Dark Photons, Kinetic Mixing & Light Dark Matter From 5-D

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Before 5-D: Lightning 4-D 'Dark Photon' Model Review

→ 3 Ingredients

· New $U(1)_D$, not coupled to SM, kinetically mixes with hypercharge field:

$$\mathcal{L} \subset -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{Z}_{D\mu\nu}\hat{Z}_D^{\mu\nu} + \underline{\frac{1}{2}\frac{\epsilon}{\cos\theta}\hat{Z}_{D\mu\nu}\hat{B}^{\mu\nu}} + \frac{1}{2}m_{D,0}^2\hat{Z}_D^{\mu}\hat{Z}_{D\mu}$$

$\varepsilon <\sim 10^{-3}$

1412.0018

· Break $U(1)_D$ symmetry w/ a Dark, SM singlet Higgs boson S:

$$V_0(H,S) = -\mu^2|H|^2 + \lambda|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|S|^2|H|^2$$

· Add SM singlet DM field (as yet unspecified) coupled to Z_D (=V)

Then →

1: Make field redefinitions to bring L to canonical form

2: Diagonalize Higgs-induced mass-mixing between Z_{SM} & VC

3: 'Light' (~100 MeV) V couples to ~eεQ, V is the 'Dark Photon'

→ SM-DM interaction mediated by V

Light DM Properties: CMB → no s-wave annihilation

CMB limits

..otherwise $\sigma v_{FO} \approx \sigma v_{CMB}$!

(~ T & v independent)

LIU, SLATYER, and ZAVALA

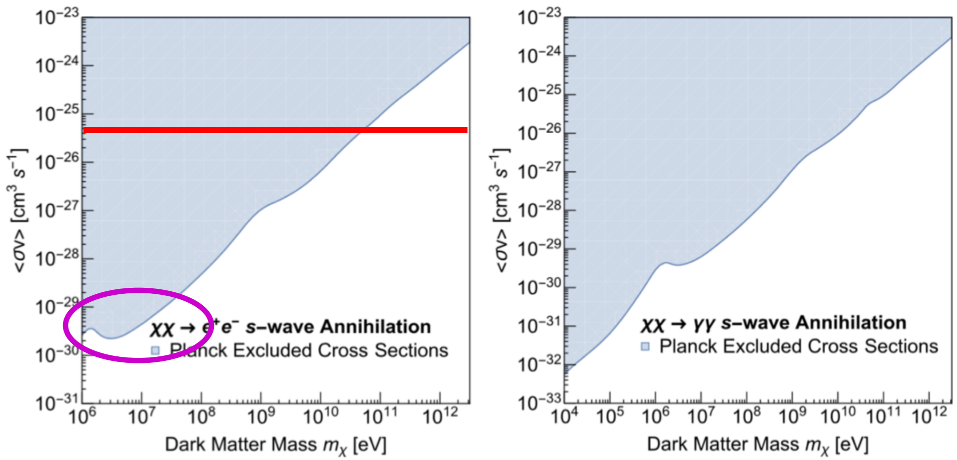


FIG. 1. The 95% excluded cross section based on Planck's upper limit given by Eq. (8) for (left) $\chi\chi \to e^+e^-$ and (right) $\chi\chi \to \gamma\gamma$ s-wave annihilation.

→ DM is NOT Dirac

→ $m_{DM} < m_V$

→ No ID signal today!

Model	Mass terms	J^μ_D	scattering $\mathcal{M} \propto$	scattering $\sigma \propto$	Annihilation $\sigma v \propto$	CMB-viable?
Fermion DM – Direct Annihilation						
Majorana	$\not{U}(1)_D$	$\bar{\Psi}\gamma^\mu\gamma_5\Psi$	$\vec{\sigma}\cdot\vec{v}$	v^2	p-wave $\propto v^2$	Y
Dirac	$U(1)_D$-inv.	$\bar{\Psi}\gamma^\mu\Psi$	1	1	s-wave $\propto v^0$	N
Pseudo-Dirac	$U(1)_D$-inv. & $\not{U}(1)_D$	$\bar{\Psi}_L\gamma^\mu\Psi_H$	1 (inelastic)	kin. forbidden [a]	kin. forbidden	Y
Scalar DM – Direct Annihilation						
Complex	$U(1)_D$-inv.	$\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*$	1	1	p-wave $\propto v^2$	Y
Pseudo-complex	$U(1)_D$-inv. & $\not{U}(1)_D$	$\phi_L\partial^\mu\phi_H - \phi_H\partial^\mu\phi_L$	v^2 (inelastic)	kin. forbidden	kin. forbidden [b]	Y

→ $m_V \sim m_{DM}$ for model to work. If $m_v > 2m_{DM}$ then V→ DM, otherwise V→e^+e^-

Some possible questions…

1: Why is $m_{DM} \sim m_V$? These masses have different origins…

2: No symmetry can forbid Dark Higgs coupling to the SM Higgs & mixing.. Shift in Higgs properties? Active Higgs portal?

3: Can we make DM a Dirac fermion somehow??

4: Is there another framework for the DP-DM model ? Etc..

- Can we modify the model to address (some of) these ??

Have fun by extending this model to EDs which provide a powerful model-building tool

Setup :

- A flat ED - interval bounded by 2 branes with an inverse size $R^{-1} \sim 10\text{-}10^3$ MeV

- SM fields are localized on one of the 4-D branes. DM & V mediator 'Dark Fields' live in the 5-D bulk. (i.e., ED's are DARK !)

- Repeat 4-D procedure above…

Kinetic Mixing? Relevant part of 5-D gauge action is

$$S = \int d^4x \int_{y_1}^{y_2} dy \left[-\frac{1}{4}\hat{V}_{AB}\hat{V}^{AB} \left(-\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} + \frac{\epsilon_5}{2c_w}\hat{V}_{\mu\nu}\hat{B}^{\mu\nu} + L_{SM} \right) \underline{\delta(y-y_{SM})} \right]$$

Note: KM takes place on the SM brane between brane-localized hypercharge B and bulk field V

- KM now involves an infinite tower of KK modes of the Dark gauge field, V, determined by their wavefunctions on the SM brane:

$$\int_{y_1}^{y_2} dy \left| \frac{\epsilon_5}{2c_w} \hat{V}_{\mu\nu} \hat{B}^{\mu\nu} \delta(y - y_{SM}) \right.$$

insert KK expansion

$$\hat{V}^A(x,y) = \sum f_n(y) \hat{V}_n^A(x)$$

→ $\epsilon_n = \epsilon_5 f_n(y_{SM})$ thus → $\sum_n \frac{\epsilon_n}{2c_w} \hat{V}_n^{\mu\nu} \hat{B}_{\mu\nu}$ Infinite sum of KK tower of states !

1 Field redefinitions to bring 4-D Lagrangian into canonical form → $\hat{B} = B + \sum_n \alpha_n V_n$, etc.

→ Self-consistency requires ϵ_n to *decrease* with n

2 Next: diagonalize (infinite) Z-V_n mass matrix..

$$\mathcal{M}^2 = \begin{pmatrix} M_Z^2 & -t_w\epsilon_1 M_Z^2 & -t_w\epsilon_2 M_Z^2 & \cdots \\ -t_w\epsilon_1 M_Z^2 & M_1^2 + t_w^2\epsilon_1^2 M_Z^2 & t_w^2\epsilon_1\epsilon_2 M_Z^2 & \cdots. \\ -t_w\epsilon_2 M_Z^2 & t_w^2\epsilon_1\epsilon_2 M_Z^2 & M_2^2 + t_w^2\epsilon_2^2 M_Z^2 & \cdots. \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} .$$

M_i determined by KK equation of motion

Small ε's → use an expansion…

$$V_i \rightarrow V_i + t_w \frac{\epsilon_i M_Z^2}{M_i^2 - M_Z^2} Z$$

$$Z \rightarrow Z - t_w \sum_i \frac{\epsilon_i M_i^2}{M_i^2 - M_Z^2} V_i ,$$

The Z & V_i masses also shift at $O(\varepsilon^2)$

The physical V_i then couple to

$$\frac{g}{c_w} t_w \epsilon_i \left[T_{3L} \frac{M_i^2}{M_Z^2 - M_i^2} + Q \frac{c_w^2 M_Z^2 - M_i^2}{M_Z^2 - M_i^2} \right] ;$$

For $M_i \rightarrow 0$ this is $e\varepsilon_i Q$ ... For $M_i \rightarrow \infty$ this is $g'\varepsilon_i Y$.

Check: Shifts in the SM Z mass & couplings → S,T ~ $O(\varepsilon^2)$ ✓

DM Models? - Follow the 4-D Table

- Complex scalar w/ no vev: DM is lightest scalar in a KK tower. $U(1)_D$ broken by BCs - no Dark Higgs field needed in 5-D! V_5 are the Goldstone bosons. Simplest possibility w/ only 2 KK towers

- Complex scalar w/ vev..breaks into real CP even scalar KK tower (the lightest being DM) + a CP odd field which mixes w/ V_5 to generate the Goldstones + a CP-odd KK tower → 3 towers

- Majorana/Pseudo Dirac: Most complex w/ 5 KK towers !

- Dirac Fermion: Excluded in 4-D..can we be tricky in 5-D?

Unfortunately time permits only an examination of the first case ..the others are more complex (& more interesting !)

Model 1: Scalar DM (=S) w/o vev

$$S_{5D} = \int d^4x \int_{y_1}^{y_2} dy \left[-\frac{1}{4} V_{AB} V^{AB} + (D_A S)^\dagger (D^A S) - V(S^\dagger S) \right]$$

$$D_A = \partial_A + i g_{5D} Q_D V_A,$$

S MUST be complex to carry charge (Q_D=1); ignore possible bulk mass for S & assume no kinetic or potential terms on either brane for V,S (for now)

- $f_n(y)$ satisfy: $\partial^2_y f_n = -m_n^2 f_n$ so $f_n = A_n \cos m_n y + B_n \sin m_n y$

$f_m \partial_y f_n |_{y_1}^{y_2} = 0$ required for consistent KK decomposition: only few choices

Now we take BCs → which satisfy requirement $\partial_y v_n(\pi R) = v_n(0) = 0$ while $\partial_y s_n(0) = s_n(\pi R) = 0.$

- Then: $m^{V,S}{}_n = (n+1/2)/R$ V,S form degenerate KK towers

So $v_n \sim \sin x_n y/R$ & $s_n \sim \cos x_n y/R$ with $x_n = n + 1/2$. $M_n = x_n/R$

→ There are no massless states! V_5's become eaten Goldstones. BCs break $U(1)_D$ w/o Higgs! There is no S-H mixing either..

→ Note this term is ZERO as the s_n vanish on the SM brane. NO symmetry can do this but we can w/ ED BCs ! ✓

$$S_{HS} = \int d^4x \int_{y_1}^{y_2} dy \; \lambda_{HS} H^\dagger H S^\dagger S \; \delta(y - y_{SM})$$

→ Trivially, the DM & the Dark Photon (the n=0 modes) have comparable (i.e., the same) masses w/o tuning. ✓

However some phenomenological problems remain :

→ As is, all the $|\varepsilon_n|$ have the same value X

→ We require that $m(S_1) < m(V_1)$ but here they're equal X

· Simultaneously solve both problems: add a common element for both the V & S fields = a Brane Localized Kinetic Term(BLKT) on the brane where the field doesn't already vanish, e.g., for V:

$\int dy\ \delta(y-y_{SM}) \cdot \delta_A R \cdot \frac{-1}{4} V_{\alpha\beta} V^{\alpha\beta}$ δ_A is a dimensionless, positive O(1) parameter

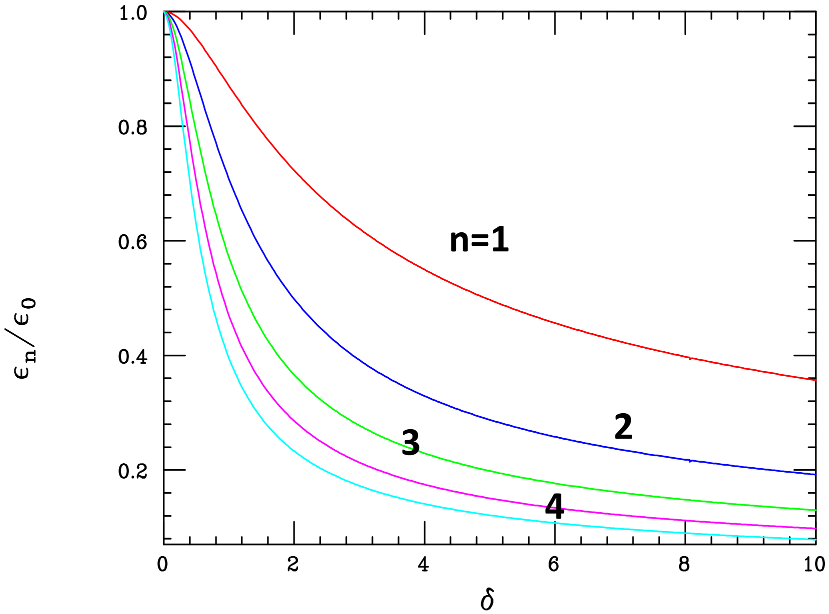
similar to the kinetic mixing term.. Also add a δ_S for S.

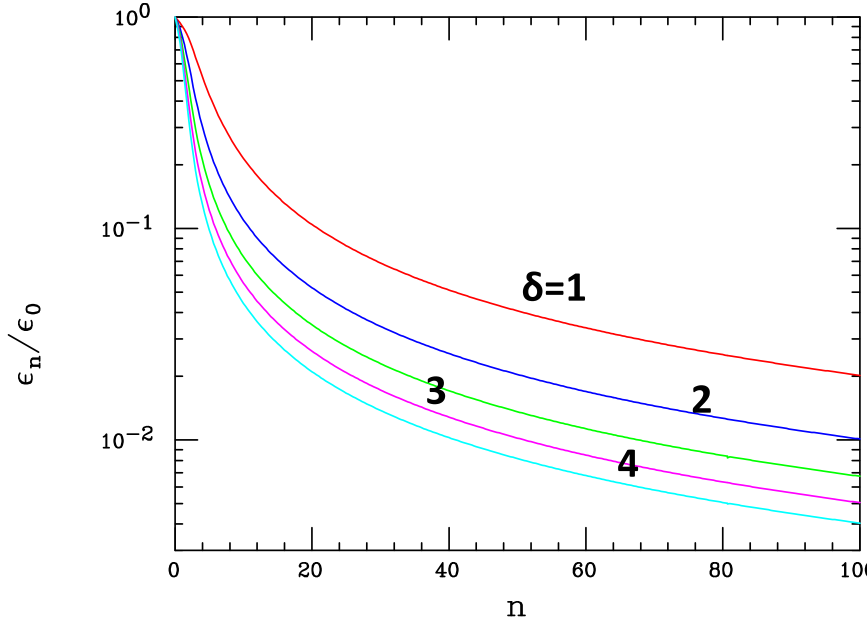
The BLKT induces a discontinuity in $\partial_y f$ at the relevant brane:

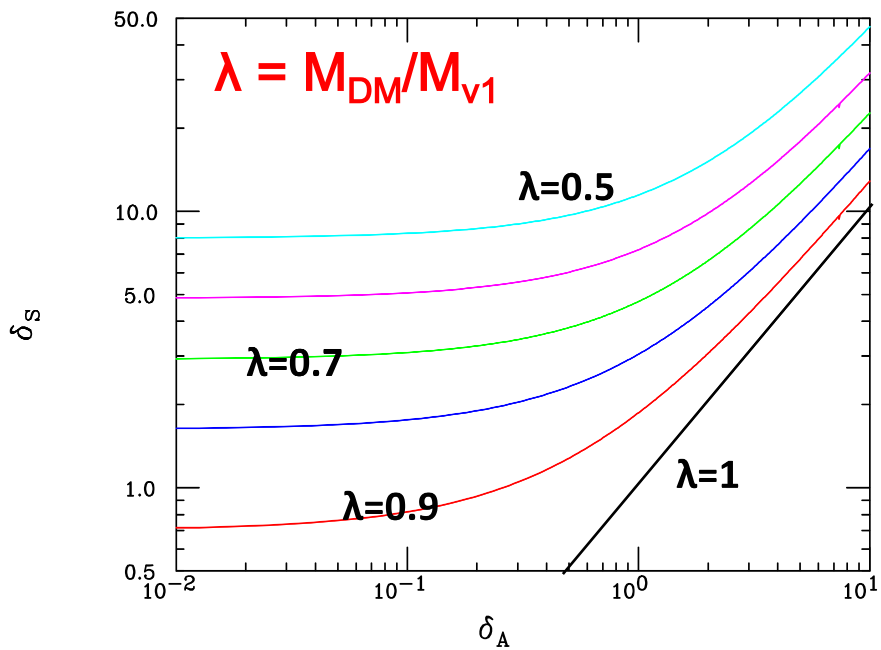
$\partial_y f(y_{br}+) - \partial_y f(y_{br}^-) = -\delta_A R\, m_n^2 f(y_{br})$ modifying the BCs.

This alters: masses, wavefunctions & normalization factors, ie, ε_n's

The ε_n's fall off rapidly with increasing n as well as increasing δ_A's







↑ O(1) BLKTs will make the ε_n fall fast enough to decouple heavier KK states from LHC mono-jet searches & invisible Z/H decays

← BLKTs will split the DM $=S_1$ & V_1 masses by an O(1) factor

Looking For Signals & Distinguishing 5-D Traits

→ Finding a DP &/or light DM is insufficient to establish 5-D (clearly).. we must observe the effects of the KK states! We need observables that can't be mimicked by shifts in the 4-D model parameters. Tough!

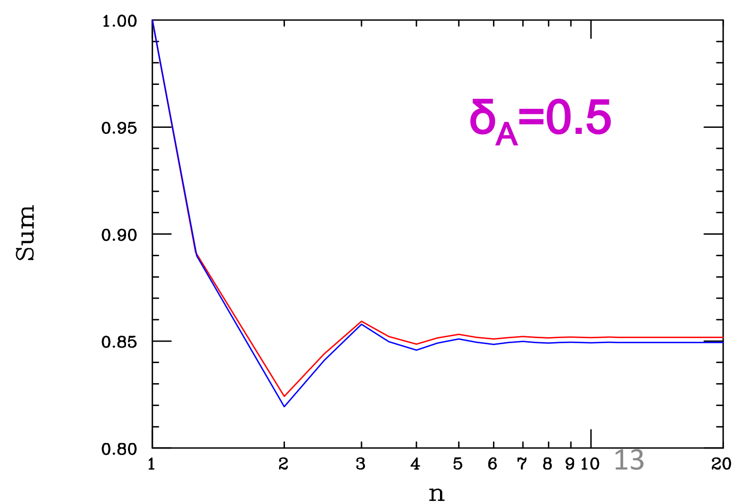
- Look at DM Direct Detection rate & relict density first:

The e-DM elastic scattering cross section:

$$\sigma_e = \frac{4\alpha\mu^2 g_D^2 \epsilon_1^2}{(m_1^V)^4}\left[\sum_n (-1)^{n+1}\frac{\epsilon_n}{\epsilon_1}\,\tilde{c}_{11}^n\,\frac{(m_1^V)^2}{(m_n^V)^2}\right]^2 \rightarrow \sigma_e \simeq 2.97\cdot 10^{-40}\mathrm{cm}^2\left(\frac{100\mathrm{MeV}}{\mathrm{m}_1^{\mathrm{V}}}\right)^4\left(\frac{\mathrm{g}_{\mathrm{D}}\epsilon_1}{10^{-4}}\right)^2 \times \mathrm{Sum}$$

$$c_{mn}^i = \int_0^{\pi R} s_n(y)s_m(y)v_i(y)\,dy \rightarrow \tilde{c}_{nm}^i = c_{nm}^i/c_{11}^1$$

BM1: λ=0.8, δ_S =2.38 + BM2: λ=0.6, δ_S =6.03



These small effects can be easily hid by a shift in 4-D ε parameter

DM annihilation cross section for DM → e^+e^-

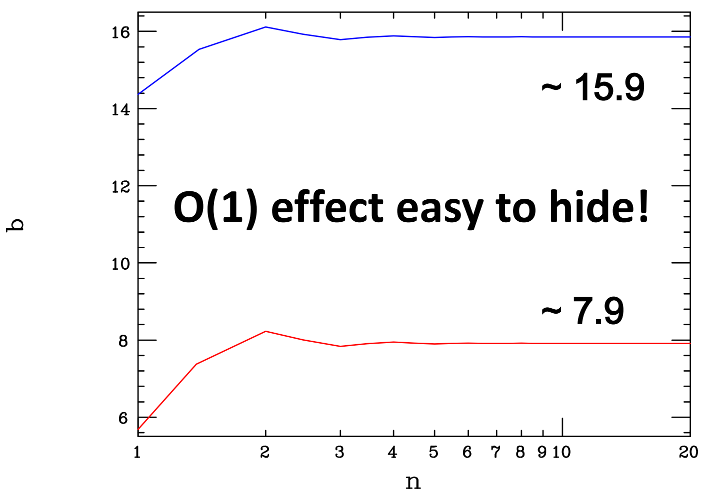
$$\sigma v_{rel} = \tilde{b} v_{rel}^2$$ **(p-wave)**

$$\tilde{b} = \frac{g_D^2 \epsilon_1^2}{192\pi m_{DM}^2} \frac{\gamma^4}{\gamma^2 - 1} \sum_{n,m} (-1)^{n+m} \left[\frac{(\epsilon_n \epsilon_m / \epsilon_1^2)\ \tilde{c}_{11}^n \tilde{c}_{11}^m}{(\gamma^2 - r_n)(\gamma^2 - r_m)} \right]$$

$$\gamma^2 = s/4m_{DM}^2$$

$$r_n = (m_n^V)^2/4m_{DM}^2$$

Even when combined with DD the small deviations from 4-D are likely too subtle X



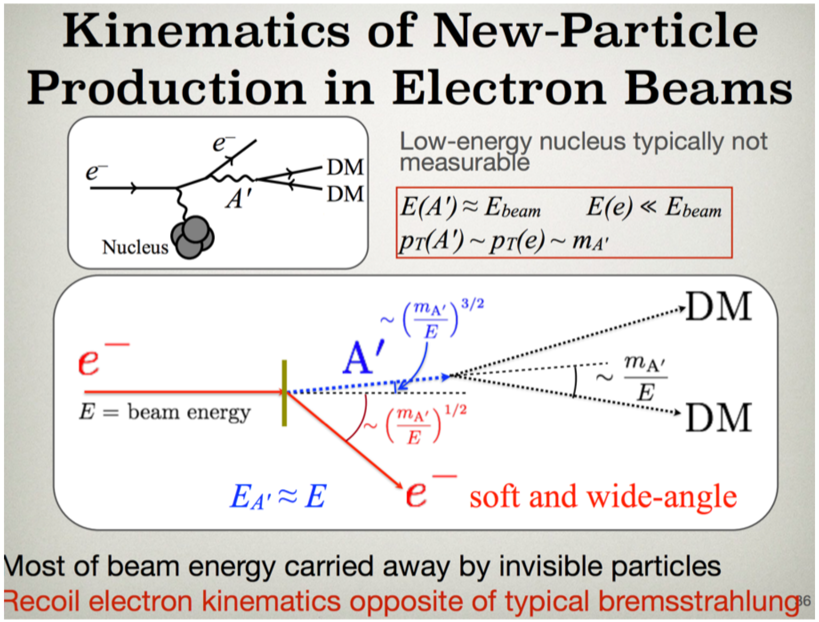
$$\tilde{b} = b \left[\frac{g_D^2 \epsilon_1^2}{m_{DM}^2 (\mathrm{GeV}^2)} \right] 10^{-20} \mathrm{cm}^3 \mathrm{s}^{-1} .$$

→ We need to PRODUCE the KK states!

Key observation: if $\lambda < 0.5$, $V_1 \rightarrow$ DM & furthermore the entire KK tower of V_n will end up as DM!

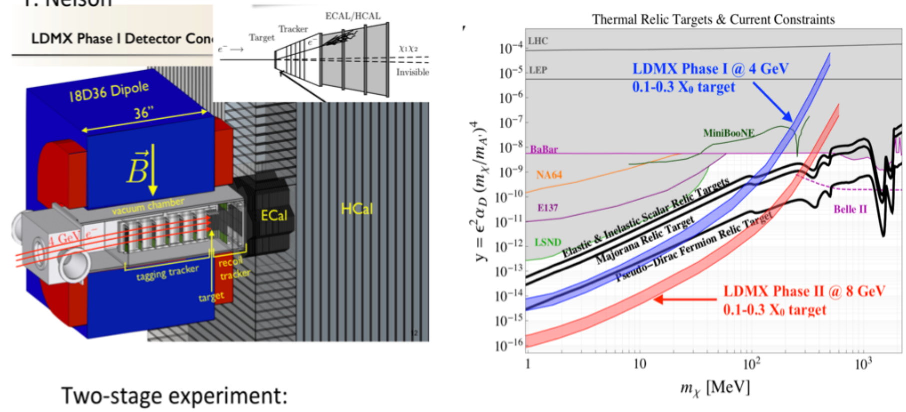
If multiple V_n are produced the missing E &/or recoil spectrum shapes for a given experiment can be modified. Shape changes can't be easily matched by 4-D parameter shifts

Accelerator Production of V_n



LDMX

T. Nelson

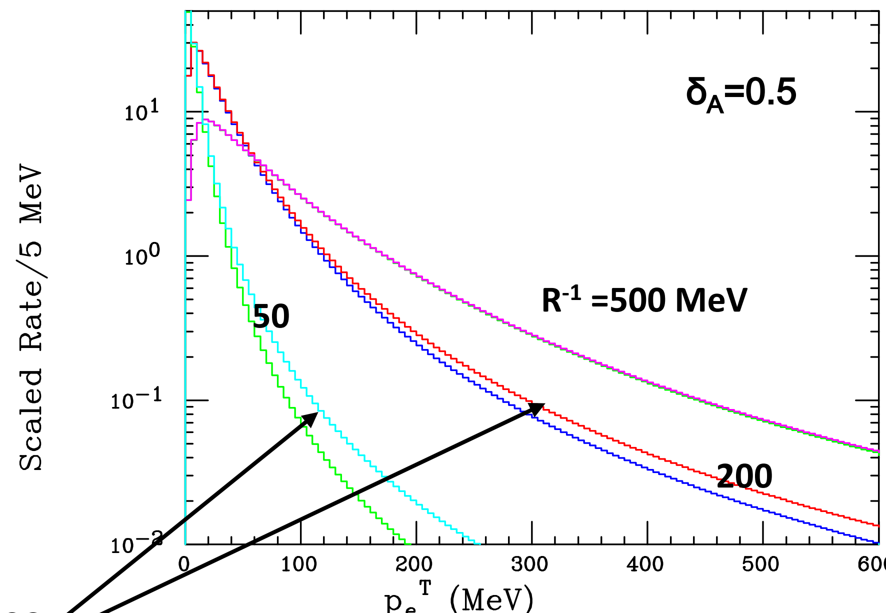


Two-stage experiment:

- Start at 4 GeV towards the end of 2021 -- sensitivity to 10^{-14}
- BES plans accelerator upgrade (LCLS-II HE / 8 GeV) -- sensitivity to 10^{-16}

- Another possibility is BELLE-II where $e^+e^- \to \gamma V_n$ may yield **multiple** mono-γ peaks depending upon the values of R & ε_n.

- Similar effect possible in meson decays, eg, $\eta \to \gamma V_n$ but w/ limited phase space available



these differences will require detector study

→ λ > 0.5 is very different & more interesting..

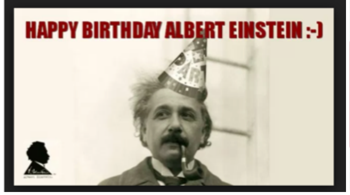
· Now $V_1 \to e^+ e^-$, $V_2 \to$ DM & $S_2 \to S_1 V_1$ (w/ BFs of ~100%) & the higher KKs have complex decay patterns involving both towers & produce lengthening cascades. BFs will be a bit model-dependent

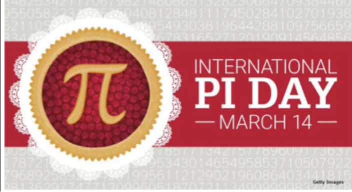
BFs in % for Heavier KKs

Process	BF(BM1)	BF(BM2)
$S_3 \to V_2 S_1$	1.20	0.62
$S_3 \to V_1 S_1$	5.10	1.78
$S_3 \to V_1 S_2$	93.7	97.6
$V_3 \to S_1^\dagger S_1$	74.9	97.3
$V_3 \to S_1^\dagger S_2$+h.c.	25.1	2.71
$V_4 \to S_1^\dagger S_1$	45.9	39.5
$V_4 \to S_1^\dagger S_2$+h.c.	51.5	18.9
$V_4 \to S_2^\dagger S_2$	1.67	38.8
$V_4 \to S_3^\dagger S_1$+h.c.	0.95	2.81

· Cascades will end up with lots of ME from multiple branches & a set of resonant $e^+ e^-$ peaks from multiple V_1 (displaced ?) decays.

· These are unique signals of 5-D





Summary & Conclusions

- Generalizing the 4-D DP model to 5-D can lead to many different & interesting scenarios addressing 4-D issues. EDs are a powerful model-building tool.. we can do things not possible in 4-D !

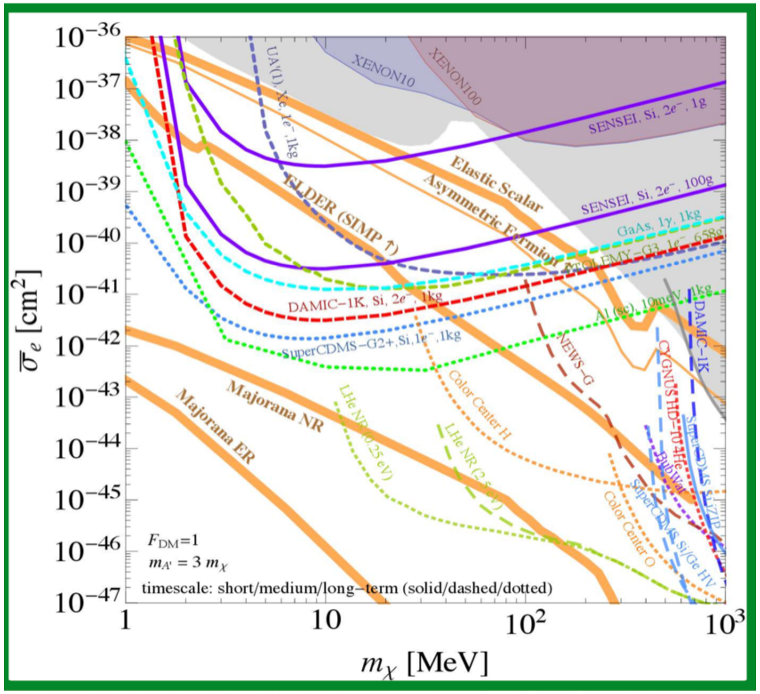
- 5-D model building of DP extensions is not trivial or straightforward..5-D restrictions can be quite strong

- The 5-D models can lead to complex & interesting phenomenology w/ unique signals in searches

- More work is needed to more fully understand these types of models

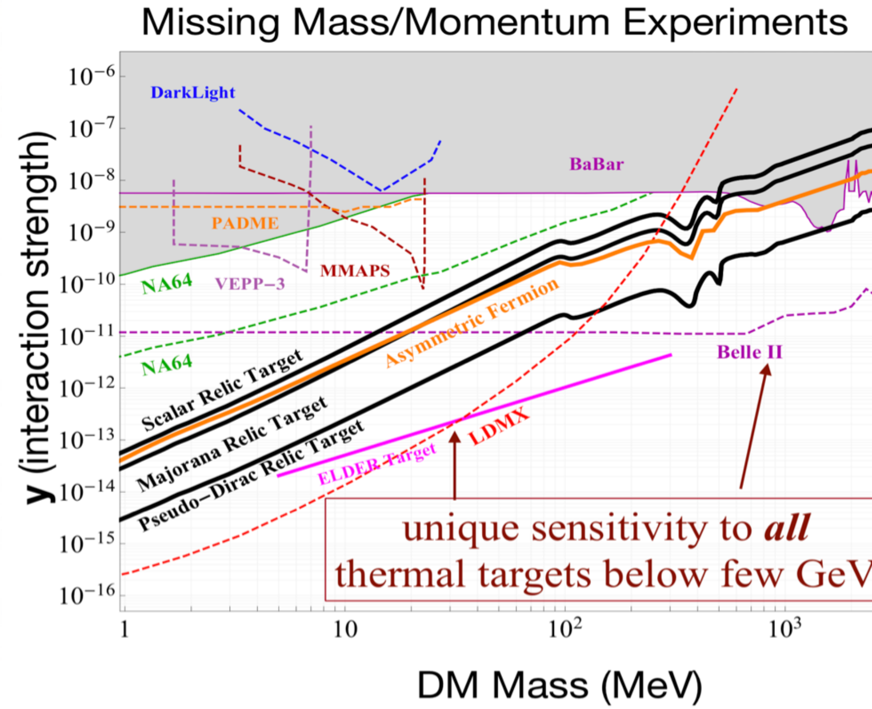
Backup

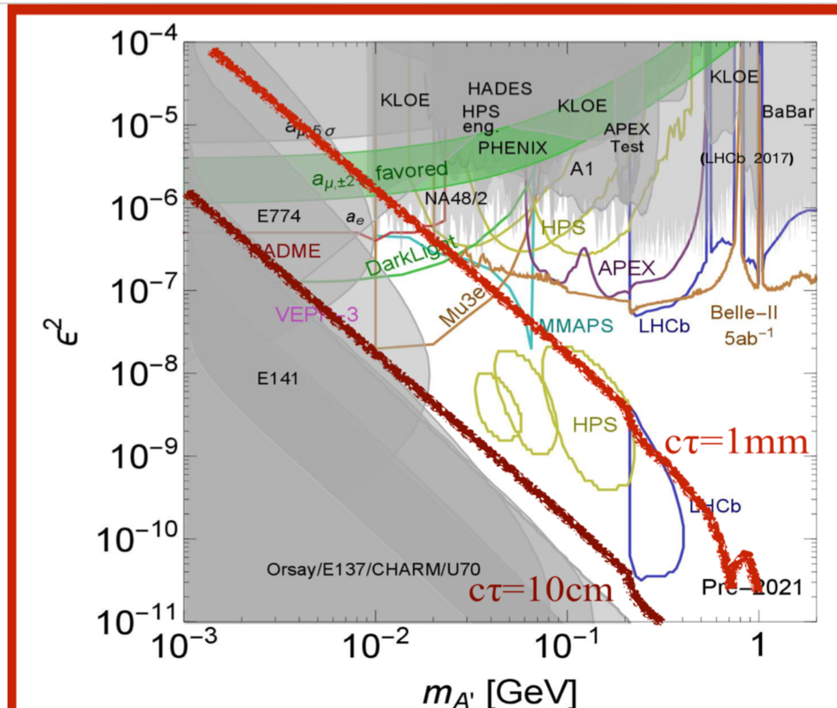
Queue the plots…



Lots of work by many people..

→ $\varepsilon \lesssim 10^{-3}$ & $m_{DM} \sim m_V$ needed





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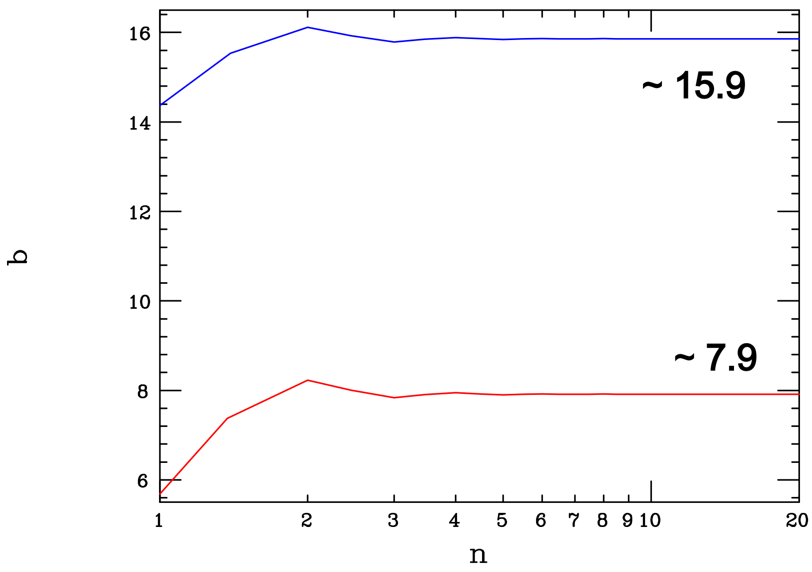
Some similarities but many significant differences for the two BMs due to coupling & PS variations.

The production of the heavier KKs can initiate long cascades with model-dependent contents

Interesting signatures!!

Maybe a few comments about other scenarios if time permits..



These again converge rapidly but they differ by a factor of ~2

What are the decays of these KK states? Depends on couplings & PS available. S_1 is DM so is stable, $V_1 \to$ SM only & $S_2 \to S_1 V_1$ only. For the others:

KK masses in R^{-1} units

KK level	V	S(BM1)	S(BM2)
1	0.463	0.371	0.278
2	1.393	1.198	1.094
3	2.332	2.123	2.051
4	3.281	3.087	3.035

$$S_n(m) \to S_m(m')V_l(m_V)$$

$$\Gamma_S = \frac{g_D^2(\tilde{c}^l_{nm})^2 m^5}{16\pi m_V^4}\left[1 - 2\frac{m_V^2 + m'2}{m^2} + \frac{(m_V^2 - m'^2)^2}{m^4}\right]^{3/2}$$

$$V_i(m_V) \to S_j^\dagger(m_j)S_k(m_k) + \text{h.c.},$$

$$\Gamma_V = \frac{g_D^2(\tilde{c}^i_{jk})^2 m_V}{24(1+\delta_{jk})\pi}\left[1 - 2\frac{m_j^2 + m_k^2}{m_V^2} + \frac{(m_j^2 - m_k^2)^2}{m_V^4}\right]^{3/2}$$

Model 2: Complex Scalar w/ vev, $S \sim v_s+h+i\chi$

- Non-orbifold BCs are again employed and χ + V_5 mix to form the CP-odd field a + unphysical Goldstones level by level. V still has BLKT but none for S. $g_D v_s R$ naturally is ~ O(1).

- h_1 or a_1 is DM.. BUT m_{DM} must be < m_{v1} However one finds

$$(m_n^h)^2 = (\tfrac{n+1/2}{R})^2 + 2\lambda_S v_s^2 \qquad (m_n^V)^2 = \left(\frac{x_n^V}{R}\right)^2 + g_D^2 v_s^2 \qquad (m_n^a)^2 = \left(\frac{n+1/2}{R}\right)^2 + g_D^2 v_s^2$$

  so that $m_{v1} < m_{a1}$ & thus h_1 is the DM with $2\lambda_s < g_D^2$

- $h_1 - a_1$ fractional mass splitting, δ, must be small as they can only co-annihilate via V_n to get relic density → the entire spectrum at a given level is compressed! Resonance enhancement can occur.

- Tree-level DD is absent due to δ, loop-level ~<10^{-51} cm^2 tiny!

- $a_1 \to h_1 e^+e^-$ unboosted lifetimes ~10-1000 cm due to small δ

· Small δ_A → ε_n's at low n's remain largish. Small R^{-1} → many KKs contribute, hence, the greatest sensitivity.

· Meson decays/colliders may do better looking for multiple γ peaks with ME recoiling with shrinking rates due to falling ε_n

→ Interesting possibility: via Z-V_i mixing, we have Z→$S_iS_j^\dagger$ +h.c. w/ the S_i decaying down to DM. In one of our BMs below this is ~763k decay modes!!! Violation of the Γ(Z→ inv) <~1 MeV bound?

Amazingly, no! Including 2k gauge KKs to determine the mixings & taking R^{-1} =100 MeV w/ $g_D\varepsilon_1$ =10^{-4} we get Γ(Z→ inv) ~0.02 MeV

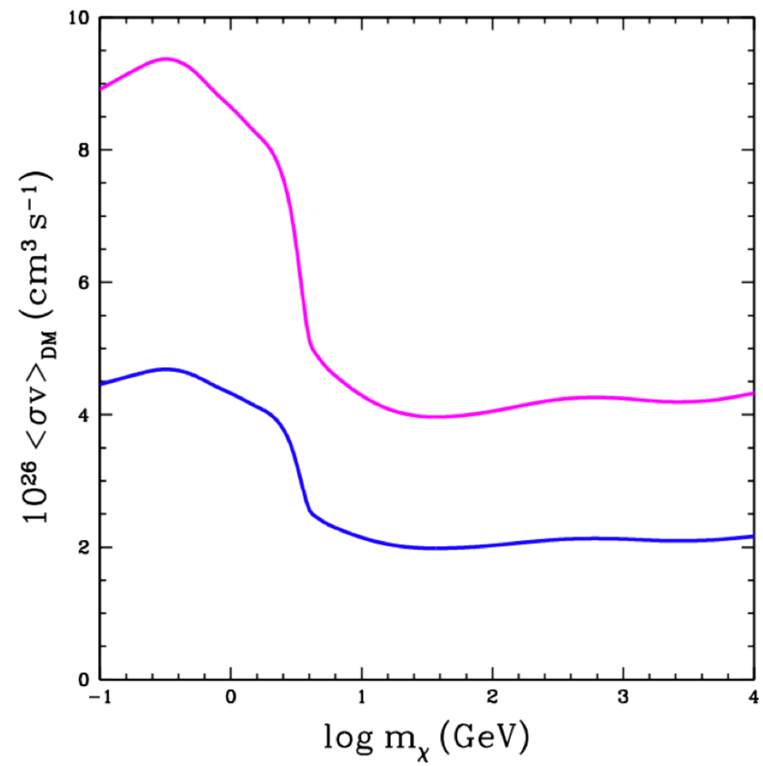
This is the result of the couplings falling off quickly as we go up the KK towers & gives us a little room for other parameter choices

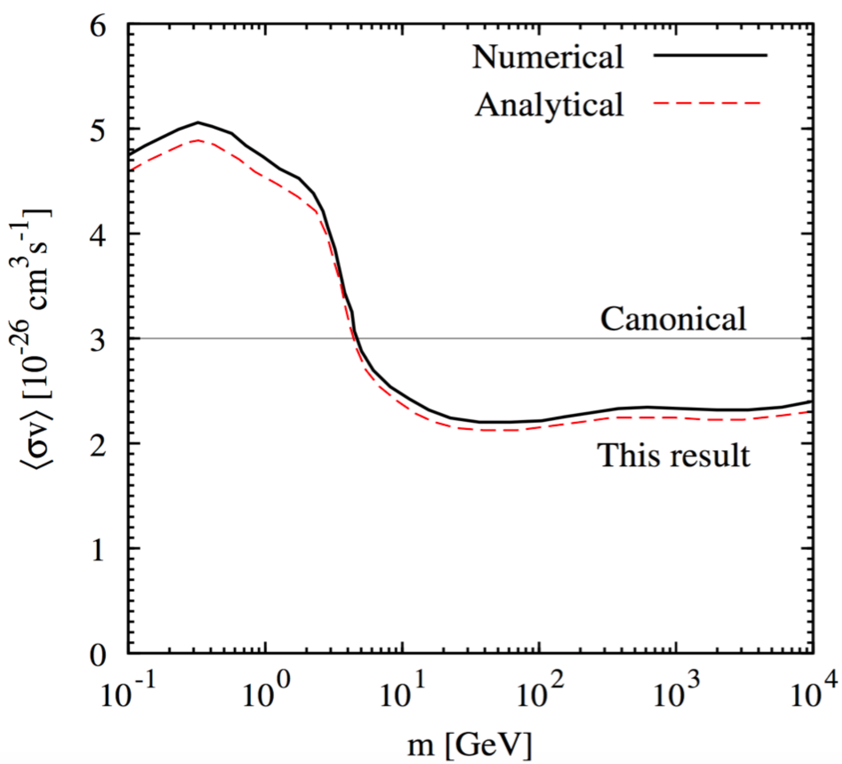
Model 3: Majorana/Pseudo-Dirac Fermion

Many moving parts here… but again NOT an orbifold

- Gauge pieces as above w/ BLKT
- Bulk SM singlet fermion w/ bulk mass m_D
- Complex bulk SM Higgs S getting vev for fermion Majorana mass but contributes to gauge masses as in model 2 → h,a
- Fermions form two relatively close mass Majorana towers → another pair of close-mass objects, one long-lived like a_1. $F_{1,2}F_{1,2}V$ & F_1F_2V+h.c. interactions both exist due to..
- Fermion BCs induce $g_L \neq g_R$ → DP has PV interactions with Dark Sector... an additional complexity
- Interesting new interactions between h,a & $F_{1,2}$.
- …

Still making more plots for this case ! Fun stuff !

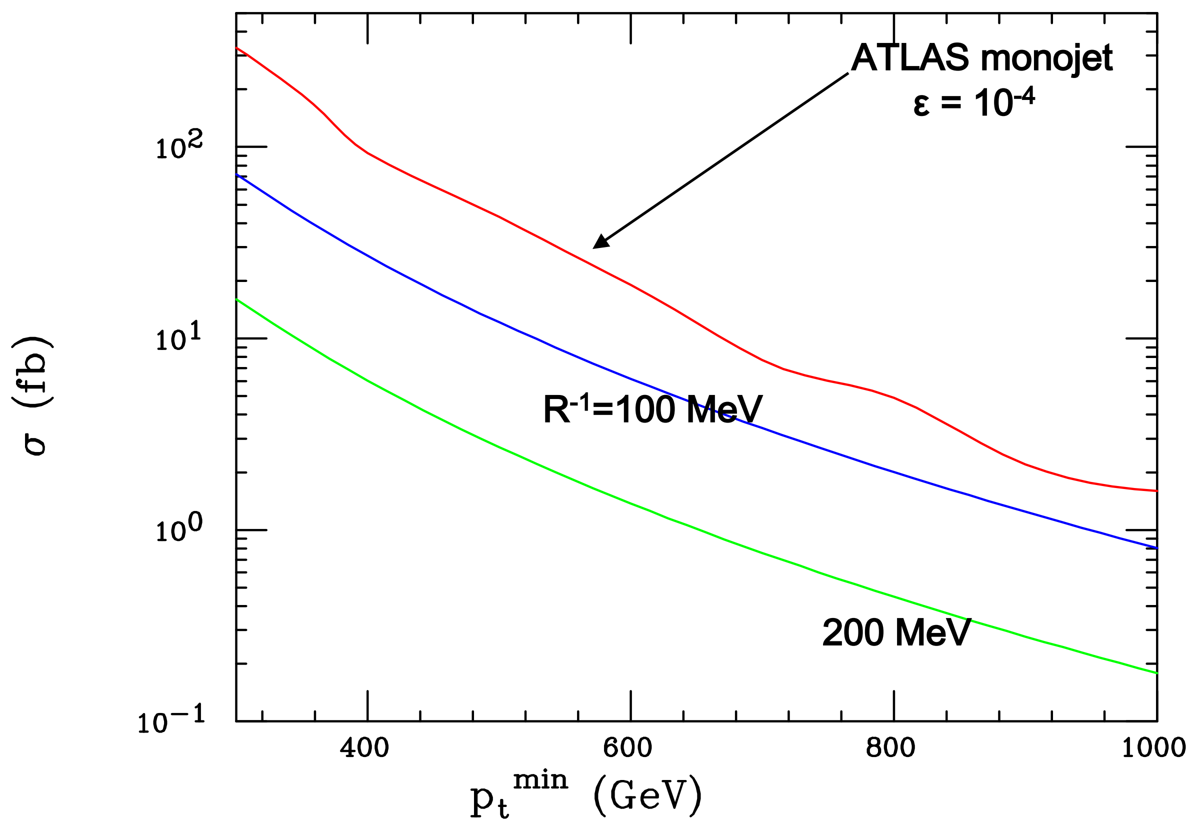




Steigman 1502.01884 ↑

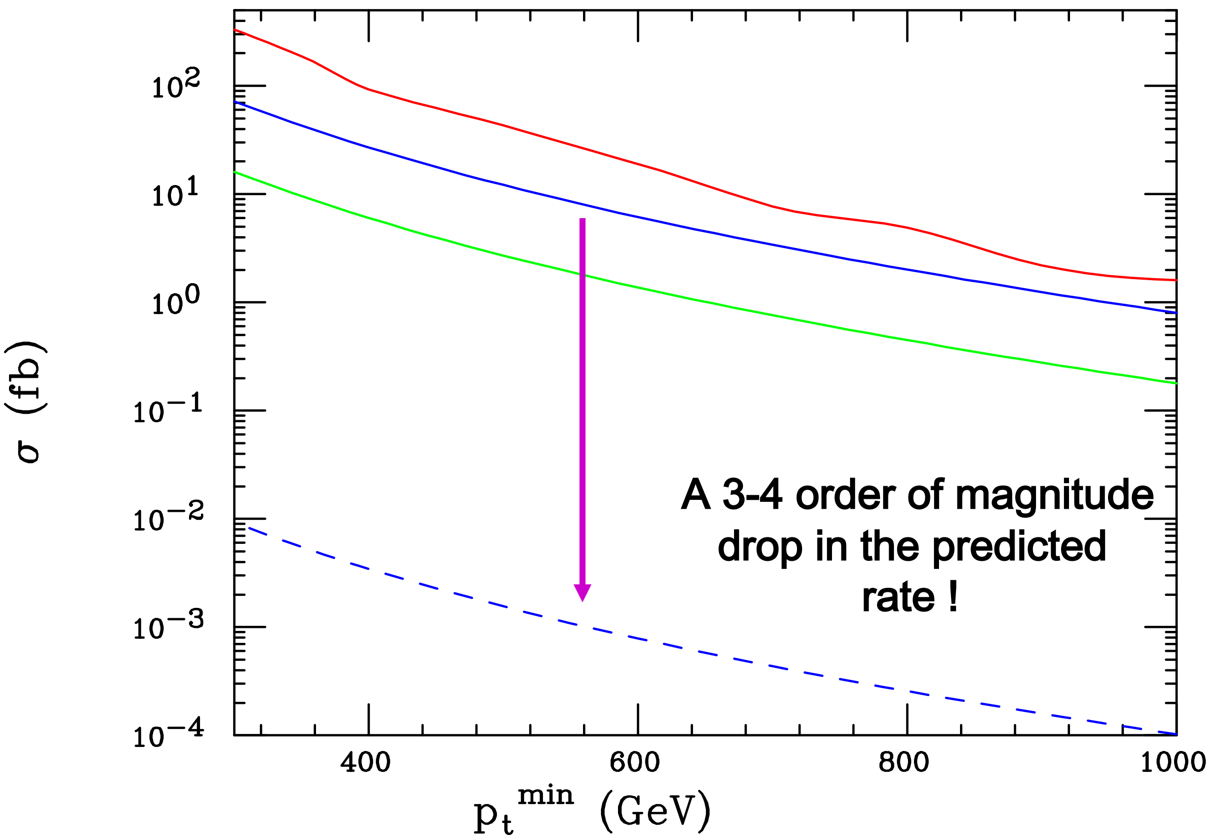
← update 2017

Can the ‘monojet’ searches probe these models? No..even the constant ε_n =10^{-4} case survives !



~10^4 gauge KKs contributing

What if we employed realistic BM1 couplings?



10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
σ (fb)
400
600
800
1000
p_t^{min} (GeV)
A 3-4 order of magnitude
drop in the predicted
rate !

Define the sums: $\Sigma_i = (1 - \sum_{a=1}^{i} s_a^2)^{1/2}$; then

→ $\alpha_1 = -s_1/(\Sigma_1\Sigma_0)$, $\alpha_2 = -s_2/(\Sigma_2\Sigma_1)$ ... $\alpha_n = -s_n/(\Sigma_n\Sigma_{n-1})$

These sums must converge or a canonical basis won't exist !

$$\Sigma_n^2 = 1 - \frac{\epsilon_1^2}{c_w^2} \sum_{a=1}^{n} \frac{\epsilon_a^2}{\epsilon_1^2}$$

(n → ∞)

→ The ε's must shrink with increasing n ..they can't be n-independent!

This imposes a non-trivial constraint on the eigenfunctions $f_n(y)$ independent of the nature of the DM -- as does the by-parts integration requirement on applied BCs w/o orbifiolding

$$f_m \partial_y f_n|_{y_1}^{y_2} = 0$$

· Next: all the V_i couple to hypercharge & so will mix with the Z & each other via the Higgs vev producing an ∞ x ∞ matrix

There is also a shift in the SM Z couplings:

$$\frac{e}{s_w c_w}\left[(1+F)T_{3L} - (s_w^2 + F)Q\right], \quad \text{where} \quad F = \sum_i \frac{(t_w \epsilon_i)^2 M_Z^2}{M_Z^2 - M_i^2}$$

Which results in non-zero oblique parameters:

$$\left\{ \begin{array}{lcl} T & = & \dfrac{2F}{\alpha_w} \\ S & = & \dfrac{4c_w^2 F}{\alpha_w} \\ U & = & 0\,, \end{array} \right. \qquad \leq \text{ ~0.05}$$

...& other couplings are induced & to LO are given by

$$K_{HZV_i} = \frac{2M_Z^2}{v_H}\left[\frac{t_w \epsilon_i M_i^2}{M_Z^2 - M_i^2}\right] \qquad K_{HV_iV_j} \simeq \frac{2M_Z^2}{v_H}\left[\frac{t_w \epsilon_i M_i^2}{M_Z^2 - M_i^2}\right]\left[i \to j\right]$$

All this happens <u>before</u> any introduction of the specific DM model !

$$
\begin{aligned}
M_i^2 &\rightarrow M_i^2\left[1+\frac{t_w^2\epsilon_i^2 M_Z^2}{M_i^2-M_Z^2}\right] \\
M_Z^2 &\rightarrow M_Z^2\left[1-t_w^2 M_Z^2\sum_n \frac{\epsilon_n^2}{M_n^2-M_Z^2}+(t_w^2 M_Z^2)^2\sum_{n,m}\frac{\epsilon_n^2\epsilon_m^2 M_n^2}{(M_n^2-M_Z^2)^2(M_m^2-M_Z^2)}\right](
\end{aligned}
$$