Ab initio calculations for non-strange and strange few-baryon systems

- Applied ab initio methods

- Non-strange sector: continuum observables with LIT
  - Resonances
  - S-Factor in presence of Coulomb potential

- Strange sector: hypernuclear bound states with NSHH
  - Benchmark calculation with other ab initio methods
  - Baryon resonances ($N\leftrightarrow\Delta$, $\Lambda\leftrightarrow\Sigma$)

Applied ab initio methods

- A-body system: A position vectors $\mathbf{r}_i$, removal of center of mass coordinate leads to (A-1) Jacobi vectors $\boldsymbol{\eta}_i$ (η_i , θ_i, ϕ_i)
- Expansion of ground-state wave function or LIT state on a complete set: Hyperspherical Harmonics (HH)
- 3(A-1) coordinates of HH basis: hyperradius ρ, 2(A-1) angular coordinates Θ_i and ϕ_i , (A-2) hyperspherical angles: 1 hyperradius + (3A – 4) angles
- HH basis states: eigen states of grand-angular momentum operator depending on the (3A – 4) angles times a hyperradial basis state
- Different HH versions: normally symmetrized basis states, but also a nonsymmetrized HH (NSHH) basis is possible
- Acceleration of convergence: effective interaction (EIHH)

  Short-range two-body correlations (CHH)

Next step

- Solve Schrödinger or LIT equation with N basis states
  Increase N up to the point that a sufficient convergence is obtained

LIT method

The LIT of a function $R(E)$ is defined as follows

$$\Rightarrow \quad L(\sigma) = \int dE\, \mathcal{L}(E,\sigma)\, R(E)\,,$$

where the kernel $\mathcal{L}$ is a Lorentzian,

$$\Rightarrow \quad \mathcal{L}(E,\sigma) = \frac{1}{(E-\sigma_R)^2 + \sigma_I^2}$$

For inclusive reactions the LIT $L(\sigma)$ is calculated by solving an equation of the form

$$\boxed{(H-\sigma)\,\tilde{\Psi} = S\,,}$$

where H is the Hamiltonian of the system under consideration and S is an asymptotically vanishing source term related to the operator inducing the specific reaction.

The solution $\tilde{\Psi}$ is localized and the LIT is given by

$$\boxed{L(\sigma) = \langle\tilde{\Psi}|\tilde{\Psi}\rangle\,.}$$

Alternative way:

$$L(\sigma) = -\frac{1}{\sigma_I} Im(\langle S|\frac{1}{\sigma_R + i\sigma_I - H}|S\rangle)\,.$$

The source term S for inclusive reactions has the form

$$\Rightarrow \quad |S\rangle = \theta|0\rangle \, ,$$

where the operator θ induces a specific electroweak reaction.

The corresponding response function is given by

$$\Rightarrow \quad R(E_f) = \int dE_f |\langle f|\theta|0\rangle|^2 \delta(E_f - E_0 - \omega)$$

Ingredients of the solution of the LIT equation via an expansion on a basis of dimension N:

N eigenstates ϕ_n with eigenenergies E_n

LIT states

and strength

$$S_n = |\langle \phi_n|\theta|0\rangle|^2$$ $\Leftarrow$ strength to a LIT state

leading to the following LIT

$$\Rightarrow \quad L(\sigma) = \sum_{i=1}^{N} \frac{S_n}{(\sigma_R - E_n)^2 + \sigma_I^2}$$

Inversion of the LIT

- LIT is calculated for a fixed σ_I in many σ_R points

- Express the searched response function formally on a basis set with M basis basis functions $f_m(E)$ and open coefficients c_m with correct threshold behaviour for the $f_m(E)$ (e.g., $f_m = f_{thr}(E) \exp(-\alpha E/m)$)

- Make a LIT transform of the basis functions and determine coefficents c_m by a fit to the calculated LIT

- Increase M up to the point that a sufficient convergence is obtained (uncontrolled oscillations should not be present)

A regularization method is needed for the inversion

Resonances

^{4}He isoscalar monopole resonance

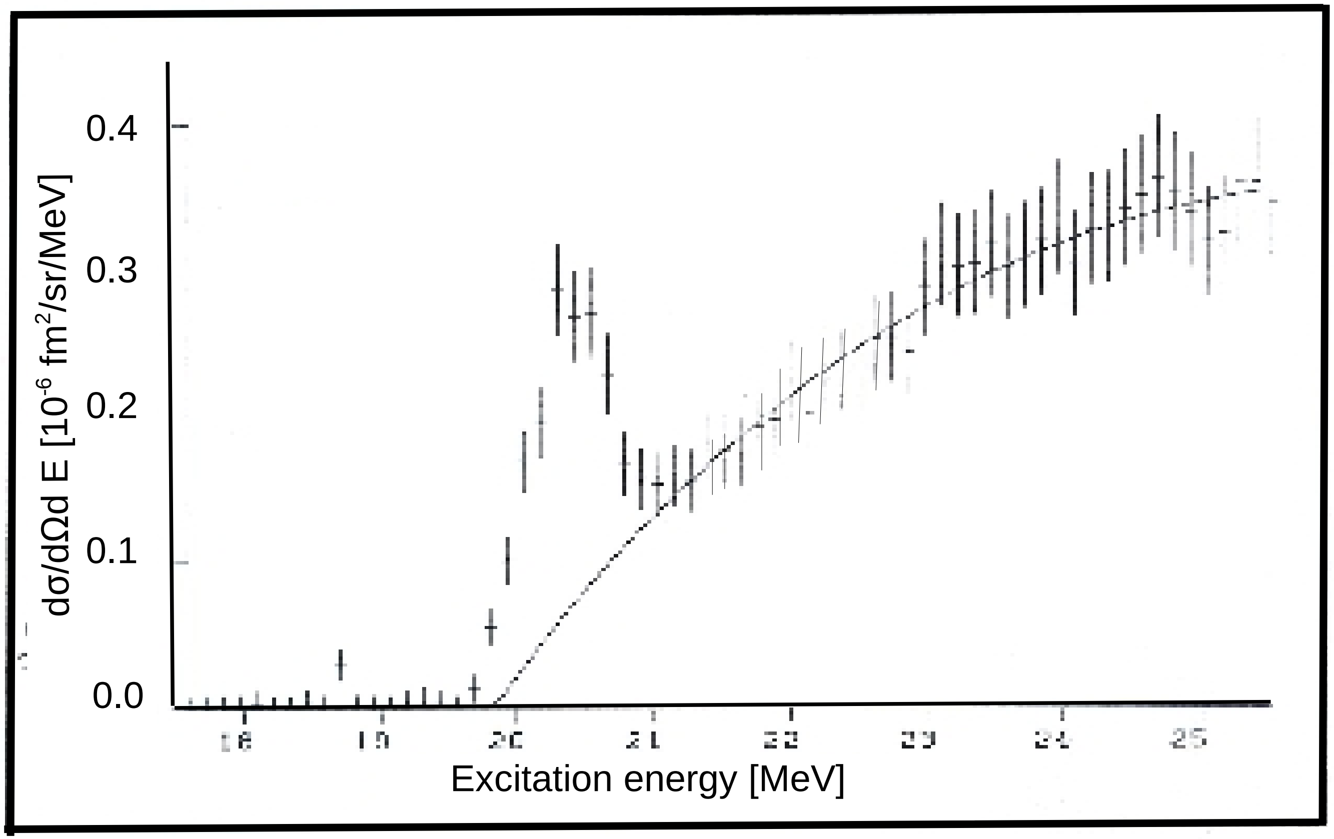
Isoscalar monopole response function $M(q, E_f = E_0 + \omega)$

with transition operator $\boxed{\theta(q) = \frac{G_E^s(q^2)}{2} \sum_{i=1}^{A} j_0(qr_i)}$

$G_E^s(q^2)$: nucleon isoscalar electric form factor
j_0: spherical Bessel function of 0^{th} order

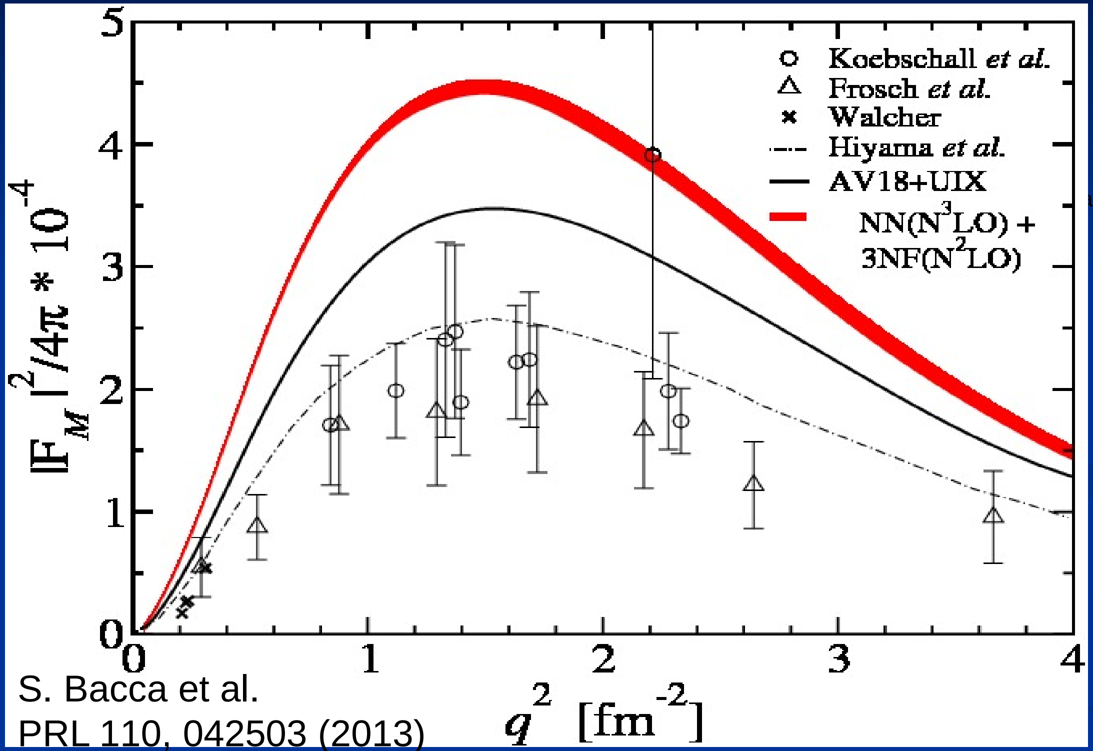
0^+ Resonance in the ^{4}He compound system

G. Köbschall et al./ Quasi bound state in ^{4}He - Nucl. Phys. A405, 648 (1983)



Resonance at E_R = -8.2 MeV, i.e. above the ^{3}H-p threshold. Strong evidence in electron scattering off ^{4}He, Γ = 270±50 keV

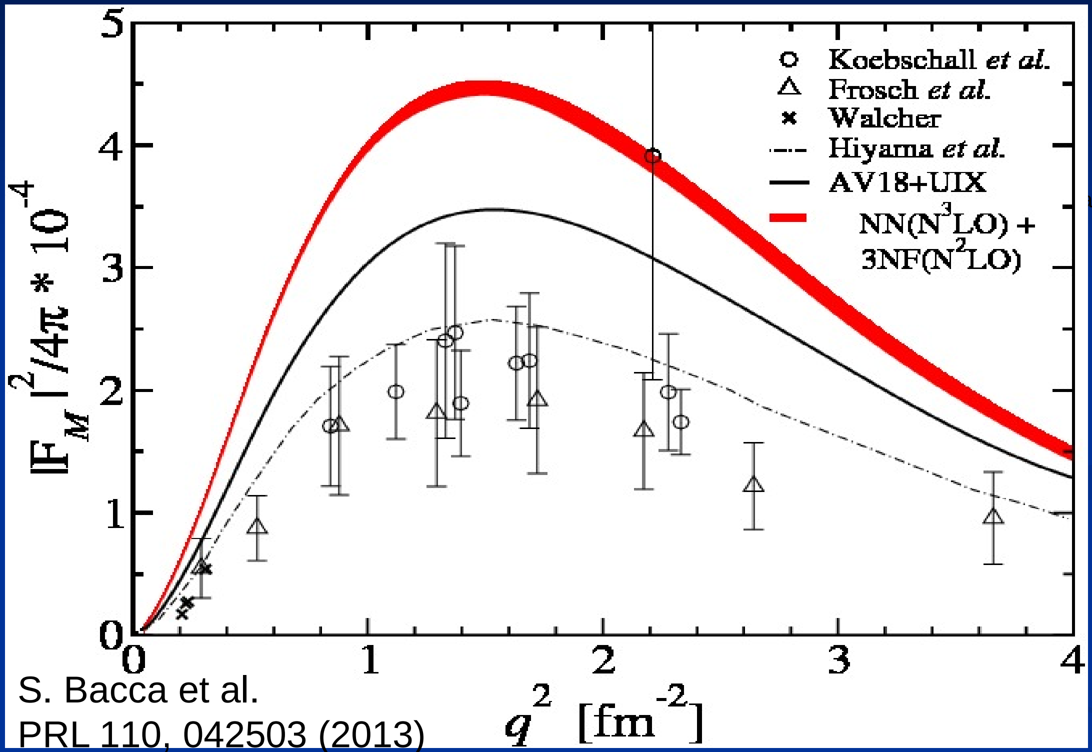
Comparison to experimental results



LIT/EIHH Calculation for AV18+UIX and Idaho-N3LO+N2LO

dash-dotted: AV8' + central 3NF (Hiyama et al.)

Comparison to experimental results



Observable is strongly dependent on potential model

Breathing Mode? (S. Bacca et al., PRC 91, 024303 (2015))

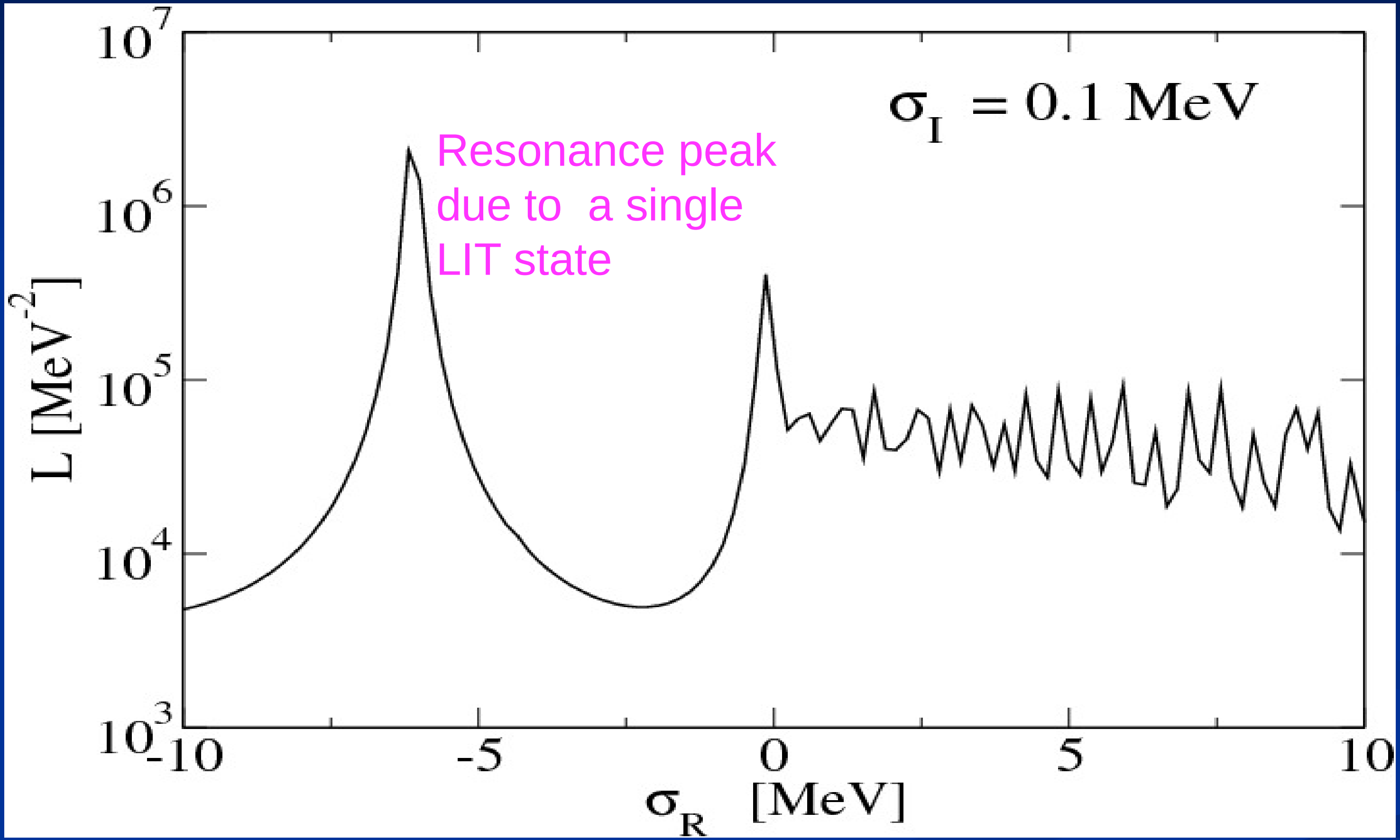
Why were we unable to determine the width of the 4He isoscalar monopole resonance?

To answer this let us check our very first LIT calculation from 1997:

^{4}He(e,e') inelastic longitudinal response function

with a central NN potential

Unpublished result from a CHH calculation with the TN potential (V. Efros, WL, G. Orlandini, PRL 78,432 (1997))



To study the problem better let us consider first instead of a four-body reaction a simpler three-body reaction:

$$^3He + \gamma \longrightarrow d + p$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

Aim: Increase low-energy density of LIT states

To study the problem better let us consider first instead of a four-body reaction a **simpler three-body reaction**:

$$^{3}He + \gamma \longrightarrow d + p$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

How: Increase number of basis states, both, hyperradial and hyperspherical

To study the problem better let us consider first instead of a four-body reaction a simpler three-body reaction:

$$^3\mathrm{He} + \gamma \longrightarrow d + p$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

Also note: hyperradial basis states consist in an expansion on Laguerre polynomials times a spatial cutoff $\exp(-\rho/b)$

Increase of b shifts spectrum to lower energies

To study the problem better let us consider first instead of a four-body reaction a **simpler three-body reaction**:

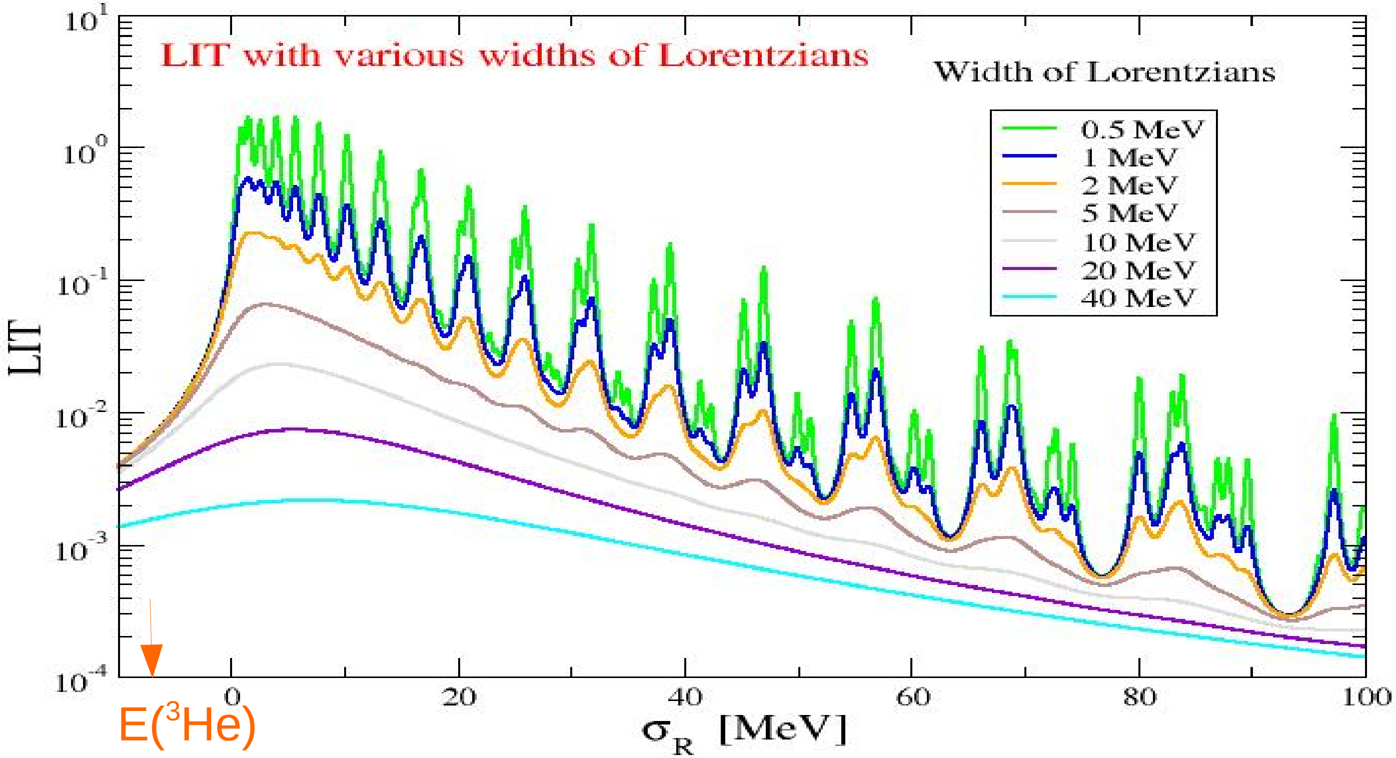
$$^{3}He + \gamma \longrightarrow d + p$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

Next slide: LIT with 30 hyperspherical and 31 hyperradial basis functions ⇒ **930 basis states** with b = 0.6 fm

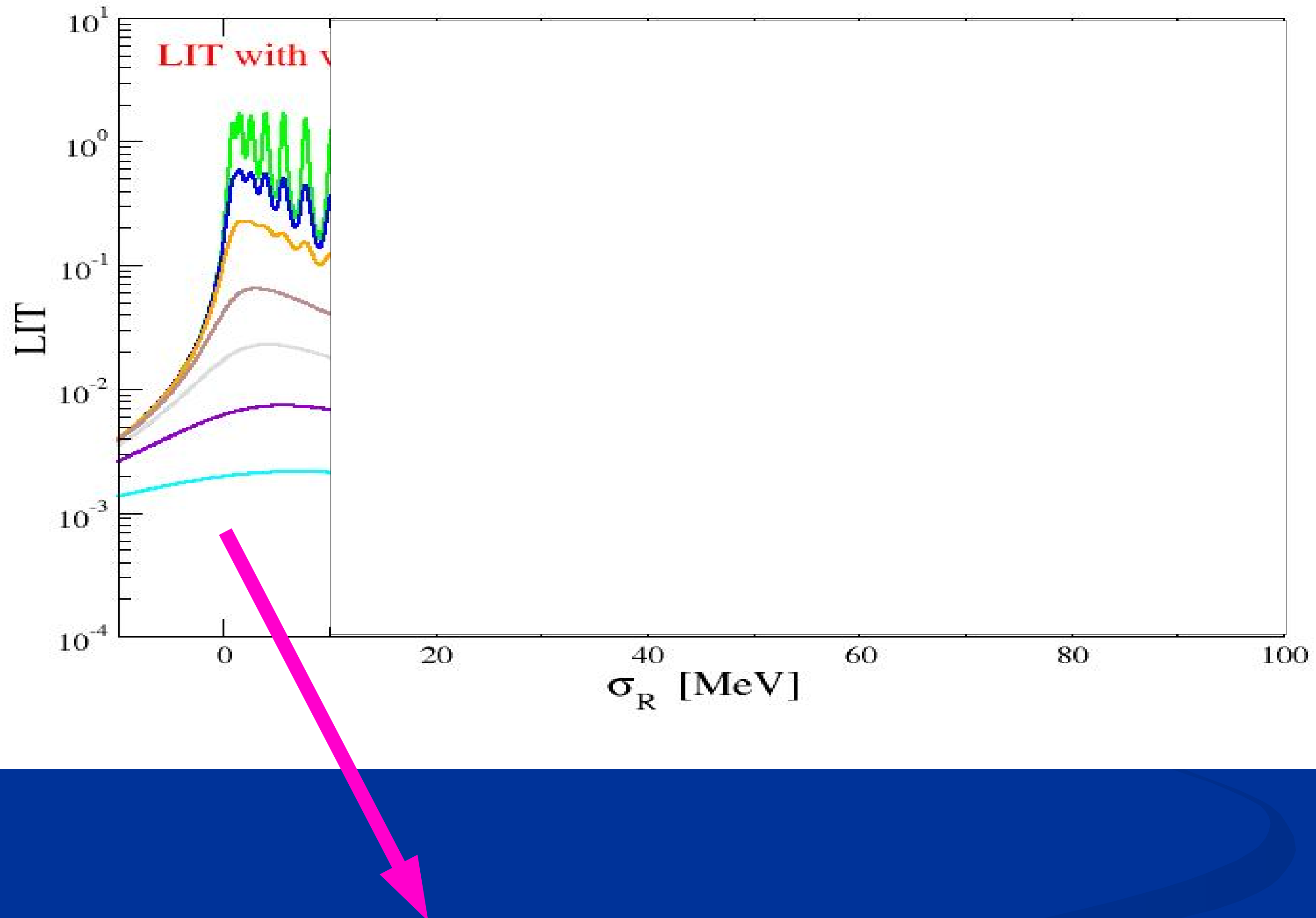
LIT with various widths of Lorentzians



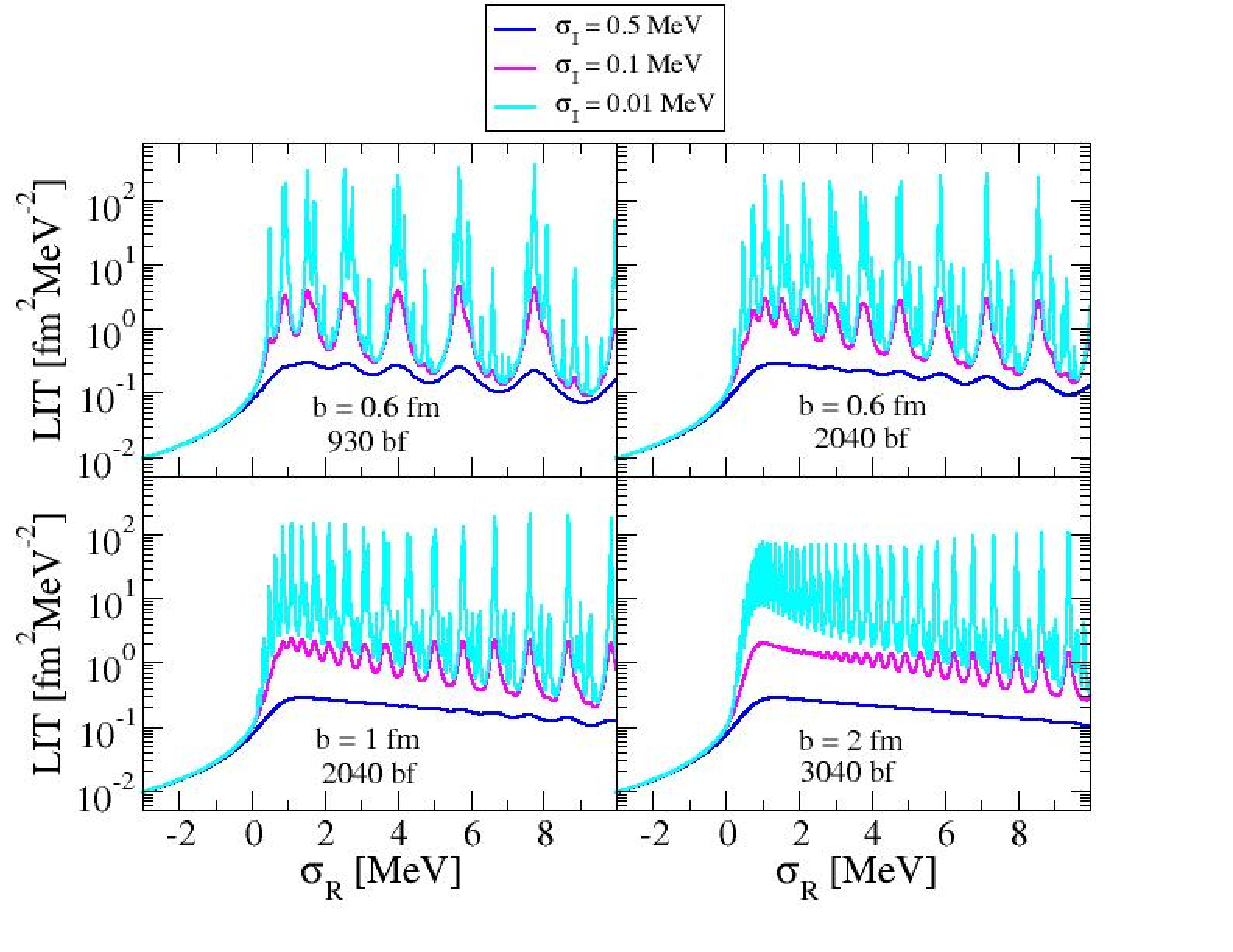
30 hyperspherical and 31 hyperradial basis functions

⇒ 930 basis states

b = 0.6 fm



Increase LIT state density and zOOM in



$\sigma_I = 0.5$ MeV
$\sigma_I = 0.1$ MeV
$\sigma_I = 0.01$ MeV
LIT [fm^{2}MeV^{-2}]
b = 0.6 fm
930 bf
b = 0.6 fm
2040 bf
b = 1 fm
2040 bf
b = 2 fm
3040 bf
σ_R [MeV]

Observation

The LIT is a method with a controlled resolution

But not a single LIT state below three-body breakup threshold
In present LIT calculation! Similar problem as in the previous four-body case

Solution: use instead of the HH basis a somewhat modified basis

New A-body basis

Note one of the (A-1) Jacobi vectors can be written in the following form:

$$\boldsymbol{\eta} = \mathbf{r}_A - \mathbf{R}_{cm}(1,2,\ldots,A\text{-}1)$$

This is the coordinate one would use for the scattering of a nucleon with a (A-1)-nucleon system. In other words the relevant coordinate for a two-body breakup. Therefore

A-body HH basis → (A-1)-body HH basis times expansion on $\boldsymbol{\eta}$

radial part: Laguerre polynomials

angular part: $Y_{LM}(\theta_\eta,\phi_\eta)$

Four-body system: HH for 3 particles plus 4-th particle coordinate $\boldsymbol{\eta}$

New A-body basis

Note one of the (A-1) Jacobi vectors can be written in the following form:

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A-body HH basis → (A-1)-body HH basis times expansion on $\boldsymbol{\eta}$

radial part: Laguerre polynomials

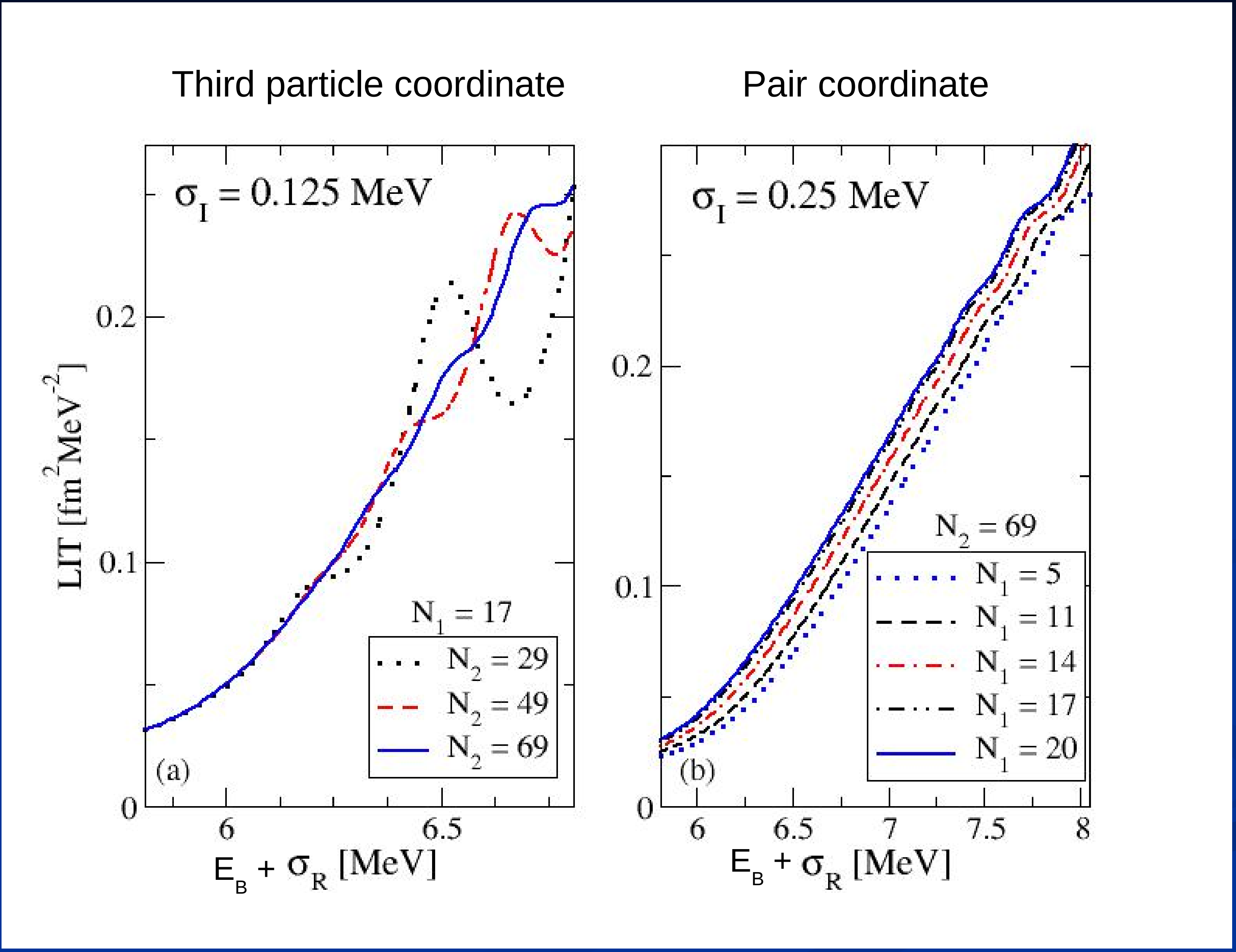
angular part: $Y_{LM}(\theta_\eta,\phi_\eta)$

Three-body system: pair coordinate for two particles plus 3rd particle coordinate $\boldsymbol{\eta}$

First three-body case

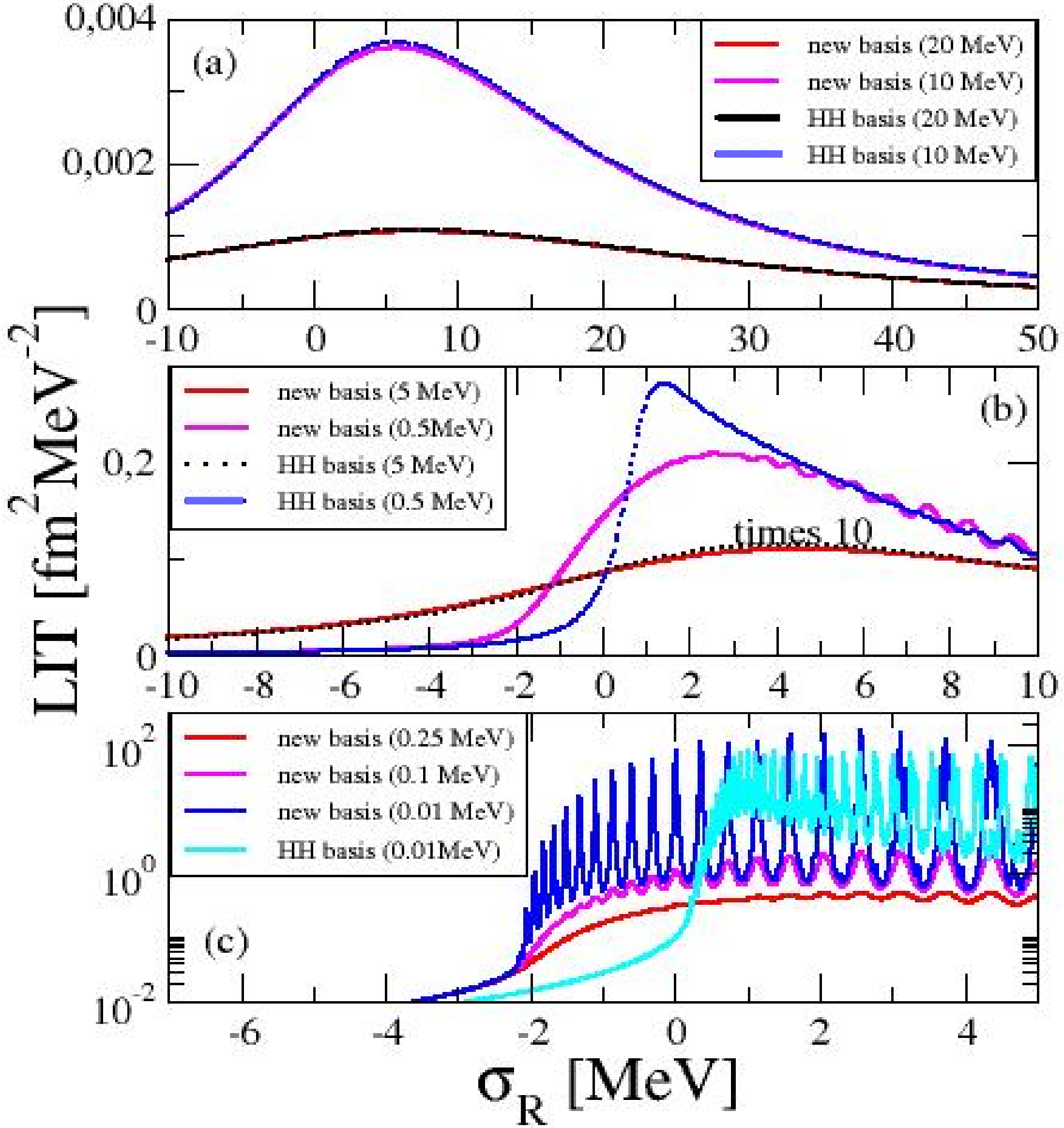
$$^3He + \gamma \longrightarrow d + p$$

With convergence for expansions in pair and third particle coordinate



Third particle coordinate
Pair coordinate
σI = 0.125 MeV
LIT [fm2MeV-2]
0.2
0.1
0
6
6.5
EB + σR [MeV]
N1 = 17
N2 = 29
N2 = 49
N2 = 69
(a)
σI = 0.25 MeV
N2 = 69
N1 = 5
N1 = 11
N1 = 14
N1 = 17
N1 = 20
(b)
7
7.5
8

LIT results with HH and new basis

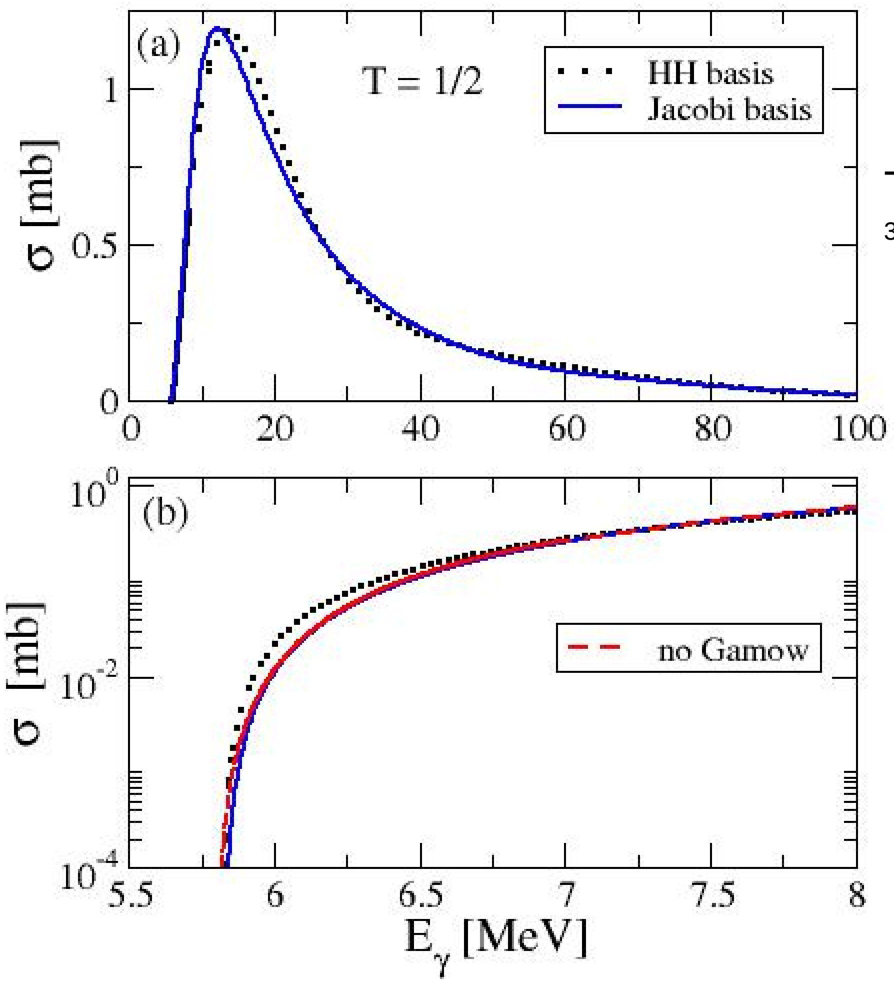


Inversions

Implement correct threshold behaviour for ^{3}He + γ ⟶ d + p

Due to Coulomb potential: usual Gamow factor

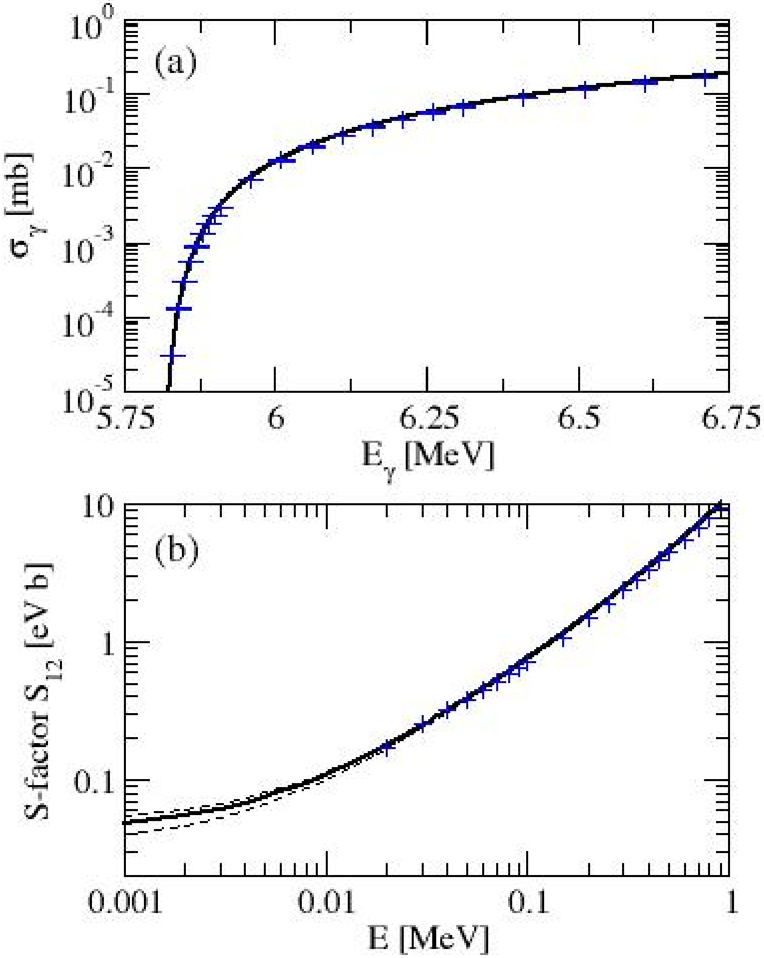
Comparison: HH and Jacobi basis



T = 1/2 cross section:
$^3He + \gamma \longrightarrow d + p$

Cross section at
Low energy

Comparison with explicit calculation of continuum state



Cross section
$^3He + \gamma \longrightarrow d + p$

LIT: full curves
cont. wf: +

S-factor
$d + p \longrightarrow {}^3He + \gamma$

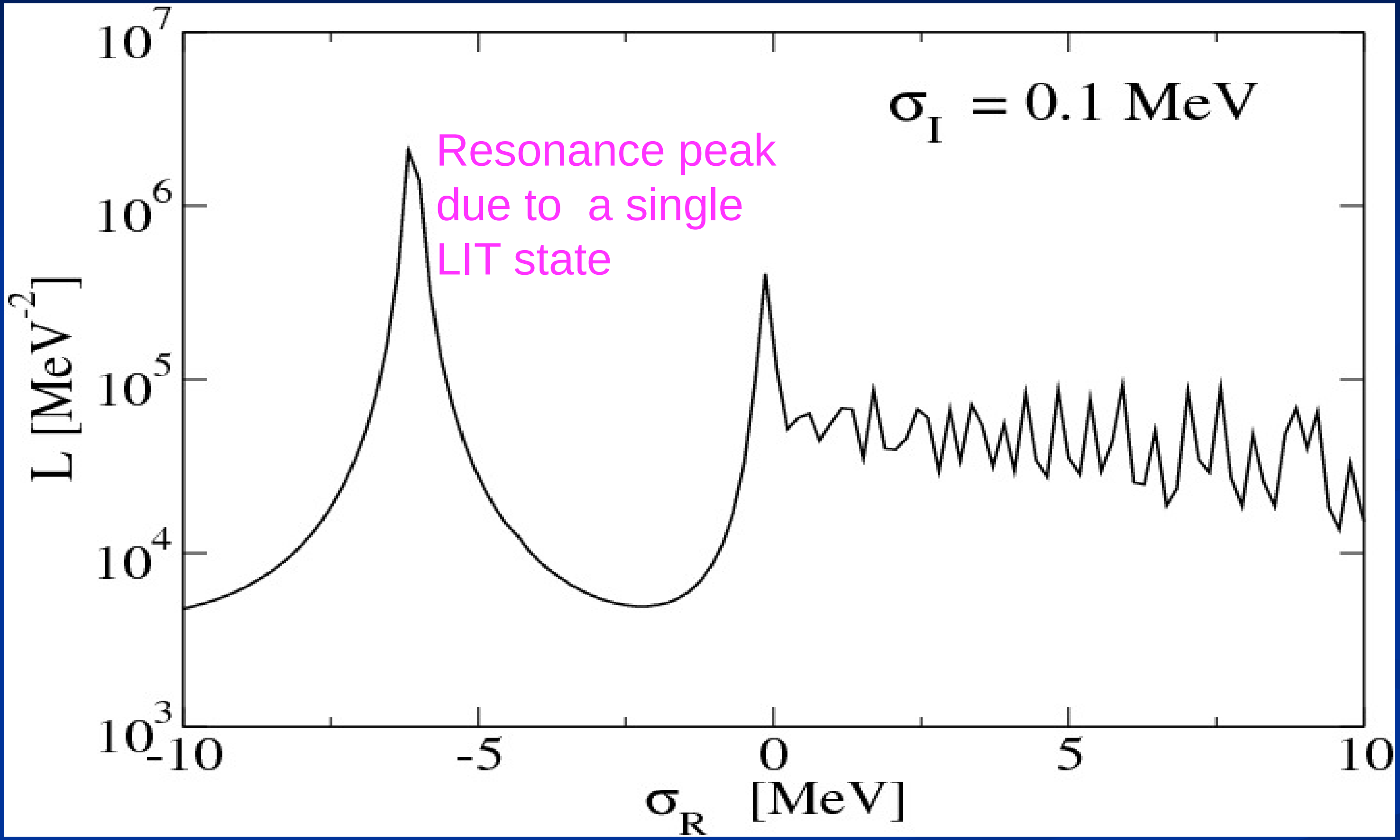
Error due to
inversion: dashed

(Standard deviation from inversions with 11-18 basis functions)

S. Deflorian, V. Efros, WL, FBS 58:3 (2017)

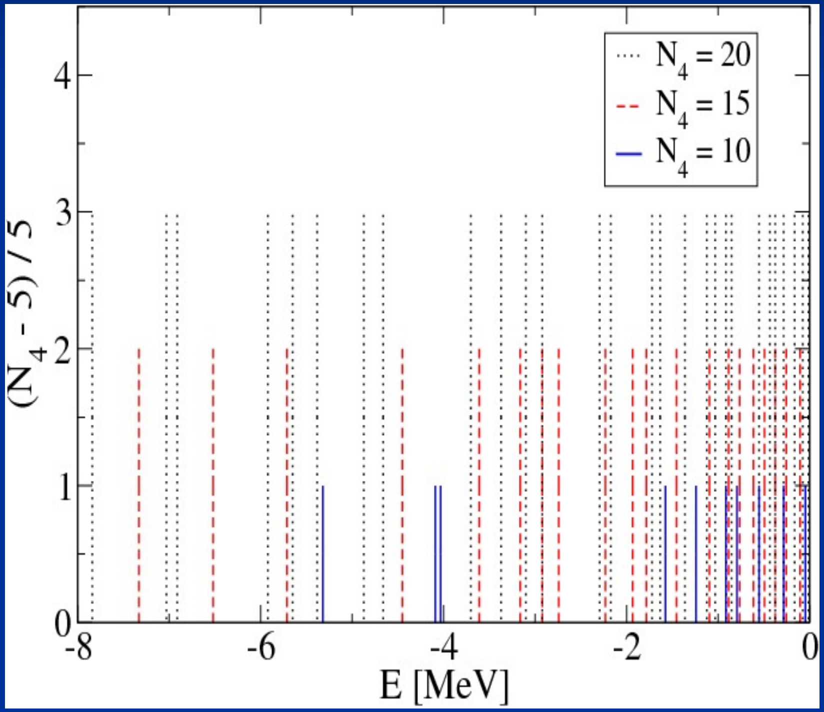
Back to the ^{4}He resonance

Unpublished result from a CHH calculation with the TN potential (V. Efros, WL, G. Orlandini, PRL 78,432 (1997))

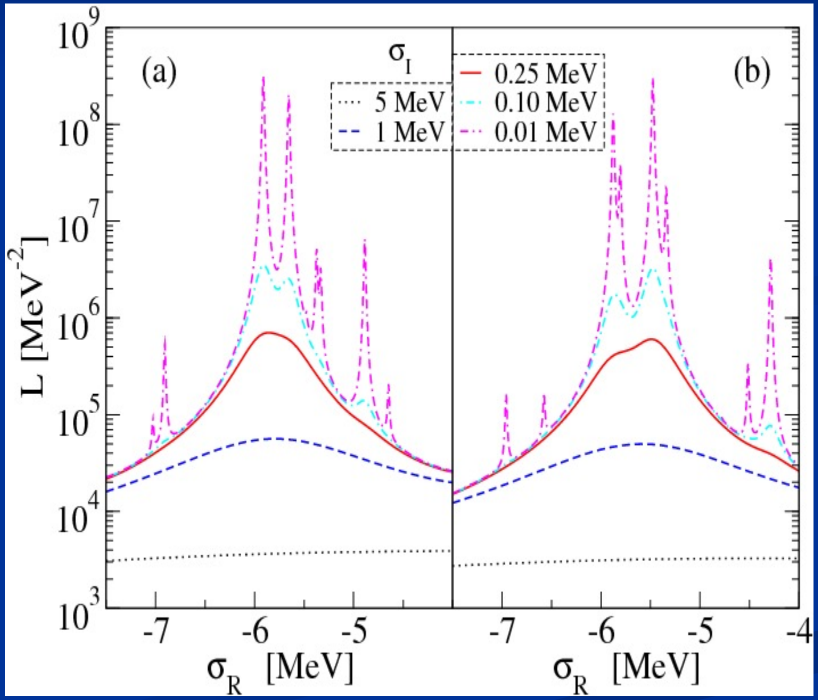


Results with new basis

Number of basis functions in 4-th particle coordinate

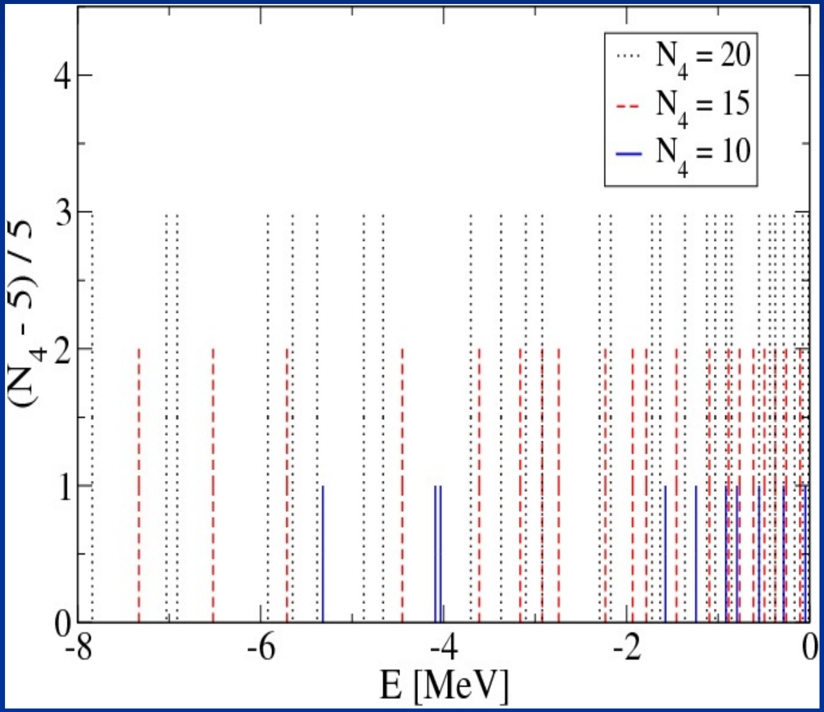


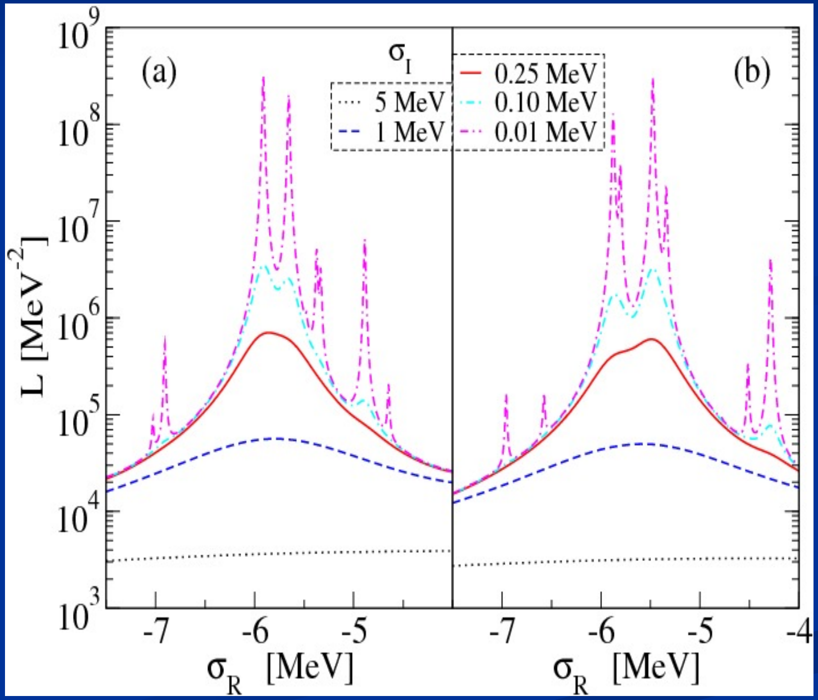
$N_4 = 20$ $N_4 = 21$



LIT

Results with new basis





Inversion: Γ = 180(70) keV

WL, PRC 91, 054001 (2015)

Hypernuclei

- Quick introduction to hypernuclei
- Short outline of our NSHH method
- some benchmark results: comparison with

  AFDMC (D. Lonardoni, F. Pederiva)

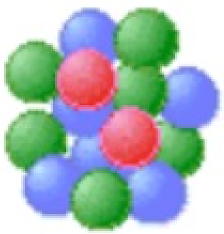
  Faddeev (A. Nogga)

  (GEM: E. Hiyama)

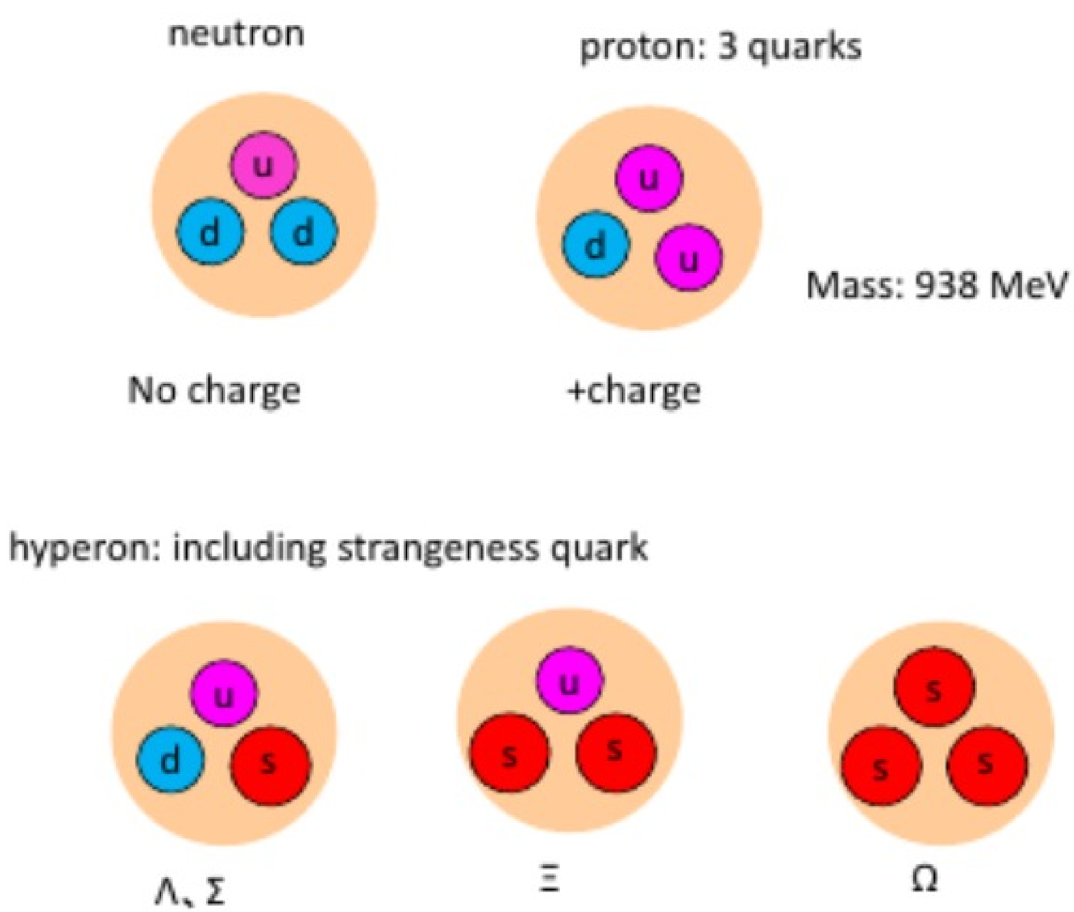
Next slides thanks to Fabrizio Ferrari-Rufino

Nuclei with Strangeness

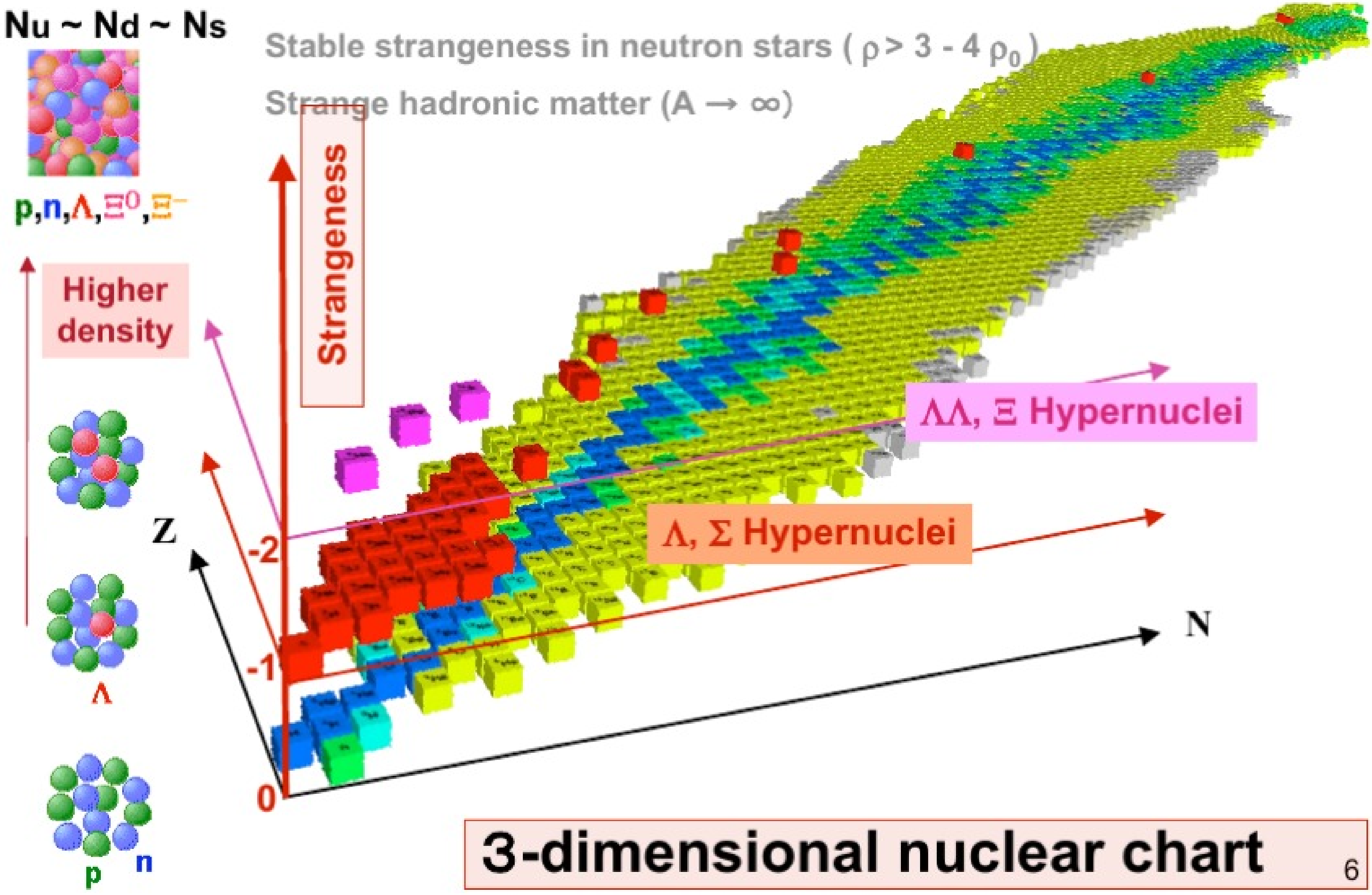
$$m_\Lambda = 1116 \text{ MeV}$$
$$m_{\Sigma^+} = 1189$$
$$m_{\Sigma^0} = 1193$$
$$m_{\Omega^-} = 1673$$



$$\tau_\Lambda = 263\ ps$$
$$\tau_{\Sigma^+} = 80\ ps$$
$$\tau_{\Sigma^0} = 7.4 \cdot 10^{-20} s$$
$$\tau_{\Omega^-} = 82\ ps$$



Hypernuclear Chart



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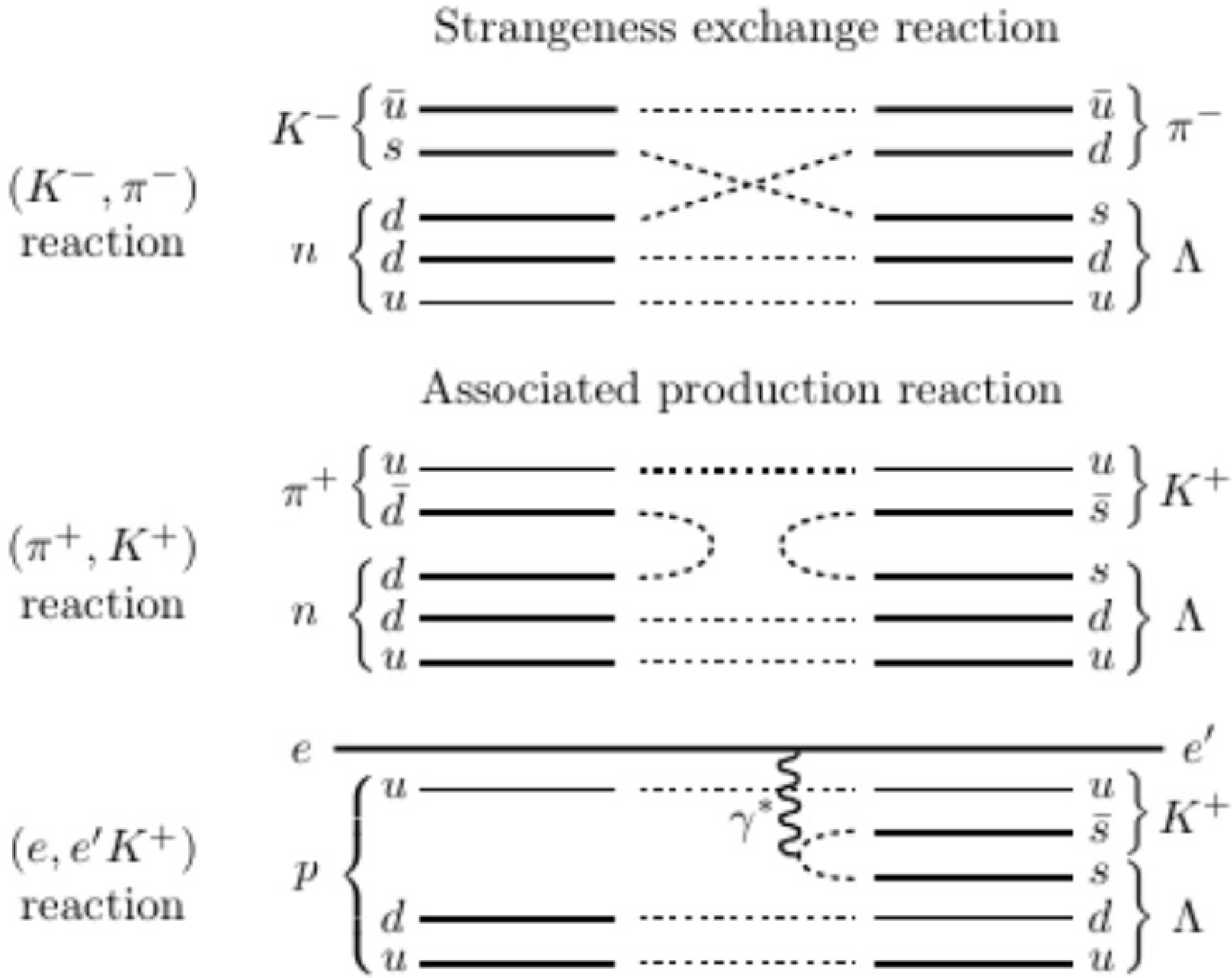
[1] by M. Kaneta inspired by HYP06 conference poster

Production of Hypernuclei



Experimental Present and Future Perspectives

- Despite exensive investigations, single Λ hypernuclei knowledge is **far** from that of ordinary nuclei;
- Only **one** bound Σ-hypernucleus detected!
- **No** Ξ hypernuclei detected (some indications of weak attraction);
- **No** experimental information about Ω hypernuclei;
- **Four** $\Lambda\Lambda$-hypernuclei energies measured (${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, ${}^{12}_{\Lambda\Lambda}$Be, ${}^{13}_{\Lambda\Lambda}$B);

$\Rightarrow$ Main goal: **extension** of nuclear chart in all directions!

Non-Symmetrized HH method

Problem: selection of **antisymmetric** states (we deal with fermions):

$\Rightarrow$ We add to Ĥ the **Casimir operator** of the **permutation group $\mathbf{S}_N$**, which selects "by himself" the interesting states:

$$\hat{H}' = \hat{H} + \gamma \hat{C}(A) \quad ; \quad \hat{C}(A) = \sum_{i>j} \hat{P}_{ij}$$

Its action on the vectors:

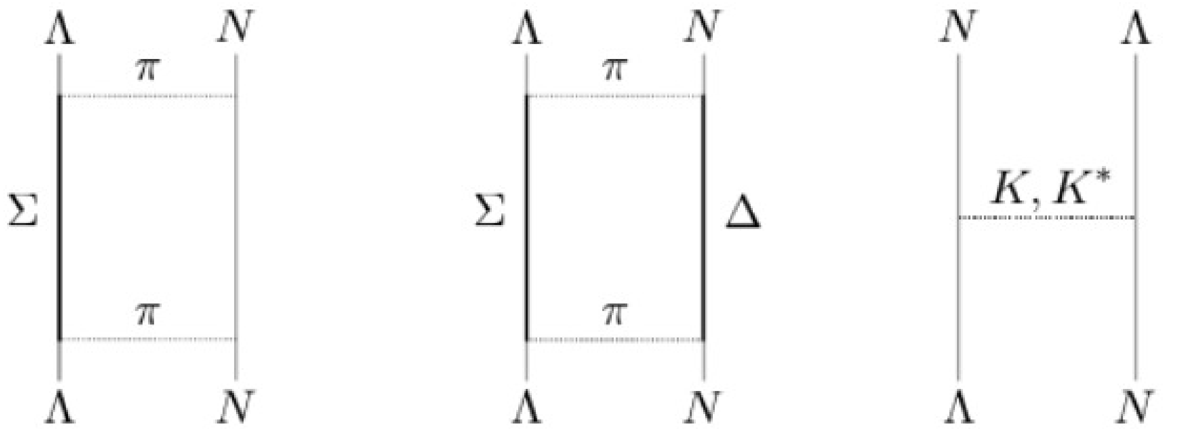
$$\hat{C}(A)\Psi_s = \frac{A(A-1)}{2}\Psi_s = \lambda_s \Psi_s \ ;$$
$$\hat{C}(A)\Psi_m = \lambda_m \Psi_m \ ;$$
$$\hat{C}(A)\Psi_a = -\frac{A(A-1)}{2}\Psi_a = \lambda_a \Psi_a \ ,$$

$\Rightarrow$ with a proper choice of γ the g.s. energy $\mathbf{E}_A^0$ becomes the **lowest eigenvalue of $\mathbf{H}'$** (similar procedure for excited states).

2-body Bodmer Usmani interaction

The Λ *particle has* $T = 0$ *so there is no OPE term*:

$$v_{\Lambda N}(r) = v_0(r) + \frac{v_\sigma}{4} T_\pi^2(r)\, \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N \,,$$



(a) (b) (c)

$$v_0(r) = \frac{W_c}{1 + e^{\frac{r-\bar{r}}{a}}} - \bar{v}\, T_\pi^2(r)$$

$$T_\pi(r) = \left[1 + \frac{3}{\mu_\pi r} + \frac{3}{(\mu_\pi r)^2}\right] \frac{e^{-\mu_\pi r}}{\mu_\pi r} (1 - e^{-cr^2})^2$$

Some Results I

Interaction	System	NSHH	AFDMC	FY
AV4'	^{2}H		-2.245(15)	-2.245(1)
AV4'+B.U.$^{2b'}$	$^{3}_{\Lambda}$H	-2.539(2)	-2.42(6)	-2.537(1)
	B_{Λ}	0.294(2)	0.18(6)	0.292(1)
AV4'	^{3}H	-8.98(1)	-8.92(4)	
AV4'+B.U.$^{2b'}$	$^{4}_{\Lambda}$H	-12.02(1)	-11.94(6)	
	B_{Λ}	3.04(1)	3.02(7)	
AV4'	^{4}He	-32.89(1)	-32.84(4)	
AV4'+B.U.$^{2b'}$	$^{5}_{\Lambda}$He	-39.54(1)	-39.51(5)	
	B_{Λ}	6.65(1)	6.67(6)	

NSC97f realistic interaction

We employed the **NSC97f realistic potential**[3] which simulates the Nijmegen scattering phase shifts:

$$
\begin{aligned}
{}^{S}V_{NY-NY'}(r) = \sum_i \Big(& {}^{S}V^{C}_{NY-NY'}\; e^{-(r/\beta_i)^2} \\
& + {}^{S}V^{T}_{NY-NY'}\; S_{12}\; e^{-(r/\beta_i)^2} \\
& + {}^{S}V^{LS}_{NY-NY'}\; \mathbf{LS}\; e^{-(r/\beta_i)^2} \Big)
\end{aligned}
$$

- Y$\rightarrow \Lambda, \Sigma$;
- C $\rightarrow$ central, T $\rightarrow$ tensor, LS $\rightarrow$ spin-orbit;
- gaussian radial functions with fitted paramenters.

Explicit use of **Σ degree of freedom** $\Rightarrow$ need for **extension** of the HYP-NSHH method.

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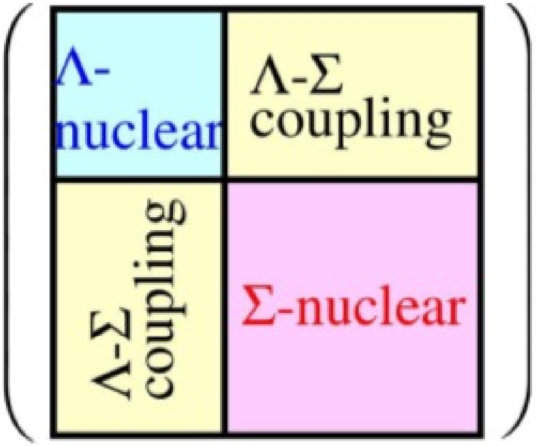
[3] E.Hiyama et al., Th.A.Rijken, Phys. Rev. C89, 061302 (2014).

Lambda-Sigma mixing

$\Rightarrow$ extension of the basis including Λ/Σ degree of freedom:



- definition of transformation between two Jacobi sets **differing by one mass**;
- **extension of Lee-Suzuki** procedure including Λ/Σ degree of freedom.

Some Results

Interaction	System	NSHH	FY	GEM
AV8'	^{2}H		-2.226(1)	
AV8'+gNSC97f	$^3_\Lambda$H	-2.413(3)	-2.415(1)	-2.42(1)
	B_Λ	0.187(3)	0.189(1)	0.19(1)
AV8'	^{3}H	-7.76(0)	-7.76(0)	-7.77(1)
AV8'+gNSC97f	$^4_\Lambda$H	-10.08(2)		-10.10(1)
	B_Λ	2.32(2)		2.33(1)

In Progress:

1. completion of the mass transformations formalism ($\mathcal{W}$ **coefficients**);
2. bound state calculation of ^{3}H with full **NN-NΔ-ΔΔ** channels;

Results were obtained in collaboration with

S. Deflorian and F. Ferrari-Rufino (ex Trento PhD students)

N. Barnea (Jerusalem), V. Efros (Moscow), G. Orlandini (Trento)