ZH\eta\text{-}vertex in the Simplest Little Higgs Model

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Talk at Moriond 2018 (EW); mainly based on the paper:
S.-P. He, Y.-N. Mao, C. Zhang, and S.-H. Zhu, [1709.08929]
I. INTRODUCTION

• In some extensions of the standard model (SM), there exist additional scalars;

• For a pseudoscalar $\eta$, it can couple to the Higgs boson ($H$) and gauge boson $Z$ through a new vertex, $Z^\mu(\eta \partial_\mu H - H \partial_\mu \eta)$, such as in 2HDM etc.;

• The vertex can lead to new phenomenologies at colliders, such as associated production of two scalars, or cascade decay of one scalar (the heavier one);

• In the simplest little Higgs (SLH) model, this vertex was known as $\sim O(v/f)$, where $f$ is a higher scale for the breaking of a new global symmetry [W. Kilian, D. Rainwater, and J. Reuter, Phys. Rev. D 71, 015008 (2005); Phys. Rev. D74, 095003 (2006)];

• We re-derived the vertex and corrected the mistake appearing for a long time.
II. EFT ANALYSIS OF $ZH\eta$-VERTEX

Consider the effective field theory (EFT) at EW scale: SM particles+$\eta$

- $\eta$ is a pure pseudoscalar and SM gauge singlet without mass mixing with other SM
gauge multiplets, assuming all other degrees of freedom are integrated out;

- Consider dim-5 and dim-6 gauge- and CP-invariant operators with $\eta$ shift symmetry
  which are possible to contribute to $ZH\eta$ vertex as:

$$O_1 = i(\partial^\mu \eta) \phi^\dagger D_\mu \phi + \text{H.c.}, \quad O_2 = (\phi^\dagger D^\mu \phi)(\phi^\dagger D_\mu \phi);$$

- $\phi \equiv ((v + H - i\chi)/\sqrt{2}, G^-)^T$ is the usual Higgs doublet, $\mathcal{L} \supset \mathcal{L}_{\text{SM}} + c_1 O_1/f + c_2 O_2/f^2$, where $f$ is a higher new scale.
Expand the term $c_1 O_1 / f$ and define $\xi \equiv v / f$, we have

$$\mathcal{L} \supset \frac{1}{2} \left[ (\partial H)^2 + (\partial \chi)^2 + (\partial \eta)^2 + 2c_1 \xi (\partial_\mu \eta)(\partial^\mu \chi) \right] - m_Z Z_\mu \partial^\mu (\chi + c_1 \xi \eta)$$

$$+ \frac{g}{2c_W} Z_\mu (\chi \partial^\mu H - H \partial^\mu \chi) - \frac{g}{c_W} c_1 \xi H Z_\mu \partial^\mu \eta.$$

- Naively, according to term D, the $Z \chi \eta$ appear at $O(\xi)$ as expected;
- However, there are additional two-point transitions in terms A and B, especially the two-point transitions in B should be exactly canceled by gauge-fixing term;
- They must be removed through field re-definition of $\chi$ and $\eta$. 
To the leading order of $\xi$, we have $\tilde{\chi} = \chi + c_1 \xi \eta + O(\xi^2), \tilde{\eta} = \eta + O(\xi^2)$.

- The new basis is canonically-normalized, the re-definition $\chi \to \tilde{\chi}$ changes the gauge-fixing term, and two-point transitions are removed;

- $\tilde{\chi}$ is the exact Goldstone field, thus term C induces another contribution to $ZH\eta$ vertex, which exactly cancels the anti-symmetric part in term D at $O(\xi)$;

- The $ZH\eta$ vertex induced by $O_1$ cannot appear before $O(\xi^3)$;

- No direct $ZH\eta$ vertex induced by $O_2$, however, redefinition $\tilde{\chi} = \chi + c_1 \xi \eta + O(\xi^2)$ can introduce such vertex at $O(\xi^3)$, no other operators contribute at this order.
Short summary and examples for EFT analysis:

- For a pure SM gauge singlet pseudoscalar field $\eta$, effective operators $O_{1,2}$ can contribute to the $ZH\eta$ vertex, which may finally appear at $O(\xi^3)$.

<table>
<thead>
<tr>
<th>Model</th>
<th>$ZH\eta$-vertex</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM+Complex Singlet</td>
<td>0</td>
<td>$\eta$ is pure SM gauge singlet, $f \to \infty$</td>
</tr>
<tr>
<td>2HDM</td>
<td>$O(1)$</td>
<td>$\eta$ is a component of SM gauge doublet</td>
</tr>
<tr>
<td>(SU(4)/SU(3))^4 LH Model</td>
<td>$O(\xi)$</td>
<td>$\eta$ contains $O(\xi)$ SM gauge doublet component</td>
</tr>
<tr>
<td>SLH [(U(3)/U(2))^2] Model</td>
<td>$O(\xi^3)$</td>
<td>$\eta$ is pure SM gauge singlet</td>
</tr>
</tbody>
</table>

[* D. E. Kaplan and M. Schmaltz, JHEP 0310, 039 (2003)]
III. \textit{ZH}_\eta\text-VERTEX IN THE SLH MODEL

- We discuss the SLH model in details as an example of EFT analysis;
- Only two scalars, and $\eta$ is a pure SM gauge singlet in this model;
- Naively calculate the $ZH\eta$ vertex, it behaves like the EFT analysis;
- We should also perform a complete formalism in kinetic diagonalization.
A. The SLH Model

- A Global symmetry breaking \((\text{SU}(3) \times \text{U}(1))^2 \rightarrow (\text{SU}(2) \times \text{U}(1))^2\) happens at a high scale \(f\), gauge group is enlarged to \(\text{SU}(3) \times \text{U}(1)\) which breaks to SM gauge group;

- Two scalar triplets \(\Phi_{1,2}\) are nonlinear realized as:

\[
\Phi_1 = e^{i\Theta} e^{it/4} \begin{pmatrix} 0_{1 \times 2} \\ f c_{\beta} \end{pmatrix}, \quad \Phi_2 = e^{i\Theta'} e^{-i\Theta/4} \begin{pmatrix} 0_{1 \times 2} \\ f s_{\beta} \end{pmatrix};
\]

with the definitions of the matrix fields

\[
\Theta \equiv \frac{1}{f} \left( \frac{\eta I_{3 \times 3}}{\sqrt{2}} + \begin{pmatrix} 0_{2 \times 2} & \phi \\ \phi^\dagger & 0 \end{pmatrix} \right), \quad \text{and} \quad \Theta' \equiv \frac{1}{f} \left( \frac{\zeta I_{3 \times 3}}{\sqrt{2}} + \begin{pmatrix} 0_{2 \times 2} & \phi \\ \phi^\dagger & 0 \end{pmatrix} \right).
\]
• There are 10 Goldstones and 8 of which are eaten by massive gauge bosons;

• $t_\beta$ means the ratio between the VEVs of the two triplets;

• $\eta$ is the pseudoscalar and $\phi$ is the Higgs doublet defined as above;

• $\zeta$ and $\varphi \equiv ((\sigma - i\omega)/\sqrt{2}, x^-)^T$ are expected to be eaten by heavy gauge bosons;

• $\eta$ can acquire its mass through $\mu^2 \Phi_1^\dagger \Phi_2$ term, and EWSB can be induced by loop corrections, heavy gauge bosons ($Z', X^\pm, Y, \bar{Y}$) can acquire their masses before EWSB, while EW gauge bosons ($W^\pm, Z$) acquire their masses after EWSB;

• All fermion doublets should be enlarged to triplets as well, and the heavy top $T$ domains the EWSB through loop corrections;
Some properties of the SLH parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Allowed Region</th>
<th>Constraints</th>
</tr>
</thead>
</table>
| $f$       | (7.5 − 84.5) TeV | Lower: LHC direct search\(^*\),\(^*\)  
Upper: Goldstone scattering unitarity\(^*\)\(^*\) |
| $t_\beta$ | 1-8.9          | Lower: Convention       
Upper: Goldstone scattering unitarity\(^*\)\(^*\) |
| $m_\eta$  | (0 − 1.5) TeV  | Theoretical EWSB conditions\(^*\)\(^*\) |
| $m_T$     | (1.7-18.7)     | Lower: Goldstone scattering unitarity\(^*\)\(^*\)  
Upper: Theoretical EWSB conditions\(^*\)\(^*\) |

\(^*\) ATLAS Collaboration, JHEP 1710, 182 (2017); \(^*\) Y.-N. Mao, 1703.10123;  
\(^*\) K. Cheung, S.-P. He, Y.-N. Mao, C. Zhang, and Y. Zhou, 1801.10066]
B. General Diagonalization Procedure

- When we expand \((D_\mu \Phi_1)^2 + (D_\mu \Phi_2)^2\), the lagrangian contains

\[
\mathcal{L} \supset \frac{1}{2} \kappa_{ij} (\partial G_i)(\partial G_j) + \mathbb{F}_{pi} V_\mu^p \partial_\mu G_i + \frac{1}{2} (M^2_{V})_{pq} V_\mu^p V_\mu^q.
\]

- Like in the EFT analysis, there are two-point transitions in term A and B, and the two-point transition in B should be exactly canceled by the gauge fixing term;

- For term A, there are two-point kinetic mixing in CP-odd scalar sector \((G_i = \eta, \zeta, \chi, \omega)\) and we need a new basis \(S_i \equiv U_{ij} G_j\) which can give \(\mathcal{L} \supset (\partial S_i)^2/2\);
• Introduce the inner product in the linear space spanned by $S_i$: $\langle S_i | S_j \rangle = \delta_{ij}$, it can be derived $\langle G_i | G_j \rangle = (K^{-1})_{ij}$;

• With this relation, for $\bar{G}_p = F_{pi} G_i$, we have $\langle \bar{G}_p | \bar{G}_q \rangle = (M^2_V)_{pq}$;

• $M^2_V$ can be diagonalized through a matrix $R$ as $(RM^2_V R^T)_{pq} = m^2_p \delta_{pq}$;

• Thus we have the Goldstone basis $\tilde{G}_p = R_{pq} F_{qi} G_i / m_p$, together with the pseudoscalar $\tilde{\eta} = \eta / \sqrt{(K^{-1})_{11}}$, the basis $(\tilde{\eta}, \tilde{G}_i)$ is canonically normalized;

• Choose the gauge-fixing term $- \left( \partial^\mu \tilde{V}_p^\mu / \sqrt{\xi_p} - \sqrt{\xi_p} m_p \tilde{G}_p \right)^2 / 2$, $\tilde{V}_p$ are in mass eigenstates, $\tilde{G}_p$ are exactly eaten by $\tilde{V}_p$ thus they are the corresponding Goldstone fields.
C. Detailed Calculation and Results

• First, divide the matrix $F$ into two parts as $F_{4 \times 3} = \left( \tilde{f}_{1 \times 3}, \tilde{F}_{3 \times 3} \right)$, with the vector component $\tilde{f}_q = F_{qq}$, we have the new relation

$$G_p = \mathbb{R}_{pq} \left( \eta \tilde{f}_q + \tilde{F}_{jk} G_k \right) / m_p, \quad \text{or} \quad G_i = (\tilde{F}^{-1})_{iq} m_q G_q - (\tilde{F}^{-1})_i \sqrt{(K^{-1})_{11}} \tilde{\eta}.$$

• $\eta$ component appears in the original Goldstone degrees of freedom as shown above;

• The antisymmetric $VHG_i$-vertex is parameterized as $L \supset \mathbb{C}_{pi} V^\mu_p (G_i \partial_\mu H - H \partial_\mu G_i)$, all $\mathbb{C}_{pi}$ are elements of a $4 \times 3$ matrix $C$;

• In the previous papers, choose $\xi \equiv v / f$ as usual and using the original fields, the $ZH\eta$ vertex was derived as $-g\xi / (\sqrt{2}c_W t_2) Z^\mu (\eta \partial_\mu H - H \partial_\mu \eta) \sim O(\xi) \quad \rightarrow \text{Incorrect!}$
• Following our diagonalization procedure, we should re-derive the $ZH\eta$-vertex, define a $1 \times 4$ vector as $\Upsilon \equiv \sqrt{\left(K^{-1}\right)_{ij}(1, -\tilde{f}^T(\tilde{F}^{-1})^T)^T}$, the $ZH\eta$ vertex should be

\[
\tilde{c}_{ZH\eta} \tilde{Z}^\mu(\bar{\eta}\partial_\mu H - H\partial_\mu \bar{\eta}) \equiv (\mathbb{R} \odot \Upsilon) \tilde{Z}^\mu(\bar{\eta}\partial_\mu H - H\partial_\mu \bar{\eta})
\]

• Straightforward calculation gives $\tilde{c}_{ZH\eta} = -g\xi^3/(4\sqrt{2}c_\rho t_2^\beta) \sim \mathcal{O}(\xi^3)$;

• That’s because $\tilde{\chi} \sim \chi + \mathcal{O}(\xi)\eta$, a corresponding contribution to $ZH\eta$-vertex then arises and at $\mathcal{O}(\xi)$ it exactly cancels the naively derived $ZH\eta$-vertex as shown in last page, just like the behavior of the EFT analysis above;

• Mass mixing between neutral gauge bosons also contribute to this vertex at $\mathcal{O}(\xi^3)$.
IV. CONCLUSIONS AND DISCUSSIONS

- Based on the EFT analysis, for a pure SM gauge singlet pseudoscalar $\eta$, the $ZH\eta$-vertex cannot arise until $O(\xi^3)$ with $\xi \equiv v/f$;

- We derived this vertex in the SLH model as an example, and showed the coefficient should be $\tilde{c}_{ZH\eta} = -g\xi^3/(4\sqrt{2}c_Wt_{23}) \sim O(\xi^3)$, instead of $-g\xi/(\sqrt{2}c_Wt_{23}) \sim O(\xi)$, which has already existed in other papers for a long time;

- All corresponding phenomenology in SLH model should also be re-considered, which will appear in my forthcoming papers soon;

- The calculation procedure can also be performed to other similar models.

Thank you! My Email: maoyn@ihep.ac.cn
Appendix

Up to $\mathcal{O}(\xi^3)$, more details during calculation are listed here:

$$K = \begin{pmatrix}
1 & 0 & \frac{\sqrt{2}\xi}{t_{2,\beta}} & \frac{7c_{2,\beta} + c_{4,\beta}}{6\sqrt{2}s_{2,\beta}} \xi^3 & -\sqrt{2}\xi + \frac{5 + 3c_{4,\beta}}{3\sqrt{2}s_{2,\beta}} \xi^3 \\
0 & 1 & -\frac{\xi}{\sqrt{2}} & \frac{5 + 3c_{4,\beta}}{12\sqrt{2}s_{2,\beta}} \xi^3 & -\frac{2\sqrt{2}\xi^3}{3s_{2,\beta}} \\
\frac{\sqrt{2}\xi}{t_{2,\beta}} - \frac{7c_{2,\beta} + c_{4,\beta}}{6\sqrt{2}s_{2,\beta}} \xi^3 & -\frac{\xi}{\sqrt{2}} + \frac{5 + 3c_{4,\beta}}{12\sqrt{2}s_{2,\beta}} \xi^3 & 1 & -\frac{5 + 3c_{4,\beta}}{12s_{2,\beta}} \xi^2 & \frac{2\xi^2}{3t_{2,\beta}} \\
-\sqrt{2}\xi + \frac{5 + 3c_{4,\beta}}{3\sqrt{2}s_{2,\beta}} \xi^3 & \frac{2\xi^2}{3t_{2,\beta}} & \frac{2\xi^2}{3t_{2,\beta}} & 1 & 0 \\
\end{pmatrix};$$

and thus $(K^{-1})_{11} = 1 + 2\xi^2/s_{2,\beta}^2$.
\[
\hat{f} = g f \left( \frac{1}{\sqrt{2c_W t_{2\beta}}} \xi^2, \frac{\rho}{t_{2\beta}} \xi^2, -\xi + \frac{5 + 3c_{4\beta}}{6s_{2\beta}} \zeta^3 \right)^T;
\]

\[
\hat{F} = g f \left( \sqrt{\frac{2}{3-c_W}} - \frac{\kappa(1+c_{2W})}{\sqrt{2c_{2W}}} \frac{\xi^2}{2c_W} - \frac{\xi \zeta^3}{24c_{2W} s_{2\beta}} - \frac{(5+3c_{4\beta}) \xi^3}{3c_W t_{2\beta}} \right);
\]

with \( \rho \equiv \sqrt{\frac{1+2c_{2W}}{1+c_{2W}}} \) and \( \kappa \equiv \frac{c_{2W}}{2c_W \sqrt{3 - t_W^2}} \);
\[
\Upsilon = c_{\gamma+\delta}^{-1} \begin{pmatrix}
1 \\
\frac{s_{\gamma} t_{\beta}^{-1} - s_{\delta}^2 t_{\beta}}{c_{2\delta} t_{\beta} - c_{2\gamma} t_{\beta}^{-1}} \\
\frac{1}{2} (s_{2\delta} t_{\beta} + s_{2\gamma} t_{\beta}^{-1})
\end{pmatrix} = \begin{pmatrix}
1 + \frac{\xi^2}{s_{\gamma}^2} \\
-\frac{\xi^2}{t_{2\delta}} \\
\frac{-\sqrt{2} \xi}{t_{2\beta}} - \frac{3 - c_{4\beta}}{3\sqrt{2}s_{2\beta}^2} \xi^3 \\
\frac{1}{\sqrt{2} \xi} + \frac{3 - c_{4\beta}}{3\sqrt{2}s_{2\beta}^3} \xi^3
\end{pmatrix};
\]

with \( \gamma \equiv \xi t_{\beta}/\sqrt{2} \) and \( \delta \equiv \xi/(\sqrt{2} t_{\beta}); \)
\[ R = \begin{pmatrix}
1 & -\frac{\kappa \rho^2 \xi^2}{2W} & -\frac{\sqrt{2} \xi^3}{3cW \xi_{2,\beta}} \\
\frac{\kappa \rho^2 \xi^2}{2W} & 1 & \frac{2 \sqrt{2} (1 + 2cW) \kappa \xi^3}{3cW \xi_{2,\beta}} \\
\frac{\sqrt{2} \xi^3}{3cW \xi_{2,\beta}} & \frac{2 \sqrt{2} (1 + 2cW) \kappa \xi^3}{3cW \xi_{2,\beta}} & 1
\end{pmatrix}; \]
\[ C = g \left( \begin{array}{cccc}
-\frac{\xi}{\sqrt{2}c_{\alpha}t_{2\beta}} + \frac{(7c_{2\beta}+c_{4\beta})\xi^3}{6\sqrt{2}c_{\alpha}w_{2\beta}} & \frac{\xi}{2\sqrt{2}c_{\alpha}w} - \frac{(5+3c_{4\beta})\xi^3}{12\sqrt{2}c_{\alpha}w^2s_{2\beta}} & -\frac{1}{2c_{\alpha}w} + \frac{(5+3c_{4\beta})\xi^2}{12c_{\alpha}w^2s_{2\beta}} & -\frac{\xi^2}{2c_{\alpha}w t_{2\beta}} \\
-\frac{\rho\xi}{t_{2\beta}} + \frac{\rho(7c_{2\beta}+c_{4\beta})\xi}{6s_{2\beta}} & \frac{\rho\xi}{2} - \frac{\rho(5+3c_{4\beta})\xi^3}{12s_{2\beta}} & \rho + \frac{\kappa(5+3c_{4\beta})\xi^2}{6s_{2\beta}} & \frac{\kappa\xi^2}{c_{2\alpha}w t_{2\beta}} \\
\frac{1}{2} - \frac{(5+3c_{4\beta})\xi^2}{4s_{2\beta}} & \frac{\xi^2}{t_{2\beta}} - \frac{\xi}{\sqrt{2}t_{2\beta}} + \frac{(7c_{2\beta}+c_{4\beta})\xi^3}{12\sqrt{2}s_{2\beta}} & 0 & 0 \\
\end{array} \right) \]