

Perturbative perspectives for nuclear effective field theories

Sebastian König

GANIL Topical Meeting

“Nuclear Structure and Reaction Theories: Building Together for the Future”

Caen, France

October 10, 2017



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Outline

- **Introduction**

- **Part I –**

Effective theory of ^3H and ^3He

SK, H.W. Grießhammer, H.-W. Hammer, U. van Kolck, J. Phys. G 42 045101 (2015)

- **Part II –**

Nuclear physics around the unitarity limit

SK, H.W. Grießhammer, H.-W. Hammer, U. van Kolck, PRL 118 202501 (2017)

- **Part III –**

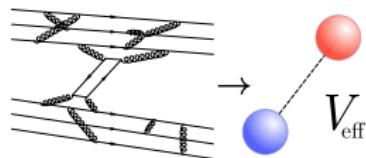
Second-order perturbation theory

SK, J Phys. G 44 064007 (2017)

- **Summary and outlook**

Effective field theories in nuclear physics

- **QCD** = underlying theory of strong interaction
- **EFT** = effective description in terms of hadrons



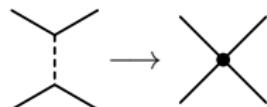
Effective field theory

- separation of scales → expansion parameter
- symmetries restrict possible terms
- order by size → **power counting**

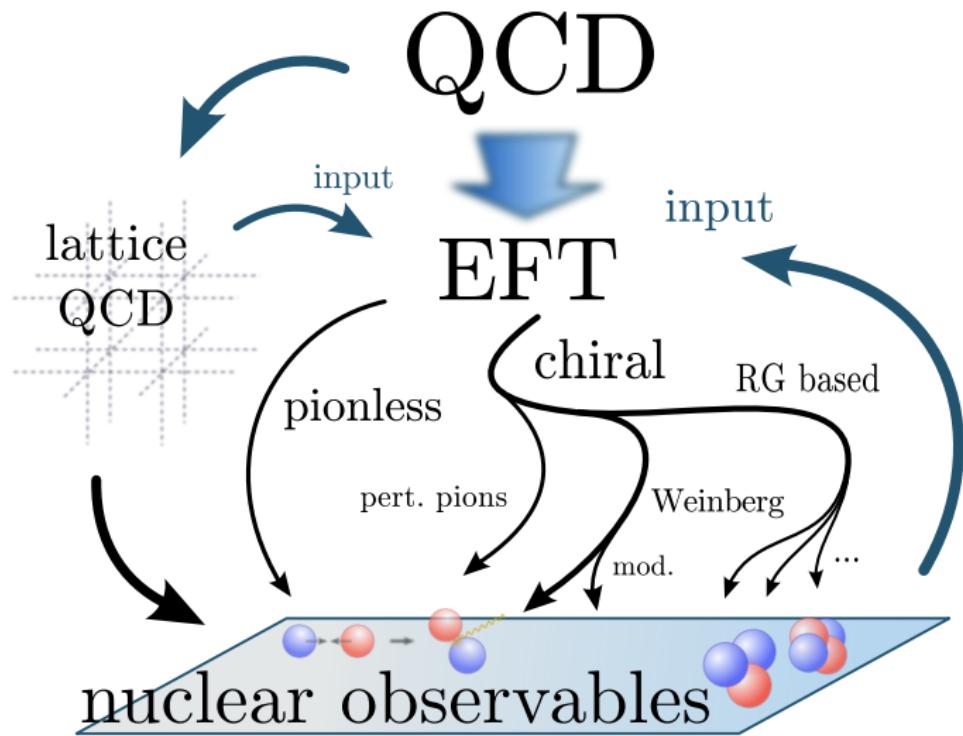
physical properties

↔ small number of low-energy parameters

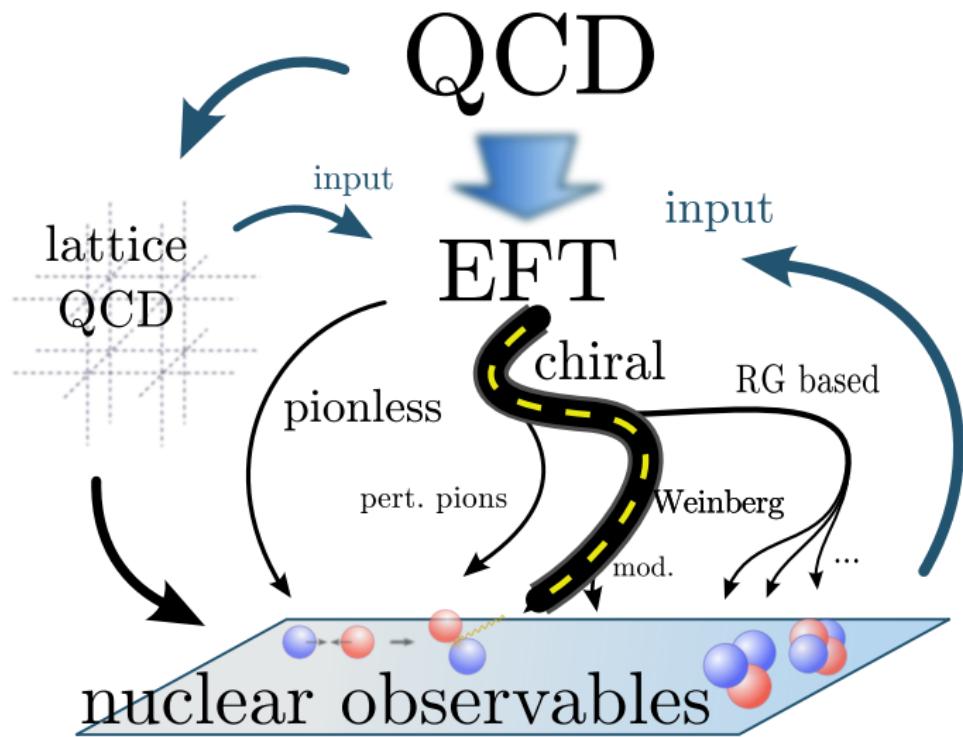
- momentum $q \sim$ pion mass m_π : chiral EFT (nucleons and pions)
- $q \ll m_\pi$: pionless EFT (only nucleons left)



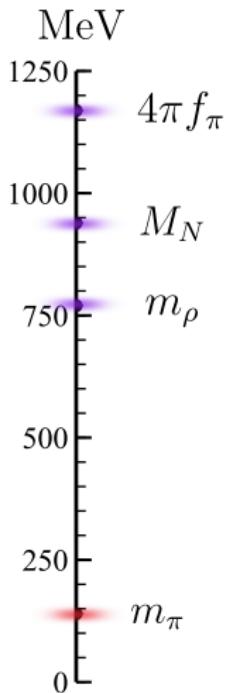
Nuclear EFT overview



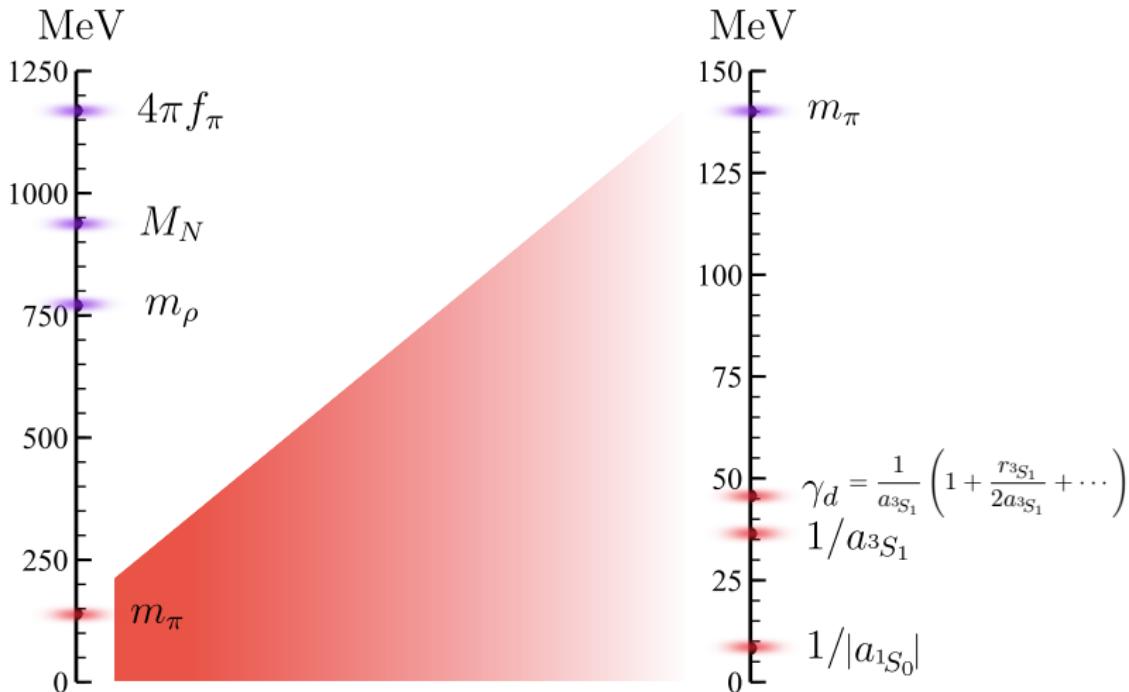
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Nuclear scales



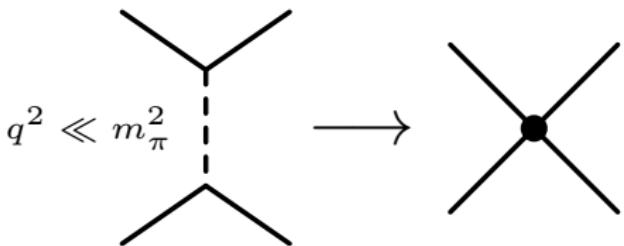
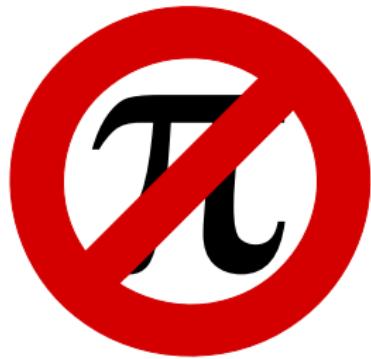
Nuclear scales



- $a_{3S_1} \approx 5.4$ fm, $a_{1S_0} \approx -23.7$ fm $\gg 1/m_\pi \approx 1.4$ fm
- effective ranges (1.8 fm, 2.7 fm) are natural

No pions at low energy!

- derivative coupling of pions!
 → no one-pion exchange contribution to NN scattering lengths!
- chiral power counting designed for momenta $q \sim m_\pi$
- relevant symmetries: spatial (rot., Galilei boost), discrete, isospin
- only contact interactions left – **plus Coulomb**



Coulomb contributions

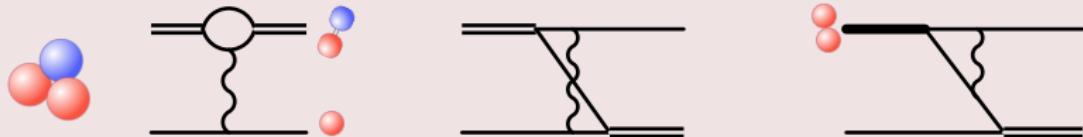
Coulomb photons:  $\sim (\text{ie}) \frac{i}{q^2} (\text{ie}) \longrightarrow (\text{ie}) \frac{i}{q^2 + \lambda^2} (\text{ie})$

- long (infinite) range: $1/r \leftrightarrow 1/q^2$, \mathbf{q} = momentum transfer
- infrared singularity regulated with photon mass λ
- two-body sector can be solved analytically

Coulomb diagrams

$$\text{Diagram with shaded loop} = \text{Diagram with red blob} + \text{Diagram with wavy line} + \text{Diagram with wavy line} + \dots$$

$$\text{Diagram with two red blobs} = \dots + \dots + \dots + \dots + \dots$$



Helium-3 energy

$$\begin{aligned} \text{Diagram A} &= \text{Diagram B} + \text{Diagram C} + \text{Diagram D} + \text{Diagram E} \times (\text{Diagram F} + \text{Diagram G} + \text{Diagram H}) \\ &+ \text{Diagram I} \times (\text{Diagram J} + \text{Diagram K}) + \text{Diagram L} \times (\text{Diagram M} + \text{Diagram N}) \end{aligned}$$



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- iterate $\mathcal{O}(\alpha)$ diagrams...

- get ${}^3\text{He}$ pole directly

- \rightsquigarrow **isospin breaking 3N force at NLO**

Vanasse et al., PRC 89 064003 (2014)

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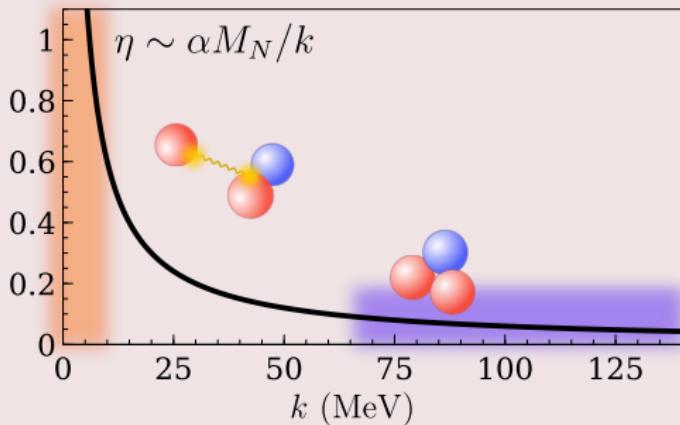
Coulomb corrections for nuclei



How much Coulomb should we really iterate?

Coulomb regimes

trinucleon binding momentum ~ 80 MeV ...



→ should Coulomb not be a small perturbative correction?

Helium-3 energy

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nonperturbative!

Helium-3 energy

$$\begin{aligned} \text{Diagram 1} &= \text{Diagram A} + \text{Diagram B} + \text{Diagram C} + \text{Diagram D} \times (\text{Diagram E} + \text{Diagram F} + \text{Diagram G}) \\ &+ \text{Diagram H} \times (\text{Diagram I} + \text{Diagram J}) + \text{Diagram K} \times (\text{Diagram L} + \text{Diagram M}) \end{aligned}$$

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$$\begin{aligned} \text{Diagram 3} &= \text{Diagram A} + \text{Diagram B} + \text{Diagram C} + \text{Diagram D} \\ &+ \text{Diagram E} \times (\text{Diagram F} + \text{Diagram G}) \end{aligned}$$

- use trinucleon wavefunctions
- fully perturbative in α !

$$\Delta E : \quad \text{Diagram A} - \text{Diagram B}$$



- nonperturbative!
- iterate $\mathcal{O}(\alpha)$ diagrams...
 - get ${}^3\text{He}$ pole directly
 - isospin breaking 3N force at NLO**

Vanassee et al., PRC 89 064003 (2014)

$$\text{Diagram 1} = \text{Diagram A} + \text{Diagram B} + \text{Diagram C}$$

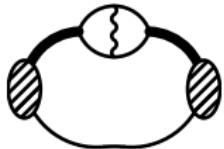
$$\text{Diagram 2} = \text{Diagram A} + \text{Diagram B} + \text{Diagram C}$$

$$\Delta E = \langle \psi | V_C | \psi \rangle$$



SK, Grießhammer, Hammer, J. Phys. G 42 045101 (2015)

New perturbative expansion



- an additional diagram is logarithmically divergent...
- ...but this divergence comes from the photon-bubble subdiagram!

Strategy

SK, Grießhammer, Hammer, van Kolck, JPG 43 055106 (2016)

- ① isolate divergence:

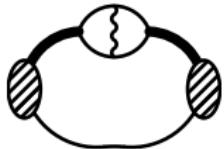
- ② take the leading-order 1S_0 in the unitarity limit

$$a_{^1S_0} = -23.7 \approx \infty \rightsquigarrow 1/a_{^1S_0} \approx 0$$

- ③ include divergent diagram together with finite $a_{^1S_0}$

$$\text{---} \bigcirc \text{---} + \text{---} \diamond \text{---} = \text{finite}$$

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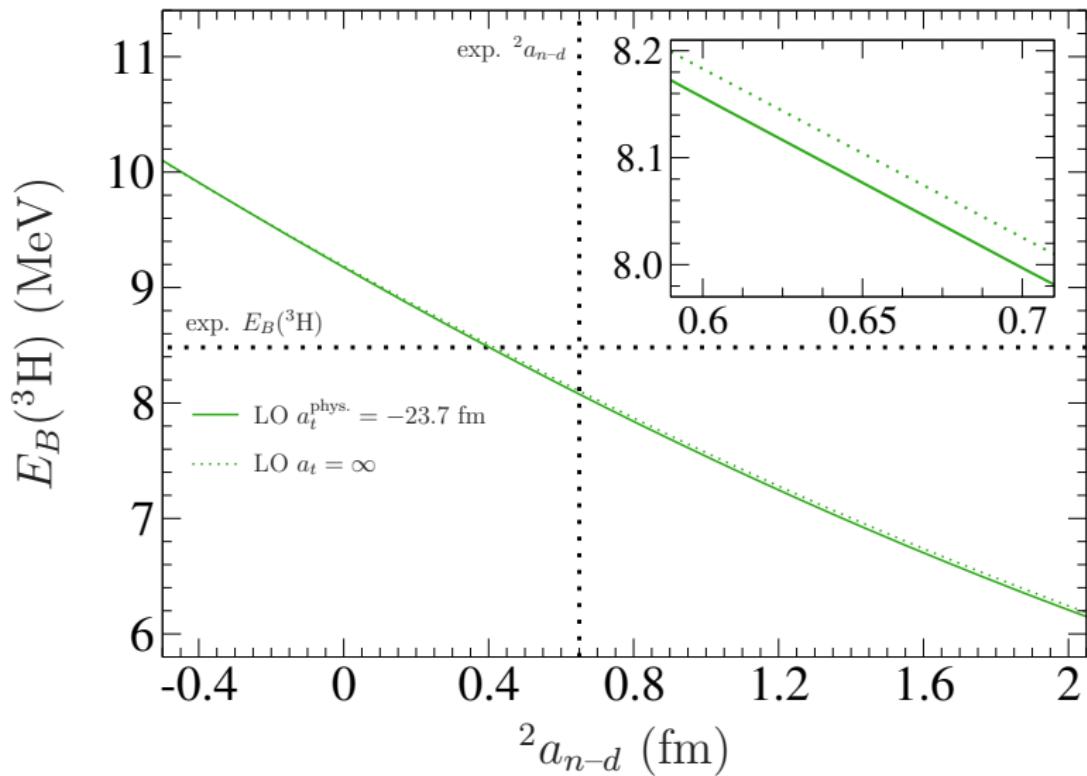
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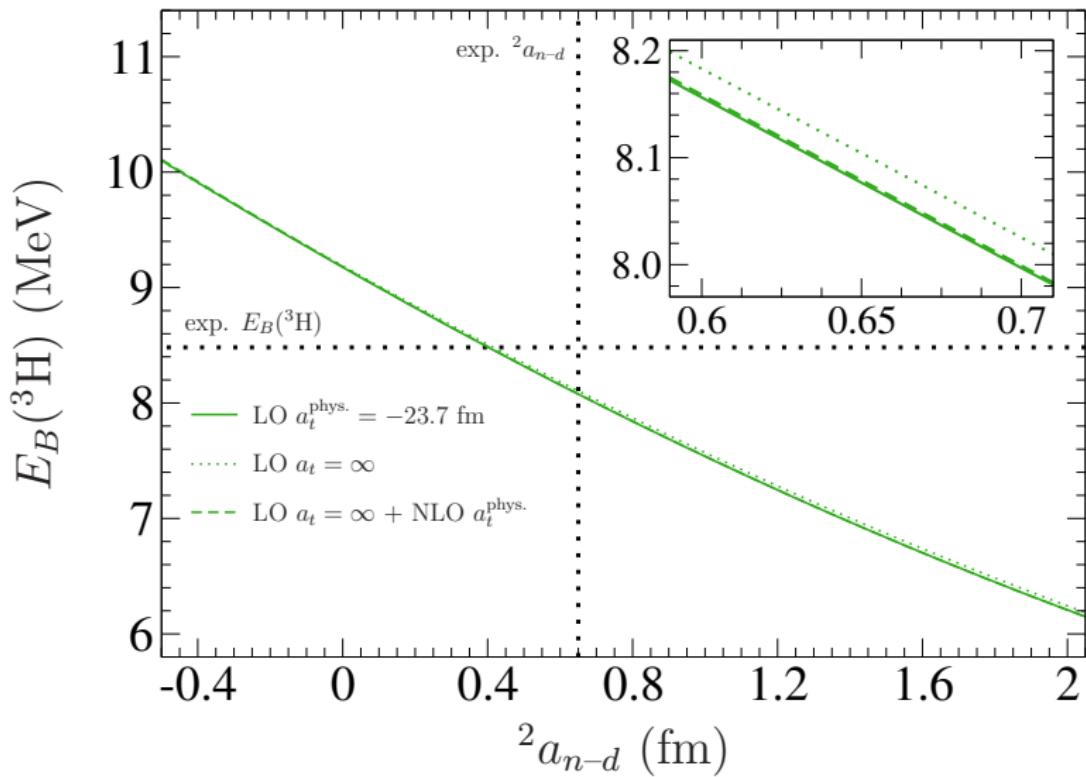
$$\text{---} \bigcirc \text{---} + \text{---} \diamond \text{---} = \text{finite}$$

- new 1S_0 LO is isospin-symmetric and parameter-free
- allows matching between perturbative and non-perturbative regimes

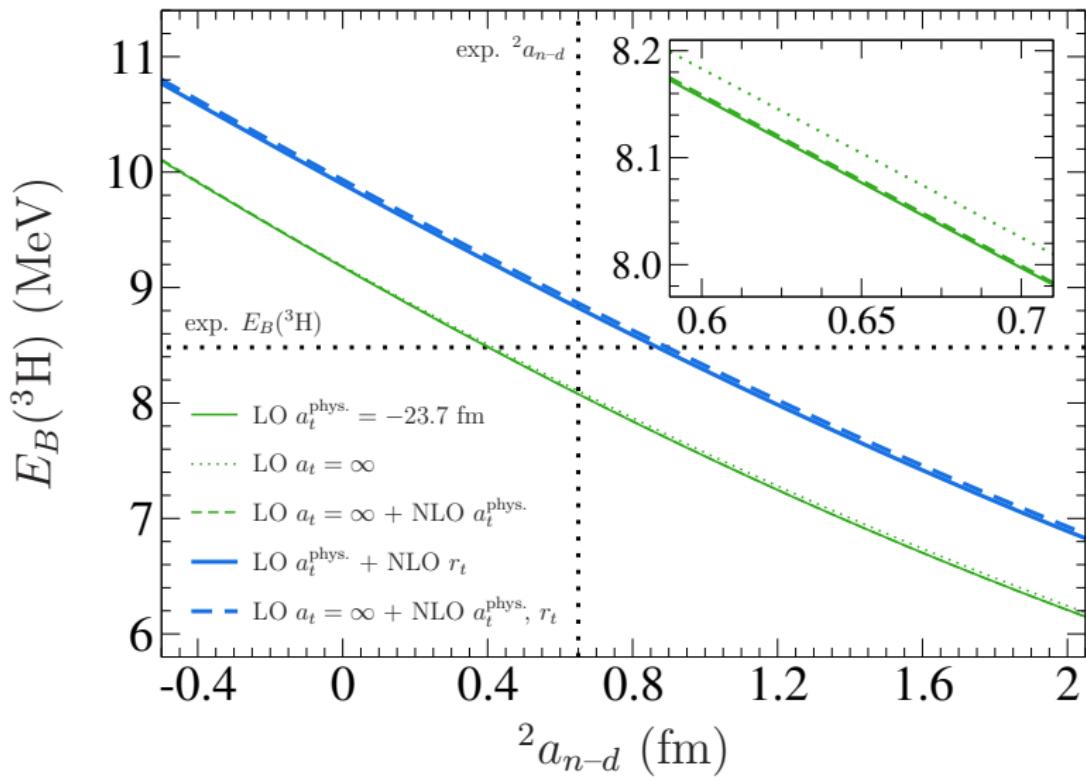
Phillips line



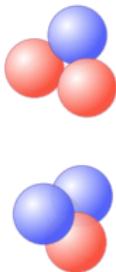
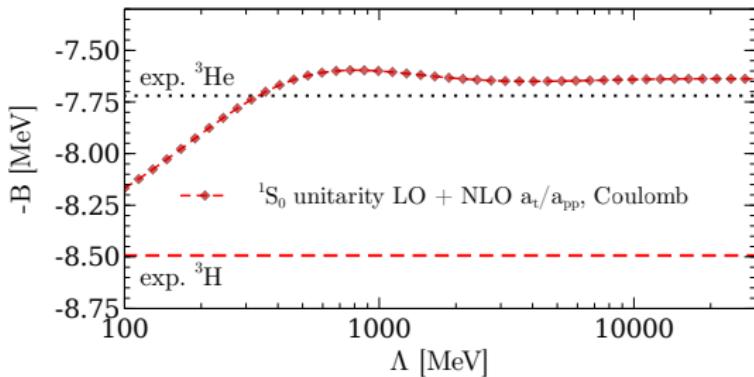
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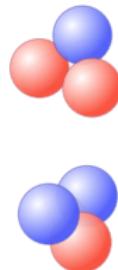
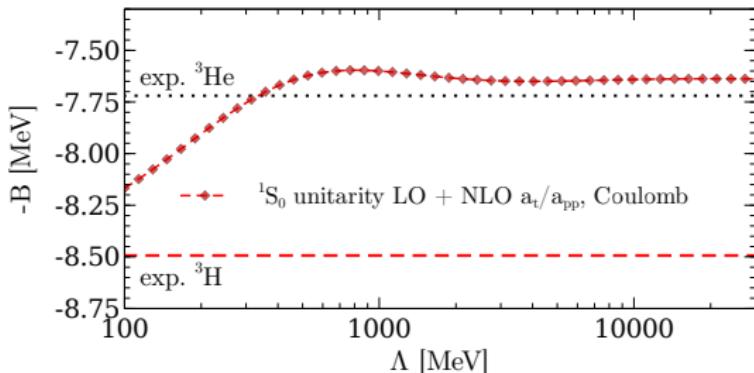


Main result



- result stable for cutoff $\Lambda \rightarrow \infty$: RG invariance ✓
- $\Delta E({}^3\text{He}, {}^3\text{H})^{\text{NLO}} = (-0.86 \pm 0.17) \text{ MeV}$
- ↗ good agreement with experimental value

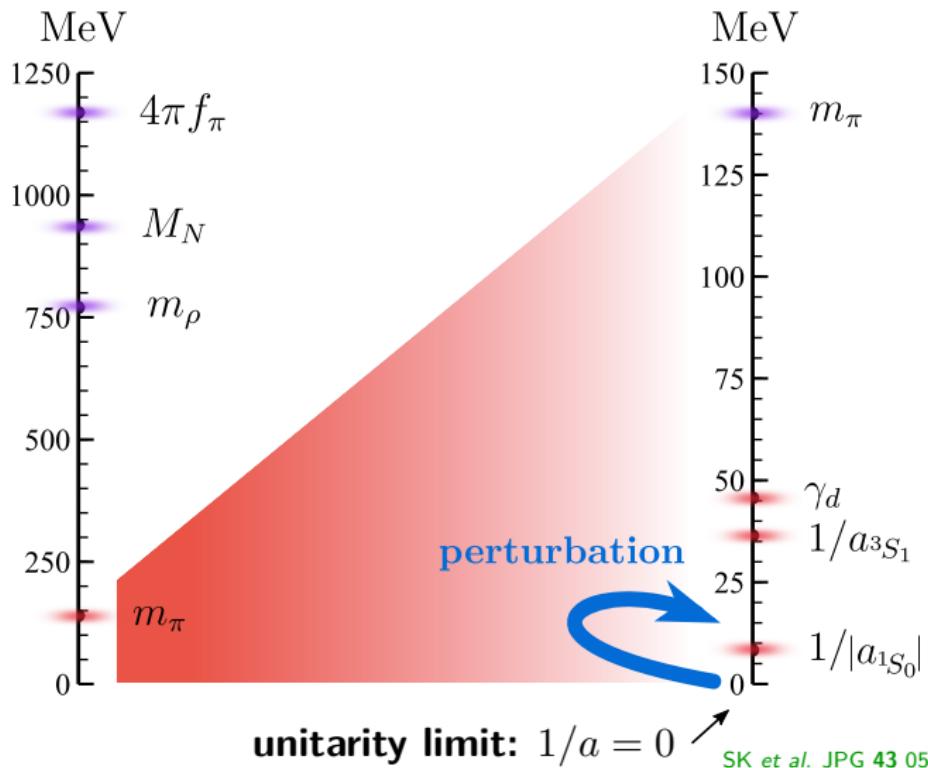
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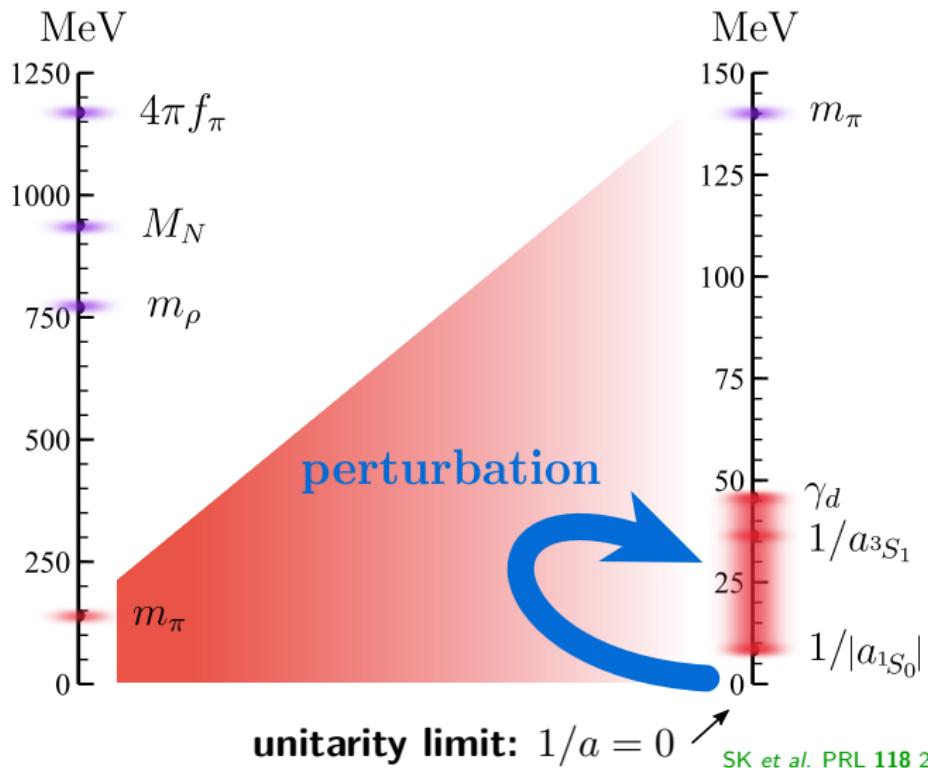
- ① Coulomb perturbative in ${}^3\text{He}$ ✓
- ② consistent renormalization crucial to achieve this!
- ③ ${}^1\text{S}_0$ NN channel can be expanded around unitarity limit

Nuclear scales revisited



SK et al. JPG 43 055106 (2016)

Nuclear scales revisited



SK et al. PRL 118 202501 (2017)

Some context

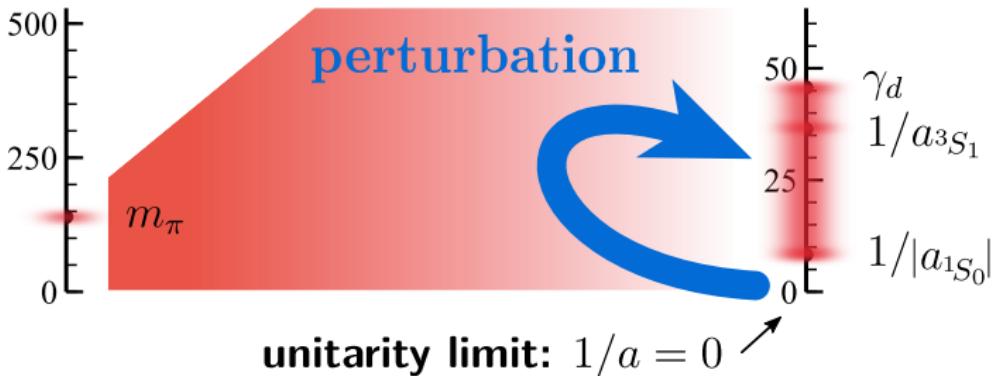
hierarchy of forces (natural in EFT)

many-body forces \leftrightarrow two-body off-shell tuning

Various approaches depart from focusing on two-body input...

- **JISP16** Shirokov *et al.*, PLB **644** 33 (2007)
 \hookrightarrow two-body only, but input from nuclei up to ^{16}O
- **N2LO_{opt}, N2LO_{sat}** Ekstöm *et al.*, PRL **110** 192502 (2013), PRC **91** 051301 (2015)
 simultaneous fit to NN + light nuclei, saturation properties
- **SRG-evolved 2N + N2LO 3N** Simonis *et al.*, PRC **93** (2016)
 \hookrightarrow predict realistic saturation properties
- **nuclear lattice calculations** Elhatisari *et al.*, PRL **117** 132501 (2016)
 \hookrightarrow use input from α - α scattering
- ...

The unitarity expansion



Basic setup

- two-body physics (LECs) \leftrightarrow effective range expansion
- assume $a_{s=^1S_0,t=^3S_1} = \infty \iff 1/a_{s,t} = 0$ at leading order
- **need pionless LO three-body force!**
 \hookrightarrow reproduce triton energy exactly
- finite a , Coulomb, ranges \rightarrow **perturbative corrections!**

The unitarity expansion

Capture gross features at leading order, build up the rest as perturbative “fine structure!”

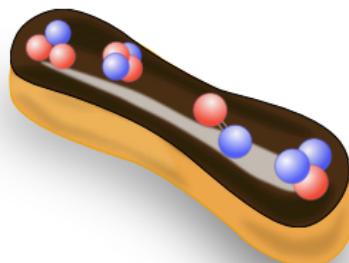
- shift focus away from two-body details
- note: zero-energy deuteron at LO and NLO
- exact $SU(4)_W$ symmetry at LO cf. Vanasse+Phillips, FB Syst. 58 26 (2017)
- universality regime: Efimov effect, bosonic clusters, . . .

Conjecture

Nuclear sweet spot

$$1/a_{s,t} < Q_A < 1/R$$

$$Q_A \sim \sqrt{2M_N B_A/A}$$

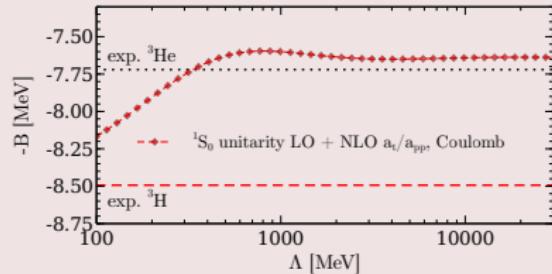


original eclair by Herve1729 (via Wikimedia Commons)

Helium results

^3He at 1S_0 and full unitarity

- good NLO established for 1S_0 unitarity



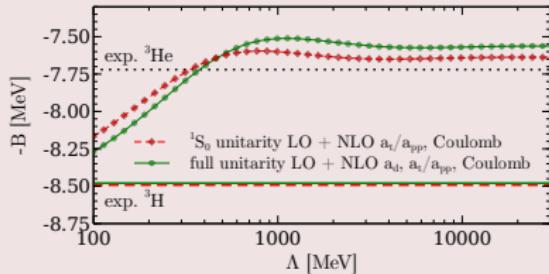
SK, Hammer, Grießhammer, van Kolck (2015/16, 2016/17)

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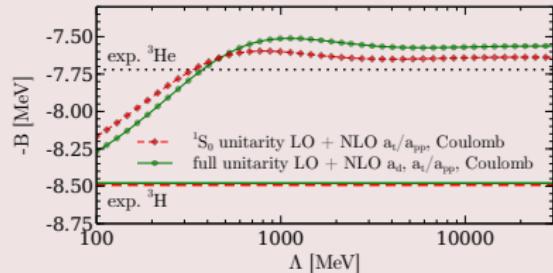
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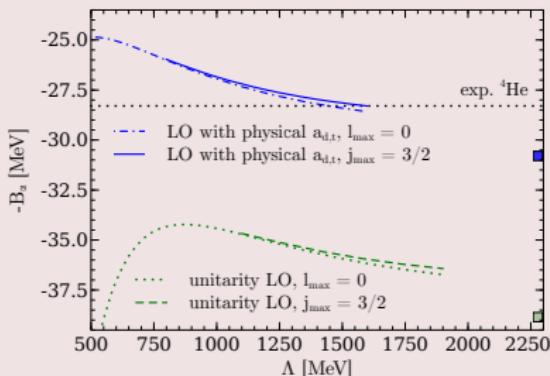
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${}^4\text{He}$ (zero-range, no Coulomb)

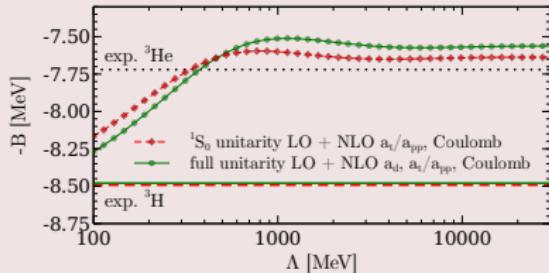


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cf. also Platter (2004)

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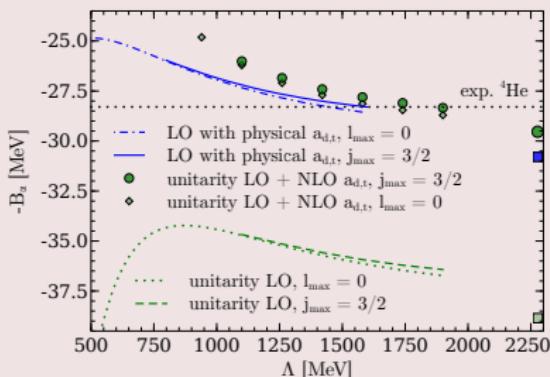
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Some details

- binding energies at LO: find zeros of $\det(\mathbf{1} - K(E))$,
 $K(E)$ = Faddeev(-Yakubowsky) kernel
- NLO energy shift: $\Delta E = \langle \Psi | V^{(1)} | \Psi \rangle$, $|\Psi\rangle$ = LO wavefunction
$$|\Psi\rangle = (\mathbf{1} - P_{34} - PP_{34})(1 + P)|\psi_A\rangle + (\mathbf{1} + P)(\mathbf{1} + \tilde{P})|\psi_B\rangle$$

wavefunction convergence slower than eigenvalue convergence!
→ need more mesh points and partial-wave components...

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Energy balance

- sample calculation with physical scattering lengths at LO:

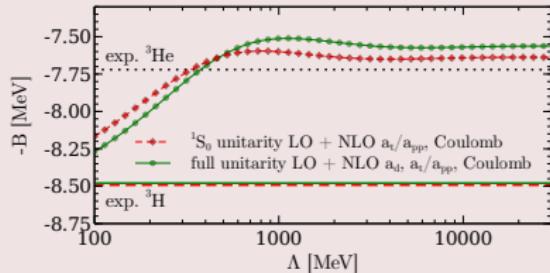
Λ / MeV	800	1000	1200	1400
$E_{\text{kin}} / \text{MeV}$	113.67	140.58	168.44	197.09
$E_{\text{pot}} / \text{MeV}$	-139.77	-167.41	-195.76	-224.62

- E_{kin} and E_{pot} not observable
- sum converges as cutoff is increased, individual values do not!

Helium results

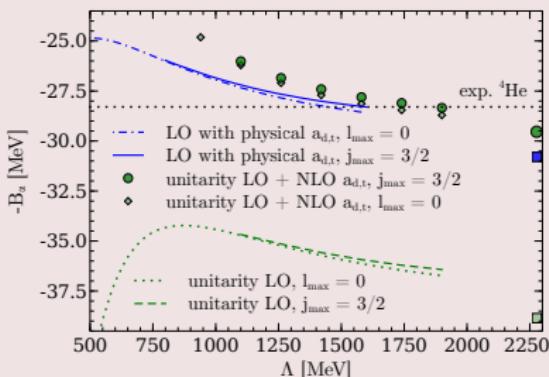
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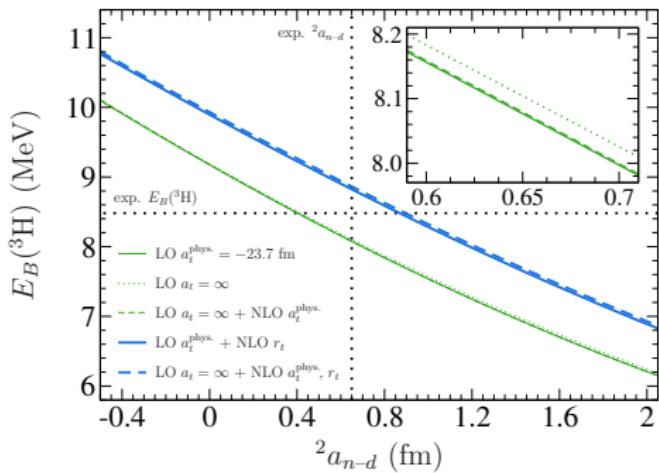
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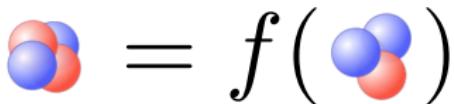


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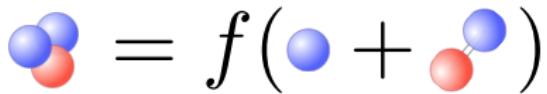
Few-nucleon correlations



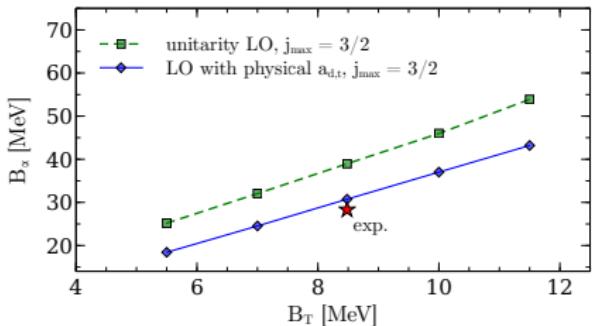
Tjon line



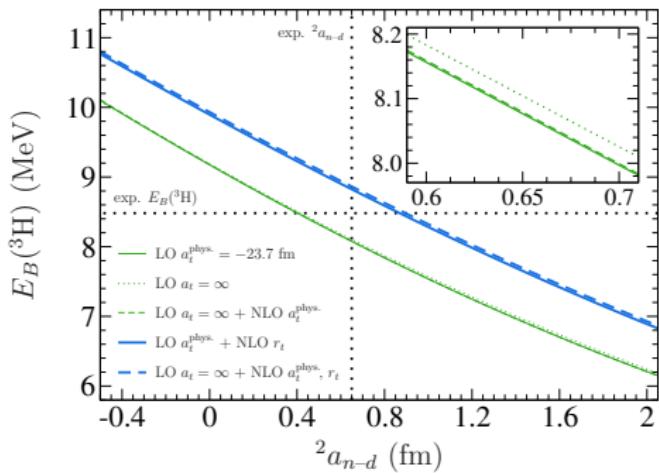
Phillips line



(1S_0 unitarity only)



Few-nucleon correlations

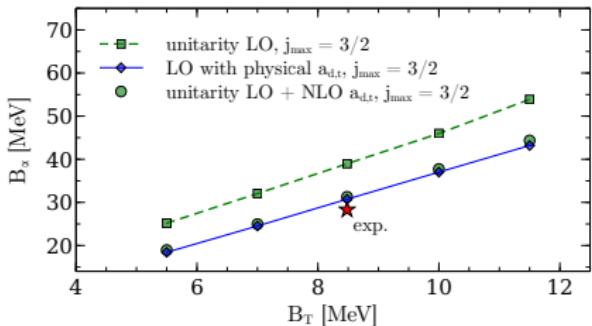
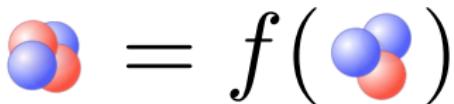


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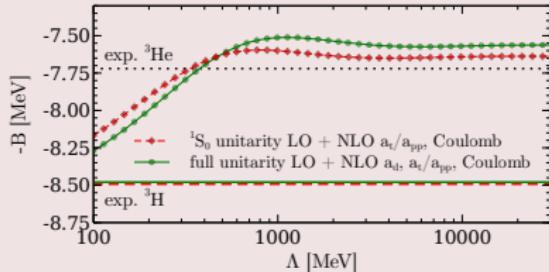


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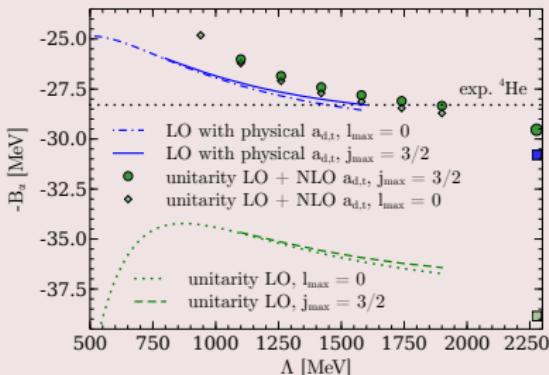
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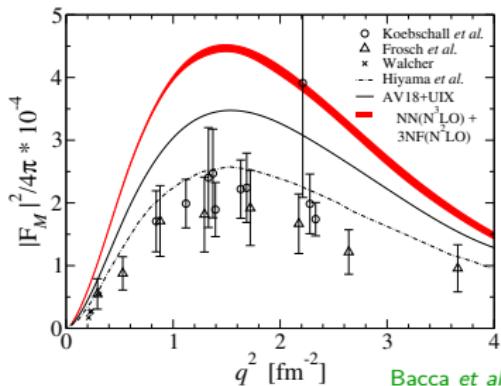
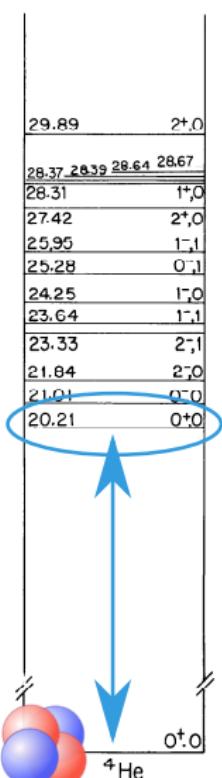


- ${}^4\text{He}$ resonance state ~ 0.3 MeV above ${}^3\text{H} + p$ threshold
- just below threshold at unitarity LO
- boson calculations with nuclear scales
~~ shift by about $0.2 - 0.5$ MeV

SK, Hammer, Grießhammer, van Kolck (2016/17)

cf. also Platter (2004)

^4He monopole resonance



theory A \neq theory B
 \neq experiment!

"a prism to nuclear Hamiltonians"

Bacca et al., PRL 110 042503 (2013)

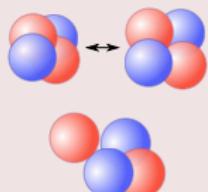
Structure of the 0^+ resonance

- suggested to be a “breathing mode”

Bacca et al., PRC 91 024303 (2015)

- indications for $p+^3\text{H}$ cluster structure

this work



TUNL nuclear data

Unitarity expansion(s) at second order

Various contributions at $N^2LO\dots$

SK, J Phys. G 44 064007 (2017)

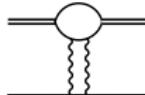
- ① quadratic scattering-length corrections

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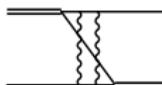
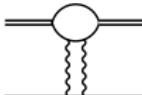
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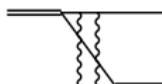
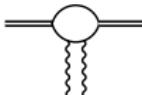
~~ log. divergence, new *pd* counterterm!

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Energy shift from T-matrix

$$B_2 = \lim_{E \rightarrow -B_0} \frac{(E + B_0)^2 \mathcal{T}^{(2)}(E; k, p) + B_1(E + B_0) \mathcal{T}^{(1)}(E; k, p)}{\mathcal{B}^{(0)}(k) \mathcal{B}^{(0)}(p)}$$

cf. Ji+Phillips (2013), Vanasse (2013)

The perturbative deuteron

Efficient method to calculate T-matrix in pert. theory: Vanasse, PRC 88 044001 (2013)

T-matrix perturbation theory for $C_0 = C_0^{(0)} + C_0^{(1)} + C_0^{(2)} + \dots$

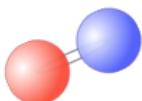
$$\text{Diagram 0} = \text{Diagram X} + \text{Diagram O}$$

$$\text{Diagram 1} = \text{Diagram X} + \text{Diagram O} + \text{Diagram 1}$$

$$\text{Diagram 2} = \text{Diagram X} + \text{Diagram O} + \text{Diagram 1} + \text{Diagram 2}$$

↪ same integral kernel at each order!

- at NLO, the deuteron remains at zero energy...
- ...but it moves to $\kappa^{(1)} = 1/a_t$ at $N^2\text{LO}$



↔ expansion in momentum, not energy

$$B_0 = \frac{(\kappa^{(0)})^2}{M_N}, \quad B_1 = \frac{2\kappa^{(0)}\kappa^{(1)}}{M_N}, \quad B_2 = \frac{(\kappa^{(1)})^2}{M_N}, \quad \kappa^{(0)} \rightarrow 0$$

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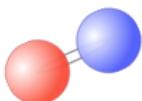
$$\text{Diagram 0} = \text{Diagram X} + \boxed{\text{Diagram Y}}$$

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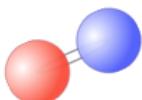
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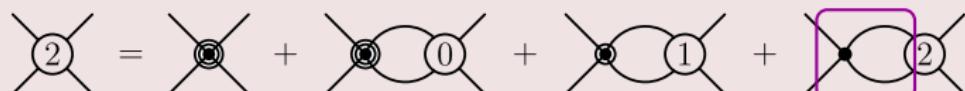
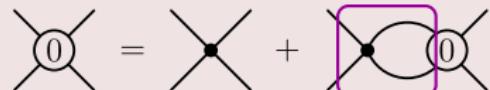
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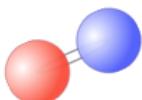
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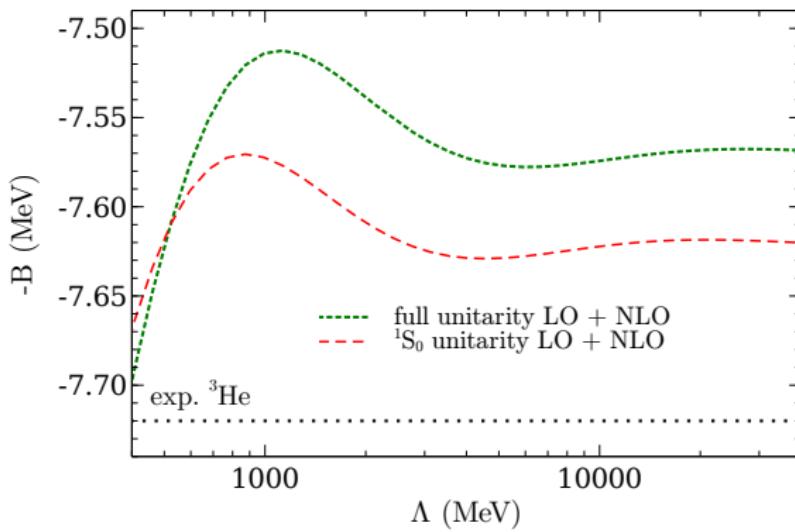
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More ^3He results

SK, J Phys. G 44 064007 (2017)

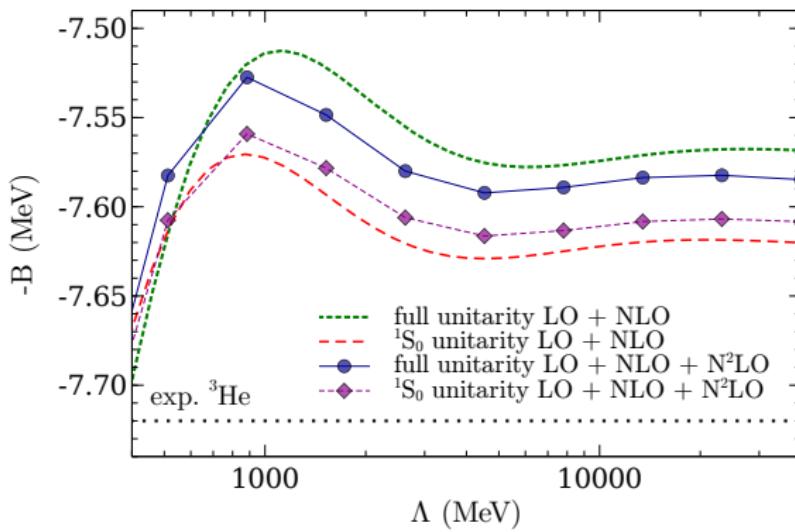
- with range corrections, there is a new pd three-body force at $\text{N}^2\text{LO}\dots$
- ... but the convergence of the unitarity expansions can be checked for the zero-range case



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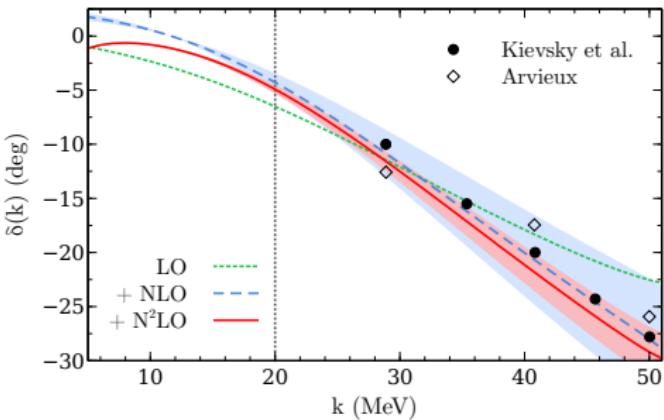
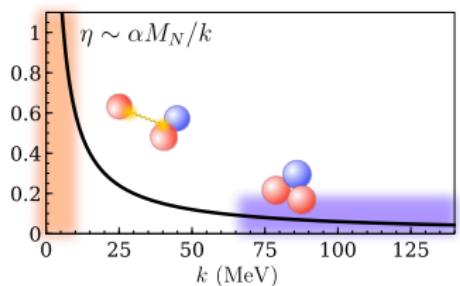


↪ good convergence of half- and full-unitarity expansions

Perturbative p - d phase shifts

At intermediate energies, Coulomb is **perturbative** for pp/pd scattering!

SK *et al.* (2015); SK (2017)



Perturbative subtracted phase shifts

$$\delta(k) \equiv \delta_{\text{full}}(k) - \delta_c(k)$$

$$= \delta_{\text{full}}^{(0)}(k) - \cancel{\delta_c^{(0)}(k)} + \delta_{\text{full}}^{(1)}(k) - \delta_c^{(1)}(k) + \delta_{\text{full}}^{(2)}(k) - \delta_c^{(2)}(k) + \dots$$

cf. also SK, Hammer (2014)

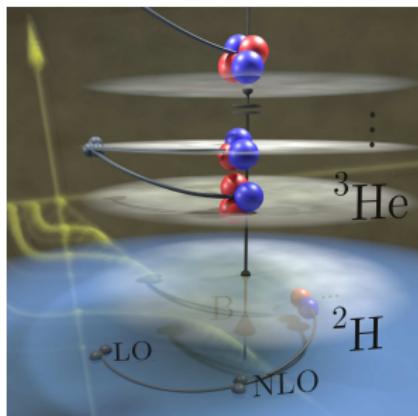
Summary and outlook

Novel approach to few-nucleon systems

SK *et al.*, PRL 118 202501 (2017)

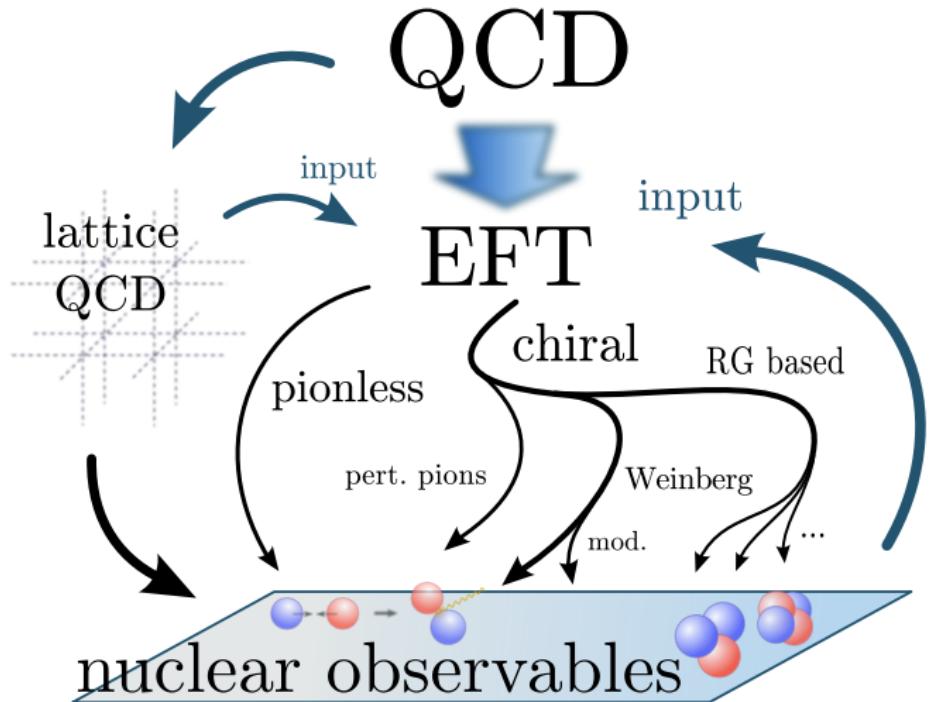
	LO	NLO	N^2LO	exp.
2H	0	0	1.41	2.22
3H	8.48	8.48	8.48	8.48
3He	8.48	7.56		7.72
4He	38.86	29.50		28.30

four-body: no Coulomb, zero-range
NLO uncertainties: 0.2 MeV (3He), 9 MeV (4He)



- emphasize **three-body sector** over two-body precision
- enhanced symmetry and **only one parameter** at leading order
- **conjecture:** unitarity expansion useful beyond four nucleons
- **in progress:** radii, charge form factors *cf.* Vanasse+Phillips, FB Syst. 58 26 (2017)

Further outlook



Further outlook

