

# **Toward an EFT approach to nuclear system**

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With M. Grasso, D. Lacroix, U. van Kolck, A. Boulet

Workshop: Nuclear Structure and Reactions: Building  
Together for the Future

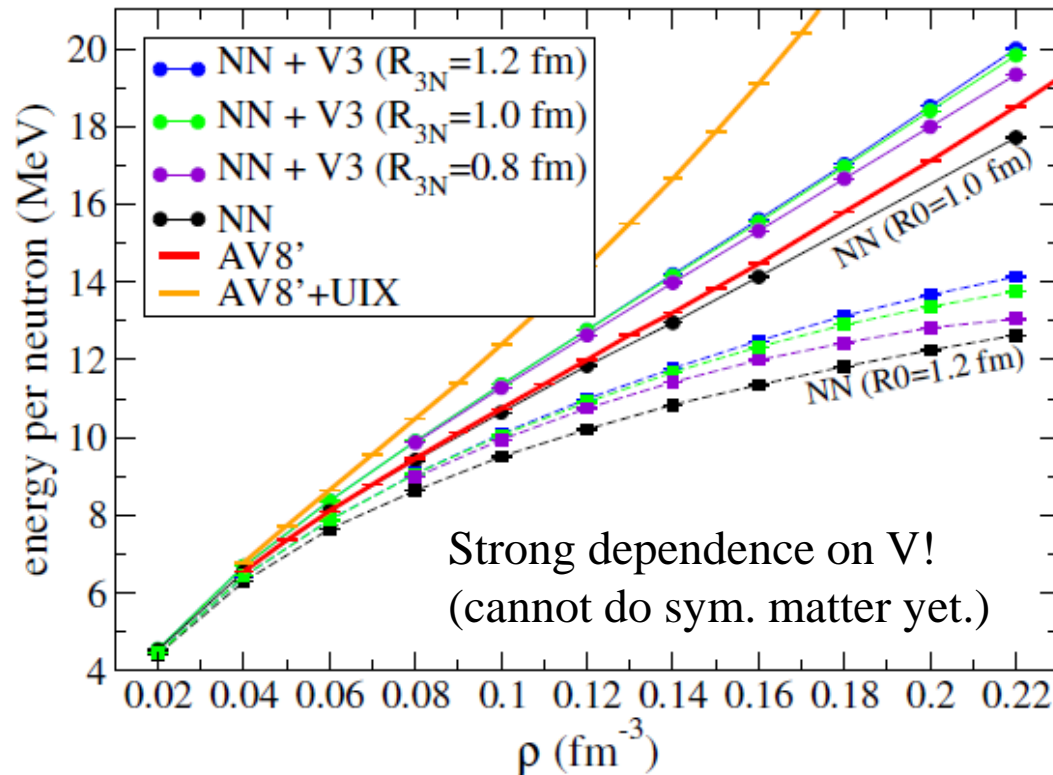
09/10/2017 Caen



# Motivation (to do EDF)

## Nuclear matter: ab-initio

Equation of state of neutron matter at  $N^2LO$ .



S. Gandolfi, talk in ESNT workshop, 2017

- Assuming no problem in the ab-initio method, the same interaction (e.g.,  $N^2\text{LO}$  under  $\text{WPC}^{[1]}$ ) with different fits/cutoffs give quite different EoS.
- A small uncertainty at 2-,3-body level seems to propagate to larger value in many-body system.

[1] Weinberg power counting (WPC) is not RG-invariant!

For details see: Nogga, Timmerman, van Kolck (2005), Yang, Elster, Phillips (2009), Ch. Zeoli R. Machleidt D. R. Entem (2012)

- Even with the correct power counting, it could be that one needs to go to very high order for the  $N^i\text{LO}$  interaction to have small enough theoretical error for many-body system.

- Assuming no problem in the ab-initio method, the same interaction (e.g.,  $N^2\text{LO}$  under  $\text{WPC}^{[1]}$ ) with different fits/cutoffs give quite different EoS.
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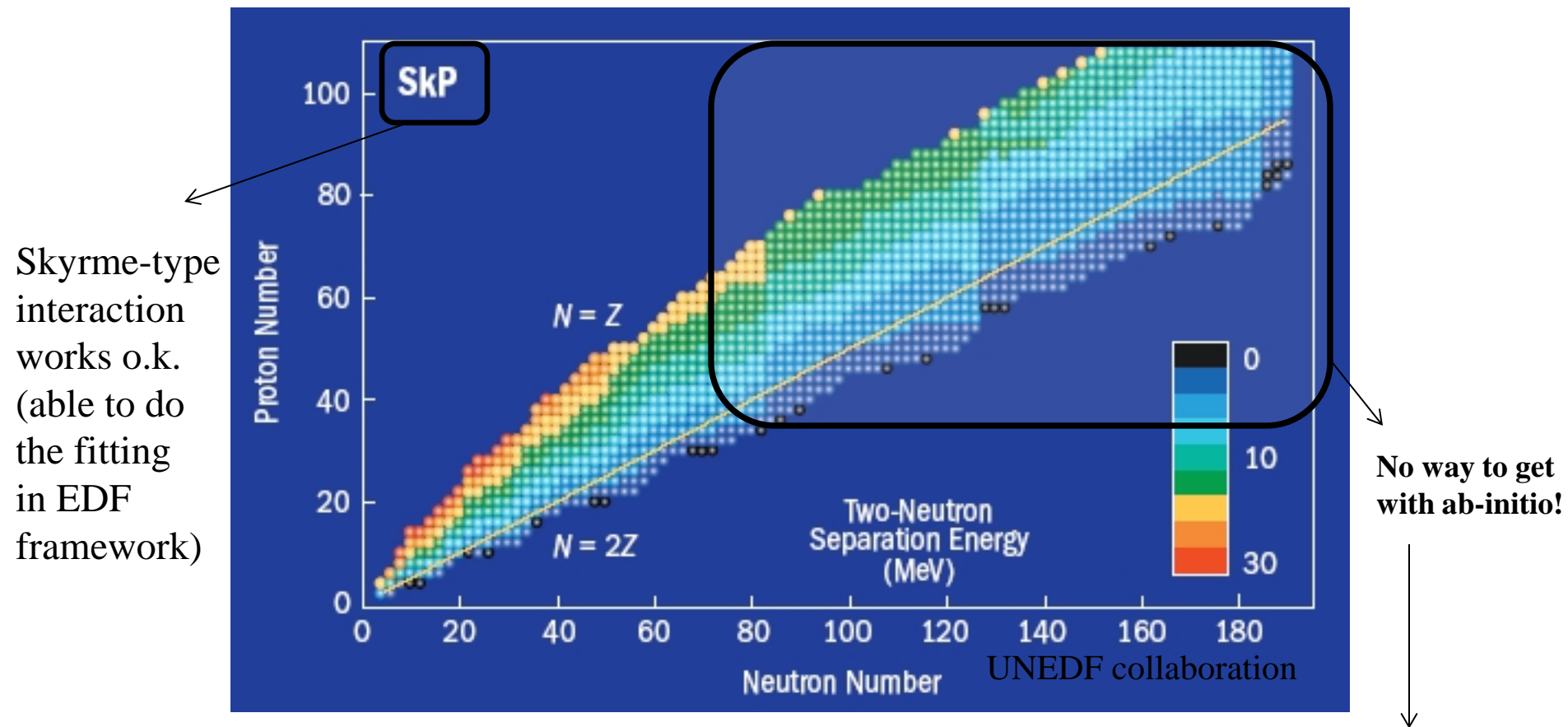
[1] Weinberg power counting (WPC) is **wrong !** Or, to be confirmed

For details see: Nogga, Timmerman, van Kolck (2005), Yang, Elster, Phillips (2009), Ch. Zeoli R. Machleidt D. R. Entem (2012)

- Even with the correct power counting, it could be that one needs to go to very high order for the  $N^i\text{LO}$  interaction to have small enough theoretical error for many-body system.

On the other hand...

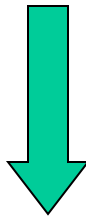
# Mean field with Skyrme-type



Need to think about other expansion (than on NN d.o.f.).

# Disadvantages of current EDF approach

- The effective interaction is model-dep. (versions of Skyrme >20) => lack of predictive power.
- Divergence occurs when goes beyond MF.



It would be good if one can find  
an EFT for it

# DFT

J. Dobaczewski's talk at IPN 2017 workshop

20	21	23	25	21	29	22	23	25	20
31	22	24	21	34	27	23	24	26	29
29	23	42	19	23	26	25	32	28	28
22	26	33	21	45	23	28	30	21	27
28	27	21	25	23	35	21	29	22	23
26	23	34	29	23	23	20	28	34	21
25	34	41	28	21	19	30	23	23	29
19	45	36	26	24	23	31	24	21	27
28	23	32	24	20	24	35	26	20	25
18	22	31	23	28	25	32	24	25	21



# DFT

J. Dobaczewski's at IPN 2017 workshop

20	21	23	25	21	29	22	23	25	20	20
31	22	24	21	34	27	23	24	26	29	21
29	23	42	19	23	26	25	32	28	28	19
22	26	33	21	45	23	28	30	21	27	21
28	27	21	25	23	35	21	29	22	19	21
26	23	34	29	23	23	20	28	34	21	20
25	34	41	28	21	19	30	23	23	29	19
19	45	36	26	24	23	31	24	21	27	19
28	23	32	24	20	24	35	26	20	25	20
18	22	31	23	28	25	32	24	25	21	18

=>18

# But life is difficult...

23	25	20
24	26	29
32	28	28

Limitations:

$V_{\text{bare}}, V_{\text{in medium}}$  not fully  
known, cannot do the full  
job due to computational  
time, etc..

# Need Strategy

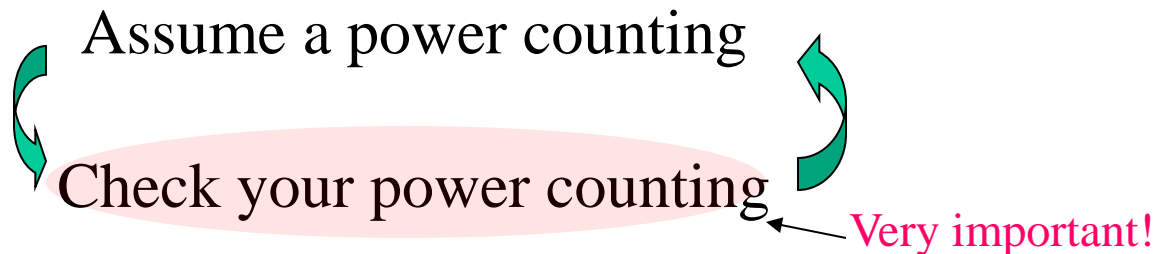
	22	23	25	20
	23	24	26	29
	25	32	28	28
	28	30	21	27

# Need Strategy

18	29	22	23	25	20
	27	23	24	26	29
	26	25	32	28	28
	23	28	30	21	27
	35	21	29	22	19

# Effective Field Theory

- Guidance: Underlying symmetries (if any)
- You always live with uncertainty/errors= $\Rightarrow$ to control/reduce it, establish power counting.



- EFT breaks down at some point (because we ignored something, d.o.f., etc... ).

# What we already knew (expansion on $k_N a$ )

Could do ‘strict’ EFT:

Pure neutron matter at **very low density** ( $k_N a < 1$ ,  $\rho < 10^{-6} \text{ fm}^{-3}$ ).

Lee & Yang formula (1957) describes the dilute system.

=> Can be re-derived by EFT with matching to ERE

E.g., L. Platter, H. Hammer, Ulf. Meissner, Nucl.Phys. A714 (2003), 250-264,

H. Hammer and R.J. Furnstahl, Nucl.Phys. A678 (2000) 277-294.

↓  
Skyrme completely wrong here!

Only ‘EFT-inspired’

Tricks to extend to higher  $\rho$  (up to  $0.3 \text{ fm}^{-3}$ )

Already discussed in Marcella’s talk

See also: P.Papakonstantinou et al, arXiv:1606.04219.

# What we already knew (expansion on $1/(k_N a)$ )

## Unitarity limit

- For  $a \rightarrow \infty$ , scale invariance gives  $\frac{E}{E_{FG}} = \xi(a_s k_F, r_e k_F)$
- Nuclear system not far from unitarity.  
 $|a_s = -18.9 \text{ fm}| \gg \text{range of interaction}$

## ‘EFT-inspired’ treatment

Neutron matter only

Expansion in  $(a_s k_F)^{-1} + \text{resum} + \text{input from ab-initio (QMC) calculations.}$

D Lacroix, Phys. Rev. A 94, 043614 (2016).

D Lacroix, A. Boulet, M. Grasso, C. J. Yang, PRC 95, 054306 (2017).

A. Boulet and D Lacroix, arXiv:1709.05160

For details: See Denis Lacroix’s talk Friday.

**Strict EFT maybe possible (within certain range of  $\rho$ )**

C.J. Yang and U. van Kolck, in preparation.

# Unitarity limit: Formula

D Lacroix, A. Boulet, M. Grasso, C. J. Yang, PRC 95, 054306 (2017) .

The proposed functional for Neutron matter

$$\frac{E}{E_{\text{FG}}} = 1 - \frac{U_0}{1 - (a_s k_F)^{-1} U_1} + \frac{R_0(r_e k_F)}{[1 - R_1(a k_F)^{-1}] [1 - R_1(a_s k_F)^{-1} + R_2(r_e k_F)]}$$

No free parameters:  $U_i$ ,  $R_i$  from QMC data (with  $V_{\text{unitarity}}$ )

**Validity:**  $\frac{1}{|a_s|} < k_F < \frac{1}{R} \Rightarrow 4 \cdot 10^{-6} < \rho < 0.002 [\text{fm}^{-3}]$ , or higher if there's an extra suppression in the coefficient in front of the range.



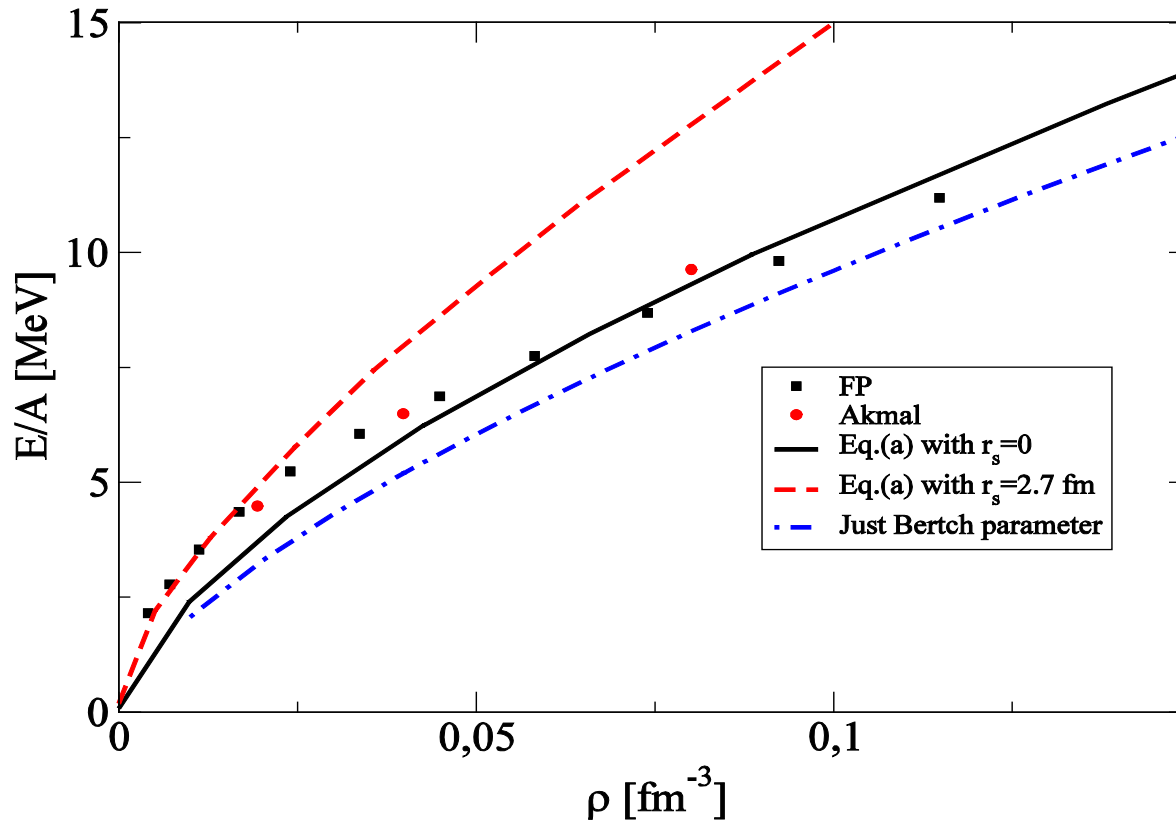
The lower limit ( $4 \cdot 10^{-6}$ ) is exactly where Skyrme breakdown.

**Hint: Skyrme is an UT-like expansion.**



# Unitarity limit: Results

D Lacroix, A. Boulet, M. Grasso, C. J. Yang, PRC 95, 054306 (2017) .



- Nuclear systems are not too far from the unitarity limit.
- Just a few more parameters might be sufficient to describe data up to  $\rho=0.3 \text{ fm}^{-3}$ , this explains why Skyrme works!

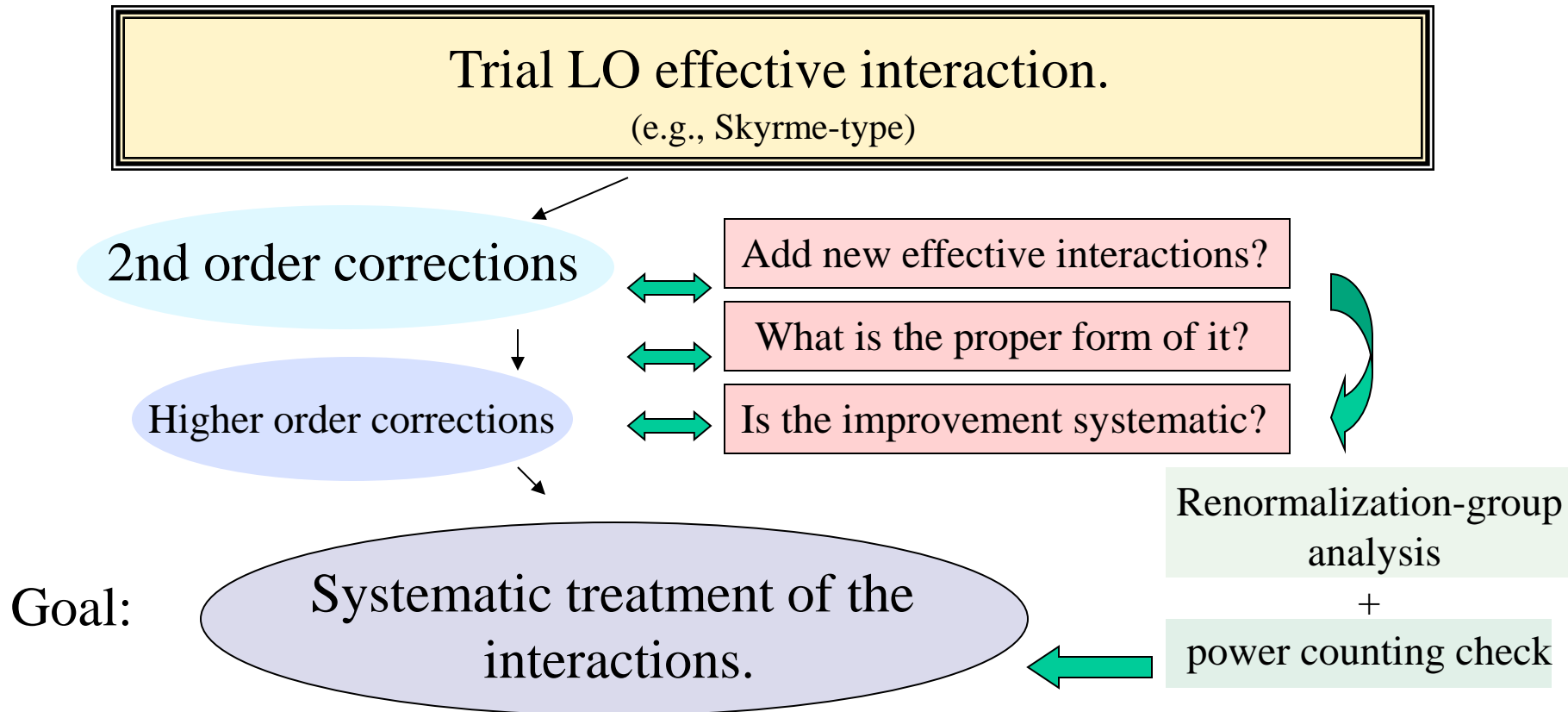
# Choose Skyrme-like interaction as the starting point for EFT approach

- Include **more parameters won't necessarily help.**  
→ Limited predictive power.
- Maybe the correct theory has a structure where **different terms appears at different order.**  
→ Need to go beyond mean field to perform the test.

# Scheme for EFT in EDF<sup>→</sup>

or whatever the name it is

**Try to bridge EFT ideas/techniques to mean field (and beyond) within EDF framework.**



I know ***NOTHING*** about the exact form of LO, NLO, etc.  
 But, for any EFT the following must be true:

$$\mathcal{O}(k, p_{typ}; \Lambda; \bar{\Lambda}_{EFT}) = \sum_i^n \left( \frac{k, p_{typ}}{\bar{\Lambda}_{EFT}} \right)^i \mathcal{O}_i(k, p_{typ}; \bar{\Lambda}_{EFT}) + \mathcal{C}_n(\Lambda; k, p_{typ}, \bar{\Lambda}_{EFT}) \left( \frac{k, p_{typ}}{\bar{\Lambda}_{EFT}} \right)^{n+1}$$

Annotations for the equation:

- observables**: points to  $\mathcal{O}(k, p_{typ}; \Lambda; \bar{\Lambda}_{EFT})$
- cutoff**: points to  $\Lambda$
- order**: points to the summation index  $i$
- Breakdown scale** (given by 1<sup>st</sup> meson not included): points to  $\bar{\Lambda}_{EFT}$
- Residual,  $\sim O(1)$  if:**
  - 1. EFT works
  - 2.  $\Lambda \geq \bar{\Lambda}_{EFT}$
- residual cutoff-dep.**: points to the residual term  $\mathcal{C}_n(\Lambda; k, p_{typ}, \bar{\Lambda}_{EFT})$

**No cutoff here!  $\Rightarrow$  physics cannot dep. on cutoff !**

H. W. Griesshammer, arXiv:1511.00490v3 [nucl-th].

Lepage plot: subtract at two  $\Lambda$ 's to extract "n+1"

# What will an EFT-based force look like?

- Leading order (LO): Need to make a guess.  
=> Since Skyrme-type works so well, try it first!

## Estimation of Breakdown scale

If require  $O\left(\left(\frac{k_F}{M_{hi}}\right)^1\right) > O\left(\left(\frac{k_F}{M_{hi}}\right)^2\right)$

to be valid up to  $\rho=0.3 \text{ fm}^{-3}$ .

Then  $M_{hi}$  need to be at least 400 MeV.

Also, the low bound cannot do better than the unitarity limit.

Then, only applicable for  $\rho > 4 * 10^{-6} [\text{fm}^{-3}]$ .

# Next-to-leading order (NLO) or higher:

## 1. Check renormalizability

C.J. Yang, M. Grasso, U. van Kolck, and K. Moghrabi, PRC 95, 054325 (2017)

## 2. Check power counting

Converging pattern

Lepage plot

## 3. Check reproduction of empirical result

C.J. Yang, M. Grasso, D. Lacroix, PRC 96, 034318 (2017)



Check renormalizability

# 2nd order results for nuclear matter

$$\frac{\Delta E_{sym(l=0)}^{(2)}}{A} = -\frac{mk_F^4}{110880\hbar^2\pi^4} \left\{ \begin{aligned} & \left[ \begin{aligned} & -6534 + 1188\ln[2] + 3564\lambda - 19602\lambda^3 - 5940\lambda^5 \\ & + (1782 - 20790\lambda^4)\ln\left[\frac{\lambda-1}{\lambda+1}\right] \\ & + (24948\lambda^5 - 5940\lambda^7)\ln\left[\frac{\lambda^2-1}{\lambda^2+1}\right] \end{aligned} \right] \tilde{T}_{03}^2 \\ & - \left[ \begin{aligned} & 14696 + 2112\ln[2] + 5280\lambda - 2860\lambda^3 \\ & - 48840\lambda^5 - 18480\lambda^7 + (2640 - 55440\lambda^6)\ln\left[\frac{\lambda-1}{\lambda+1}\right] \\ & + (71280\lambda^7 - 18480\lambda^9)\ln\left[\frac{\lambda^2-1}{\lambda^2+1}\right] \end{aligned} \right] k_F^2 \tilde{T}_{03} \tilde{T}_1 \\ & + \left[ \begin{aligned} & -9886 + 1128\ln[2] + 2520\lambda + 147\lambda^3 - 3654\lambda^5 \\ & - 35280\lambda^7 - 15120\lambda^9 + (1260 - 41580\lambda^8)\ln\left[\frac{\lambda-1}{\lambda+1}\right] \\ & + (55440\lambda^9 - 15120\lambda^{11})\ln\left[\frac{\lambda^2-1}{\lambda^2+1}\right] \end{aligned} \right] k_F^4 \tilde{T}_1^2 \end{aligned} \right\}$$

Diverge as  $\Lambda^5$

$$\frac{\Delta E_{sym(l=1)}^{(2)}}{A} = -\frac{mk_F^8}{73920\hbar^2\pi^4} \left\{ \left[ \begin{aligned} & -1033 + 156\ln[2] + 420\lambda + 140\lambda^3 - 840\lambda^5 \\ & - 5880\lambda^7 - 2520\lambda^9 + (-210 + 6930\lambda^8)\ln\left[\frac{\lambda-1}{\lambda+1}\right] \\ & + (9240\lambda^9 - 2520\lambda^{11})\ln\left[\frac{\lambda^2-1}{\lambda^2+1}\right] \end{aligned} \right] \tilde{T}_2^2 \right\},$$

$$\frac{\Delta E_{neutr(l=0)}^{(2)}}{A} = -\frac{mk_{F_N}^4}{166320\hbar^2\pi^4} \left\{ \begin{aligned} & \left[ \begin{aligned} & -6534 + 1188\ln[2] + 3564\lambda - 19602\lambda^3 - 5940\lambda^5 \\ & + (1782 - 20790\lambda^4)\ln\left[\frac{\lambda-1}{\lambda+1}\right] \\ & + (24948\lambda^5 - 5940\lambda^7)\ln\left[\frac{\lambda^2-1}{\lambda^2+1}\right] \end{aligned} \right] T_{03}^2 \\ & - \left[ \begin{aligned} & 14696 + 2112\ln[2] + 5280\lambda - 2860\lambda^3 \\ & - 48840\lambda^5 - 18480\lambda^7 + (2640 - 55440\lambda^6)\ln\left[\frac{\lambda-1}{\lambda+1}\right] \\ & + (71280\lambda^7 - 18480\lambda^9)\ln\left[\frac{\lambda^2-1}{\lambda^2+1}\right] \end{aligned} \right] k_{F_N}^2 T_{03} T_1 \\ & + \left[ \begin{aligned} & -9886 + 1128\ln[2] + 2520\lambda + 147\lambda^3 - 3654\lambda^5 \\ & - 35280\lambda^7 - 15120\lambda^9 + (1260 - 41580\lambda^8)\ln\left[\frac{\lambda-1}{\lambda+1}\right] \\ & + (55440\lambda^9 - 15120\lambda^{11})\ln\left[\frac{\lambda^2-1}{\lambda^2+1}\right] \end{aligned} \right] k_{F_N}^4 T_1^2 \end{aligned} \right\}$$

Diverge as  $\Lambda^5$

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- When  $\Lambda \rightarrow \infty$ , how the 2<sup>nd</sup> order terms behaves?

$$\frac{\Delta E_f^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4 \hbar^2} k_F^4 [A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4], \quad \text{finite terms}$$

$$\frac{\Delta E_a^{(2)}(k_F, \lambda)}{A} = -\frac{m}{8\pi^4 \hbar^2} \lambda k_F^3 [B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2], \quad \text{Diverge, } k_F\text{-dep appears in MF}$$

$$\frac{\Delta E_d^{(2)}(k_F, \lambda)}{A} = -\frac{m}{8\pi^4 \hbar^2} \lambda k_F^3 [C_0 T_3^2 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4], \quad \text{Diverge, } k_F\text{-dep } \textit{not} \text{ in MF}$$

Note that the above are regulator-dependent, except for the finite terms.

- Treatment I:

Absorb divergence into redefinition of parameters.

- Treatment II:

Add counter terms correspond to the divergences.

# Treatment I:

No new term added, use special cases of  $\alpha$  and  $t_i$

C.J. Yang, M. Grasso, K. Moghrabi, U van Kolck, PRC 95, 054325 (2017)

- Idea: Absorb the  $\Lambda$ -divergence in 2<sup>nd</sup> order into mean field terms with the same  $k_F$ -dependence.

$$\frac{\Delta E_f^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4\hbar^2} k_F^4 [A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4], \quad \text{converge}$$

$$\frac{\Delta E_a^{(2)}(k_F, \lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 [B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2], \quad \text{Diverge, } k_F\text{-dep appears in MF}$$

~~$$\frac{\Delta E_d^{(2)}(k_F, \lambda)}{A} = \frac{m}{8\pi^4\hbar^2} \lambda k_F^3 [C_0 T_3^2 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4], \quad \text{Diverge, } k_F\text{-dep not in MF}$$~~



Eliminate or re-absorb into first two lines by setting:

1.  $\alpha=1/3$  and  $t_1=t_2=0$ .
2.  $\alpha=-1/6$  and  $t_1=t_2=0$ ,  $m=m^R$ .
3.  $\alpha=2/3$  and  $t_1=t_2=t_3=0$ .

# Results: $\alpha=2/3$

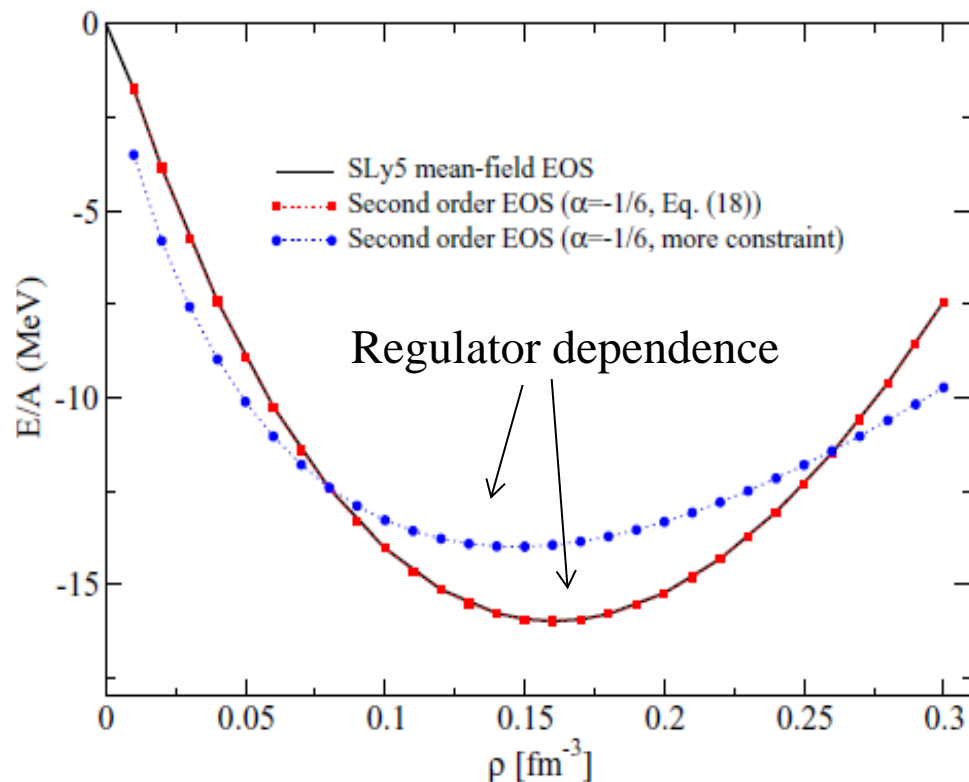
$m$ (MeV)	$t_0^R$ (MeV fm <sup>3</sup> )	$ x_0^R $	$\chi^2$
939	-358.16	$< 10^{-4}$	346 850
-969.55	212.28	$< 10^{-4}$	15 989

No saturation! Only  $t_0$  !



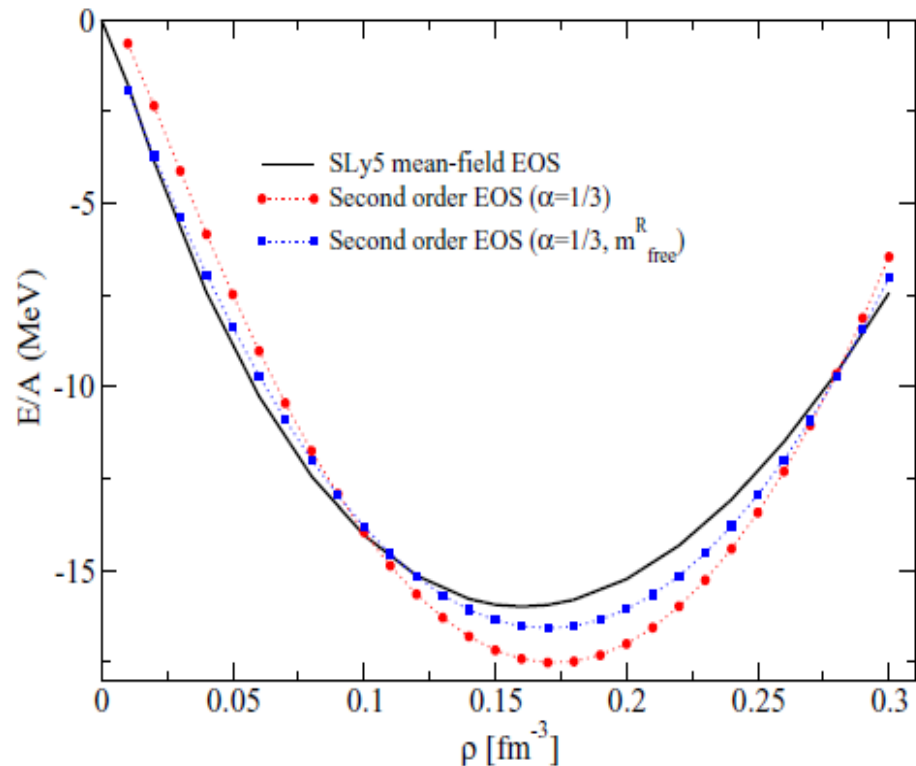
# Results: $\alpha=-1/6$

$m^R$ (MeV)	$t_0^R$ (MeV fm <sup>3</sup> )	$T_3^R$ (MeV fm <sup>5/2</sup> )	$x_0^R$	$x_3^R$	$\chi^2$
591.9	793.15	-1570.8	1.465	-0.1759	<0.1



# Results: $\alpha=1/3$

$m$ (MeV)	$t_0^R$ (MeV fm <sup>3</sup> )	$T_3^R$ (MeV fm <sup>4</sup> )	$ x_3^R $	$\chi^2$
939	-1244.1	247.11	$<10^{-4}$	1364
23845	-580.16	46.248	$<10^{-2}$	188



# Lessons

1. The leading order quite possible just contains only  $t_0$ - $t_3$  terms.
2. However, the **regulator dependence** tells us the power counting cannot be established in this way.

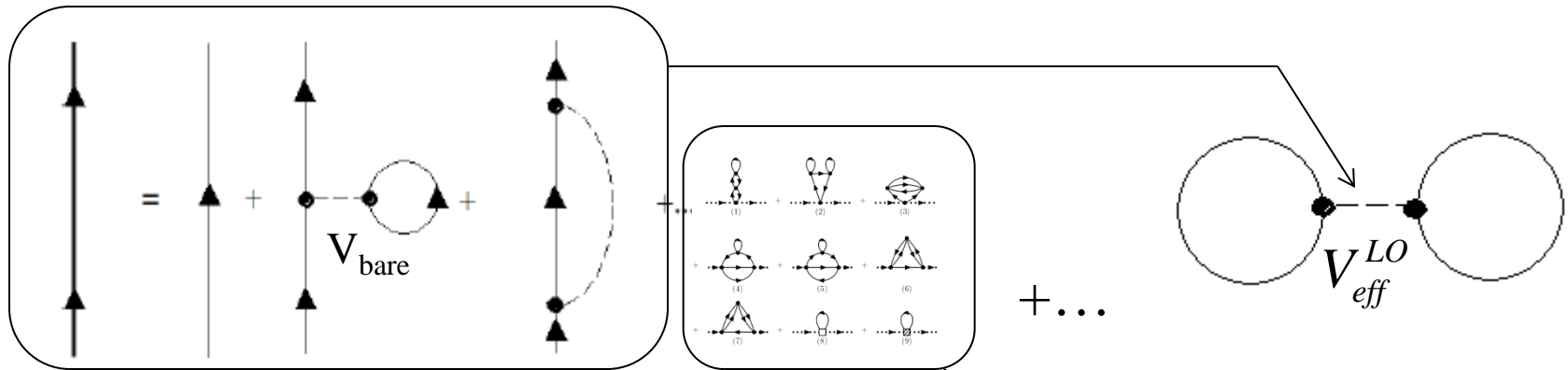
# **More general consideration** **(adding counter terms at NLO):**

C.J. Yang, M. Grasso, D. Lacroix, PRC 96, 034318 (2017)

Diagrammatic explanation of  
the idea

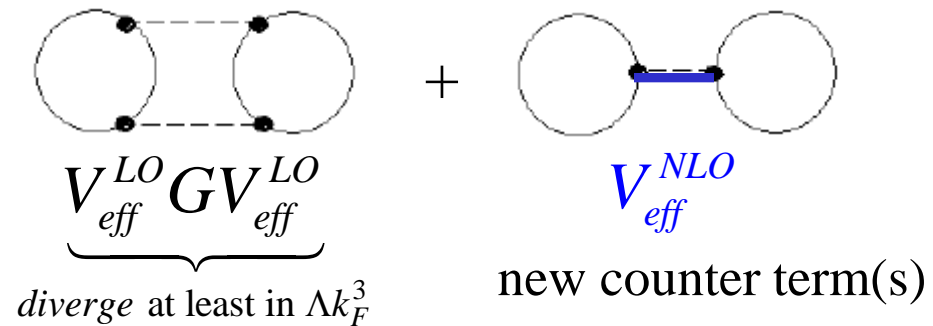


# Dressing of propagator $\rightarrow V_{\text{eff}}$

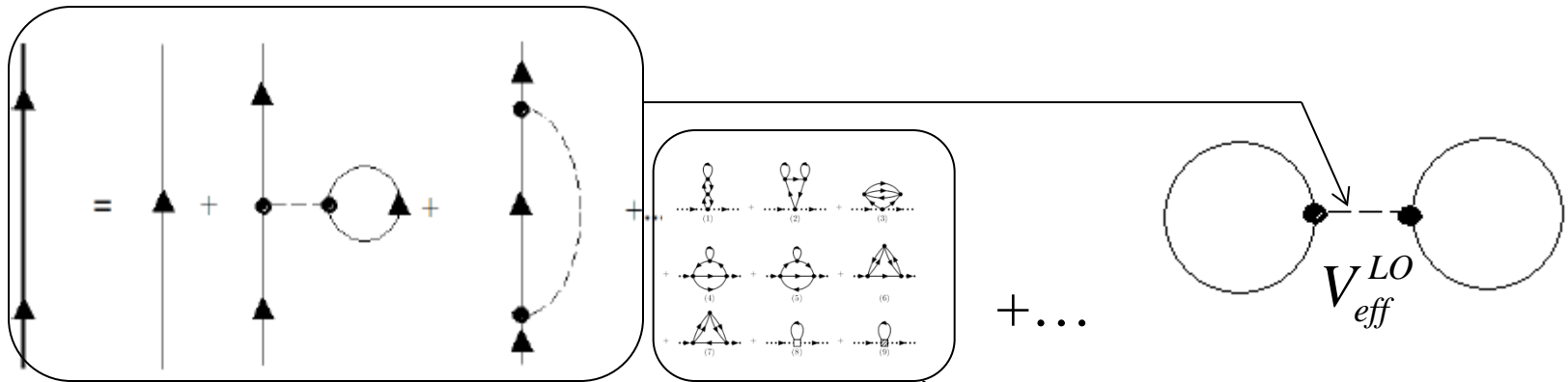


Leading order (LO)

Then, NLO includes (at least):

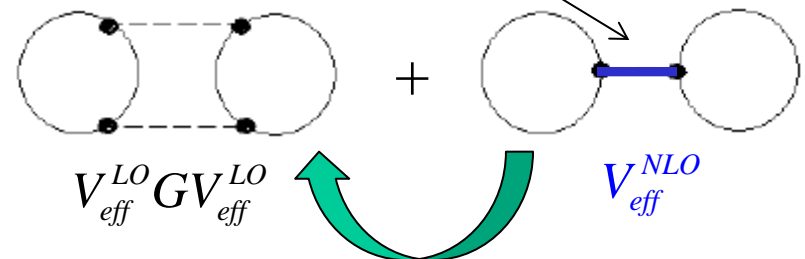


# Dressing of propagator $\rightarrow V_{\text{eff}}$



Leading order (LO)

Then, NLO includes:



\*  $V_{\text{eff}}^{NLO}$  contains (at least) contact terms to renormalize  $V_{\text{eff}}^{LO} G V_{\text{eff}}^{LO}$ .

# Counter term part of the NLO potential

$V_{\text{eff}}^{\text{NLO}}$  : For  $t_0$ - $t_3$  model, the divergence from  $V_{\text{eff}}^{\text{LO}} G V_{\text{eff}}^{\text{LO}}$  is:

$$\underbrace{O(k_F^3), O(k_F^{3+3\alpha})}_{k_F^n\text{-dep. appears in MF}}, \underbrace{O(k_F^{3+6\alpha})}_{\text{new } k_F^n\text{-dep.}}. \quad \text{3 different } k_F\text{-dep.}$$

If want to keep  $\alpha$  free, => Minimum contact term required:  $Ck_F^{3+6\alpha}$ .

Most general case:  $Ak_F^3, Bk_F^{3+3\alpha}, Ck_F^{3+6\alpha}$ .

In infinite matter,  $k_F^{3n}$  in-distinguishable with  $3\pi^2\rho$

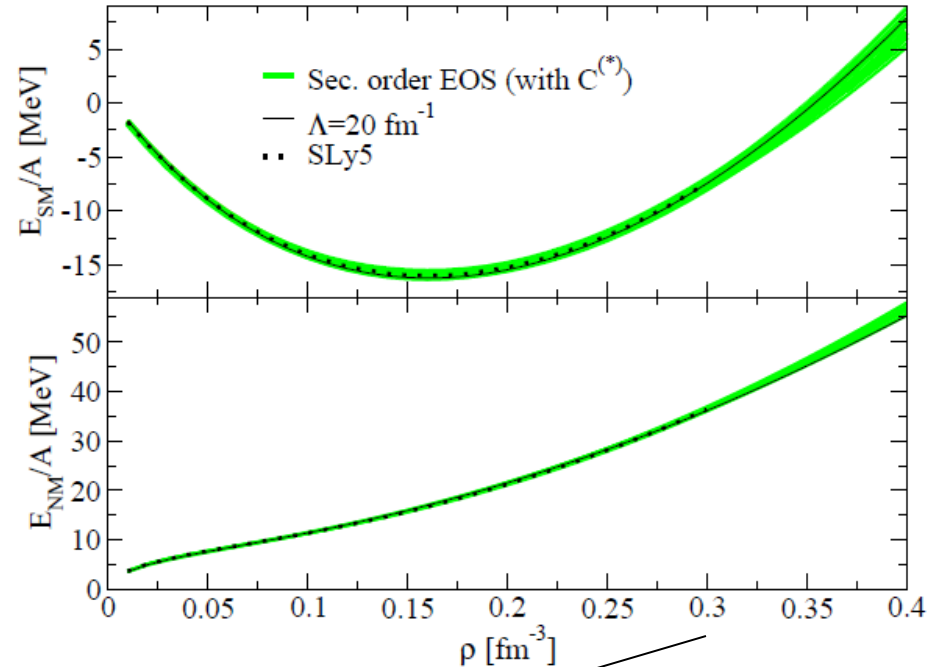
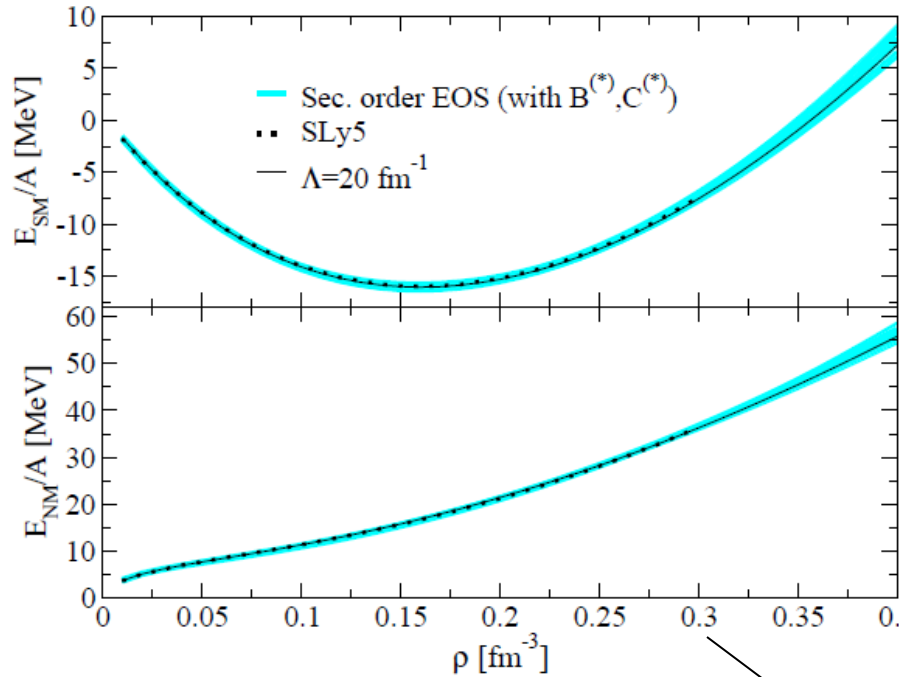
=>  $k_F^n$ -term in EOS *could* originated (at interaction level) from:

$$(k - k')^{n-3\nu-3} \rho^\nu,$$

where  $\nu$  is an extra parameter to be decided in the fitting to finite nuclei.

NLO results (based on  $t_0$ - $t_3$  as LO)  
 $\alpha < 1/6$  case

Color band:  $\Lambda=1.2\sim 20\text{ fm}^{-1}$

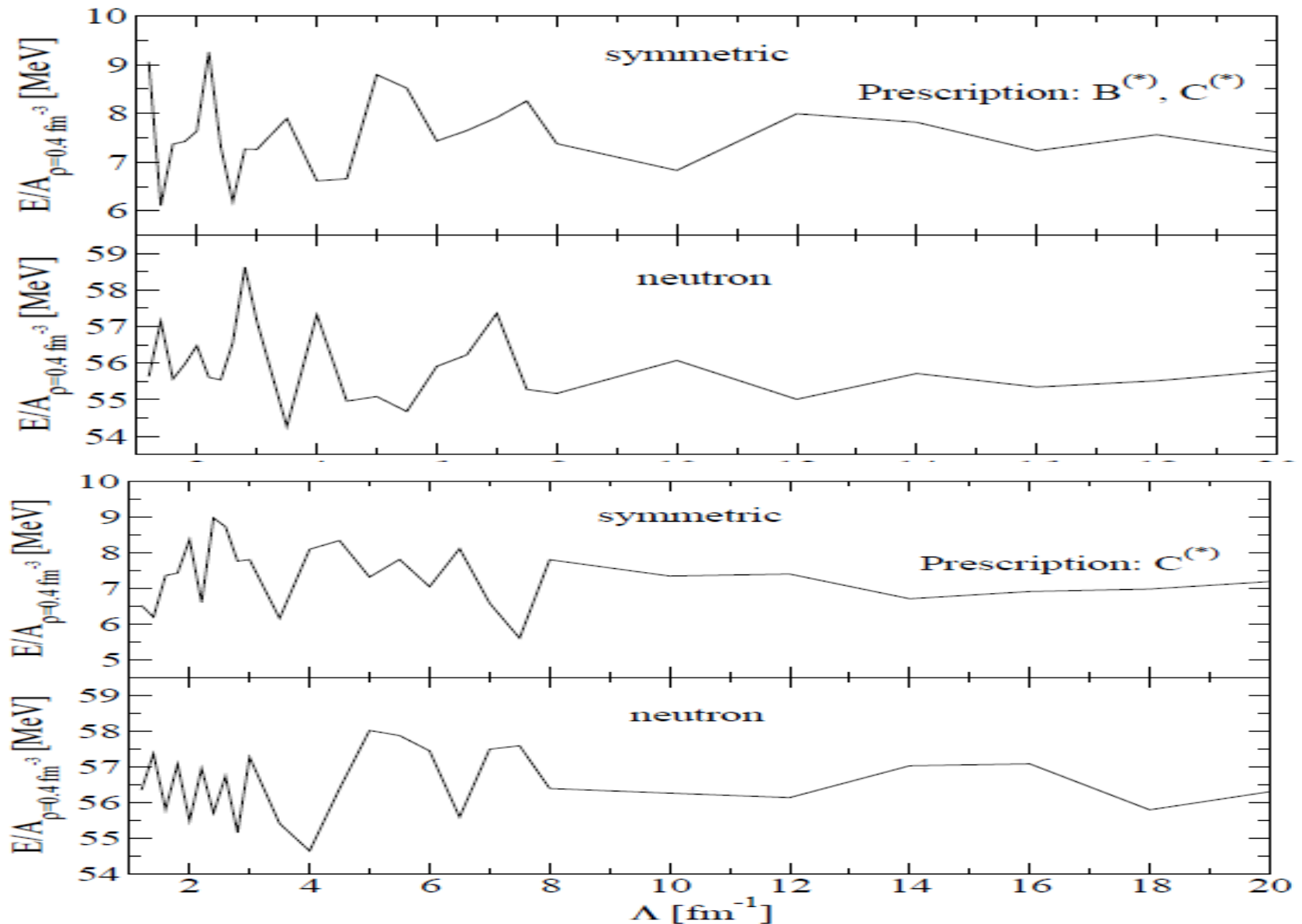


LECs fitted up to  $0.3\text{ fm}^{-1}$

Similar results (with different counter terms) tell us that the regulator-dependence is eliminated by adding counter terms!



# Renormalization group (RG) check at $\rho=0.4 \text{ fm}^{-3}$



# Future work

- So far only perform calculations at EOS level, and has (too) many parameters and limited observables to fit.  $\Rightarrow$  Many sets of LECs fit equally well.

- Need to go to:

1. finite nuclei  $\longrightarrow$

2. NNLO  $\longrightarrow$

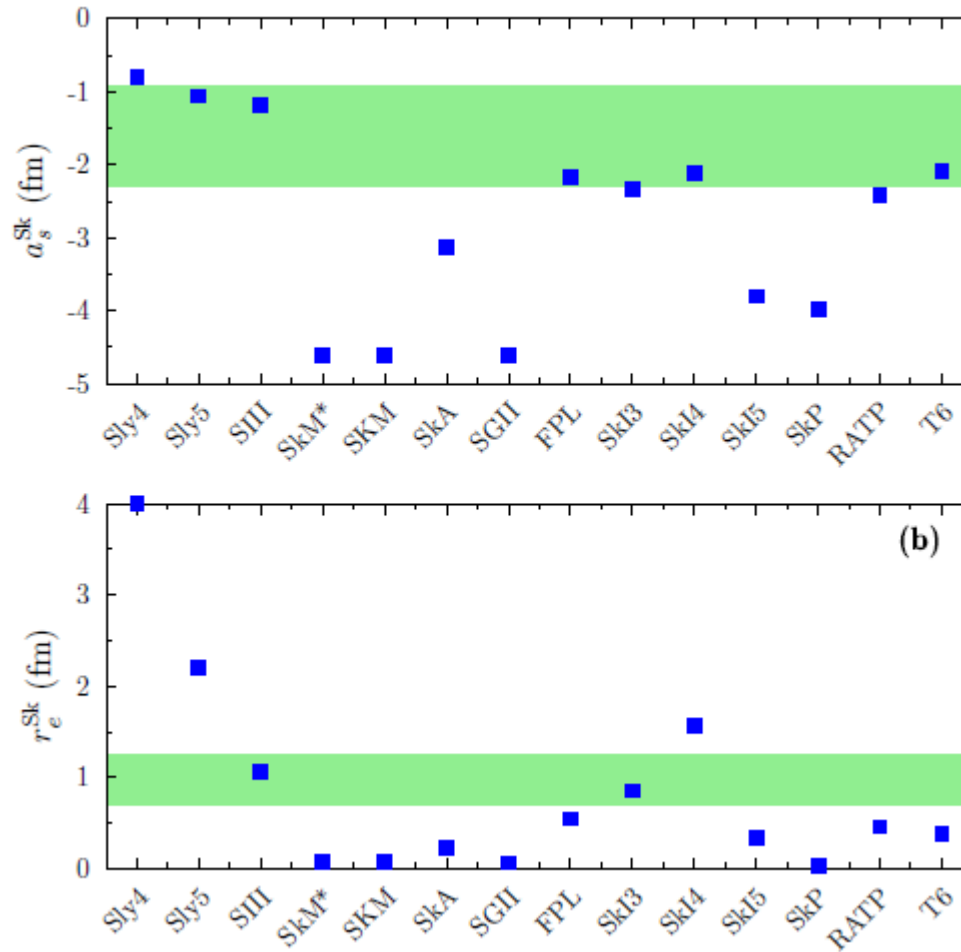


Power counting check  
(e.g., Lepage plot)

Thank you!



Compare  $\tilde{a}_s(k_F)$ ,  $\tilde{r}_e(k_F)$  generated by QMC and by Skyrme  $t_i$ ,  $x_i$ :



**Skyrme-like approaches are not far from the unitarity expansion!**