Nuclear Structure and Reactions: Building Together for the Future

9 October 2017 GANIL

Marcella Grasso

From dilute matter to the equilibrium point in the energy-density-functional theory

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Present collaborators along this research line:

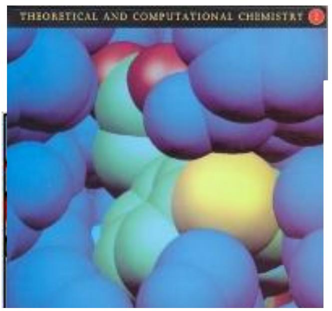
- ENSAR2, JRA TheoS (Theoretical Support for Nuclear Facilities in Europe), Task: Development of suitable effective interactions in mean-field and BMF theories

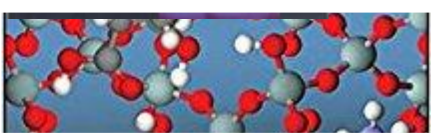
- Laboratoire Internationale LIA COLL-AGAIN

- J. Bonnard, A. Boulet,,U. van Kolck, D. Lacroix, O. Vasseur, J. Yang (IPN Orsay)

- G. Colò, X. Roca-Maza (Univ. of Milano)

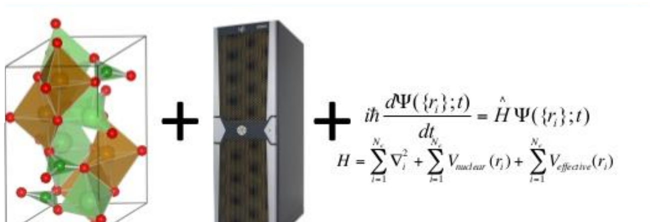
Density Functional Theory in chemistry and solid state physics





Nuclear many-body problem with effective interactions

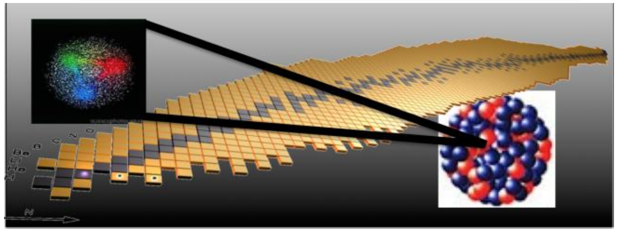
Energy Density Functional (EDF) theory (functionals derived in most cases from effective phenomenological interactions)
… since several decades



Mean-field models and beyond

Double counting …

Work on EDF designed for beyond-mean-field models Nuclear matter
Bridging with EFT/ ab initio



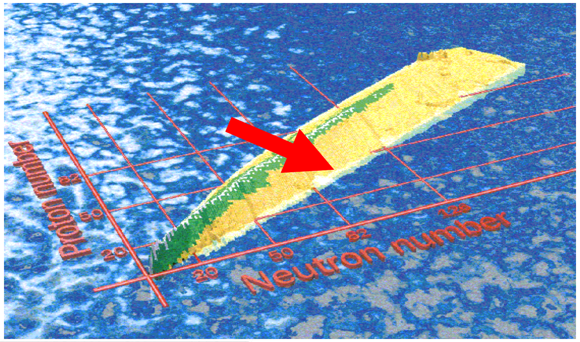
OUTLINE

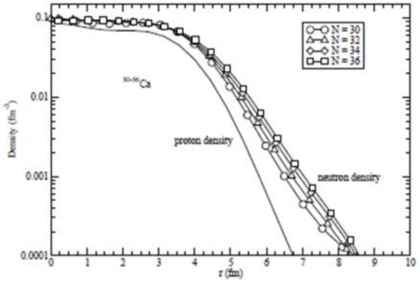
- **Perturbative many-body problem (beyond mean field)**

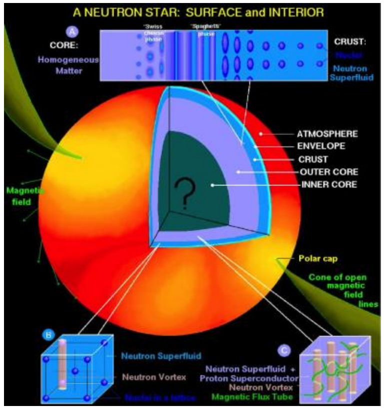
Work on EDF designed for beyond-mean-field models

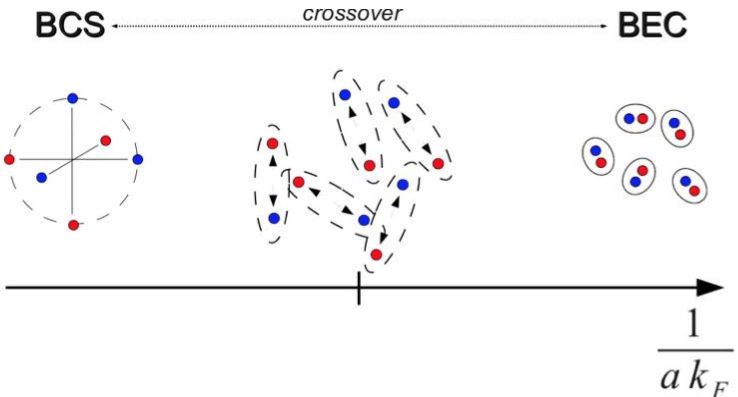
Divergences, instabilities, double counting -> bridging with EFT -> regularization procedures and parameters adjustment (around the saturation density)

Properties of matter: relevant for constraining energy density functionals









Properties of matter: relevant for constraining energy density functionals

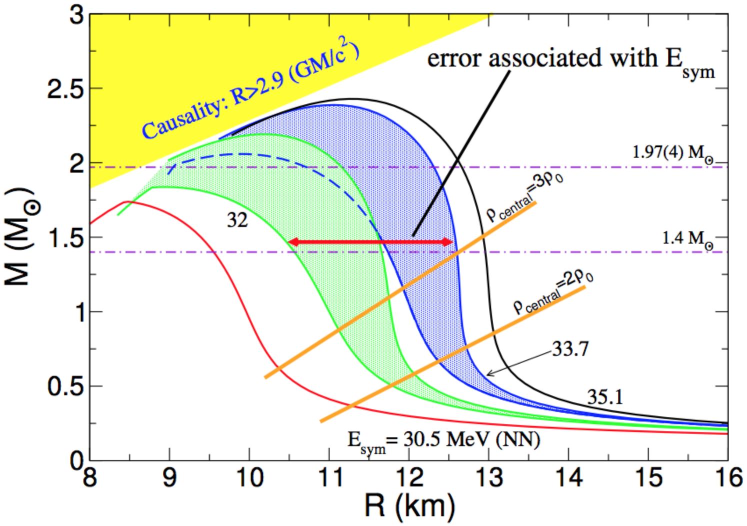
- **Around the saturation point and beyond.**

The structure of neutron stars may be calculated by solving the Tolmann-Oppenheimer-Volkoff equations.

Pressure (first derivative of the EOS) enters in such equations. The total radius is provided by the point where the pressure vanishes:

neutron star mass/radius,

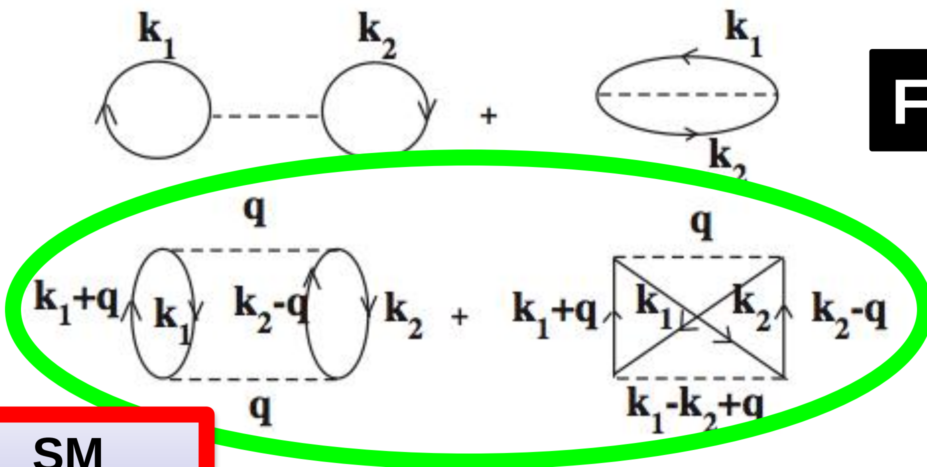
symmetry energy and its density dependence



- Moghrabi, Grasso, Colo', Van Giai, PRL 105, 262501 (2010)
- First application to finite nuclei: Brenna et al. PRC 90, 044316 (2014)
- Yang, Grasso, Roca-Maza, et al., PRC 94, 034311 (2016)
- Grasso, Lacroix, van Kolck, Invited Comment, Phys. Scr. 91, 063005 (2016)

Equation of state of nuclear matter with a Skyrme-type interaction.

The perturbative many-body problem:



First order

Second order

$k_F = (3/2\ \pi^2\ \rho)^{1/3}$ SM

$k_F = k_N = (3\pi^2\ \rho)^{1/3}$ NM

k_F -> Fermi momentum

This second-order contribution diverges ->

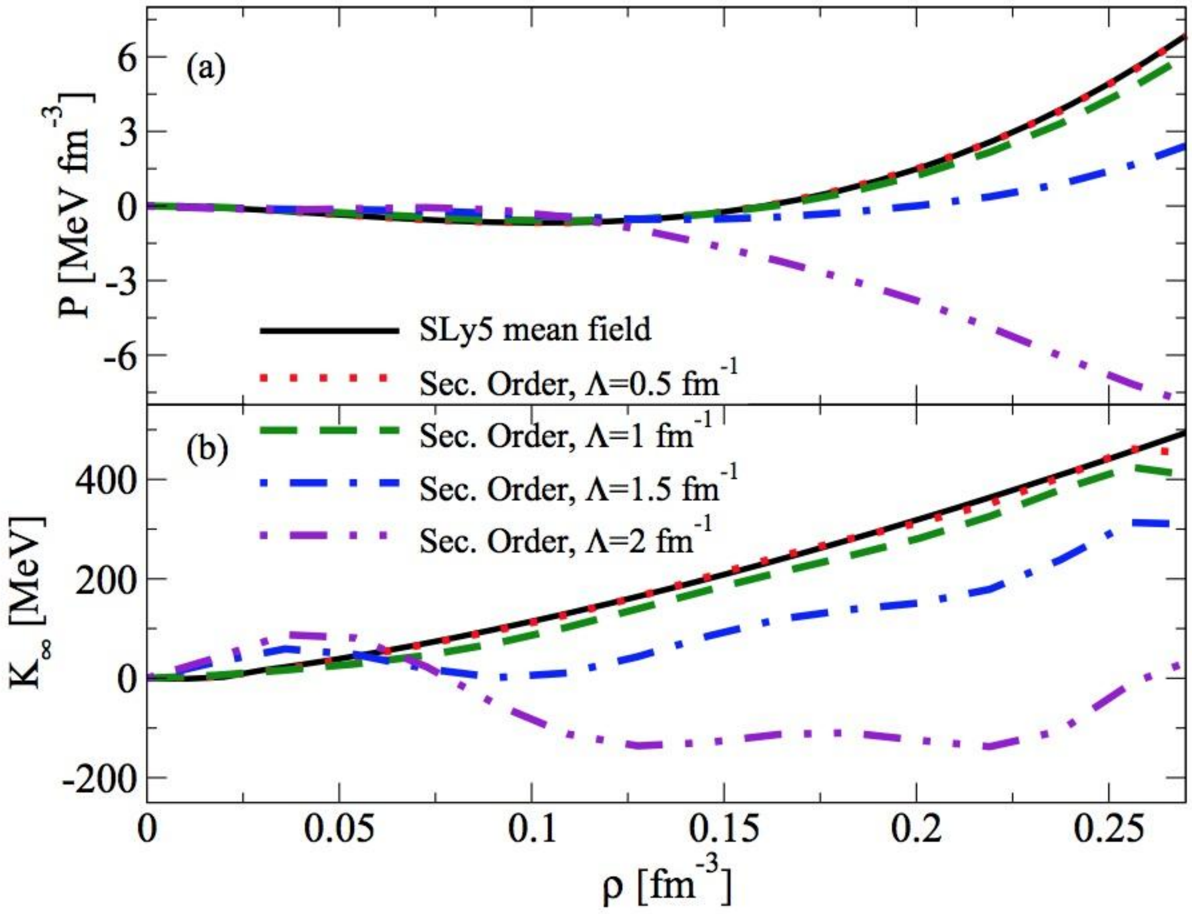
For ex. the square of the Skyrme t_0 term has a k_F^4 dependence and is the sum of a finite part plus a term linearly dependent on the cutoff (on the transferred momentum q) -> REGULARIZATION

Skyrme interaction for matter (no spin orbit at the mean-field level)

$$v = t_0(1 + x_0 P_\sigma) + \frac{1}{2} t_1 (1 + x_1 P_\sigma)(\mathbf{k}'^2 + \mathbf{k}^2) + t_2(1 + x_2 P_\sigma)\mathbf{k}' \cdot \mathbf{k} + \frac{1}{6} t_3 (1 + x_3 P_\sigma)\rho^\alpha$$

Spin-exchange operator ➡ $P_\sigma = \frac{1}{2}(1 + \sigma_1 \cdot \sigma_2)$

Double counting and ultraviolet divergence (pressure and incompressibility)



(a) Second-order pressure; (b) second-order incompressibility modulus. The used parameters are those of SLy5.

PRESSURE

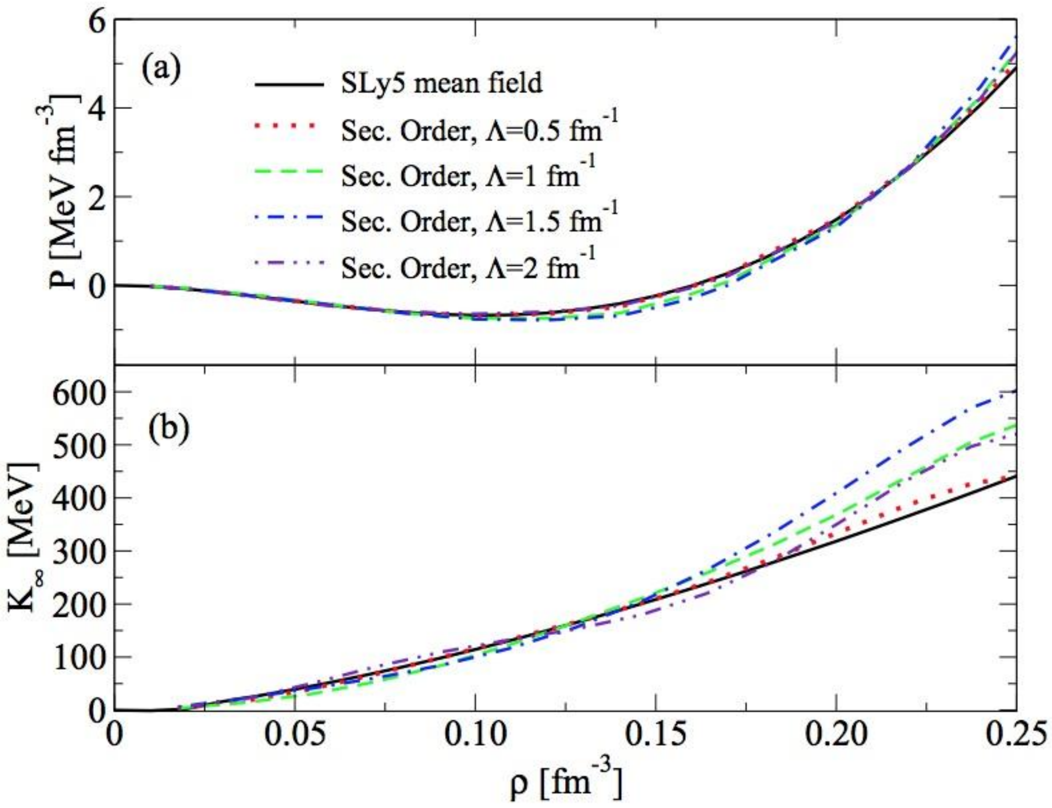
$$P(\rho,\Lambda) = \rho^2 \frac{d}{d\rho}\frac{E}{A}(\rho,\Lambda)$$

$$K(\rho,\Lambda) = 9\rho^2 \frac{d^2}{d\rho^2}\frac{E}{A}(\rho,\Lambda)$$

INCOMPR.

Yang, Grasso, Roca-Maza, et al, PRC 94, 034311 (2016)

Pressure and incompressibility (not entering in the fit)



Pressure (a) and incompressibility modulus (b) computed with the parameters of the simultaneous fit and compared with the mean-field curves.

PRESSURE

INCOMPRESSIBILITY:

BENCHMARK: 229.9 MeV

Max deviation: 25 MeV

Yang, Grasso, Roca-Maza, et al, PRC 94, 034311 (2016)

Low-density regime …
at leading order

Low-density for neutron matter (EFT satisfies this regime -> Hammer, Furnstahl, NPA 678, 277 (2000))

Lee-Yang expansion in (ak_N). Low-density EOS

Lee and Yang, Phys. Rev. 105, 1119 (1957)

$$\frac{E}{N} = \frac{\hbar^2 k_N^2}{2m}\left[\frac{3}{5} + \frac{2}{3\pi}(k_N a) + \frac{4}{35\pi^2}(11 - 2ln2)(k_N a)^2\right]$$

α = 1/3

Skyrme term	kN dependence in the EOS
t_0	k_N^3
t_3	$k_N^{3\alpha+3}$
t_1 and t_2	k_N^5

We have to constrain the parameters in the following way:

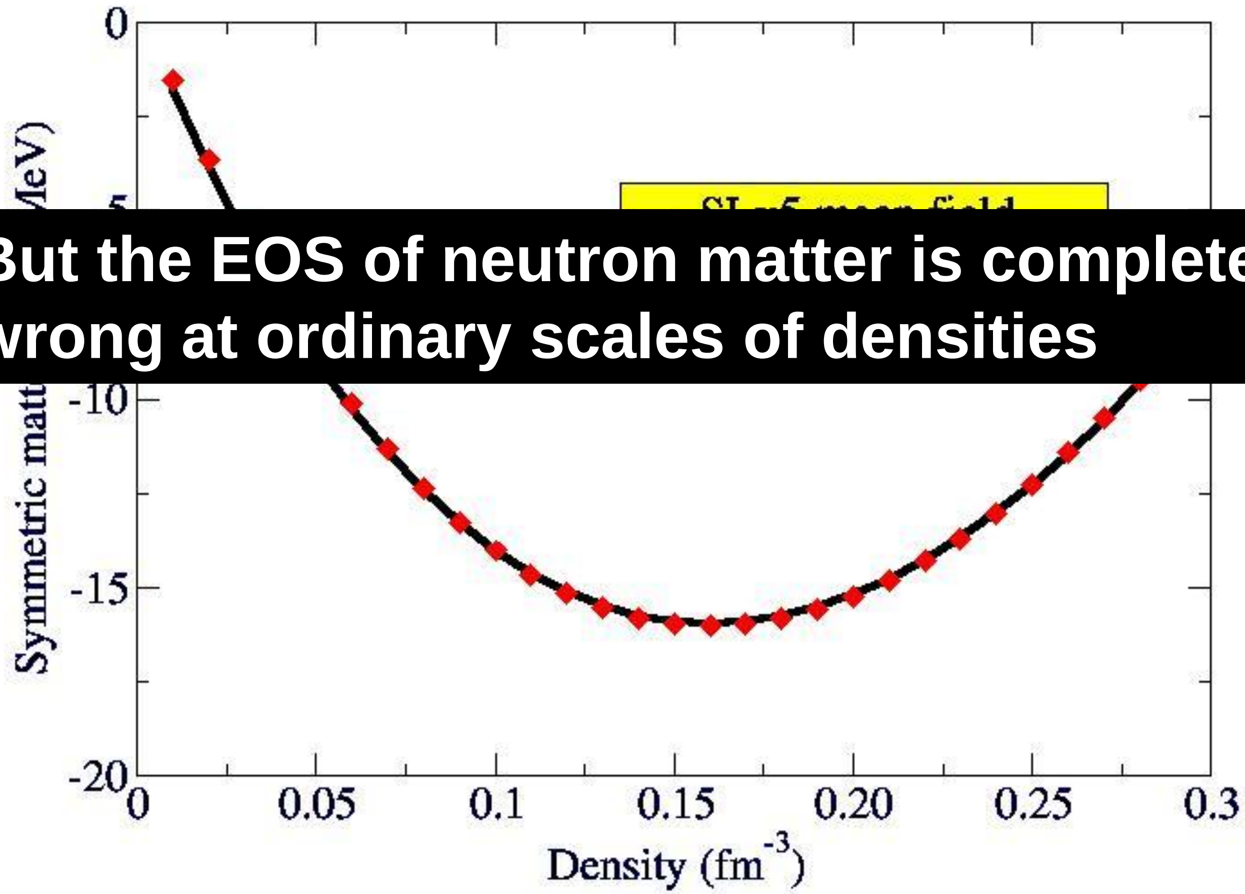
$$t_0(1-x_0) = 4\pi\hbar^2 a/m.$$

$$t_3(1-x_3) = \frac{\hbar^2}{m}\frac{144}{35}(3\pi^2)^{1/3}(11-2ln2)a^2.$$

$$t_0 = \frac{-9849.45}{1-x_0} MeV fm^3,$$

$$t_3 = \frac{1812705.02}{1-x_3} MeV fm^4.$$

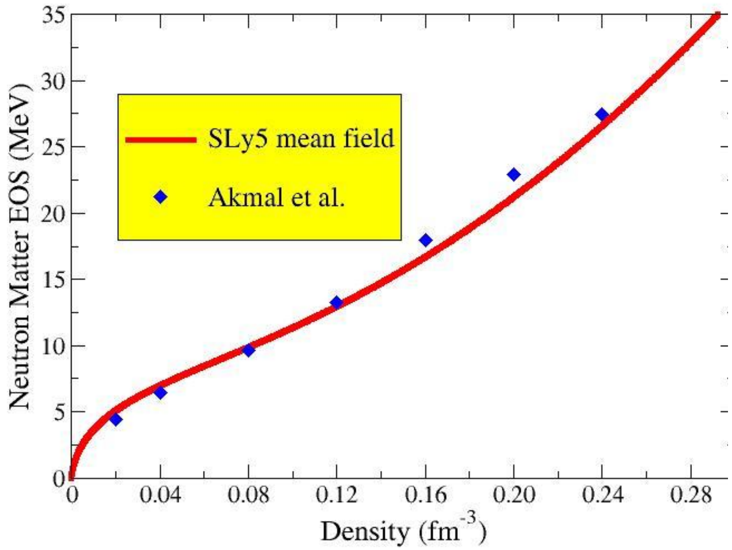
It is possible to constrain the low-density behavior of neutron matter, with α=1/3, and to adjust x0 and x3 for reproducing a reasonable EOS for symmetric matter (at ordinary densities)



But the EOS of neutron matter is completely wrong at ordinary scales of densities

Yang, Grasso, Lacroix, PRC 94 , 031301(R) (2016)

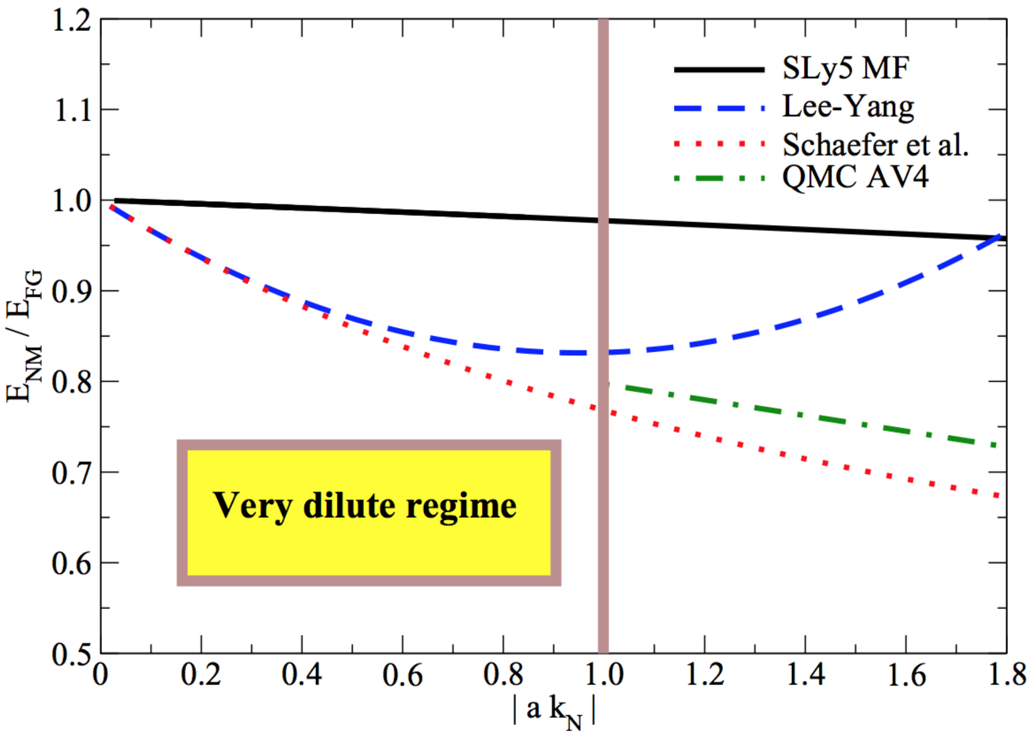
Neutron matter at 'usual' density scales. Example of Lyon-Saclay forces adjusted on the neutron EOS



SLy5 -> Chabanat et al. NPA 627, 710 (1997); 635, 231 (1998), 643, 441 (1998)
Akmal et al. -> PRC 58, 1804 (1998)

Low-density regime

$$\frac{E}{N} = \frac{\hbar^2 k_N^2}{2m}\left[\frac{3}{5} + \frac{2}{3\pi}(k_N a) + \frac{4}{35\pi^2}(11 - 2ln2)(k_N a)^2\right]$$



Neutron matter energy divided by the free gas energy

The second-order contribution has the required k_F^4 term

… but only second order is not enough (if one wants to keep the correct value of the scattering length)

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NUCLEAR PHYSICS A

Many body methods and effective field theory

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Available online 30 August 2005

Resummation techniques

- Steele, arXiv: nucl-th/0010066v2
- Kaiser, NPA 860, 41 (2011)
- Schaefer, NPA 762, 82 (2005)

Effective field theories capable of describing systems with anomalously large scattering lengths require summing an infinite number of Feynman diagrams at leading order …

Guided by:

1) **The fact that the second-order t_0 contribution leads to the correct dependence on the Fermi momentum in neutron matter;**
2) **The resumed formulae;**
3) **Good properties of Skyrme functionals**

A hybrid functional, YGLO (Yang, Grasso, Lacroix, Orsay): Matching the low-density limit with the Lee-Yang expansion of the energy (at leading order)

$$\mathcal{V} = \frac{B_\beta \rho^2}{1 - R_\beta \rho^{1/3} + C_\beta \rho^{2/3}} + D_\beta \rho^{8/3} + F_\beta \rho^{\alpha+2}$$

Beta = 1 - > symmetric matter
Beta =0 -> neutron matter

YGLO functional: Inspired by resumed expressions (resumed functional) (EFT)

EOS for symmetric and

Constrained by the first two terms of Lee Yang formula (scattering length)

$$\frac{E}{A} = K_\beta + \frac{B_\beta \rho}{1 - R_\beta \rho^{1/3} + C_\beta \rho^{2/3}} + D_\beta \rho^{5/3} + F_\beta \rho^{\alpha+1}$$

Resumed expression

Mimic velocity- and density-dependent terms

Other parameters adjusted on QMC results at extremely low densities and on Friedman et al. or Akmal et al. EOSs at higher densities

Yang, Grasso, Lacroix, PRC 94 , 031301(R) (2016)

B and R are fixed by imposing to recover the Lee-Yang formula (the analog for symmetric matter may be found in Fetter-Walecka book)

$$B_\beta = 2\pi \frac{\hbar^2}{m} \frac{(\nu - 1)}{\nu} a, \quad R_\beta = \frac{6}{35\pi} \left(\frac{6\pi^2}{\nu} \right)^{\frac{1}{3}} (11 - 2\ln 2) a,$$

$\nu = 2\ (4)$ is the degeneracy for $\beta = 0\ (1)$

The other adjusted parameters

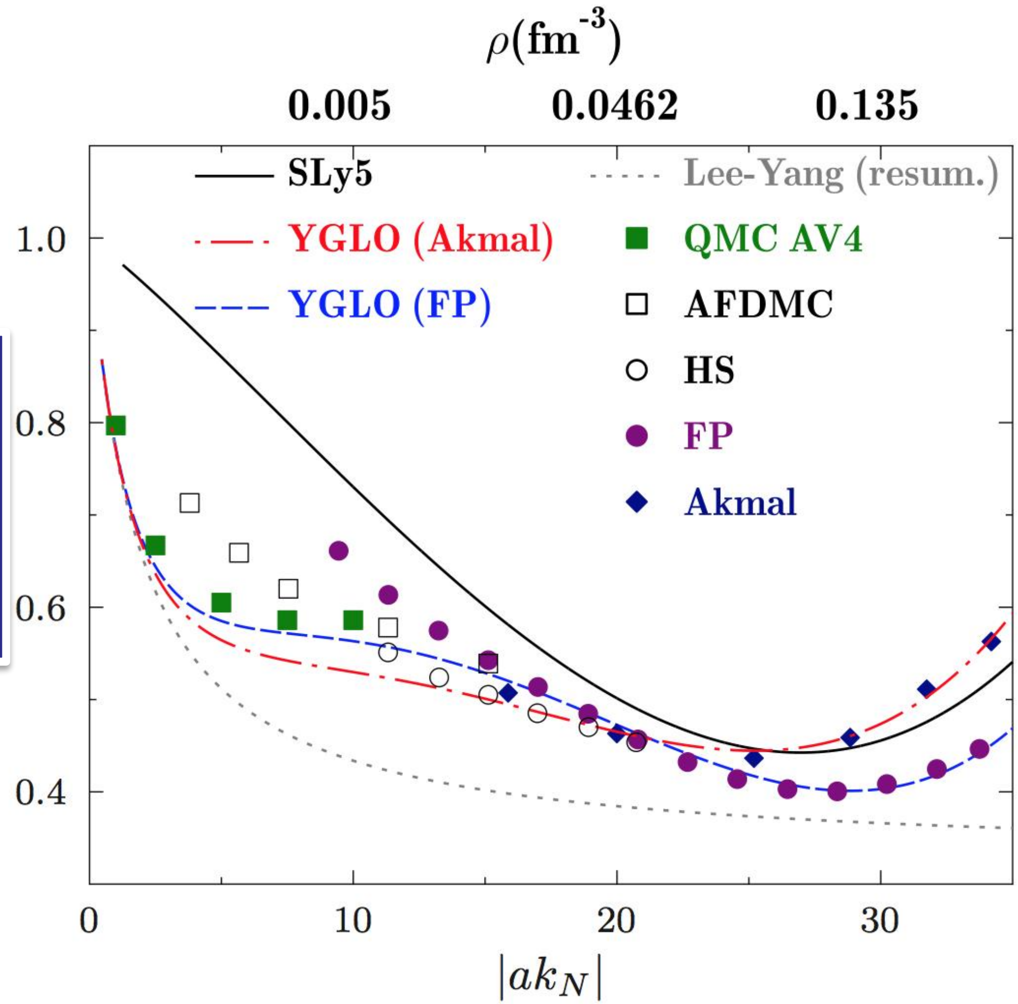
YGLO functional. In all cases, $\alpha = 0.7$.

	C_β (fm^2)	D_β (MeV fm^5)	F_β (MeV $\mathrm{fm}^{3+3\alpha}$)
$\beta = 0$ (FP)	100.87	−9264.18	9571.90
$\beta = 0$ (Akmal)	70.19	−8377.83	8743.85
$\beta = 1$ (FP)	8.188	−6624.87	6995.46

Benchmark data are (i) for neutron matter, the QMC AV4 results of Ref. [17] for values of $|ak_N| < 10$ ($\rho < 0.05\ \mathrm{fm}^{-3}$), and two different sets of results for $|ak_N| > 10$: the Friedman *et al.* results (FP) of Ref. [18] or the Akmal *et al.* results (stiffer EOS) of Ref. [19] [we call here the corresponding parameter sets YGLO (FP) and YGLO (Akmal), respectively]. (ii) For symmetric matter, the FP results and those of Akmal *et al.* are very close from each others and we made a fit using only the FP points.

YGLO. Very low-density behavior of neutron matter

Energy divided by the free gas energy



Yang, Grasso, Lacroix, PRC 94, 031301(R) (2016)

Asymmetric matter

Parabolic approximation

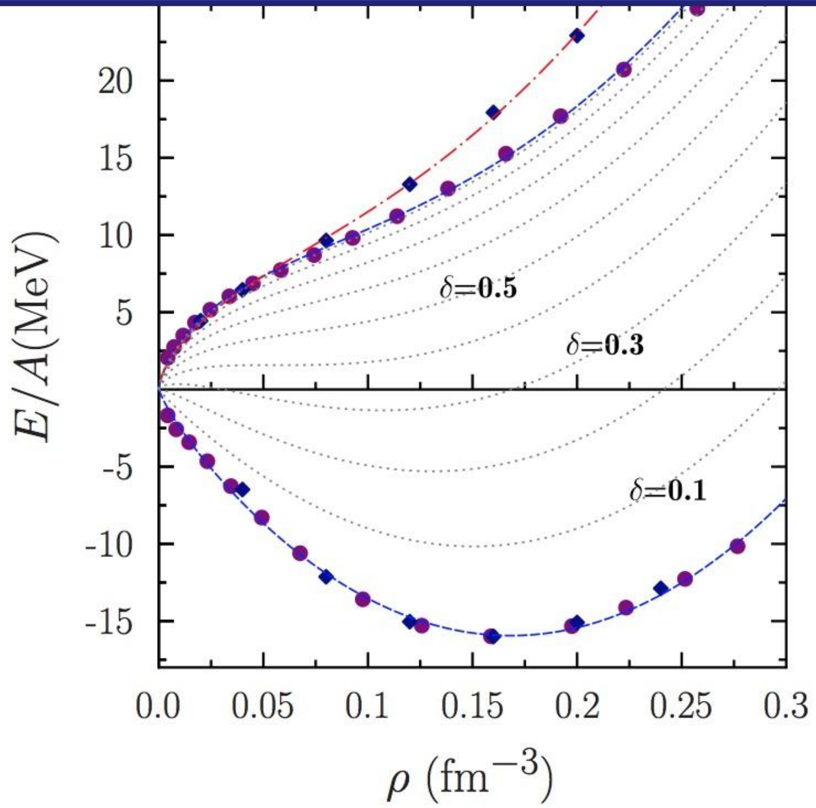
$$\frac{E_\delta}{A}(\rho) = \frac{E_{SM}}{A}(\rho) + S(\rho)\delta^2$$

$$\delta = (\rho_N - \rho_P)/(\rho_N + \rho_P)$$

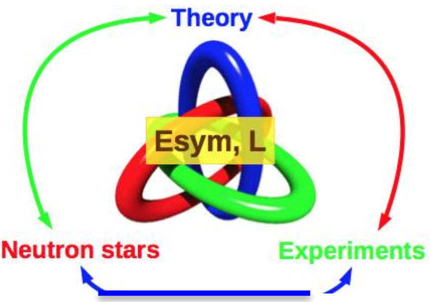
Neutron density

Proton density

Asymmetric matter



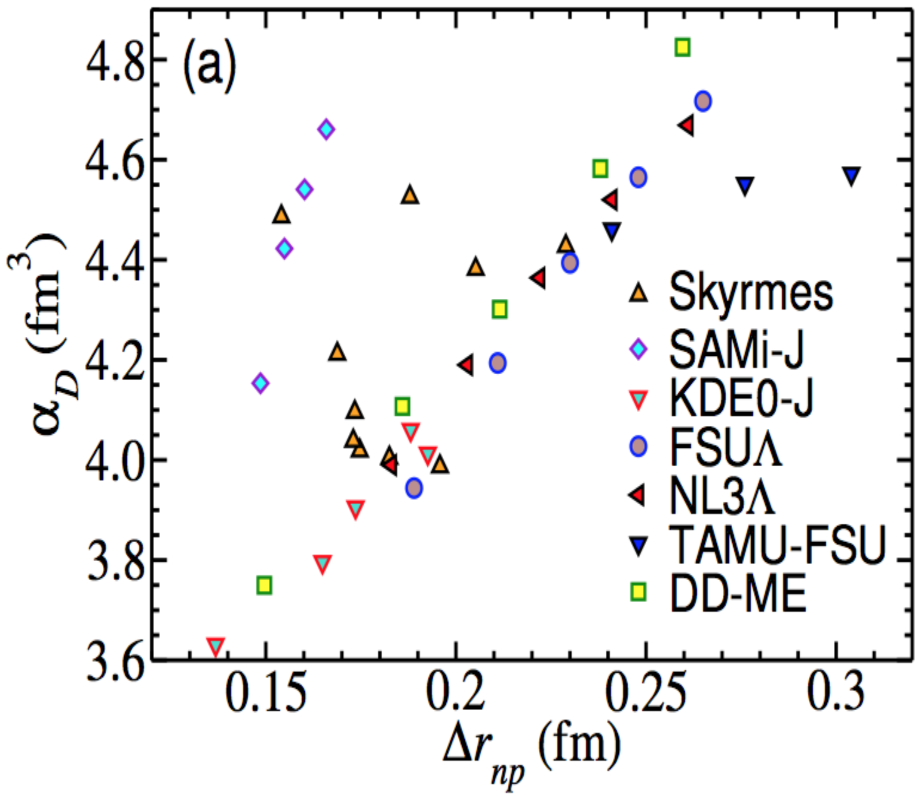
EOSs from Akmal *et al.* [19] (blue diamonds) and Friedman *et al.* (purple circles) in symmetric and neutron matter compared to the YGLO (Akmal) (red dot-dashed curve) and YGLO (FP) (blue dashed curve) results. The different gray dotted curves correspond to the YGLO(FP) EOSs obtained for different asymmetry δ from 0.1 to 0.9 by steps of 0.1 (see text).

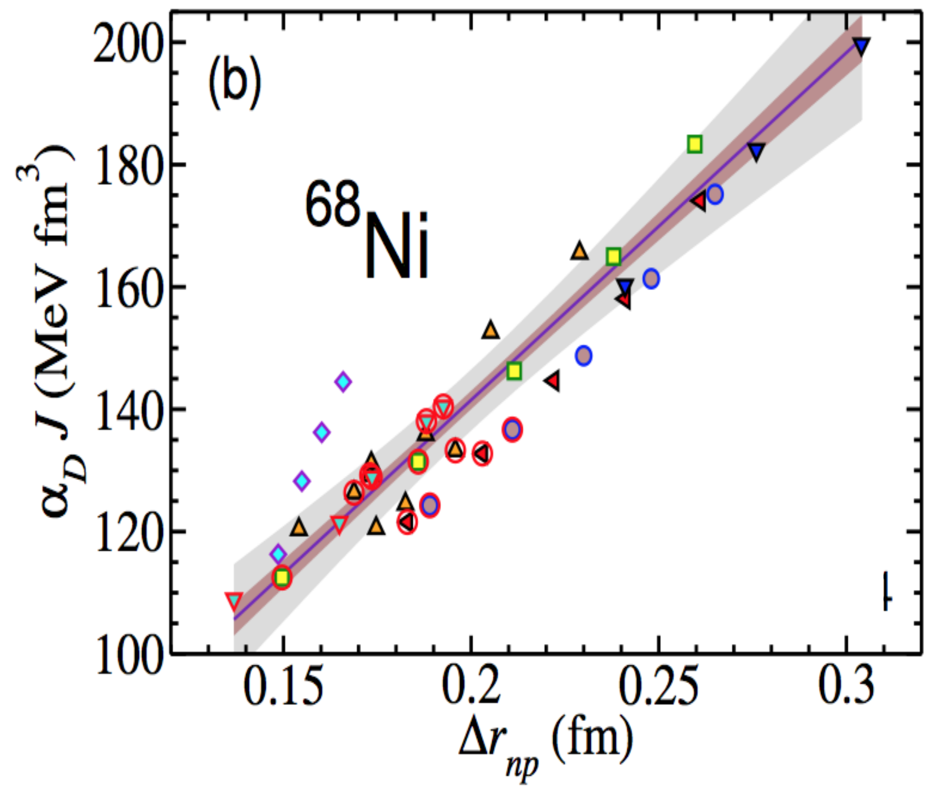


Neutron skin thickness (difference between rms radii of neutrons and protons)

Dipole polarizability versus neutron skin thickness

Dipole polarizability times symmetry energy versus neutron skin thickness





Roca-Maza et al, PRC 92, 064304 (2015)

Symmetry energy and its slope $L=3\rho_0 \, (dS/d\rho)_{\rho=\rho_0}$

Strong correlation observed between the neutron skin thickness and the slope L of the symmetry energy (see for instance: Warda et al. PRC 80, 024316 (2009), Centelles et al. PRL 102, 122502 (2009) ->

This correlation is thus expected to exist between the electric dipole polarizability times the symmetry energy and the slope of the symmetry energy

Recent experimental determinations of the electric dipole polarizability:

- ^{208}Pb (polarized proton inelastic scattering at forward angles, RCNP) (Tamii et al. PRL107, 062502 (2011)). Combining all available data: $\alpha_D = 20.1 \pm 0.6$ fm^3

- - ^{120}Sn (polarized proton inelastic scattering at forward angles, RCNP) (Hashimoto et al. PRC 92, 031305 (2015)). Combining all available data: $\alpha_D = 8.93 \pm 0.36$ fm^3

- ^{68}Ni (Coulomb excitation in inverse kinematics and invariant mass in one- and two-neutron decay channels, GSI) (Wieland et al, PRL 102, 092502 (2009); Rossi et al. PRL 111, 242503 (2013)). $\alpha_D = 3.40 \pm 0.23$ fm^3

Using the experimental values of the electric dipole polarizability in the three nuclei

^{208}Pb -> $J=(24.5 \pm 0.8)+(0.168 \pm 0.007)\,L$

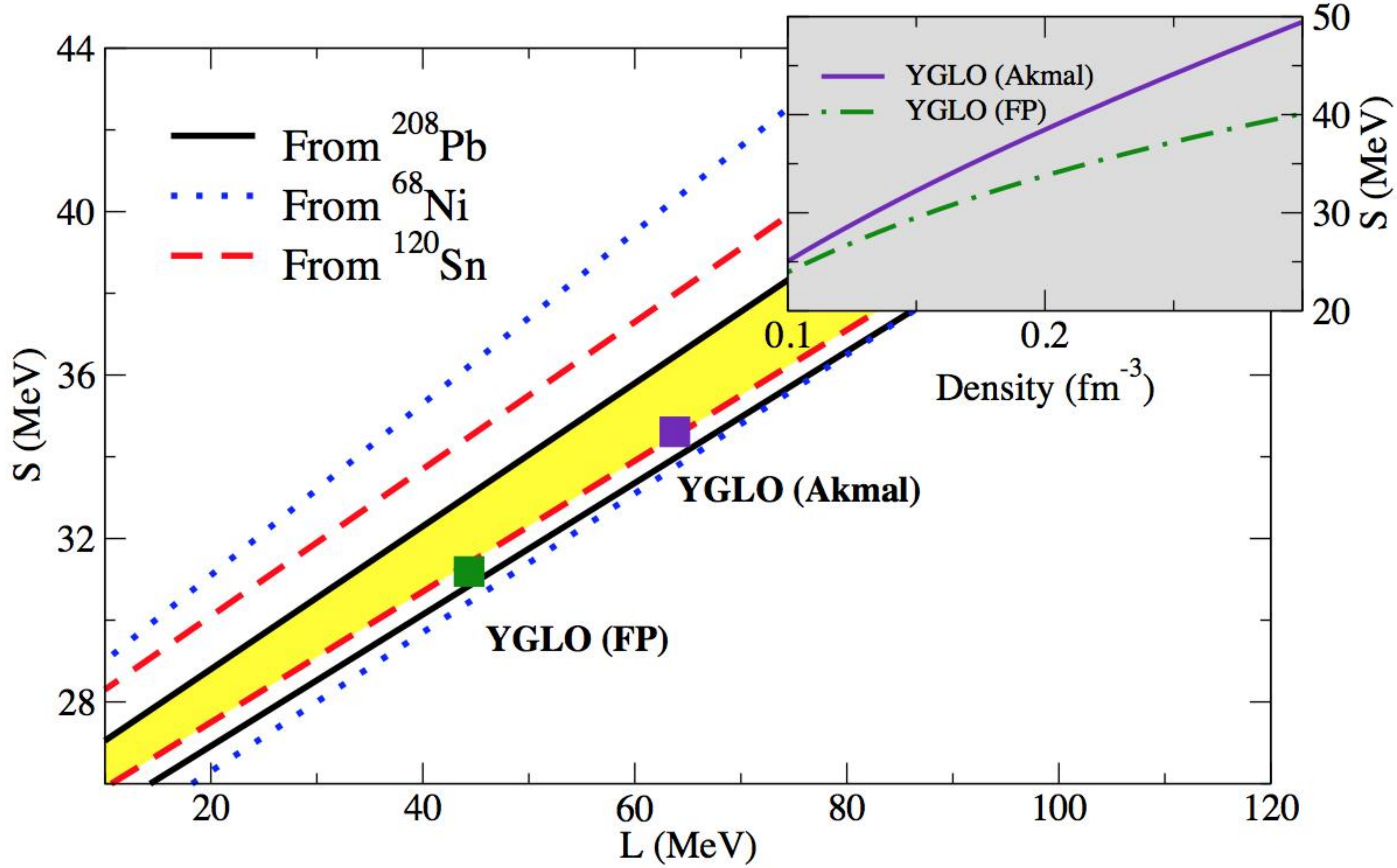
^{68}Ni -> $J=(24.9 \pm 2.0)+(0.19 \pm 0.02)\,L$

^{120}Sn -> $J=(25.4 \pm 1.1)+(0.17 \pm 0.01)\,L$

Roca-Maza et al, PRC 92, 064304 (2015)

Symmetry energy and its slope

Lines delimit the phenomenological areas constrained by the exp. determination of the electric dipole polarizability



Yang, Grasso, Lacroix, PRC 94, 031301(R) (2016)

… Without resummation, how to handle the different density scales ?

… A Lee-Yang type expression for the EOS of neutron matter-> if the low-density regime is always satisfied

$$\frac{E}{N} = \frac{\hbar^2 k_F^2}{2m}\left[\frac{3}{5} + \frac{2}{3\pi}(k_F a) + \frac{4}{35\pi^2}(11 - 2\ln 2)(k_F a)^2 + \frac{1}{10\pi}(k_F r_s)(k_F a)^2 + 0.019(k_F a)^3\right]$$

s-wave scattering length

Associated effective range

We choose to keep terms containing only the s-wave scattering length. The next term in the Lee-Yang expansion contains the p-wave scattering length

Grasso, Lacroix, Yang,Phys. Rev. C 95, 054327 (2017)

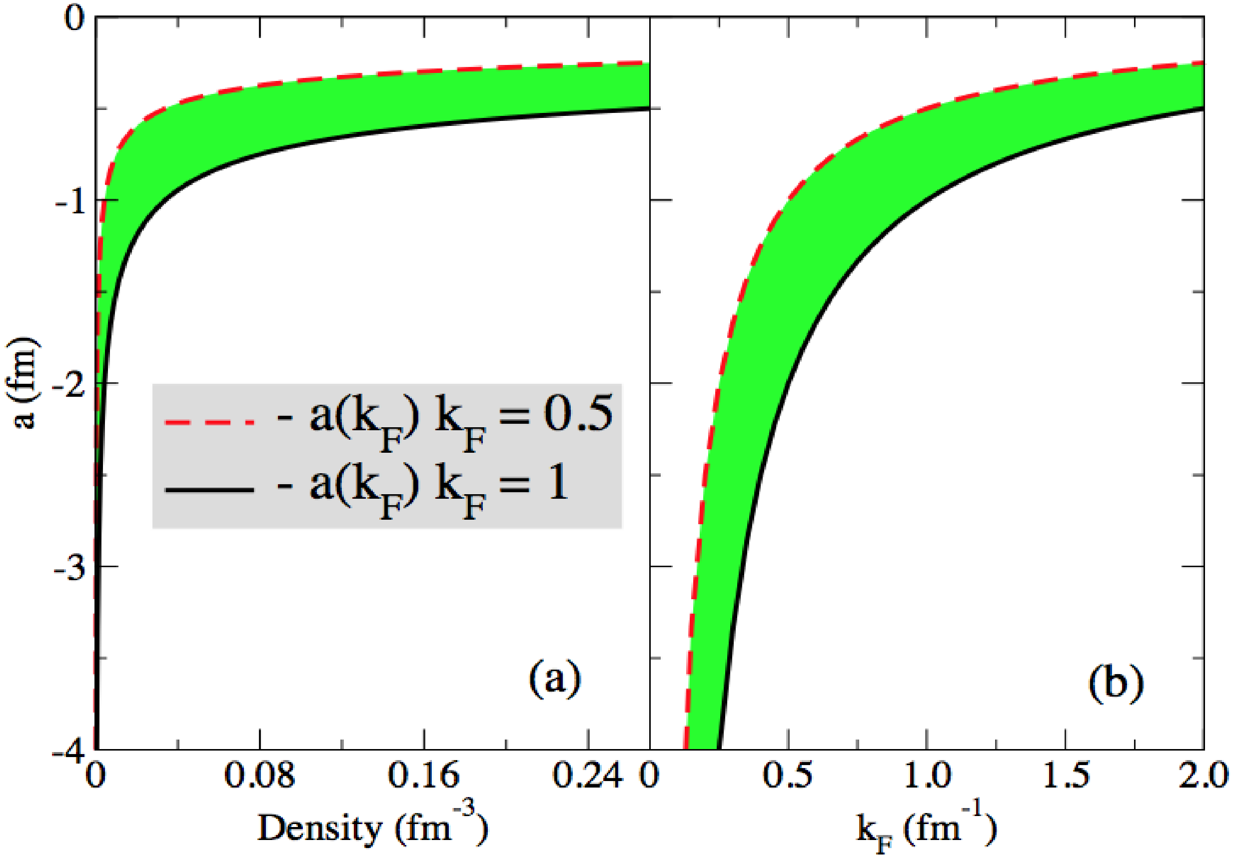
Neutron-neutron scattering length.

Low-density regime: $|ak_F| < 1.$

We impose a low-density constraint: - a k_F=1 ->

- **a =-18.9 fm up to a max momentum so that 18.9 k_F=1.**
- **Beyond this value, we tune the scattering length so that –a = 1/k_F**

Neutron-neutron scattering length



Grasso, Lacroix, Yang, PRC 95, 054327 (2017).

We introduce a Skyrme-type functional containing only s-wave terms and leading, at the mean-field level, to a neutron matter EOS given by the LY expression, with the relations :

$$
\begin{aligned}
t_0(1-x_0) &= \frac{4\pi\hbar^2}{m}a, \\
t_3(1-x_3) &= \frac{144\hbar^2}{35m}(3\pi^2)^{1/3}(11-2\ln 2)a^2 \\
t_1(1-x_1) &= \frac{2\pi\hbar^2}{m}(a^2 r_s + 0.19\pi a^3),
\end{aligned}
$$

The power of the density-dependent term is chosen equal to 1/3

Grasso, Lacroix, Yang, PRC 95, 054327 (2017).

We require that:

(i) **The functional correctly describes neutron matter at all density scales**

(ii) **The functional leads to a reasonable EOS for symmetric matter around the equilibrium point**

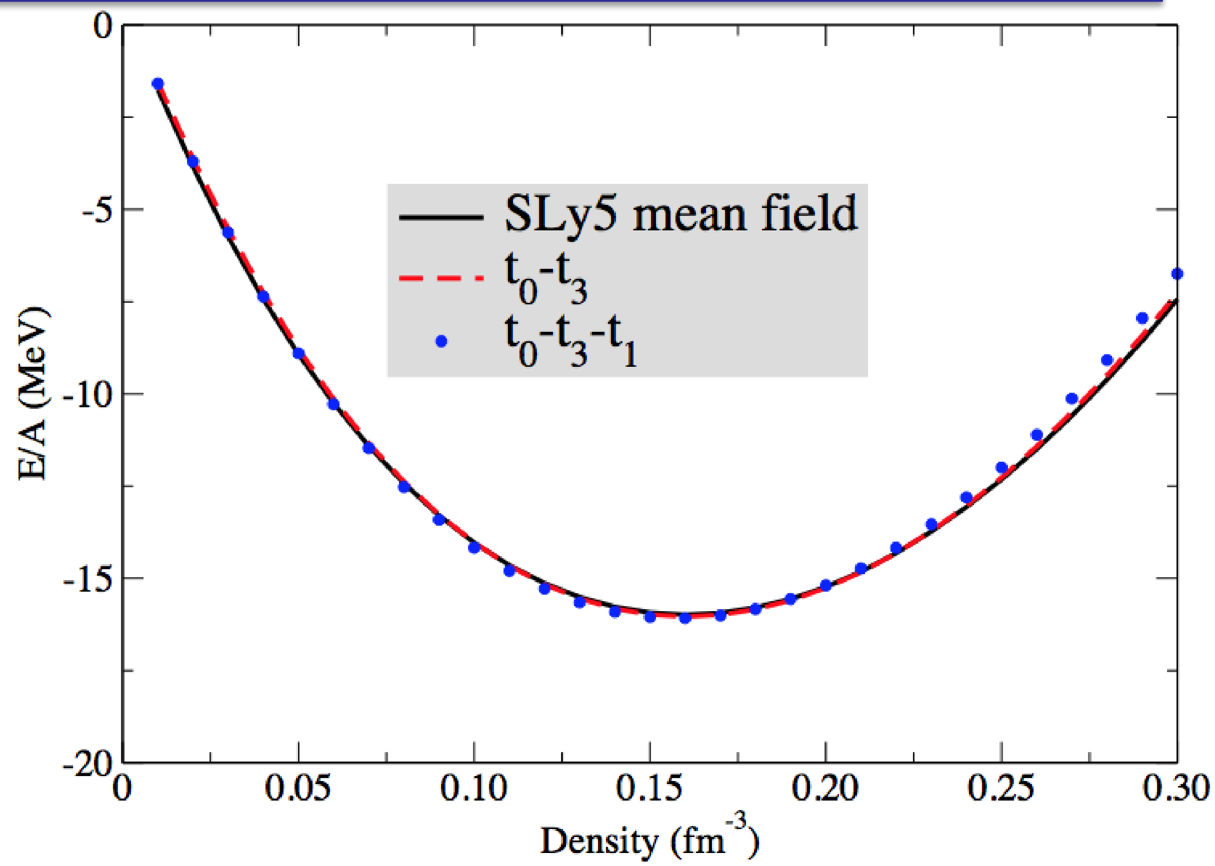
This may be obtained by imposing a low-density regime everywhere (with a density-dependent neutron-neutron scattering length)

The parameters x_i do not enter in the EOS of symmetric matter.

We may thus adjust the parameters t_i to have a reasonable EOS of symmetric matter and tune the neutron-neutron scattering length by imposing, at each density scale, a low-density constraint

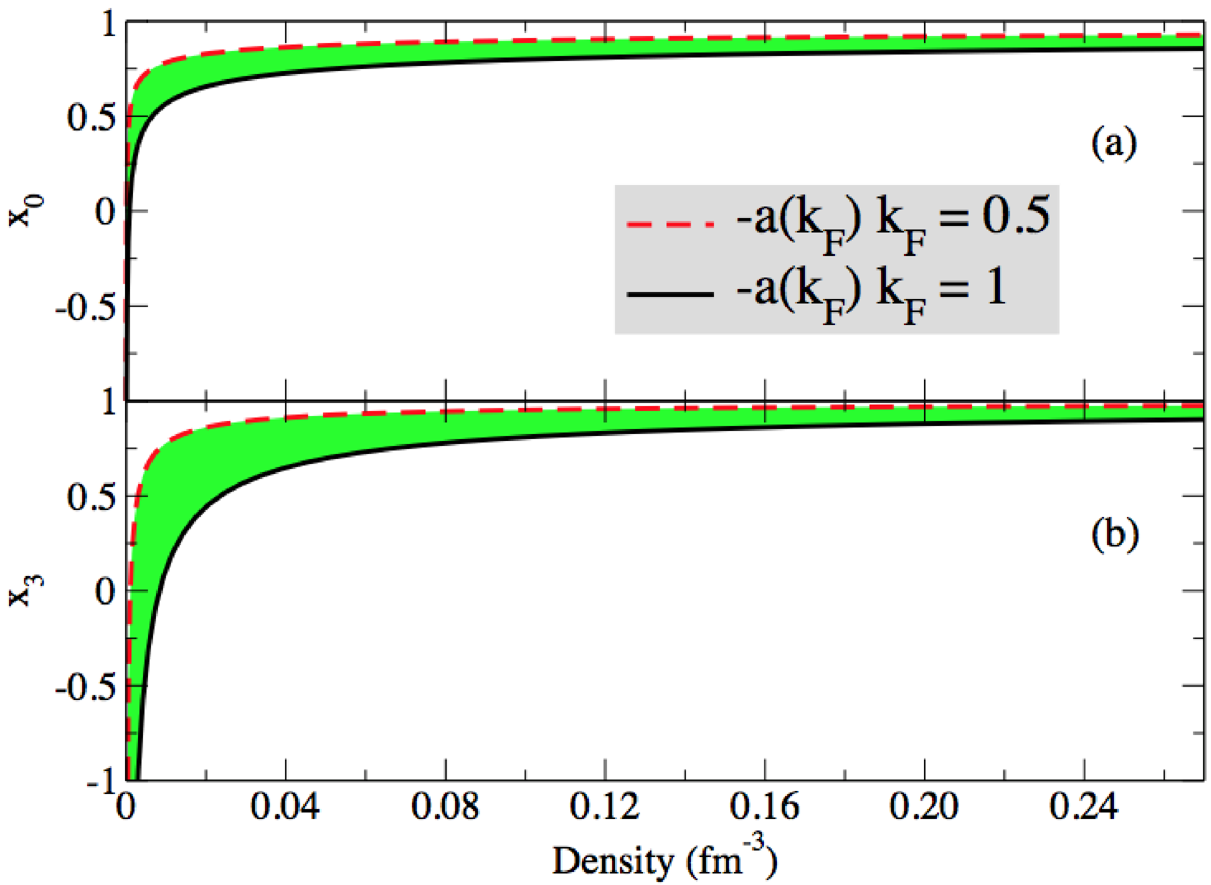
Symmetric matter, two cases

- t_0-t_3
- t_0-t_3-t_1



Grasso, Lacroix, Yang,Phys. Rev. C 95, 054327 (2017)

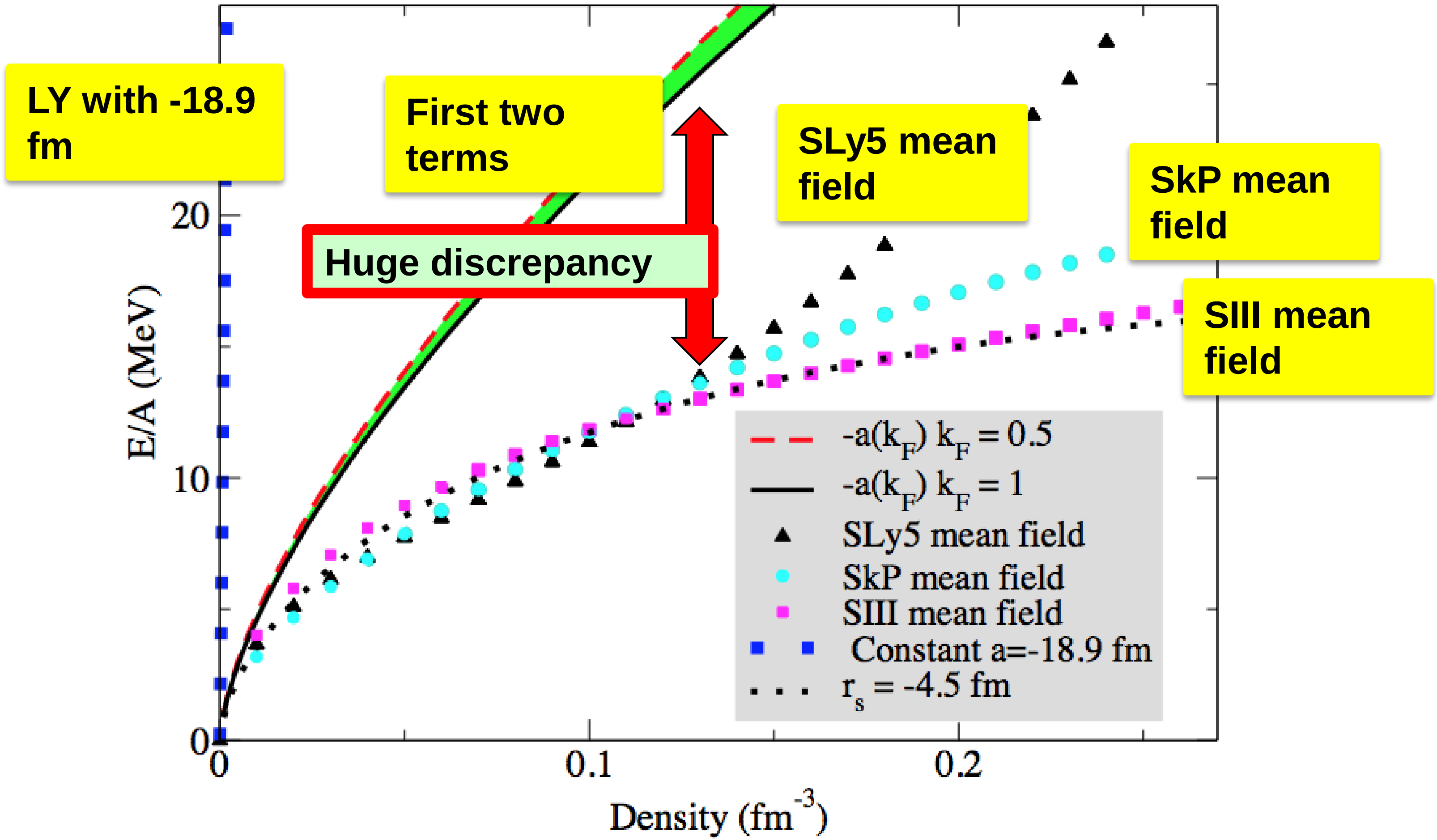
Density dependence of the parameters x_0 and x_3



Typical Skyrme values at densities around the saturation point

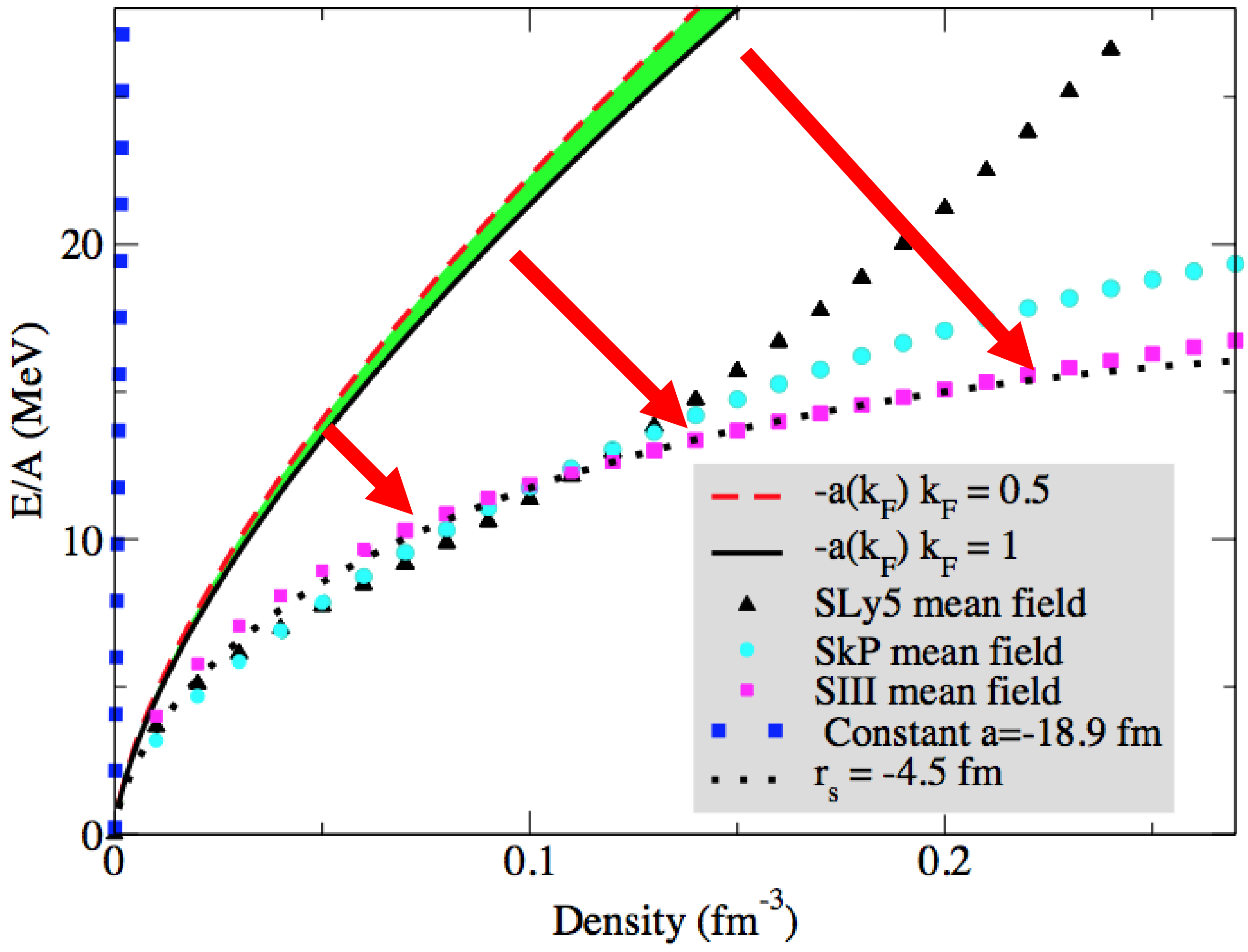
Grasso, Lacroix, Yang,Phys. Rev. C 95, 054327 (2017)

First two terms of the Lee-Yang expansion. EOS of neutron matter



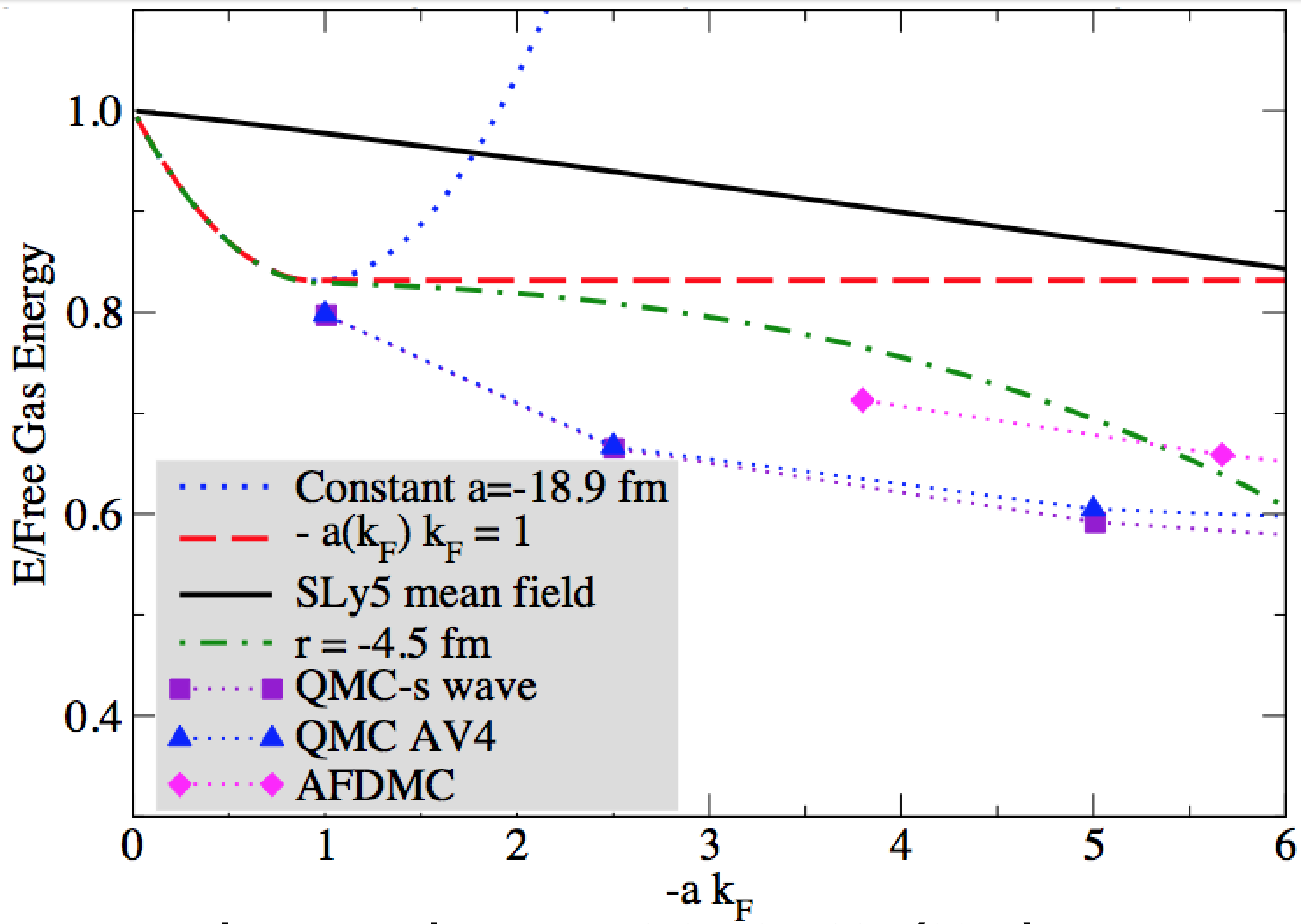
Grasso, Lacroix, Yang,Phys. Rev. C 95, 054327 (2017)

Including the s-wave k_F^5 terms and adjusting the effective range



Grasso, Lacroix, Yang, PRC 95, 054327 (2017).

Low-density behavior



Grasso, Lacroix, Yang,Phys. Rev. C 95, 054327 (2017)

Conclusions

- **Beyond-mean-field tailored interactions -> second-order calculations for matter**

- **New functionals valid at all density scales for neutron matter and at densities around saturation for symmetric matter**

-> YGLO (resummation from EFT and good properties of Skyrme forces : a hybrid functional)

-> without resummation -> density-dependent neutron-neutron scattering length