H-coupled Minimal Dark Matter

Laura Lopez Honorez

based on arXiv:1711.08619
in collaboration with M. Tytgat, B. Zaldivar & P. Tziveloglou

Rencontres de Moriond, La Thuile, 10-17/03/18
Fermionic Dark Matter: Gauge Portal

[Hisano’04, Hisano’06, Cirelli’07, Arkani-Hamed’08, Cohen’13, Cirelli’15, Garcia-Cely’15++, Asadi’16, Mitridate’17]

**Dark Sector**  
**Portal**  
**SM**

SM + 1 single Majorana \( n \)  
\( \text{3-plet (Z}_2 \) and 5-plet (stable) \)

"Minimal Dark Matter" (MDM) \( \Rightarrow m_3 = 2 \) \( 4 \) TeV and \( m_5 = 4.4 \) TeV

Large non-perturbative effects due to multiple exchanges of light mediators: Sommerfeld and Bound state formation \( \Rightarrow m_3 = 3 \) TeV and \( m_5 = 11.5 \) TeV

Gamma-ray searches: MDM at the verge of discovery/exclusion if DM profile is cuspy, mostly not probed if the profile is cored
Fermionic Dark Matter: Gauge Portal

[Hisano'04, Hisano'06, Cirelli'07, Arkani-Hamed'08, Cohen'13, Cirelli'15, Garcia-Cely'15++, Asadi'16, Mitridate'17]

\[
- \frac{1}{2} m_M \chi \chi
\]

Dark Sector

1 Majorana fermion

\[
\chi_{(5,0)} = \begin{pmatrix}
\chi^{++} \\
\chi^+
\chi^0
\chi^-
\chi^{--}
\end{pmatrix}
\]

Portal

\(W, Z, \gamma\)

SM

SM + 1 single Majorana \(n\)plet of SU(2)_L \(\sim \) 3-plet (\(Z_2\)) and 5-plet (stable)

“Minimal Dark Matter” (MDM) \(\sim \) \(m_3 = 2.4\) TeV and \(m_5 = 4.4\) TeV
Fermionic Dark Matter: Gauge Portal
[Hisano’04, Hisano’06, Cirelli’07, Arkani-Hamed’08, Cohen’13, Cirelli’15, Garcia-Cely’15++, Asadi’16, Mitridate’17]

- SM + 1 single Majorana nplet of SU(2)_L \rightarrow 3-plet (Z_2) and 5-plet (stable)
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  Sommerfeld and Bound state formation \rightarrow m_3 = 3 \text{ TeV} and m_5 = 11.5 \text{ TeV}
Introduction

Fermionic Dark Matter: Gauge Portal

[Hisano’04, Hisano’06, Cirelli’07, Arkani-Hamed’08, Cohen’13, Cirelli’15, Garcia-Cely’15++, Asadi’16, Mitridate’17]

- SM + 1 single Majorana \( n \)-plet of SU(2)\(_L\) \( \leadsto \) 3-plet (\( Z_2 \)) and 5-plet (stable)
  - “Minimal Dark Matter” (MDM) \( \leadsto m_3 = 2.4 \text{ TeV} \) and \( m_5 = 4.4 \text{ TeV} \)

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see also [1M2D: Mahubani’05, D’Eramo’07, Enberg’07, Cohen’11, Clifford’14, Calibi’15; 3M2D: Dedes’14, Freitas’15; 3M4D: Tait’16, etc]

≡ Integrating the Higgs portal into fermionic MDM.

- SM + 3 dark SU(2)_L n–plet \( \sim \) \( \mathbb{Z}_2 \) symmetry for DM stability
- Already well known examples in SUSY:
  1\(_M\)2\(_D\) \( \equiv \) bino-higgsino or 3\(_M\)2\(_D\) \( \equiv \) wino-higgsino systems
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  1. $1_{M2D} \equiv$ bino-higgsino or $3_{M2D} \equiv$ wino-higgsino systems
- coupling to $H \sim \Delta n = 1$
- EW perturbativity till $M_{pl}$:

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Small mass splittings expected beyond pure MDM

- 4 free parameters: \( m_M, m_D, y_1, y_2 \)

\[
\mathcal{L} \subset \mathcal{L}_K - m_D \psi \tilde{\psi} - \frac{1}{2} m_M \chi \chi - (y_1 \psi \chi^* + y_2 \tilde{\psi} \chi h + h c)
\]

\( \chi \) Majorana fermion, \( \psi, \tilde{\psi} \) are Weyl fermions of representations \( (n, 0) \) and \( (n \pm 1, 1/2), (n \pm 1, -1/2) \) of \( (\text{SU}(2), \text{U}(1)_Y) \) with \( n \) odd.
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- **Generic Mass Patterns** after EWSB at tree-level: see also e.g. [Freitas’15, Tait’16]

- Custodial sym. limit ($y_1 = \pm y_2$): $\chi_\ell^0$ degenerate with at least $\chi^\pm$ (except for $1_M 2_D$)

- Beyond custodial sym. $\chi_\ell^0$ is the lightest with compressed spectra

\[
\Delta m_{\chi^0} \chi^\pm \propto y^2 v^2 / m_M \text{ or } \propto y^4 v^4 / m_D^3
\]

Loop corrections might give $m_{\chi^\pm} < m_{\chi^0}$ [Tait’16]
Boundaries of HMDM parameter space*

For one single multiplet:

$$\sigma v_{\text{eff},n} \sim \frac{\zeta}{n^2} \frac{\alpha_2 C_n}{m_{DM}^2}$$

*perturbative level only!!
Boundaries of HMDM parameter space*

For one single multiplet:

\[ \sigma v_{\text{eff}, n} \approx \frac{\zeta}{n^2} \frac{\alpha_e^2 C_n}{m_{DM}^2} \]

For mixed Dirac-Majorana states that may coannihilate at the boundaries (small \( y \)):

\[ \sigma v_{\text{eff}} \approx \frac{1}{g_{\text{eff}}^2} \sum_{i=M,D} g_i^2 \sigma v_{\text{eff}, i} \]

\[ g_{\text{eff}} = \sum_{i=M,D} g_i \]

\[ g_i = n_i (1 + \Delta_i)^{3/2} \exp(-x_f \Delta_i) \]

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\sigma_{v, n} \simeq \frac{\zeta}{n^2} \frac{\alpha_s^2 C_n}{m_{DM}^2}
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*perturbative level only!!

confirmed at perturbative level using
micrOMEGAs

Laura Lopez Honorez (FNRS@ULB & VUB)
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*perturbative level only!!*
Sommerfeld effect on HMDM Boundaries

$\sigma_{\text{eff}} \sim 1$ and $g_{\text{eff}} \sum_{i=M, D} \sigma_{\text{eff,DM}}$.

$m_{DM} \sim \text{MultiTeV} \sim f.o. \text{SU}(2) \text{ symmetric limit}$: combination of abelian-like Sommerfeld corrections, obtained using group theory decompositions [Strumia’08].
**Sommerfeld effect on HMDM Boundaries**

[Hisano’04, Hisano’06, Cirelli’07, Arkani-Hamed’08, Cohen’13, Cirelli’15, Garcia-Cely’15++]

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<th>( n )</th>
<th>( I_a )</th>
<th>( \lambda_a )</th>
<th>( S_{I_a} )</th>
<th>( \sigma_{I_a}^{\text{pert}} )</th>
<th>( m_{DM}^{\text{pert}} ) [TeV]</th>
<th>( m_{\text{Som}}^{\text{DM}} ) [TeV]</th>
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<td>( \frac{3\pi\alpha^2}{8m_{DM}^2} )</td>
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<td>1.1</td>
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<td>( \frac{25\pi\alpha^2}{16m_{DM}^2} )</td>
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<td>( \frac{4\pi\alpha^2}{m_{DM}^2} )</td>
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<td>1</td>
<td>1.6</td>
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<td>2.4</td>
<td>3.9</td>
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<td>1.5</td>
<td>( \frac{8\pi\alpha^2}{m_{DM}^2} )</td>
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<td>( -\frac{9}{4} + \frac{1}{4} \sqrt{2} )</td>
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<td>( \frac{5\pi\alpha^2}{m_{DM}^2} )</td>
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<tr>
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<td>6</td>
<td>5.9</td>
<td>( \frac{60\pi\alpha^2}{m_{DM}^2} )</td>
<td>4.4</td>
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<td>1.</td>
<td>( \frac{7\pi\alpha^2}{m_{DM}^2} )</td>
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</table>

\( \Omega h^2 = 0.12 \) no Sommerfeld
Sommerfeld effect on HMDM Boundaries

[Hisano’04, Hisano’06, Cirelli’07, Arkani-Hamed’08, Cohen’13, Cirelli’15, Garcia-Cely’15++]

\(m_{DM} \sim \text{MultiTeV} \rightarrow \text{f.o. SU(2) symmetric limit: combination of abelian-like Sommerfeld corrections, obtained using group theory decompositions [Strumia’08].}

\[
\sigma v_{\text{eff}} \simeq \frac{1}{2} g_v^2 \sum_i \frac{m_{DM,i}}{m_{DM}} \sigma v_{\text{Som},i} \cdot \Omega h^2 = 0.12 \text{ no Sommerfeld}
\]
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\[
\sigma_{\text{eff}} \sim \frac{1}{g_{\text{eff}}} \sum_{i=M,D} g_i^2 \sigma_{\text{Som,DM},i}^\text{pert}
\]

\[
\Omega h^2 = 0.12 \text{ with Sommerfeld (\& bound states)}
\]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
n & I_a & \lambda_a & S_{I_a} & \sigma_{\text{pert}}^{I_a} & m_{DM}^{\text{pert}} \ [\text{TeV}] & m_{DM}^{\text{Som}} \ [\text{TeV}] \\
\hline
2 & 0 & \frac{3}{4} + \frac{i^2}{4} & 1.5 & \frac{3\pi \alpha_i^2}{8} & 1.1 & 1.1 \\
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\hline
3 & 0 & 2 & 2.3 & \frac{4\pi \alpha_i^2}{m_{DM}^2} & 2.4 & 3. \\
 & 1 & 1 & 1.6 & \frac{4\pi \alpha_i^2}{m_{DM}^2} & 2.4 & 3. \\
 & 2 & -1 & 0.6 & \frac{4\pi \alpha_i^2}{m_{DM}^2} & 2.4 & 3. \\
\hline
4 & 0 & \frac{15}{4} + \frac{i^2}{4} & 3.9 & \frac{75\pi \alpha_i^2}{m_{DM}^2} & 2.4 & 3.9 \\
 & 1 & \frac{11}{4} + \frac{i^2}{4} & 3. & \frac{75\pi \alpha_i^2}{m_{DM}^2} & 2.4 & 3.9 \\
 & 2 & \frac{3}{4} + \frac{i^2}{4} & 1.5 & \frac{4\pi \alpha_i^2}{m_{DM}^2} & 2.4 & 3.9 \\
 & 3 & -\frac{9}{4} + \frac{i^2}{4} & 0.3 & \frac{4\pi \alpha_i^2}{m_{DM}^2} & 2.4 & 3.9 \\
\hline
5 & 0 & 6 & 5.9 & \frac{60\pi \alpha_i^2}{m_{DM}^2} & 4.4 & 9.3 \\
 & 1 & 5 & 5. & \frac{60\pi \alpha_i^2}{m_{DM}^2} & 4.4 & 9.3 \\
 & 2 & 3 & 3.1 & \frac{60\pi \alpha_i^2}{m_{DM}^2} & 4.4 & 9.3 \\
 & 3 & 0 & 1. & \frac{60\pi \alpha_i^2}{m_{DM}^2} & 4.4 & 9.3 \\
\hline
\end{array}
\]
Prospects for DM detection

- Direct detection
  - Majorana nature of DM suppresses direct detection except for substantial $y_1, y_2$.
  - Increased sensitivity to HMDM than in MDM due to tree level $H$ exchange
  - Sommerfeld corrections shift to higher masses the allowed (tree-level) models.

![Graph showing prospects for DM detection](image)
Prospects for DM detection

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![Graph showing prospects for DM detection](image-url)
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  - Increased sensitivity to HMDM than in MDM due to tree level $H$ exchange
  - Sommerfeld corrections shift to higher masses the allowed (tree-level) models.

- Indirect detection: larger (tree-level) $\Delta m_{\chi^\pm \chi^0} \leadsto$ change the Sommerfeld effect and move the position of resonant peaks, see e.g. [Slatyer’09, Jin Chun’12] $\leadsto$ help to evade gamma-ray constraints (?)

- Colliders: disappearing tracks (up to multi TeV for 100 TeV collider); mono-X (up to multi TeV for 100 TeV collider); EWPT measurements and H-V modified couplings (up to ~ TeV)

see [Ostdiek’15, Cirelli’14, Fedderke’15, Ismail’16, Cai’16, Voigt’17, Wang’17, Xiang’17]
We extend the known viable parameter space of HMDM with $5_{M4D}$

Estimate Sommerfeld effects for EWDM in the SU(2) symmetric limit to determine the boundaries of the parameter space.

Parameter space constrained by direct searches and potentially evade indirect DM searches.
Thank you for your attention!
Backup
Fermionic Dark Matter: Gauge Portal

see e.g. [Cirelli et al’05-15, Hisano et al ’04-15, Cohen’13, Garcia-Cely et al ’15, Lefranc’15,...]

**Pure** gauge portal: **Minimal DM (MDM)** scenarios [Cirelli et al’05]

$$\mathcal{L} \subset \bar{\chi} (\Phi - m_M) \chi$$

- SM + 1 single Majorana nplet of SU(2)$_L$
  - $\sim$ 3-plet ($Z_2$) and 5-plet (stable)
- minimal fermionic WIMP DM
- loop corrections $\sim M_{\chi}^\pm - M_{\chi}^0 \sim$ 100’s MeV
  - $\sim$ the neutral $\chi^0$ is the lightest
- Gauge interactions only $\sigma v \propto \alpha_W^2 / M^2$, $\Omega h^2 = 0.12$

$\sim M_{0,3} = 2.4$ TeV and $M_{0,5} = 4.4$ TeV (at first sight!!)
Sommerfeld effect and bound state formation

[Hisano’04, Hisano’06, Cirelli’07, Arkani-Hamed’08, Cohen’13, Cirelli’15, Garcia-Cely’15++]

- The **Sommerfeld effect** is caused by distortion of the 2bdy wave function due to long range potential.

  ![Sommerfeld effect diagram]

  - For EWDM $\chi_0$, $M_{\text{med}} \ll \alpha M_{\chi_0}$ and low $\nu$: smoking gun signatures and enhance $\sigma \nu$ at freeze-out and today [Hisano’04’06, Cirelli’07]

- Formation of **bound states** from 2 DM particle can provide an extra enhancement [Asadi’16, Mitridate’17]
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$\sim M_{0,3} = 3 \text{ TeV} \& M_{0,5} = 11.5 \text{ TeV}$
Minimal Dark matter: at the verge of discovery/exclusion

![Graphs and diagrams illustrating indirect and direct detection of dark matter particles.](image-url)

- Indirect Detection
  - Fermion 5plet, indirect signals at $v_{rel}/2 = \beta = 10^{-3}$

- Direct Detection
  - Various Feynman diagrams showing different decay modes

References:
- [Mitridate 17]
- [Rinchioso 17]
- [Hisano 15]
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Are the possibilities infinite?

- coupling to $H \sim \Delta n = 1$
- EW perturbativity till $M_{Pl}$:

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What are the generic features?
(Minimal Dark Matter)$^2$: the general setup

- 4 free parameters: $m_M = m_\chi$, $m_D = m_\psi$, $y_1$, $y_2$

$$L_{\text{DARK}} = L_K - m_\psi \psi \tilde{\psi} - \frac{1}{2} m_\chi \chi \chi - (y_1 \psi \chi h^* + y_2 \tilde{\psi} \chi h + h c)$$

$\chi$ and $\psi, \tilde{\psi}$ are Weyl fermions of representations $(n, 0)$ and $(n \pm 1, 1/2), \ (n \pm 1, -1/2)$ of $(\text{SU}(2), \text{U}(1)_Y)$ with $n$ odd.
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- 4 free parameters: $m_M = m_\chi, m_D = m_\psi, y_1, y_2$

\[ \mathcal{L}_{\text{DARK}} = \mathcal{L}_K - m_\psi \psi \tilde{\psi} - \frac{1}{2} m_\chi \chi \chi - (y_1 \psi \chi h^* + y_2 \tilde{\psi} \chi h + h c) \]

$\chi$ and $\psi, \tilde{\psi}$ are Weyl fermions of representations $(n, 0)$ and $(n \pm 1, 1/2), (n \pm 1, -1/2)$ of (SU(2), U(1)$_Y$) with $n$ odd.

- After EWSB, the fermions of charge $Q = T_3 + Y$ in the bases $\{\chi^Q, \psi^Q, \tilde{\psi}^Q\}$ have mass matrices of the form:

\[
M_Q^{3 \times 3} = (-1)^Q \begin{pmatrix}
  m_\chi & a_Q y_1 v & \tilde{a}_Q y_2 v \\
  \tilde{a}_Q y_1 v & 0 & m_\psi \\
  a_Q y_2 v & m_\psi & 0 \\
\end{pmatrix},
\]

\[
M_Q^{2 \times 2} = (-1)^Q \begin{pmatrix}
  m_\chi & y_1 v \\
  y_2 v & m_\psi \\
\end{pmatrix}, \quad M_Q^{1 \times 1} = (-1)^Q m_\psi.
\]
Custodial symmetry $y_1 = \pm y_2$

\[ m_1 = \frac{1}{2} (m_M + m_D + \Delta m_\eta) \]
\[ m_2 = m_D \]
\[ m_3 = \frac{1}{2} (m_M + m_D - \Delta m_\eta) \]

\[ \Delta m_\eta = \sqrt{(m_D - m_M)^2 + 8(\eta y_\nu / \sqrt{2})^2} \]

\[
\left( \begin{array}{c}
\chi_1^0 \\
\chi_2^0 \\
\chi_3^0
\end{array} \right) = \left( \begin{array}{ccc}
c_\eta & s_\eta / \sqrt{2} & s_\eta / \sqrt{2} \\
0 & i / \sqrt{2} & -i / \sqrt{2} \\
-s_\eta & c_\eta / \sqrt{2} & c_\eta / \sqrt{2}
\end{array} \right) \left( \begin{array}{c}
\chi_0 \\
\psi_0 \\
\tilde{\psi}_0
\end{array} \right) \]

\[ \sin^2 \theta_\eta = \frac{1}{2} \left( 1 + \frac{m_D - m_M}{\Delta m_\eta} \right) \]

\[ \mathcal{L} = -\frac{g}{2} (\bar{\psi}_0 \sigma^\mu \psi_0 - \bar{\tilde{\psi}}_0 \sigma^\mu \tilde{\psi}_0) Z_\mu - y_\eta (\bar{\tilde{\psi}}_0 - \psi_0) \chi_0 h \]

\[ \mathcal{L} = \frac{g}{2} \chi_2^{0*} \sigma^\mu (s_\eta \chi_1^0 + c_\eta \chi_3^0) Z_\mu + h.c. \]

\[ -\frac{y_\eta}{2\sqrt{2}} (s_{2\eta} (\chi_1^0 \chi_1^0 - \chi_3^0 \chi_3^0) + 2c_{2\eta} \chi_1^0 \chi_3^0) \ h + h.c. \]
## Generic Mass Patterns

<table>
<thead>
<tr>
<th>M-D system</th>
<th>$m_M &lt; m_* \sim m_D$</th>
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</tr>
</thead>
<tbody>
<tr>
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<td>$\chi_l^0 \subset \begin{cases} 3_M \text{ at } y_1 = -y_2 \ 1_M \text{ at } y_1 = y_2 \end{cases}$</td>
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1M2D, $m_M > m_\tau$ and $y_1 = 1$

1M2D, $m_M < m_\tau$ and $y_1 = 1$
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3M4D, \( m_M > m_* \) and \( y_1 = 1 \)

\[ m_M = 400 \text{ GeV} \]
\[ m_D = 200 \text{ GeV} \]

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Generic Mass Patterns

5M4D, $m_M > m_*$ and $y_1 = 1$

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Sommerfeld effect and bound state formation

The Sommerfeld effect $\equiv$ NR QM effect. It is caused by the distortion of the wave function describing the relative motion of annihilating particles through the exchange of light mediators

$\sim$ significant enhancement or suppression of DM scattering by Coulomb like forces when $M_{med} < \alpha M_{DM}$
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For EWDM $\chi_0$, at low $v$ and $M_{\chi_0}$ large:

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Pheno Hints - 5M4D*

*tree level only!!

- **Envelope for** \( y_1, y_2 < 1 \):
  \[ m_\psi = m_{4-plet}, \quad m_\chi = m_{5-plet}, \]
  \[ m_\chi = m_\psi \]

- Need \( y_1, y_2 \sim \mathcal{O}(1) \) to cover other possibilities
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  $m_{\psi} = m_4$-plet, $m_{\chi} = m_5$-plet,
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- Majorana nature of DM suppresses direct detection except for substantial $y_1, y_2$.

- $\sigma_{SI}$ is maximal for $m_{\chi} \simeq m_{\psi}$ where the coupling to $H$ is maximal, see also [Freitas'15] and $\sigma_{SI}$ is supressed for $y_1 \to -y_2$

  \[ (\sigma_{SI}^{5-plet} = 2.4 \times 10^{-46} \text{ cm}^2, \ & \text{Xenon 1T reach} \ \sigma_{SI} > 10^{-44} \text{ cm}^2) \]
Sommerfeld effect in SU(2) symmetric limit

- Multi-TeV DM $\rightsquigarrow$ compute annihilations in SU(2) symmetric limit
  $\rightsquigarrow$ Isospin is conserved in annihilations
- Decompose the product of initial states as a sum of 2-particle states of definite isospin $I$: $R_i \otimes R_j = \sum_I R_I$ [Strumia’08]
  Associated an abelian-like Sommerfeld correction with $a_I = v/(2\alpha)$

$$S_I = \frac{\pi/a_I}{1 - \exp(a_I)}$$
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$$S_I = \pi/a_I \left(1 - \exp(a_I)\right)$$

- For $\Omega_{\text{DM}} h^2 \propto 1/\sigma v_{\text{eff}}$, $\sigma v_{\text{eff}} = \sum_{ij} g_i g_j / g_{\text{eff}} \sigma v_{ij}$ becomes

$$\sigma v_{\text{eff}} = \frac{1}{(2I_X+1)^2} \left(\sum_I (2I + 1) S_I \sigma v^\text{pert}_I + S_{I=1} \sigma v^\text{pert}_{gg'} + S_{I=0} \sigma v^\text{pert}_{g'}\right),$$

$\sigma v_I$: pure $SU(2)$, Clebsh-Gordan relate $|ij\rangle$ to $|I\rangle$,
$\sigma v_{gg'}$ and $\sigma v_{g'}$: involves $U(1)$ gauge boson

see also [Garcia-Cely’15] and [Mitridate’17]
Sommerfeld effect in SU(2) symmetric limit

- $m_{\text{DM}} \sim \text{MultiTeV} \leadsto \text{f.o. SU(2) symmetric limit}$
- In this limit, non-abelian Sommerfeld effect can be reduced to a combination of abelian-like Sommerfeld corrections, using group theory decompositions [Strumia'08].

Potential between particles of repres. $R_i$ and $R_j \equiv V_{Ia} = V_{Ia}^{SU(2)} + V^{U(1)}$:

$$V_{Ia}^{SU(2)}(r) = \frac{\alpha_2}{r} \frac{1}{2} (C_a - C_i - C_j); \quad V^{U(1)} = -\frac{\alpha_2 t_w^2 Y^2}{r}$$

and $R_i \otimes R_j = \sum_{k=1}^{N'} R_k$ with $C_l$ the quadratic Casimir of $R_l$
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Fermionic Minimal Dark Matter: multi-TeV EWDM

[Cirelli et al’05]

- SM +1 single nplet of SU(2)$_L$
- nplet ⊇ a neutral component ≡ DM
  \[ Y \text{ is fixed for given } n, \text{ e.g. } n = 3, Y = 0, 1/2 \]
- EW perturbativity till $M_{pl} \sim n \leq 5$ for fermion DM
- for $n < 5$, need $Z_2$ symmetry for stability
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- loop corrections $\rightsquigarrow M^\pm_\chi - M^0_\chi \sim 100$’s MeV
  $\rightsquigarrow$ the neutral $\chi^0$ is the lightest
- Gauge interactions only $\sigma v \propto \alpha^2_{W}/M^2$, $\Omega h^2 = 0.12$

$\rightsquigarrow M_{0,3} = 2.4$ TeV and $M_{0,5} = 4.3$ TeV (at first sight)
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Sommerfeld effect: Evaluation for EWDM freeze-out

- Very degenerate mass spectra $\leadsto$ all $n$-plets components play into the game!

- Need the NR EW potentials associated to $\mathcal{V} = W, Z, \gamma$ exchange
  \[
  V_{\alpha,\beta} = c_{\alpha,\beta} \frac{e^{-M_{\mathcal{V}} r}}{r}
  \]

- Need the the absorptive parts $\Gamma_{\alpha,\beta}$ associated to scattering amplitudes: $\mathcal{M}_{\alpha \rightarrow XX',} \mathcal{M}^{*}_{\beta \rightarrow XX',}$

\[\text{[Hisano'04,Hisano'06,Cirelli'07,Cohen'13, Cirelli'15,Garcia-Cely'15++]}\]

\[\text{[Cirelli'15]}\]

If $S = 1$, you recover the perturbative result $\sigma v_{\alpha} = \Gamma_{\alpha\alpha}$. 

Laura Lopez Honorez (FNRS@ULB & VUB)
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  \( V_{\alpha, \beta} \) needs the absorptive parts \( \Gamma_{\alpha, \beta} \) associated to scattering amplitudes: \( M_{\alpha \to XX'} \quad M_{\beta \to XX'}^* \),

- Evolve \( N \) initial pairs \( \leadsto N \) coupled diff. eqs.:

\[
\sigma v_{\alpha \beta} = \sum_{\beta' \beta'} S^*_{\alpha \beta \beta' \beta'} \Gamma_{\beta' \beta} S_{\beta' \alpha} ,
\]

\( S_{\alpha \beta} \) depends on \( V_{\alpha' \beta'} \) and \( M_{DM} v^2 \);

[Hisano’04, Hisano’06, Cirelli’07, Cohen’13, Cirelli’15, Garcia-Cely’15++]

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\[ \sim M_{0,3} = 2.7 \text{ TeV} & \sim M_{0,5} = 9.4 \text{ TeV} \]
SU(2) unbroken Sommerfeld on pure 4-plet

\[ \psi = \begin{pmatrix} \psi^{++} \\ \psi^+ \\ \psi^0 \\ \psi^- \end{pmatrix}, \quad \tilde{\psi} = \begin{pmatrix} \tilde{\psi}^+ \\ \tilde{\psi}^0 \\ \tilde{\psi}^- \\ \tilde{\psi}^{--} \end{pmatrix}, \]

\[ 4 \otimes \bar{4} = 1 \oplus 3 \oplus 5 \oplus 7 = \sum_{a=1}^{N'} \mathcal{R}_a \]

with \( I_a = \{0, 1, 2, 3\} \) and

\( Y_4 = -Y_{\bar{4}} = 1/2 \)

\( \Rightarrow S_I = \{3.9, 3.0, 1.5, 0.3\} \)
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\]

- \(4 \otimes \bar{4} = 1 \oplus 3 \oplus 5 \oplus 7 = \sum_{a=1}^{N'} R_a\)
- With \(I_a = \{0, 1, 2, 3\}\) and \(Y_4 = -\bar{Y}_4 = 1/2\)
- \(\sim S_I = \{3.9, 3.0, 1.5, 0.3\}\)
- From charge to Isospin e.g. \(\sigma v_{I=2}/2 = \sigma v_{\chi_1^+ \chi_2^+ \rightarrow WW}\)
- \(\sigma v_{\text{eff}} = \frac{1}{16} \left( S_{I=0} \sigma v_{I=0} + S_{I=1} 3 \sigma v_{I=1} + S_{I=2} 5 \sigma v_{I=2} + S_{I=0} \sigma v_{g'} + S_{I=1} \sigma v_{g'g} \right)\)
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\[ \sigma v_{\text{eff}} = \frac{1}{16} \left( S_{I=0} \sigma v_{I=0} + S_{I=1} 3 \sigma v_{I=1} + S_{I=2} 5 \sigma v_{I=2} + S_{I=0} \sigma v_{g'} + S_{I=1} \sigma v_{g'g} \right) \]

Perturb: \( M_{DM} = 2.4 \text{ TeV while w/ Sommerfeld } M_{DM} = 3.9 \text{ TeV} \)
Quadruplet $\sigma v_{\text{eff}}$ contribs

\[
\sigma v_0 = \frac{75}{4} \frac{\alpha_2^2 \pi}{M_{DM}^2}, \quad \sigma v_1 = \frac{125}{8} \frac{\alpha_2^2 \pi}{M_{DM}^2}, \quad \sigma v_2 = 6 \frac{\alpha_2^2 \pi}{M_{DM}^2}
\]

\[
\sigma v_{g'g} = \frac{15}{2} t_w^2 \frac{\alpha_2^2 \pi}{M_{DM}^2}
\]

\[
\sigma v_{g'} = \frac{43}{8} t_w^4 \frac{\alpha_2^2 \pi}{M_{DM}^2}
\]
Quadruplet $\sigma v_{ij}$ total contribs

$Q_{tot} = 0$

$\sigma v_{0,0} = \frac{\alpha_2^2 \pi}{32 c_w^4 M_{DM}^2} (223 - 442 s_w^2 + 262 s_w^4) = \sigma v_{1,-1}$

$\sigma v_{2,-2} = \frac{\alpha_2^2 \pi}{32 c_w^4 M_{DM}^2} (423 - 810 s_w^2 + 430 s_w^4) = \sigma v_{-1,1}$

$Q_{tot} = 1$

$\sigma v_{2,-1} = \frac{3 \alpha_2^2 \pi}{16 c_w^4 M_{DM}^2} (41 - 37 s_w^2) = \sigma v_{0,1}$

$\sigma v_{1,0} = \frac{\alpha_2^2 \pi}{4 c_w^4 M_{DM}^2} (25 - 21 s_w^2)$

$Q_{tot} = 2$

$\sigma v_{2,0} = \frac{3 \alpha_2^2 \pi}{M_{DM}^2} = \sigma v_{1,1}$
Quadruplet $\sigma v_I$ contribs (SU(2) only)

$Q_{tot} = 0$

$\sigma v_{0,0} = \sigma v_{I=0}/4 + \sigma v_{I=2}/4 + \sigma v_{I=1}/20 = \sigma v_{1,-1}$

$\sigma v_{2,-2} = \sigma v_{I=0}/4 + \sigma v_{I=2}/4 + 9\sigma v_{I=1}/20 = \sigma v_{-1,1}$

$Q_{tot} = 1$

$\sigma v_{2,-1} = \sigma v_{I=2}/2 + 3\sigma v_{I=1}/10 = \sigma v_{0,1}$

$\sigma v_{1,0} = 2/5\sigma v_{I=1}$

$Q_{tot} = 2$

$\sigma v_{2,0} = \sigma v_{I=2}/2 = \sigma v_{1,1}$
Sommerfeld $e^+e^-$

Related to $e^+e^- \rightarrow \gamma\gamma$ hep-ph/0412403

→ diverges at low $\nu$

$A_n \propto \alpha(\alpha/\nu)^n$

If $\nu<\alpha \rightarrow$ Non perturbative, the ladder diagrams have to be resummed.

NB: $\nu<\alpha \rightarrow$ The coulomb potential is larger than the kinetic energy of the incident particles.

The NR incident pair of electron has its plane wave deformed by the coulomb potential.

→ Need an improved calculation [Sommerfeld 1931]
Sommerfeld EWDM

For EWIMP hep-ph/0412403

\[ \sigma v \sim \frac{\alpha^2 \alpha_2^2}{m_W^2} \]

Unitary limit
\[ \sigma v < \frac{4\pi}{v m^2} \]

Bound exceeded for heavy m or low v

→ Higher order corr. to be included

\[ A_n \sim \alpha \left( \frac{\alpha_2 m}{m_W} \right)^n \]

→ Higher order corrections increasingly important for \( \alpha_2 m \gtrsim m_W \)

→ non perturbative regime

→ resum the ladder diag.
Sommerfeld single mediator -theory

ArXiv:0810.0713 : Single mediator

Sommerfeld enhancement is an elementary effect in NR QM: The potential may significantly distort the wave function of describing the relative motion of the initial pair of particles:

$$\frac{1}{2M} \nabla^2 \psi_k + V(r) \psi_k = \frac{k^2}{2M} \psi_k$$

$$\sigma = \sigma_0 S_k \quad S_k = \frac{|\psi_k(0)|^2}{|\psi_k^{(0)}(0)|^2}$$

Coulomb case

$$V(r) = -\frac{\alpha}{2r} \quad S_k = \left| \frac{\frac{\pi}{\epsilon_v} \frac{1}{1 - e^{-\frac{\pi}{\epsilon_v}}} \right| \quad \epsilon_v \equiv \frac{v}{\alpha}$$

$S_k \to \frac{\pi \alpha}{v}$ At small $v$

The Sommerfeld effect is caused by the distortion of the plane wave describing the relative motion of the ann. pair through the exchange of light mediators.
Yukawa case

\[ V(r) = -\frac{\alpha}{2r} e^{-m_\phi r} \]

\[ S = \frac{\pi}{\epsilon_v \cosh \left( \frac{2\pi \epsilon_v}{\pi^2 \epsilon_\phi / 6} \right)} \frac{\sinh \left( \frac{2\pi \epsilon_v}{\pi^2 \epsilon_\phi / 6} \right)}{\cosh \left( \frac{2\pi \epsilon_v}{\pi^2 \epsilon_\phi / 6} \right) - \cos \left( \frac{2\pi}{\pi^2 \epsilon_\phi / 6} - \frac{\epsilon_\phi^2}{(\pi^2 \epsilon_\phi / 6)^2} \right)} \]

Resonant behaviour at:

\[ m_\phi \simeq \frac{6\alpha_X m_X}{\pi^2 n^2}, \quad n = 1, 2, 3 \ldots \]

\[ S \simeq \frac{\pi^2 \alpha_X m_\phi}{6 m_X v^2} \]

\[ \epsilon_\phi \equiv \frac{m_\phi}{\alpha M} \]
bla
This is really the end