{-odd}}$ ($a_{CP}^{\hat{T}\text{-odd}}$), the values of the χ^2/ndf for the P-symmetry (CP-symmetry) hypothesis, represented by a dashed line, are quoted.

CPV in baryon decays

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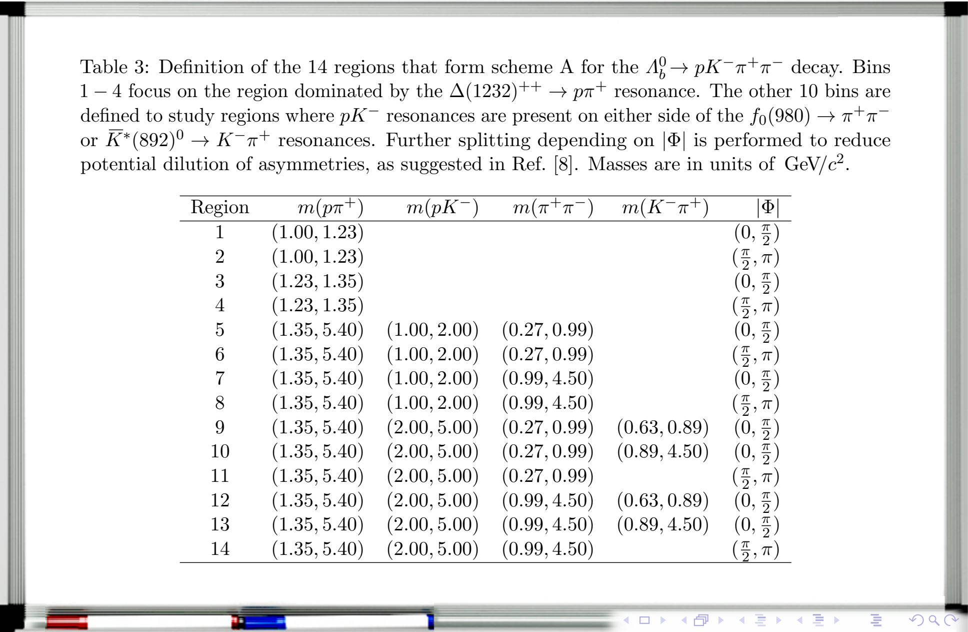

Table 3: Definition of the 14 regions that form scheme A for the $\Lambda_b^0 \to pK^-\pi^+\pi^-$ decay. Bins $1-4$ focus on the region dominated by the $\Delta(1232)^{++} \to p\pi^+$ resonance. The other 10 bins are defined to study regions where pK^- resonances are present on either side of the $f_0(980) \to \pi^+\pi^-$ or $\overline{K}^*(892)^0 \to K^-\pi^+$ resonances. Further splitting depending on $\lvert\Phi\rvert$ is performed to reduce potential dilution of asymmetries, as suggested in Ref. [8]. Masses are in units of GeV/c^2.

Region	$m(p\pi^+)$	$m(pK^-)$	$m(\pi^+\pi^-)$	$m(K^-\pi^+)$	$\lvert\Phi\rvert$
1	$(1.00, 1.23)$				$(0, \frac{\pi}{2})$
2	$(1.00, 1.23)$				$(\frac{\pi}{2}, \pi)$
3	$(1.23, 1.35)$				$(0, \frac{\pi}{2})$
4	$(1.23, 1.35)$				$(\frac{\pi}{2}, \pi)$
5	$(1.35, 5.40)$	$(1.00, 2.00)$	$(0.27, 0.99)$		$(0, \frac{\pi}{2})$
6	$(1.35, 5.40)$	$(1.00, 2.00)$	$(0.27, 0.99)$		$(\frac{\pi}{2}, \pi)$
7	$(1.35, 5.40)$	$(1.00, 2.00)$	$(0.99, 4.50)$		$(0, \frac{\pi}{2})$
8	$(1.35, 5.40)$	$(1.00, 2.00)$	$(0.99, 4.50)$		$(\frac{\pi}{2}, \pi)$
9	$(1.35, 5.40)$	$(2.00, 5.00)$	$(0.27, 0.99)$	$(0.63, 0.89)$	$(0, \frac{\pi}{2})$
10	$(1.35, 5.40)$	$(2.00, 5.00)$	$(0.27, 0.99)$	$(0.89, 4.50)$	$(0, \frac{\pi}{2})$
11	$(1.35, 5.40)$	$(2.00, 5.00)$	$(0.27, 0.99)$		$(\frac{\pi}{2}, \pi)$
12	$(1.35, 5.40)$	$(2.00, 5.00)$	$(0.99, 4.50)$	$(0.63, 0.89)$	$(0, \frac{\pi}{2})$
13	$(1.35, 5.40)$	$(2.00, 5.00)$	$(0.99, 4.50)$	$(0.89, 4.50)$	$(0, \frac{\pi}{2})$
14	$(1.35, 5.40)$	$(2.00, 5.00)$	$(0.99, 4.50)$		$(\frac{\pi}{2}, \pi)$

CPV in baryon decays

[New result! Preliminary!]

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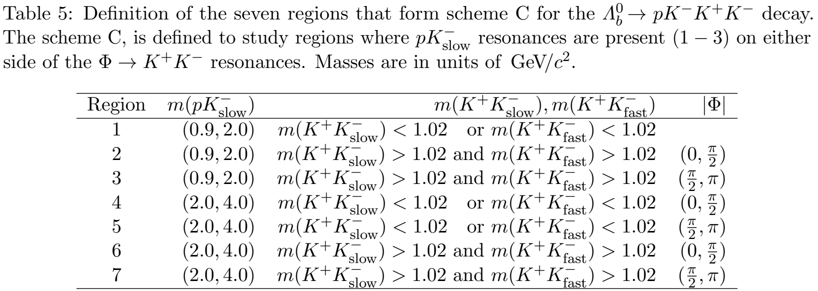

Table 5: Definition of the seven regions that form scheme C for the $\Lambda_b^0 \to pK^-K^+K^-$ decay. The scheme C, is defined to study regions where pK^-_{slow} resonances are present $(1-3)$ on either side of the $\Phi \to K^+K^-$ resonances. Masses are in units of GeV/c^2.

Region	$m(pK^-_{\text{slow}})$	$m(K^+K^-_{\text{slow}}), m(K^+K^-_{\text{fast}})$	$\lvert\Phi\rvert$
1	$(0.9, 2.0)$	$m(K^+K^-_{\text{slow}}) < 1.02$ or $m(K^+K^-_{\text{fast}}) < 1.02$	
2	$(0.9, 2.0)$	$m(K^+K^-_{\text{slow}}) > 1.02$ and $m(K^+K^-_{\text{fast}}) > 1.02$	$(0, \frac{\pi}{2})$
3	$(0.9, 2.0)$	$m(K^+K^-_{\text{slow}}) > 1.02$ and $m(K^+K^-_{\text{fast}}) > 1.02$	$(\frac{\pi}{2}, \pi)$
4	$(2.0, 4.0)$	$m(K^+K^-_{\text{slow}}) < 1.02$ or $m(K^+K^-_{\text{fast}}) < 1.02$	$(0, \frac{\pi}{2})$
5	$(2.0, 4.0)$	$m(K^+K^-_{\text{slow}}) < 1.02$ or $m(K^+K^-_{\text{fast}}) < 1.02$	$(\frac{\pi}{2}, \pi)$
6	$(2.0, 4.0)$	$m(K^+K^-_{\text{slow}}) > 1.02$ and $m(K^+K^-_{\text{fast}}) > 1.02$	$(0, \frac{\pi}{2})$
7	$(2.0, 4.0)$	$m(K^+K^-_{\text{slow}}) > 1.02$ and $m(K^+K^-_{\text{fast}}) > 1.02$	$(\frac{\pi}{2}, \pi)$