$ZH\eta$-vertex in the Simplest Little Higgs Model

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S.-P. He, **Y.-N. Mao**, C. Zhang, and S.-H. Zhu, 1709.08929.

I. INTRODUCTION

- In some extensions of the standard model (SM), there exist additional scalars;
- For a pseudoscalar η, it can couple to the Higgs boson (H) and gauge boson Z through a new vertex, $Z^\mu(\eta\partial_\mu H - H\partial_\mu \eta)$, such as in 2HDM etc.;
- The vertex can lead to new phenomenologies at colliders, such as associated production of two scalars, or cascade decay of one scalar (the heavier one);
- In the simplest little Higgs (SLH) model, this vertex was known as $\sim \mathcal{O}(v/f)$, where f is a higher scale for the breaking of a new global symmetry [W. Kilian, D. Rainwater, and J. Reuter, Phys. Rev. D 71, 015008 (2005); Phys. Rev. D74, 095003 (2006)];
- We re-derived the vertex and corrected the mistake appearing for a long time.

II. EFT ANALYSIS OF $ZH\eta$-VERTEX

Consider the effective field theory (EFT) at EW scale: SM particles+η

- η is a pure pseudoscalar and SM gauge singlet without mass mixing with other SM gauge multiplets, assuming all other degrees of freedom are integrated out;
- Consider dim-5 and dim-6 gauge- and CP-invariant operators with η shift symmetry which are possible to contribute to $ZH\eta$ vertex as:

$$\mathcal{O}_1 = \mathrm{i}(\partial^\mu \eta)\phi^\dagger D_\mu \phi + \mathrm{H.c.}, \qquad \mathcal{O}_2 = (\phi^\dagger D^\mu \phi)(\phi^\dagger D_\mu \phi);$$

- $\phi \equiv ((v+H-\mathrm{i}\chi)/\sqrt{2}, G^-)^T$ is the usual Higgs doublet, $\mathcal{L} \supset \mathcal{L}_{\mathrm{SM}} + c_1\mathcal{O}_1/f + c_2\mathcal{O}_2/f^2$, where f is a higher new scale.

Expand the term $c_1\mathcal{O}_1/f$ and define $\xi \equiv v/f$, we have

$$\mathcal{L} \supset \frac{1}{2}\left[(\partial H)^2 + (\partial\chi)^2 + (\partial\eta)^2 + \underbrace{2c_1\xi(\partial_\mu\eta)(\partial^\mu\chi)}_{\text{A}}\right] - \underbrace{m_Z Z_\mu \partial^\mu\left(\chi + c_1\xi\eta\right)}_{\text{B}}$$
$$+ \underbrace{\frac{g}{2c_W} Z_\mu\left(\chi\partial^\mu H - H\partial^\mu\chi\right)}_{\text{C}} - \underbrace{\frac{g}{c_W} c_1\xi H Z_\mu \partial^\mu\eta}_{\text{D}}\,.$$

- Naively, according to term D, the $ZH\eta$ appear at $\mathcal{O}(\xi)$ as expected;
- However, there are additional two-point transitions in terms A and B, especially the two-point transitions in B should be exactly canceled by gauge-fixing term;
- They must be removed through field re-definition of χ and η.

To the leading order of ξ, we have $\tilde{\chi} = \chi + c_1 \xi \eta + \mathcal{O}(\xi^2)$, $\tilde{\eta} = \eta + \mathcal{O}(\xi^2)$.

- The new basis is canonically-normalized, the re-definition $\chi \to \tilde{\chi}$ changes the gauge-fixing term, and two-point transitions are removed;
- $\tilde{\chi}$ is the exact Goldstone field, thus term C induces another contribution to $ZH\eta$ vertex, which exactly cancels the anti-symmetric part in term D at $\mathcal{O}(\xi)$;
- The $ZH\eta$ vertex induced by $\mathcal{O}_1$ cannot appear before $\mathcal{O}(\xi^3)$;
- No direct $ZH\eta$ vertex induced by $\mathcal{O}_2$, however, redefinition $\tilde{\chi} = \chi + c_1 \xi \eta + \mathcal{O}(\xi^2)$ can introduce such vertex at $\mathcal{O}(\xi^3)$, no other operators contribute at this order.

Short summary and examples for EFT analysis:

- For a pure SM gauge singlet pseudoscalar field η, effective operators $\mathcal{O}_{1,2}$ can contribute to the $ZH\eta$ vertex, which may finally appear at $\mathcal{O}(\xi^3)$.

Model	$ZH\eta$-vertex	Note
SM+Complex Singlet	0	η is pure SM gauge singlet, $f \to \infty$
2HDM	$\mathcal{O}(1)$	η is a component of SM gauge doublet
$(\mathrm{SU}(4)/\mathrm{SU}(3))^4$ LH Model*	$\mathcal{O}(\xi)$	η contains $\mathcal{O}(\xi)$ SM gauge doublet component
SLH $[(\mathrm{U}(3)/\mathrm{U}(2))^2]$ Model	$\mathcal{O}(\xi^3)$	η is pure SM gauge singlet

[* D. E. Kaplan and M. Schmaltz, JHEP 0310, 039 (2003).]

III. $ZH\eta$-VERTEX IN THE SLH MODEL

- We discuss the SLH model in details as an example of EFT analysis;
- Only two scalars, and η is a pure SM gauge singlet in this model;
- Naively calculate the $ZH\eta$ vertex, it behaves like the EFT analysis;
- We should also perform a complete formalism in kinetic diagonalization.

A. The SLH Model

- A Global symmetry breaking $(\mathrm{SU}(3)\times \mathrm{U}(1))^2 \to (\mathrm{SU}(2)\times \mathrm{U}(1))^2$ happens at a high scale f, gauge group is enlarged to $\mathrm{SU}(3)\times \mathrm{U}(1)$ which breaks to SM gauge group;

- Two scalar triplets $\Phi_{1,2}$ are nonlinear realized as:

$$\Phi_1 = \mathrm{e}^{\mathrm{i}\Theta'}\mathrm{e}^{\mathrm{i}t_\beta\Theta}\begin{pmatrix}\mathbf{0}_{1\times 2}\\ f c_\beta\end{pmatrix}, \qquad \Phi_2 = \mathrm{e}^{\mathrm{i}\Theta'}\mathrm{e}^{-\mathrm{i}\Theta/t_\beta}\begin{pmatrix}\mathbf{0}_{1\times 2}\\ f s_\beta\end{pmatrix};$$

with the definitions of the matrix fields

$$\Theta \equiv \frac{1}{f}\left(\frac{\eta\mathbb{I}_{3\times 3}}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times 2} & \phi\\ \phi^\dagger & 0\end{pmatrix}\right), \quad \text{and} \quad \Theta' \equiv \frac{1}{f}\left(\frac{\zeta\mathbb{I}_{3\times 3}}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times 2} & \varphi\\ \varphi^\dagger & 0\end{pmatrix}\right).$$

- There are 10 Goldstones and 8 of which are eaten by massive gauge bosons;
- t_β means the ratio between the VEVs of the two triplets;
- η is the pseudoscalar and ϕ is the Higgs doublet defined as above;
- ζ and $\varphi \equiv ((\sigma - \mathrm{i}\omega)/\sqrt{2}, x^-)^T$ are expected to be eaten by heavy gauge bosons;
- η can acquire its mass through $\mu^2 \Phi_1^\dagger \Phi_2$ term, and EWSB can be induced by loop corrections, heavy gauge bosons $(Z', X^\pm, Y, \bar{Y})$ can acquire their masses before EWSB, while EW gauge bosons $(W^\pm, Z)$ acquire their masses after EWSB;
- All fermion doublets should be enlarged to triplets as well, and the heavy top T domains the EWSB through loop corrections;

Some properties of the SLH parameters:

Parameter	Allowed Region	Constraints
f	$(7.5 - 84.5)$ TeV	Lower: LHC direct search*,* Upper: Goldstone scaterring unitarity**
t_β	1-8.9	Lower: Convention Upper: Goldstone scaterring unitarity**
m_η	$(0 - 1.5)$ TeV	Theoretical EWSB conditions**
m_T	(1.7-18.7)	Lower: Goldstone scaterring unitarity** Upper: Theoretical EWSB conditions**

[* ATLAS Collaboration, JHEP 1710, 182 (2017); * Y.-N. Mao, 1703.10123;
** K. Cheung, S.-P. He, Y.-N. Mao, C. Zhang, and Y. Zhou, 1801.10066.]

B. General Diagonalization Procedure

- When we expand $(D_\mu\Phi_1)^2 + (D_\mu\Phi_2)^2$, the lagrangian contains

$$\mathcal{L} \supset \underbrace{\frac{1}{2}\mathbb{K}_{ij}(\partial G_i)(\partial G_j)}_{\text{A}} + \underbrace{\mathbb{F}_{pi}V_p^\mu \partial_\mu G_i}_{\text{B}} + \underbrace{\frac{1}{2}(\mathbb{M}_V^2)_{pq}V_{p,\mu}V_q^\mu}_{\text{C}}\,.$$

- Like in the EFT analysis, there are two-point transitions in term A and B, and the two-point transition in B should be exactly canceled by the gauge fixing term;

- For term A, there are two-point kinetic mixing in CP-odd scalar sector ($G_i = \eta, \zeta, \chi, \omega$) and we need a new basis $S_i \equiv U_{ij}G_j$ which can give $\mathcal{L} \supset (\partial S_i)^2/2$;

- Introduce the inner product in the linear space spanned by S_i: $\langle S_i|S_j\rangle = \delta_{ij}$, it can be derived $\langle G_i|G_j\rangle = (\mathbb{K}^{-1})_{ij}$;

- With this relation, for $\bar{G}_p = \mathbb{F}_{pi}G_i$, we have $\langle \bar{G}_p|\bar{G}_q\rangle = (\mathbb{M}_V^2)_{pq}$;

- $\mathbb{M}_V^2$ can be diagonalized through a matrix $\mathbb{R}$ as $(\mathbb{R}\mathbb{M}_V^2 R^T)_{pq} = m_p^2\delta_{pq}$;

- Thus we have the Goldstone basis $\tilde{G}_p = \mathbb{R}_{pq}\mathbb{F}_{qi}G_i/m_p$, together with the pseudoscalar $\tilde{\eta} = \eta/\sqrt{(\mathbb{K}^{-1})_{11}}$, the basis $(\tilde{\eta}, \tilde{G}_i)$ is canonically normalized;

- Choose the gauge-fixing term $-\left(\partial_\mu \tilde{V}_p^\mu/\sqrt{\xi_p} - \sqrt{\xi_p}m_p\tilde{G}_p\right)^2/2$, $\tilde{V}_p$ are in mass eigenstates, $\tilde{G}_p$ are exactly eaten by $\tilde{V}_p$ thus they are the corresponding Goldstone fields.

C. Detailed Calculation and Results

- First, divide the matrix $\mathbb{F}$ into two parts as $\mathbb{F}_{4\times3} = \left(\tilde{f}_{1\times3}, \tilde{F}_{3\times3}\right)$, with the vector component $\tilde{f}_q = \mathbb{F}_{q\eta}$, we have the new relation

$$\tilde{G}_p = \mathbb{R}_{pq}\left(\eta \tilde{f}_q + \tilde{F}_{qk} G_k\right)/m_p, \quad \text{or} \quad G_i = (\tilde{F}^{-1}\mathbb{R}^T)_{iq} m_q \tilde{G}_q - (\tilde{F}^{-1}\tilde{f})_i \sqrt{(\mathbb{K}^{-1})_{11}}\tilde{\eta}.$$

- η component appears in the original Goldstone degrees of freedom as shown above;

- The antisymmetric VHG_i-vertex is parameterized as $\mathcal{L} \supset \mathbb{C}_{pi} V_p^\mu (G_i \partial_\mu H - H \partial_\mu G_i)$, all $\mathbb{C}_{pi}$ are elements of a 4×3 matrix $\mathbb{C}$;

- In the previous papers, choose $\xi \equiv v/f$ as usual and using the original fields, the $ZH\eta$ vertex was derived as $-g\xi/(\sqrt{2}c_W t_{2\beta}) Z^\mu (\eta\partial_\mu H - H\partial_\mu \eta) \sim \mathcal{O}(\xi)$ $\longrightarrow$Incorrect!

- Following our diagonalization procedure, we should re-derive the $ZH\eta$-vertex, define a 1×4 vector as $\Upsilon \equiv \sqrt{(\mathbb{K}^{-1})_{11}}(1, -\tilde{f}^T(\tilde{F}^{-1})^T)^T$, the $ZH\eta$ vertex should be

$$\tilde{c}_{ZH\eta}\tilde{Z}^{\mu}(\tilde{\eta}\partial_{\mu}H - H\partial_{\mu}\tilde{\eta}) \equiv (\mathbb{R}\mathbb{C}\Upsilon)\tilde{Z}^{\mu}(\tilde{\eta}\partial_{\mu}H - H\partial_{\mu}\tilde{\eta})$$

- Straightforward calculation gives $\tilde{c}_{ZH\eta} = -g\xi^3/(4\sqrt{2}c_W^3 t_{2\beta}) \sim \mathcal{O}(\xi^3)$;

- That's because $\tilde{\chi} \sim \chi + \mathcal{O}(\xi)\eta$, a corresponding contribution to $ZH\eta$-vertex then arises and at $\mathcal{O}(\xi)$ it exactly cancels the naively derived $ZH\eta$-vertex as shown in last page, just like the behavior of the EFT analysis above;

- Mass mixing between neutral gauge bosons also contribute to this vertex at $\mathcal{O}(\xi^3)$.

IV. CONCLUSIONS AND DISCUSSIONS

- Based on the EFT analysis, for a pure SM gauge singlet pseudoscalar η, the $ZH\eta$-vertex cannot arise until $\mathcal{O}(\xi^3)$ with $\xi \equiv v/f$;

- We derived this vertex in the SLH model as an example, and showed the coefficient should be $\tilde{c}_{ZH\eta} = -g\xi^3/(4\sqrt{2}c_W^3 t_{2\beta}) \sim \mathcal{O}(\xi^3)$, instead of $-g\xi/(\sqrt{2}c_W t_{2\beta}) \sim \mathcal{O}(\xi)$, which has already existed in other papers for a long time;

- All corresponding phenomenology in SLH model should also be re-considered, which will appear in my forthcoming papers soon;

- The calculation procedure can also be performed to other similar models.

Thank you! My Email: maoyn@ihep.ac.cn

Appendix

Up to $\mathcal{O}(\xi^3)$, more details during calculation are listed here:

$$\mathbb{K} = \begin{pmatrix} 1 & 0 & \frac{\sqrt{2}\xi}{t_{2\beta}} - \frac{7c_{2\beta}+c_{6\beta}}{6\sqrt{2}s_{2\beta}^3}\xi^3 & -\sqrt{2}\xi + \frac{5+3c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 \\ 0 & 1 & -\frac{\xi}{\sqrt{2}} + \frac{5+3c_{4\beta}}{12\sqrt{2}s_{2\beta}^2}\xi^3 & -\frac{2\sqrt{2}\xi^3}{3t_{2\beta}} \\ \frac{\sqrt{2}\xi}{t_{2\beta}} - \frac{7c_{2\beta}+c_{6\beta}}{6\sqrt{2}s_{2\beta}^3}\xi^3 & -\frac{\xi}{\sqrt{2}} + \frac{5+3c_{4\beta}}{12\sqrt{2}s_{2\beta}^2}\xi^3 & 1 - \frac{5+3c_{4\beta}}{12s_{2\beta}^2}\xi^2 & \frac{2\xi^2}{3t_{2\beta}} \\ -\sqrt{2}\xi + \frac{5+3c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 & -\frac{2\sqrt{2}\xi^3}{3t_{2\beta}} & \frac{2\xi^2}{3t_{2\beta}} & 1 \end{pmatrix};$$

and thus $(\mathbb{K}^{-1})_{11} = 1 + 2\xi^2/s_{2\beta}^2$;

$$\tilde{f} \;=\; gf \left(\frac{1}{\sqrt{2} c_W t_{2\beta}} \xi^2, \frac{\rho}{t_{2\beta}} \xi^2, -\xi + \frac{5+3c_{4\beta}}{6 s_{2\beta}^2} \xi^3 \right)^T ;$$

$$\tilde{F} \;=\; gf \begin{pmatrix} -\frac{\xi^2}{2\sqrt{2}c_W} & \frac{\xi}{2c_W} - \frac{(5+3c_{4\beta})\xi^3}{24 c_W s_{2\beta}^2} & \frac{\xi^3}{3c_W t_{2\beta}} \\ \sqrt{\frac{2}{3-t_W^2}} - \frac{\kappa(1+c_{2W})\xi^2}{\sqrt{2}c_{2W}} & \kappa\xi - \frac{\kappa(5+3c_{4\beta})\xi^3}{12 s_{2\beta}^2} & -\frac{2\kappa\xi^3}{3c_{2W} t_{2\beta}} \\ \frac{-2\xi^3}{3t_{2\beta}} & \frac{\sqrt{2}\xi^2}{3t_{2\beta}} & \frac{1}{\sqrt{2}} \end{pmatrix};$$

with $\quad \rho \equiv \sqrt{\dfrac{1+2c_{2W}}{1+c_{2W}}} \quad$ and $\quad \kappa \equiv \dfrac{c_{2W}}{2c_W^2\sqrt{3-t_W^2}};$

$$\Upsilon = c_{\gamma+\delta}^{-1} \begin{pmatrix} 1 \\ s_\gamma^2 t_\beta^{-1} - s_\delta^2 t_\beta \\ (c_{2\delta} t_\beta - c_{2\gamma} t_\beta^{-1}) \frac{\xi}{\sqrt{2}} \\ \frac{1}{2}(s_{2\delta} t_\beta + s_{2\gamma} t_\beta^{-1}) \end{pmatrix} = \begin{pmatrix} 1 + \frac{\xi^2}{s_{2\beta}^2} \\ -\frac{\xi^2}{t_{2\beta}} \\ -\frac{\sqrt{2}\xi}{t_{2\beta}} - \frac{3 - c_{4\beta}}{\sqrt{2} t_{2\beta} s_{2\beta}^2} \xi^3 \\ \sqrt{2}\xi + \frac{3 - c_{4\beta}}{3\sqrt{2} s_{2\beta}^2} \xi^3 \end{pmatrix};$$

with $\quad \gamma \equiv \xi t_\beta / \sqrt{2} \quad$ and $\quad \delta \equiv \xi / (\sqrt{2} t_\beta)$;

$$\mathbb{R} = \begin{pmatrix} 1 & -\frac{\kappa\rho^2\xi^2}{2c_W} & -\frac{\sqrt{2}\xi^3}{3c_W t_{2\beta}} \\ \frac{\kappa\rho^2\xi^2}{2c_W} & 1 & -\frac{2\sqrt{2}(1+2c_{2W})\kappa\xi^3}{3c_{2W}t_{2\beta}} \\ \frac{\sqrt{2}\xi^3}{3c_W t_{2\beta}} & \frac{2\sqrt{2}(1+2c_{2W})\kappa\xi^3}{3c_{2W}t_{2\beta}} & 1 \end{pmatrix};$$

$$\mathbb{C} = g \begin{pmatrix} -\frac{\xi}{\sqrt{2}c_W t_{2\beta}} + \frac{(7c_{2\beta}+c_{6\beta})\xi^3}{6\sqrt{2}c_W s_{2\beta}^3} & \frac{\xi}{2\sqrt{2}c_W} - \frac{(5+3c_{4\beta})\xi^3}{12\sqrt{2}c_W s_{2\beta}^2} & -\frac{1}{2c_W} + \frac{(5+3c_{4\beta})\xi^2}{12c_W s_{2\beta}^2} & -\frac{\xi^2}{2c_W t_{2\beta}} \\ -\frac{\rho\xi}{t_{2\beta}} + \frac{\rho(7c_{2\beta}+c_{6\beta})\xi^3}{6s_{2\beta}^3} & \frac{\rho\xi}{2} - \frac{\rho(5+3c_{4\beta})\xi^3}{12s_{2\beta}^2} & -\kappa + \frac{\kappa(5+3c_{4\beta})\xi^2}{6s_{2\beta}^2} & \frac{\kappa\xi^2}{c_{2W}t_{2\beta}} \\ \frac{1}{2} - \frac{(5+3c_{4\beta})\xi^2}{4s_{2\beta}^2} & \frac{\xi^2}{t_{2\beta}} & -\frac{\xi}{\sqrt{2}t_{2\beta}} + \frac{(7c_{2\beta}+c_{6\beta})\xi^3}{12\sqrt{2}s_{2\beta}^3} & 0 \end{pmatrix}.$$