EXCLUSIVE $|V_{cb}|$ and $R(D^*)$: A FRESH LOOK

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The importance of $|V_{cb}|$

The most important CKM unitarity test is the Unitarity Triangle (UT) $V_{cb}$ plays an important role in UT

$$\varepsilon_K \approx x |V_{cb}|^4 + \ldots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 \left[ 1 + O(\lambda^2) \right]$$

where it often dominates the theoretical uncertainty.

$V_{ub}/V_{cb}$ constrains directly the UT

Since several years, exclusive decays prefer smaller $|V_{ub}|$ and $|V_{cb}|$
STATUS of $V_{cb}$ and $V_{ub}$

- New $V_{ub}$ incl by Babar in agreement with exclusive
  PRD 95 (2017) 7, 072001

- New HPQCD $B \rightarrow D^*$ result at zero recoil
  arXiv:1711.11013

- New Belle $B \rightarrow D^*$ result: with FNAL
  $V_{cb} = 37.4(1.3) \times 10^{-3}$
  arXiv:1702.01521
LEPTON FLAVOUR UNIVERSALITY VIOLATION?

$R(D) = \frac{B(B \to D^(*) \tau \nu)}{B(B \to D^(*) \mu \nu)}$

$R(D^*) = 0.252(3)$ S. Fajfer et al. (2012)

$\Delta \chi^2 = 1.0$ contours

HFLAV FPCP 2017

$P(\chi^2) = 71.6\%$
Allow for the determination of $V_{cb}$, which drops out of $R(D,D^*)$. There are 1(2) and 3(4) FFs for $D$ and $D^*$ for light (heavy) leptons, for instance

$$\langle D|\bar{c}\gamma^\mu b|B\rangle \propto f_{+,0}(q^2)$$
MODEL INDEPENDENT FF PARAMETRIZATION

CROSSING + ANALYTICITY

PHYSICAL SEMILEPTONIC REGION
\[ m_c^2 \leq q^2 \leq (m_B - m_D)^2 \]

2-POINT CORRELATOR CUTS
\[ q^2 \geq (m_B + m_D)^2 \]

POLES AT \( q^2 = m_{Bc}^2 \) ETC

\[
\sum_{n} X_n = \alpha |f_i|^2
\]

USING QUARK-HADRON DUALITY

+ PERT CORR (+CONDENSATES)
UNITARITY CONSTRAINTS

\[ z = \frac{\sqrt{1 + w} - \sqrt{2}}{\sqrt{1 + w} + \sqrt{2}} \]

\[ w = \frac{m_B^2 + m_{D*}^2 - q^2}{2m_B m_{D*}} \]

\[ 0 < z < 0.056 \]

\[ f_i(z) = \frac{\sqrt{\chi_i}}{P(z) \phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n \]

**Blaschke Factors**
- Remove poles below threshold

**Phase Space Factors**

**Truncated at order N**

\[ \sum_{n=0}^{N} (a_n^i)^2 < 1 \]

**Weak Unitarity Constraints**
- Assuming saturation by single hadron channel

BGL Boyd Grinstein Lebed 1997
LATTICE + EXP FIT for $B \rightarrow D \ell \nu$

$|V_{cb}| = 40.5(1.0) \times 10^{-3}$, $R(D) = 0.299(3)$

- Babar 2009
- Belle 2015
- MILC-FNAL
- HPQCD

$BGL N=4$

$\chi^2/dof = 19/22$

Lattice determination of slopes

Form factors $f_+(z)$ (upper plot) and $f_0(z)$ (lower plot)
Strong Unitarity Bounds

Information on other channels makes the constraints tighter. HQS implies that all $B^*(\rightarrow D^*)$ ff either vanish or are prop to the Isgur-Wise function: any ff $F_j$ can be expressed as

$$F_j(z) = \left(\frac{F_j}{F_i}\right)_{\text{HQET}} F_i(z)$$

which leads to (hyper)ellipsoids in the $a_i$ space for S, P, V, A currents.

**Caprini Lellouch Neubert (CLN, 1998)** exploit NLO HQET relations between form factors + QCD sum rules to reduce parameters for ff... up to < 2% uncertainty, never included in exp analysis.

$$h_{A1}(z) = h_{A1}(1) \left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right]$$

nike: only 2 parameters! but theoretical uncertainty?
$|V_{cb}|$ from $B \rightarrow D^{*}l\nu$ (usual way)

So far LQCD gives only light lepton FF at zero recoil, $w=1$, where rate vanishes. Experimental results must therefore be extrapolated to zero-recoil.

Exp error only $\sim 1.3\%$: $F(1)\eta_{ew}|V_{cb}| = 35.61(45) \times 10^{-3}$
(extrapolation with CLN parameterization)

Two unquenched lattice calculations

$F(1) = 0.906(13)$  
Bailey et al 1403.0635 (FNAL/MILC)

$F(1) = 0.895(26)$  
Harrison et al 1711.11013 (HPQCD)

Using their average $0.904(12)$:

$|V_{cb}| = 39.13(75) \times 10^{-3}$

$\sim 2.9\sigma$ or $\sim 7\%$ from inclusive determination $42.00(65) \times 10^{-3}$

PG, Healey, Turczyk 2016
2017 preliminary Belle analysis

w and angular deconvoluted distributions (independent of parameterization). All previous analyses are CLN based.

\[ w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_Bm_{D^*}} \]

Slope and curvature are linked in CLN. Bands show two parametrizations both fitting data well, with 6% different \( V_{cb} \).
**HQS breaking in FF relations**

**HQET**: \( F_i(w) = \xi(w) \left[ 1 + c_{\alpha_s}^i \frac{\alpha_s}{\pi} + c_b^i \epsilon_b + c_c^i \epsilon_c + \ldots \right] \) \( \epsilon_{b,c} = \frac{\Lambda}{2m_{b,c}} \)

\( c_{b,c} \) can be computed using subleading IW functions from QCD sumrules
Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330

**RATIOS**

\[
\frac{F_j(w)}{V_1(w)} = A_j \left[ 1 + B_j w_1 + C_j w_1^2 + D_j w_1^3 + \ldots \right] \quad w_1 = w - 1
\]

Roughly \( \epsilon_c \sim 0.25, \quad \epsilon_c^2 \sim 0.06 \) but coefficients??

In a few cases we can compare these ratios with recent lattice results:
there are 5-13% differences, always > NLO correction. For ex.:

\[
\frac{A_1(1)}{V_1(1)} \bigg|_{\text{LQCD}} = 0.857(15), \quad \frac{A_1(1)}{V_1(1)} \bigg|_{\text{HQET}} = 0.966(28)
\]
The size of NLO corrections varies strongly. Some ff are protected by Luke's theorem (no $1/m$ corrections at zero recoil), others are linked by kinematic relations at max recoil to those protected.

NNLO corrections can be sizeable and are naturally $O(10-20)\%$

$$\frac{F_j(w)}{V_1(w)} = A_j \left[ 1 + B_j w_1 + C_j w_1^2 + D_j w_1^3 + \ldots \right]$$
Using strong unitarity bounds brings BGL closer to CLN and reduce uncertainties but 3.5-5% difference persists
Comparison of $R_{1,2}$ from BGL fit vs HQET+QCD sum rule predictions (with parametric + 15% th uncertainty)

black points from preliminary FNAL-MILC calculation according to Bernlochner et al 1708.07134 (before continuum and chiral extrapolations...)

**CONSISTENCY WITH HQET**
CALCULATION of $R(D^*)$

\[
\frac{d\Gamma_\tau}{dw} = \frac{d\Gamma_{\tau,1}}{dw} + \frac{d\Gamma_{\tau,2}}{dw}
\]

\[
\begin{align*}
\frac{d\Gamma_{\tau,1}}{dw} &= \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \left(1 + \frac{m_\tau^2}{2q^2}\right) \frac{d\Gamma}{dw}, \\
\frac{d\Gamma_{\tau,2}}{dw} &= k \frac{m_\tau^2(m_\tau^2 - q^2)^2 r^3(1+r)^2 (w^2 - 1)^{3/2}}{(q^2)^3} P_1(w)^2
\end{align*}
\]

$\pm 30\%!!$

\[
R(D^*) = R_{\tau,1}(D^*) + R_{\tau,2}(D^*)
\]

\[
R_{\tau,1}(D^*) = \frac{\int_{1}^{w_{\tau,max}} dw \frac{d\Gamma_{\tau,1}}{dw}}{\int_{1}^{w_{\max}} dw \frac{d\Gamma}{dw}}
\]

\[
R_{\tau,2}(D^*) = \frac{\int_{1}^{w_{\tau,max}} dw \frac{d\Gamma_{\tau,2}}{dw}}{\int_{1}^{w_{\max}} dw \frac{d\Gamma}{dw}}
\]

$w_{\max} \approx 1.56, \quad w_{\tau,max} \approx 1.35$

$P_1$ is a new FF, for which no lattice calculation is yet available, but its contribution is only $\sim 10\%$

$R_{\tau,1} \sim 90\% R_{\tau} \quad R_{\tau,2} \sim 10\% R_{\tau}$

Again, normalize $P_1$ to one of the FF with proper uncertainties

\[
P_1 = (P_1/V_1)_{\text{HQET}} V_1^{\text{exp}} \quad P_1 = (P_1/A_1)_{\text{HQET}} A_1^{\text{exp}} \quad P_1 = \xi(w)(1 + \ldots)_{\text{HQET}}
\]
<table>
<thead>
<tr>
<th>Ref.</th>
<th>$R(D^*)$</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment [HFLAV update]</td>
<td>0.304(13)(7)</td>
<td>—</td>
</tr>
<tr>
<td>2017 theory results, using new lattice and exp. data:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Bernlochner Ligeti Papucci Robinson 1703.05330]</td>
<td>0.257(3)</td>
<td>3.1σ</td>
</tr>
<tr>
<td>Our result [Bigi Gambino Schacht 1707.09509]</td>
<td>0.260(8)</td>
<td>2.6σ</td>
</tr>
<tr>
<td>[Jaiswal Nandi Patra 1707.09977]</td>
<td>0.257(5)</td>
<td>3.0σ</td>
</tr>
<tr>
<td>2012 theory results:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Fajfer Kamenik Nisandzic 1203.2654]</td>
<td>0.252(3)</td>
<td>3.5σ</td>
</tr>
<tr>
<td>[Celis Jung Li Pich 1210.8443]</td>
<td>0.252(2)(3)</td>
<td>3.4σ</td>
</tr>
<tr>
<td>[Tanaka Watanabe 1212.1878]</td>
<td>0.252(4)</td>
<td>3.4σ</td>
</tr>
</tbody>
</table>
SUMMARY

• **Is the $V_{cb}$ puzzle resolved?** No, but a few pieces fit together. The uncertainty of $B \rightarrow D^{*}\ell \nu$ was **underestimated** and the result was likely biased: old data should be reanalysed.

• We revisited main ideas behind CLN, using LQCD & exp results and conservative theory uncertainties, and obtained new strong unitarity bounds. We do not give a simplified parametrization. Our results provide a **framework for future exp analyses**. *Lattice will soon settle the matter* with calculations at non-zero recoil.

• For $R(D^*)$ we know little about $P_1$ and we have to rely on HQET + QCD sum rules. Hence a larger uncertainty, but the **anomaly persists**. The upcoming LQCD determination of $P_1$ at zero recoil could cut the uncertainty by $\sim 2$. 
BACKUP
Role of HQET relations in $V_{cb}$ extraction (prelim Belle data only)

- “practical” CLN: $|V_{cb}| = 38.2(1.5) \times 10^{-3}$ [1,5,6,7,8]
- CLN+QCD sumrule errors+B→D $|V_{cb}| = 38.5(1.1) \times 10^{-3}$ [2]
- same + lattice at non-zero recoil $|V_{cb}| = 39.3(1.0) \times 10^{-3}$ [2]
- BGL+HQET+B→D with nuisance $|V_{cb}| = 40.9(0.9) \times 10^{-3}$ [3]
- BGL+strong unitarity $|V_{cb}| \sim 40.8(1.5) \times 10^{-3}$ [4]
- BGL+weak unitarity $|V_{cb}| = 41.7(2.0) \times 10^{-3}$ [5,6,7,8]

References:

[1] Belle coll. 1702.01521
[2] Bernlochner et al. 1703.05330
[4] Bigi, Gambino, Schacht 1707.09509
[5] Bigi, Gambino, Schacht 1703.06124
[8] Grinstein, Kobach, 1703.08170
Important endpoint constraint

\[ P_1(w_{\text{max}}) = A_5(w_{\text{max}}) = 0.545 \pm 0.025 \]

\[ R(D^*) = 0.260(5)(6) = 0.260(8) \]

Consistent with previous estimates but with larger uncertainty