Neutron-proton pairing and double beta decay in the IBM

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Neutron-proton pairing and double beta decay in the IBM

- Motivation
- Nucleon-pair shell model
- Effective operators in a collective subspace
- Mapping to boson operators
- Application to $0\nu\beta\beta$ decay in the *pf* shell

Neutrino-less $\beta\beta$ decay

The process:

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+2}X_{N-2} + 2e^{-} $$

Importance: neutrinos are Majorana particles with mass, violation of lepton number, physics beyond the standard model,…

The half-life of this process is

$$ \frac{1}{\tau_{1/2}^{0\nu}} = G_{0\nu} \left| M_{0\nu} \right|^{2} \left| f\left(m_i, U_{ei}\right) \right|^{2} $$

Nuclear physics must provide the nuclear matrix element $M_{0\nu}$.

Observed $2\nu\beta\beta$ emitters

Isotope	isotopic abundance (%)	$Q_{\beta\beta}$ [MeV]
^{48}Ca	0.187	4.263
^{76}Ge	7.8	2.039
^{82}Se	9.2	2.998
^{96}Zr	2.8	3.348
^{100}Mo	9.6	3.035
^{116}Cd	7.6	2.813
^{130}Te	34.08	2.527
^{136}Xe	8.9	2.459
^{150}Nd	5.6	3.371

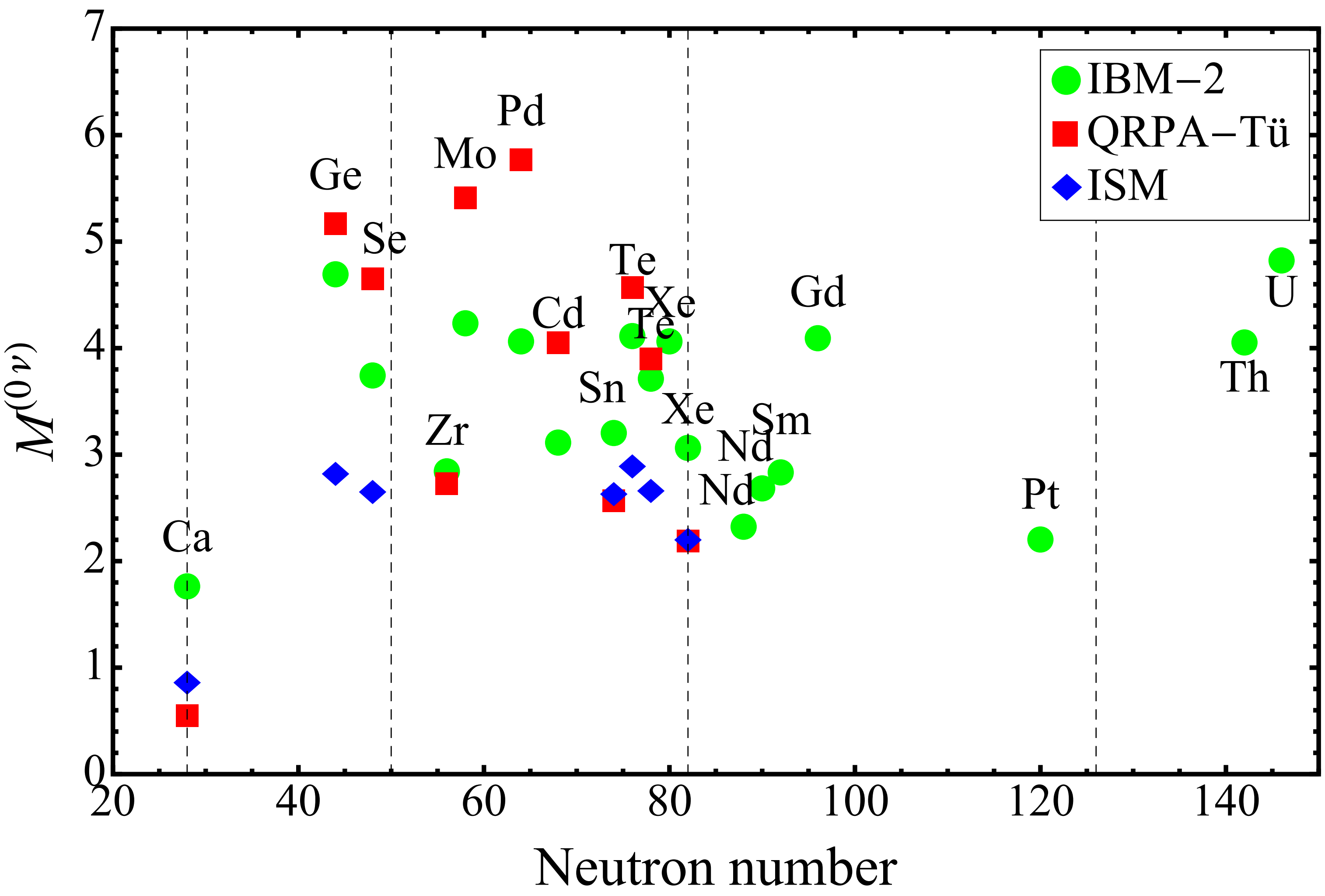
S. Dell'Oro, arXiv:1601.07512v2

$0\nu\beta\beta$ experiments

Isotope	Technique	$T_{1/2}^{0\nu}$	$\langle m_{\beta\beta}\rangle$ (eV)	Reference
^{48}Ca	CaF_2 scint. crystals	$>1.4\times10^{22}$ yr	$<7.2-44.7$	Ogawa *et al.* (2004)
^{76}Ge	^{enr}Ge det.	$>1.9\times10^{25}$ yr	<0.35	Klapdor-Kleingrothaus *et al.* (2001a)
^{76}Ge	^{enr}Ge det.	$(2.23^{+0.44}_{-0.31})\times10^{25}$ yr (1σ)	0.32 ± 0.03	Klapdor-Kleingrothaus and Krivosheina (2006)
^{76}Ge	^{enr}Ge det.	$>1.57\times10^{25}$ yr	$<0.33-1.35$	Aalseth *et al.* (2002a)
^{82}Se	Thin metal foils and tracking	$>2.1\times10^{23}$ yr	$<1.2-3.2$	Barabash (2006b)
^{100}Mo	Thin metal foils and tracking	$>5.8\times10^{23}$ yr	$<0.6-2.7$	Barabash (2006b)
^{116}Cd	$^{116}CdWO_4$ scint. crystals	$>1.7\times10^{23}$ yr	<1.7	Danevich *et al.* (2003)
^{128}Te	Geochemical	$>7.7\times10^{24}$ yr	$<1.1-1.5$	Bernatowicz *et al.* (1993)
^{130}Te	TeO_2 bolometers	$>3.0\times10^{24}$ yr	$<0.41-0.98$	Arnaboldi *et al.* (2007)
^{136}Xe	Liquid Xe scint.	$>4.5\times10^{23}$ yr[a]	$<0.8-5.6$	Bernabei *et al.* (2002)
^{150}Ne	Thin metal foils and tracking	$>3.6\times10^{21}$ yr		Barabash (2005)

F.T. Avignone III et al., Rev. Mod. Phys. **80** (2008) 481

Comparison SM-QRPA-IBM



J. Barea *et al.*, Phys. Rev. C **91** (2015) 034304

Aims of this work

To obtain a better understanding of the relation between IBM and SM.

To use an isospin-invariant version of the IBM.

To study the influence of neutron-proton pairing on double-beta decay.

Nucleon-pair shell model (NPSM)

Pairs of fermions

$$P^{+}_{j_1 j_2 JM} = \left(a^{+}_{j_1} \times a^{+}_{j_2}\right)^{(J)}_{M} \equiv P^{+}_{\alpha JM}$$

Basis states for 2*n* nucleons in NPSM

$$\left|\alpha_1 J_1 \ldots \alpha_n J_n; L_2 \ldots L_n\right\rangle \equiv \left(\cdots\left(\left(P^{+}_{\alpha_1 J_1} \times P^{+}_{\alpha_2 J_2}\right)^{(L_2)} \times P^{+}_{\alpha_3 J_3}\right)^{(L_3)} \times \cdots \times P^{+}_{\alpha_n J_n}\right)^{(L_n)} \left|\mathrm{O}\right\rangle$$

Isospin-invariant formulation: *J* -> *JT*.

Matrix elements can be calculated with a recursive technique.

Use of this overcomplete & non-orthogonal basis requires diagonalization of overlap matrix.

J.-Q. Chen, Nucl. Phys. A **562** (1993) 218; **626** (1997) 686
Y. Zhao and A. Arima, Phys. Reports **545** (2014) 1
G.J. Fu *et al.*, Phys. Rev. C **87** (2013) 044310

Collective subspace

Collective pairs of fermions

$$B^+_{JM} = \sum_{j_1 j_2} \alpha^J_{j_1 j_2} \left(a^+_{j_1} \times a^+_{j_2} \right)^{(J)}_M$$

Collective basis states for $2n$ nucleons

$$\left| J_1 \ldots J_n; L_2 \ldots L_n \right\rangle \equiv \left(\cdots \left(\left(B^+_{J_1} \times B^+_{J_2} \right)^{(L_2)} \times B^+_{J_3} \right)^{(L_3)} \times \cdots \times B^+_{J_n} \right)^{(L_n)} \left| \mathrm{O} \right\rangle$$

Dimension of the collective subspace much smaller than that of the full shell-model space, $\omega \ll \Omega$.

Effective operators

Let $\mathbf{H}_P$ be the collective subspace, $\mathbf{H}_Q$ the excluded space and $\mathbf{H}=\mathbf{H}_P+\mathbf{H}_Q$ the full SM space.

The method of Lee-Suzuki defines an operator η that maps states in $\mathbf{H}_P$ to states in $\mathbf{H}_Q$ such that

$$\hat{\eta}\left(\hat{P}\left|E_k\right\rangle\right)=\hat{Q}\left|E_k\right\rangle, \quad k=1,\ldots,\omega, \quad \left|E_k\right\rangle\in\mathbf{H}$$

For small nucleon number η can be calculated exactly

$$\eta_{ri}=\left(\left(\boldsymbol{I}_\Omega-\boldsymbol{b}^T\times\boldsymbol{b}\right)\times\tilde{\boldsymbol{E}}^T\times\boldsymbol{d}^{-1}\right)_{ri}, \quad r=1,\ldots,\Omega, \quad i=1,\ldots\omega$$

K. Suzuki & S.Y. Lee, Prog. Theor. Phys. **64** (1980)

Mapping to bosons

Map fermion pairs to bosons

$$B^{+}_{JM} = \sum_{j_1 j_2} \alpha^{J}_{j_1 j_2} \left(a^{+}_{j_1} \times a^{+}_{j_2} \right)^{(J)}_{M} \Rightarrow b^{+}_{JM}$$

Basis states for n bosons

$$\left| b^{n}_{i} \right\rangle \equiv \left(\cdots \left(\left(b^{+}_{J_1} \times b^{+}_{J_2} \right)^{(L_2)} \times b^{+}_{J_3} \right)^{(L_3)} \times \cdots \times b^{+}_{J_n} \right)^{(L_n)} \left| \mathrm{o} \right\rangle$$

Corresponding NPSM basis is not orthogonal.
Mapping is based on the diagonalization of the overlap matrix.

P. Löwdin, J. Chem. Phys. **18** (1950) 365

Mapping to boson operators

For the mapping to 1+2-body boson operators we need the correspondence for n=1 & n=2:

$$n=1: \left|B_{JM}\right\rangle \Rightarrow \left|b_{JM}\right\rangle$$

$$n=2: \left|\bar{B}_i^2\right\rangle \Rightarrow \left|\bar{b}_i^2\right\rangle = \sum_{j=1}^{\omega} c_{ij} \left|b_j^2\right\rangle$$

This is known as a \`democratic mapping' and avoids the hierarchy of states in OAI.

Boson operators are defined through

$$\left\langle \bar{b}_i^n \left| \hat{T}^{\mathrm{b}} \right| \bar{b}_j^n \right\rangle = \left\langle \bar{B}_i^n \left| \hat{T}^{\mathrm{f}} \right| \bar{B}_j^n \right\rangle$$

L.D. Skouras *et al.*, Nucl. Phys. A **516** (1990) 255

Application to $0\nu\beta\beta$ decay

Shell-model Hamiltonian in the *pf* shell:

Modified Kuo-Brown KB3G

Collective separable approximation to it

Shell-model $0\nu\beta\beta$-decay operator defined via its matrix elements:

$$M_{0\nu\beta\beta} = M_{0\nu\beta\beta}^{\mathrm{GT}} - \left(\frac{g_{\mathrm{V}}}{g_{\mathrm{A}}}\right)^2 M_{0\nu\beta\beta}^{\mathrm{F}} + M_{0\nu\beta\beta}^{\mathrm{T}}$$

A. Poves et al., Nucl. Phys. A **694** (2001) 157
M. Dufour and A.P. Zuker, Phys. Rev. C **54** (1996) 1641
J. Menéndez *et al.*, Phys. Rev. C **93** (2013) 014305

Mapping to boson models

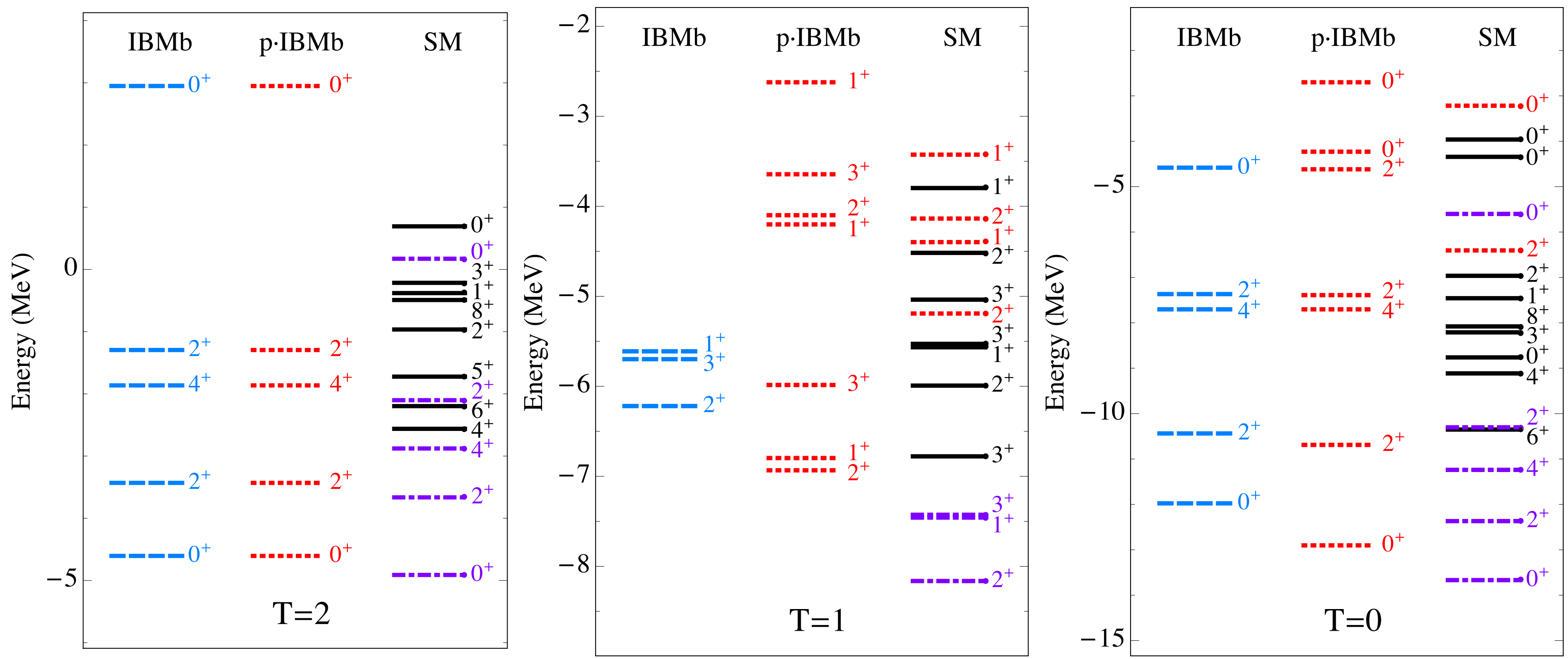
1. Bosons with J=0 (s) and J=2 (d) and isospin T=1. This is an isospin-invariant version of IBM.
2. In addition a boson with J=1 (p) and isospin T=0. This to probe the importance of isoscalar correlations in $0\nu\beta\beta$ decay. This will be referred to as p–IBM.

We map

original SM operators -> IBMb (bare)

effective SM operators -> IBMe (effective)

A=44 energy spectra



A fly in the ointment

The mapped IBM Hamiltonian is obtained from *A*=42 and *A*=44.

To obtain an *A*-dependent IBM Hamiltonian, we use the following property:

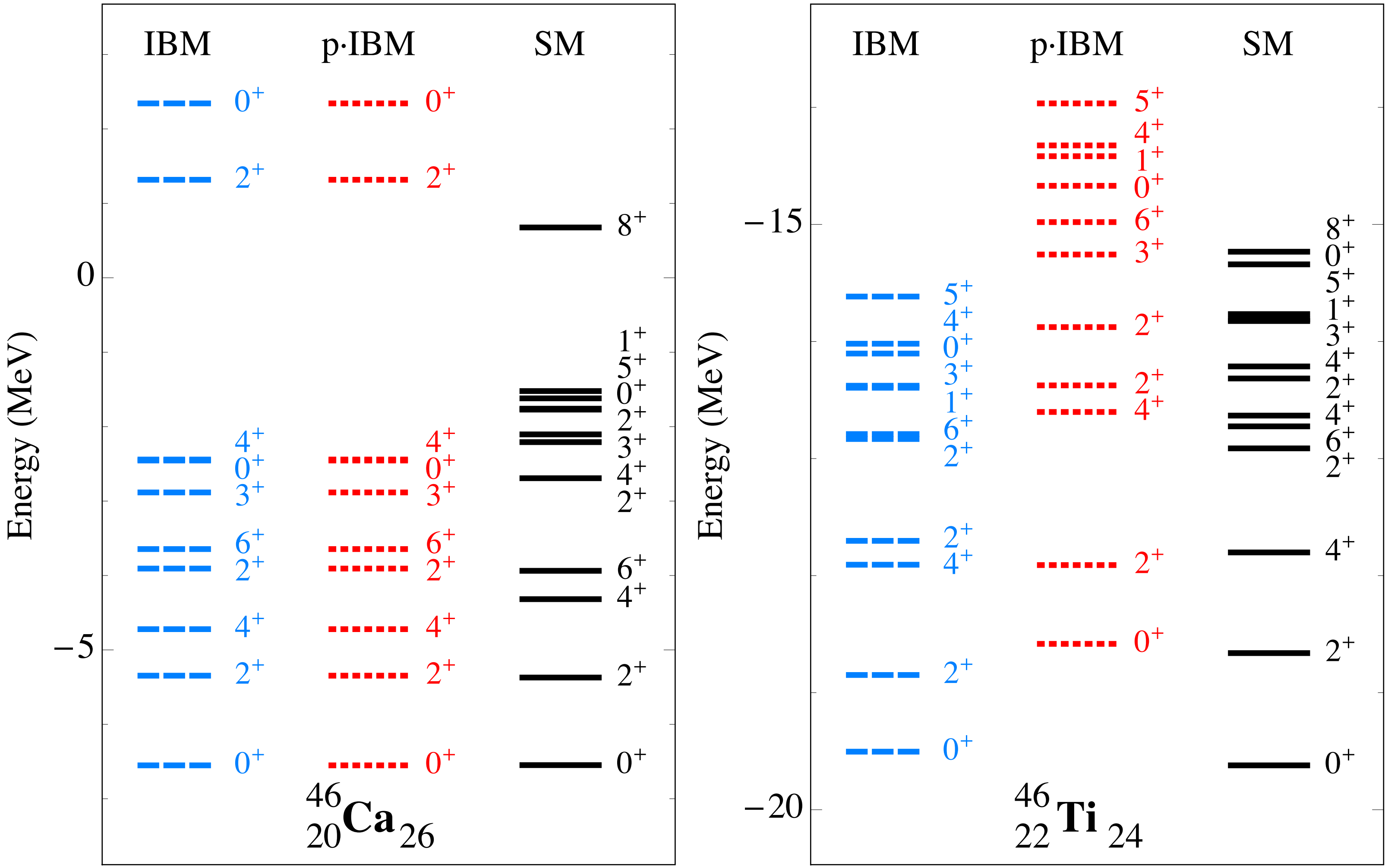
$$\left\langle \hat{H}_{\mathrm{e}}^{\mathrm{b}} \right\rangle_{n,J,T} \leq \left\langle \hat{H}^{\mathrm{f}} \right\rangle_{2n,J,T} \leq \left\langle \hat{H}_{\mathrm{b}}^{\mathrm{b}} \right\rangle_{n,J,T}$$

This suggests the use of an (*n*,*T*)-dependent boson Hamiltonian of the form

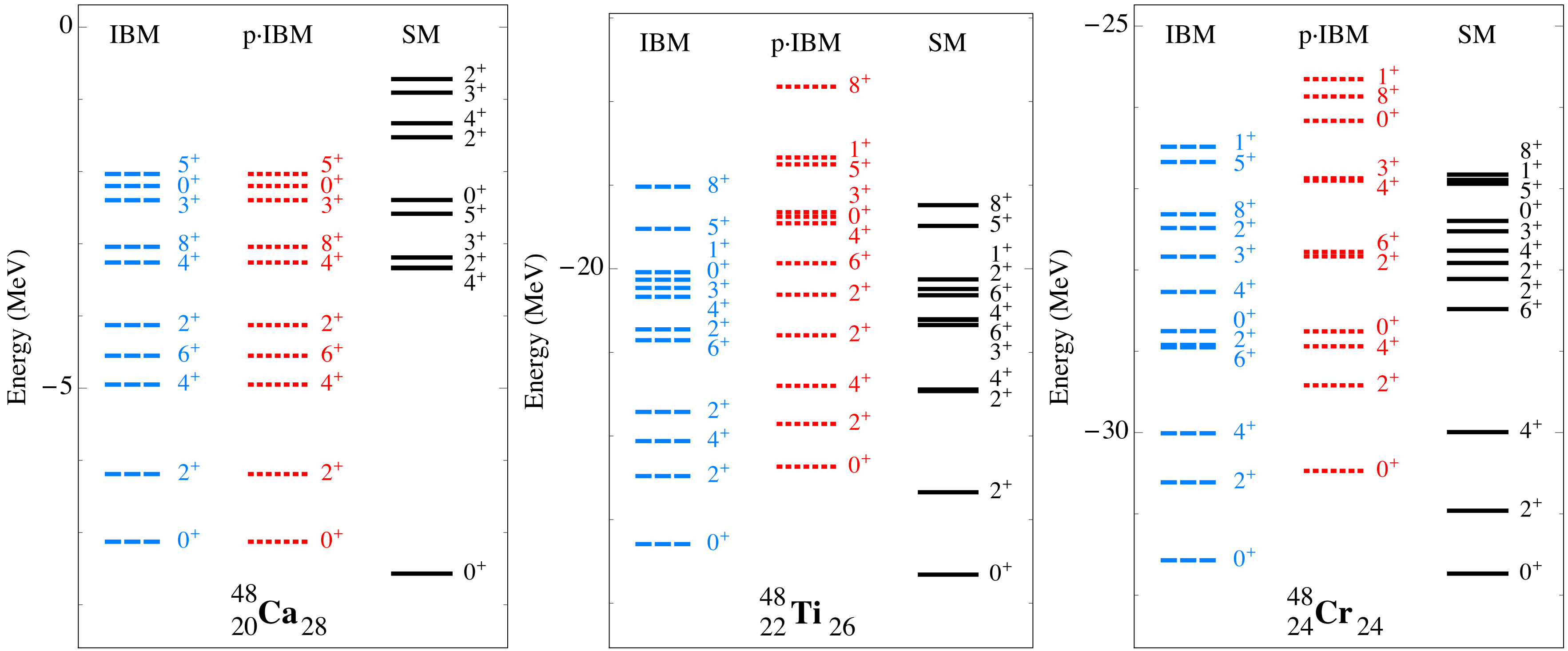
$$\hat{H}^{\mathrm{b}} = x\hat{H}_{\mathrm{e}}^{\mathrm{b}} + \left(1 - x\right)\hat{H}_{\mathrm{b}}^{\mathrm{b}}$$

with *x* an (*n*,*T*)-dependent parameter.

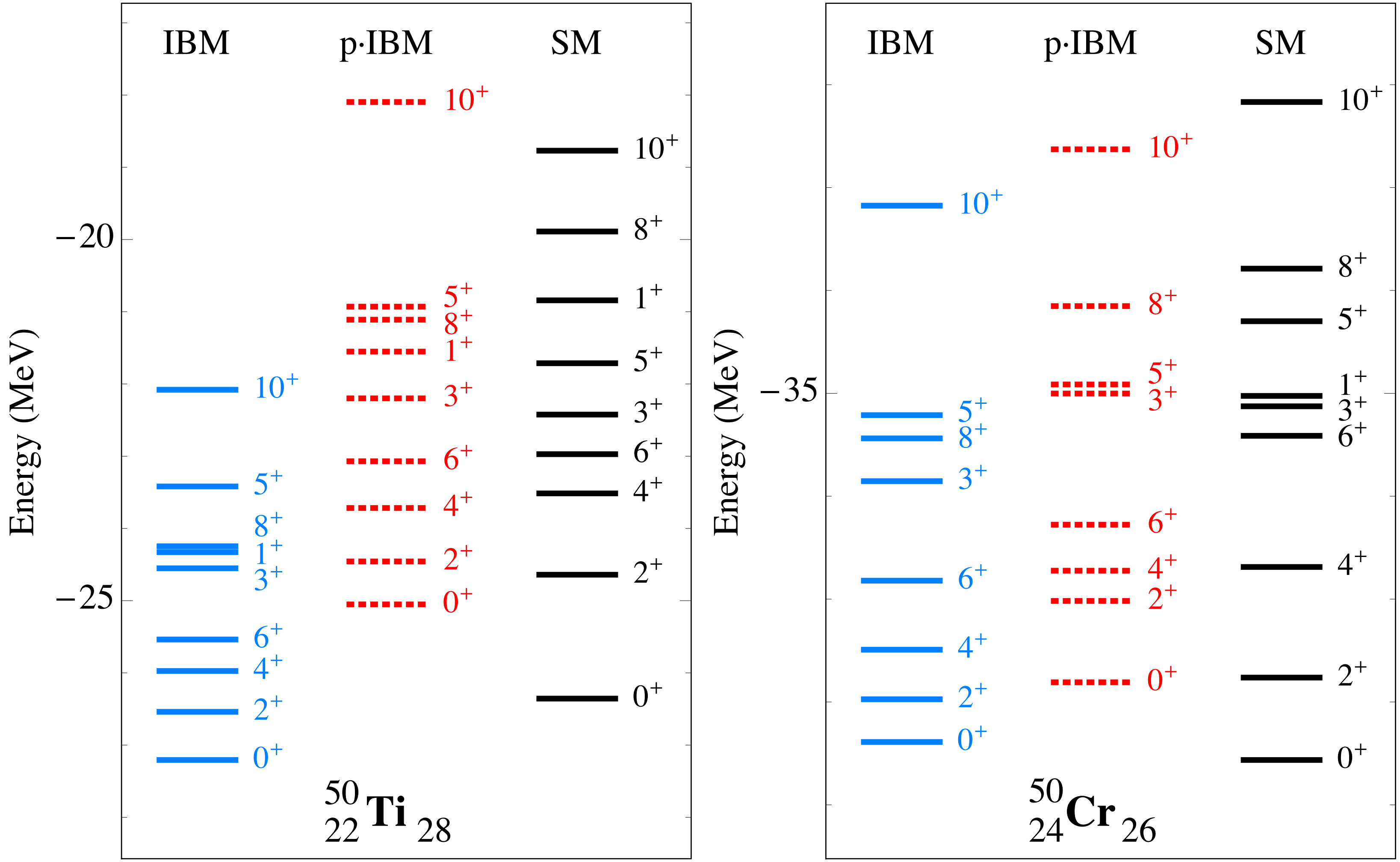
A=46 energy spectra



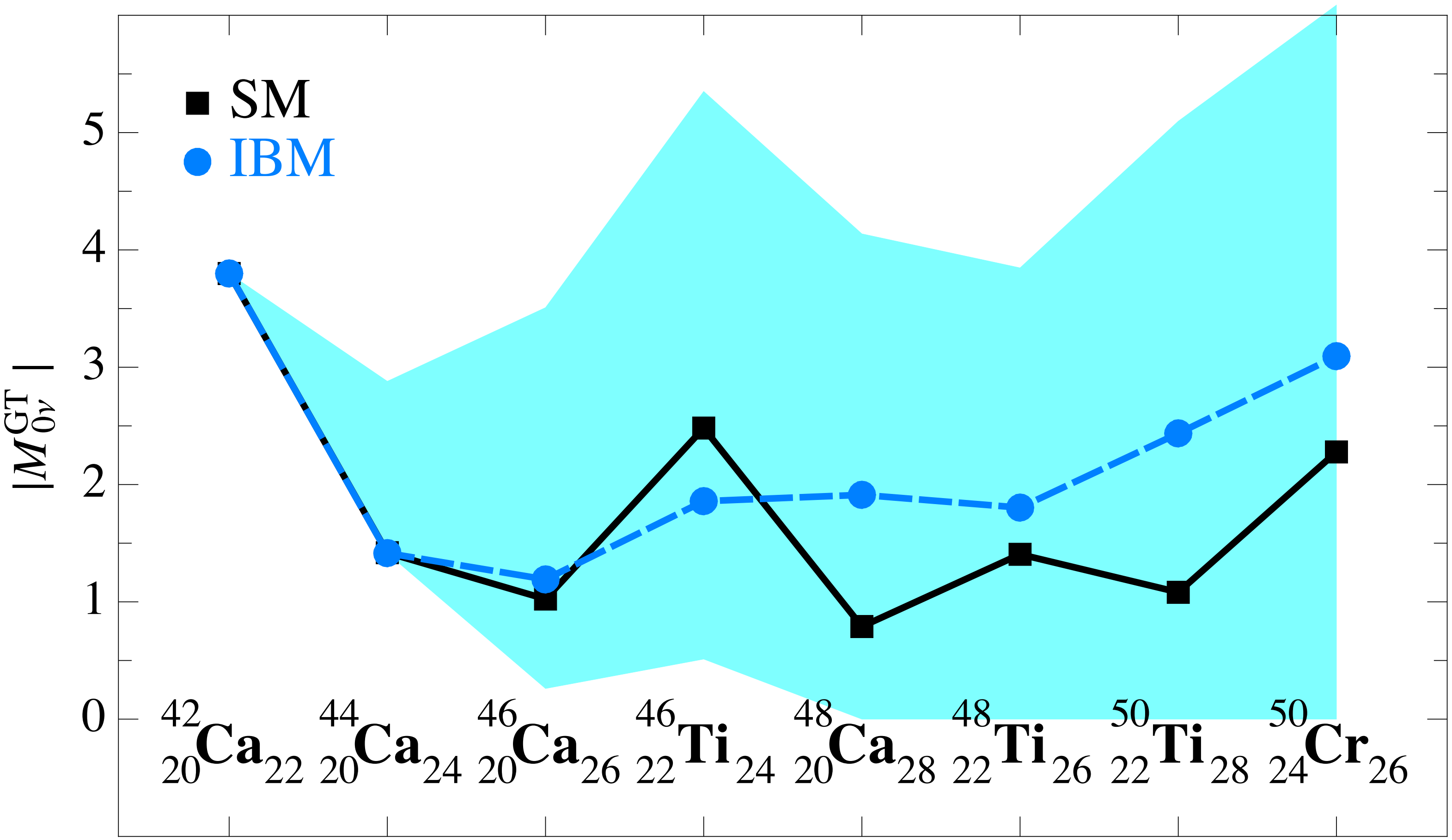
A=48 energy spectra



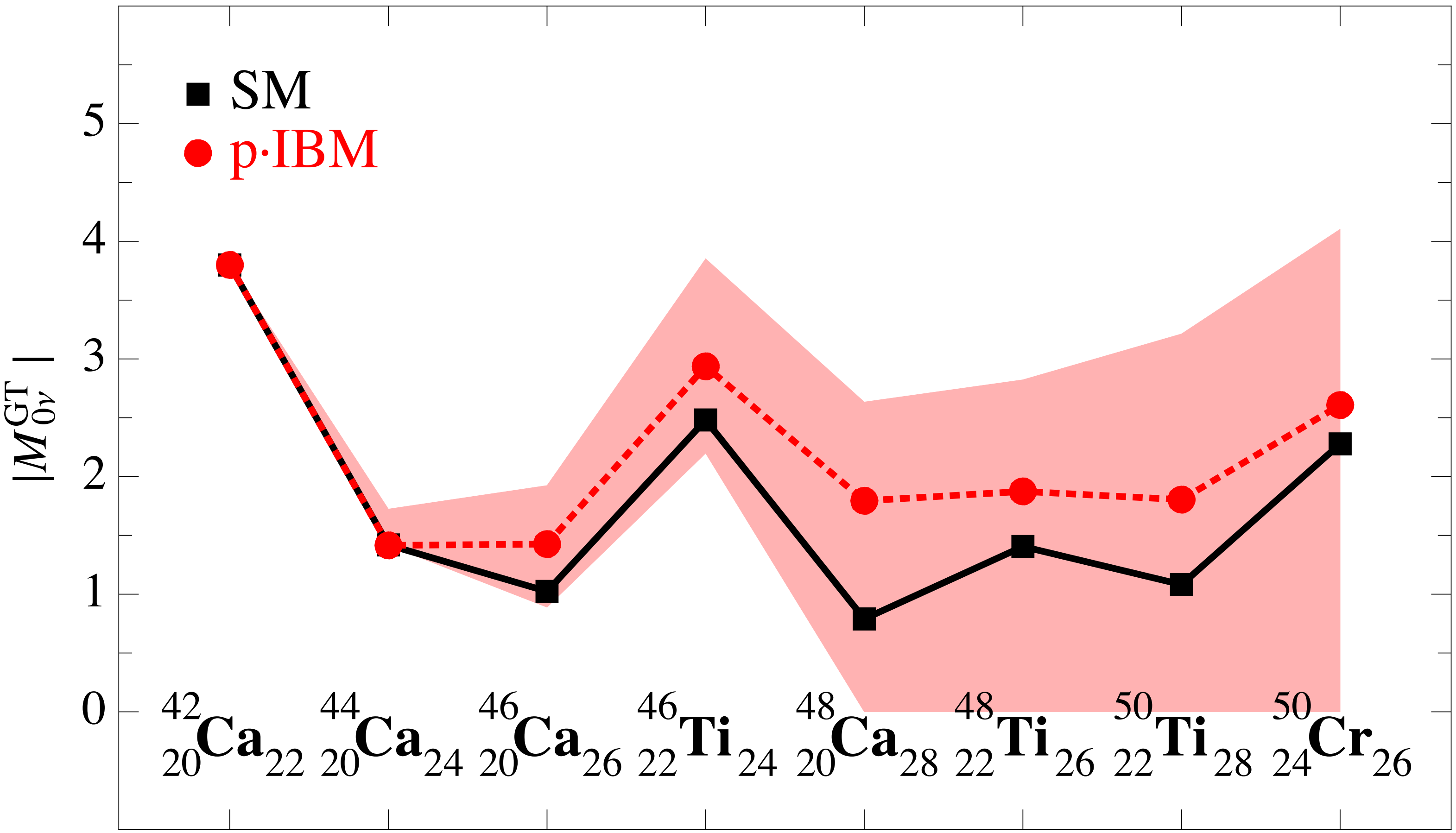
A=50 energy spectra



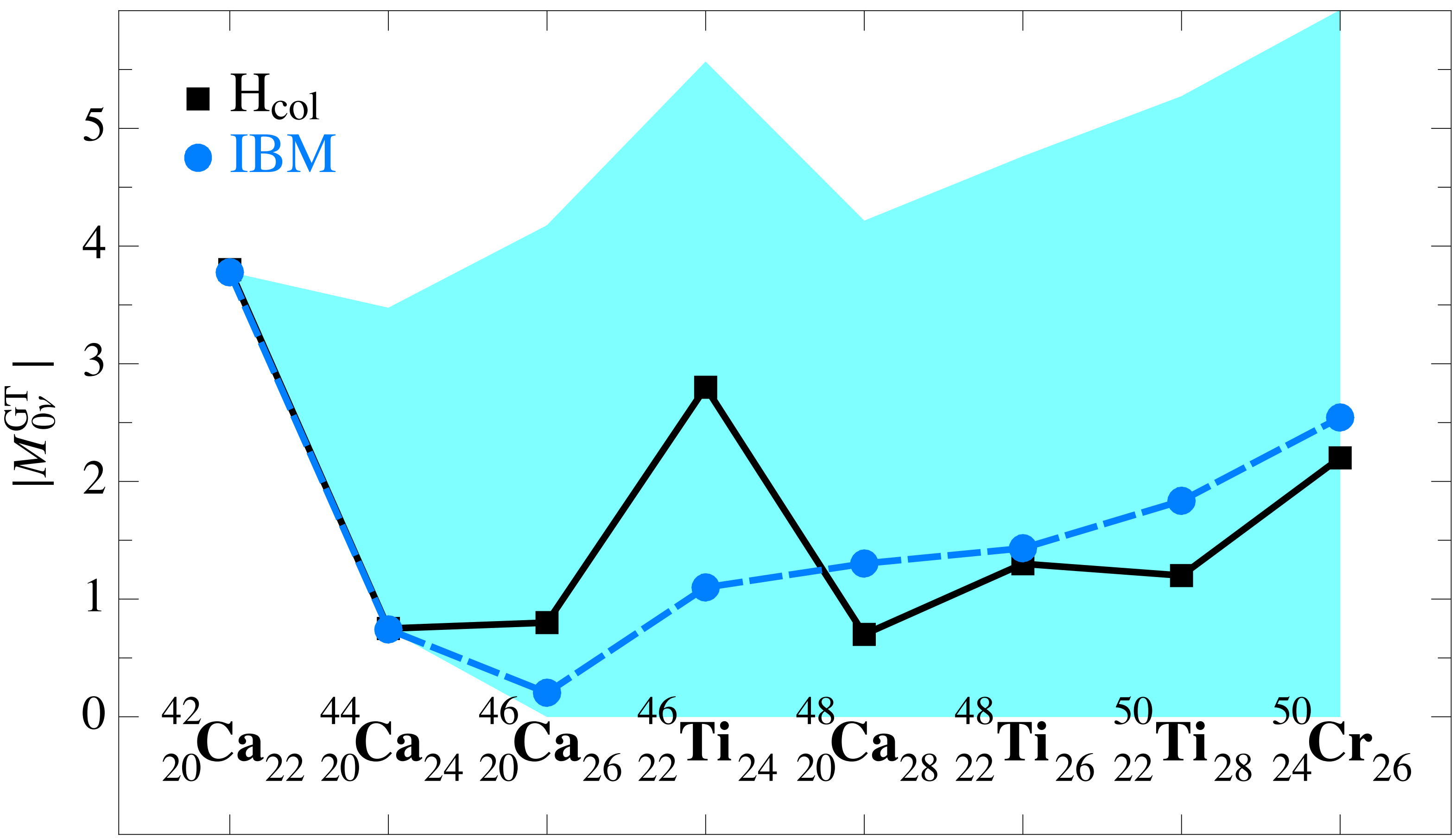
Gamow-Teller $\beta\beta$ in *sd*-IBM



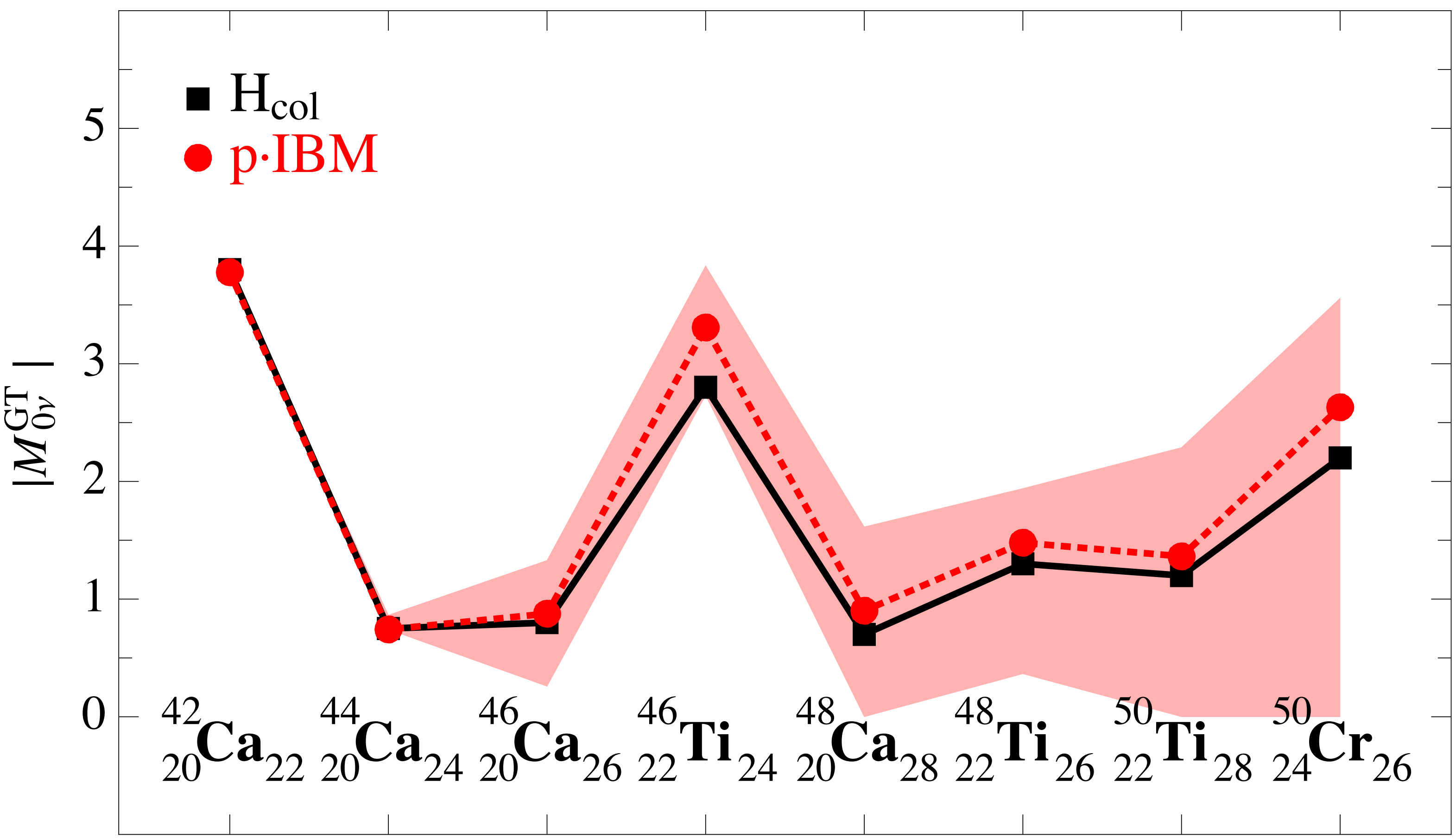
Gamow-Teller $\beta\beta$ in *sdp*-IBM



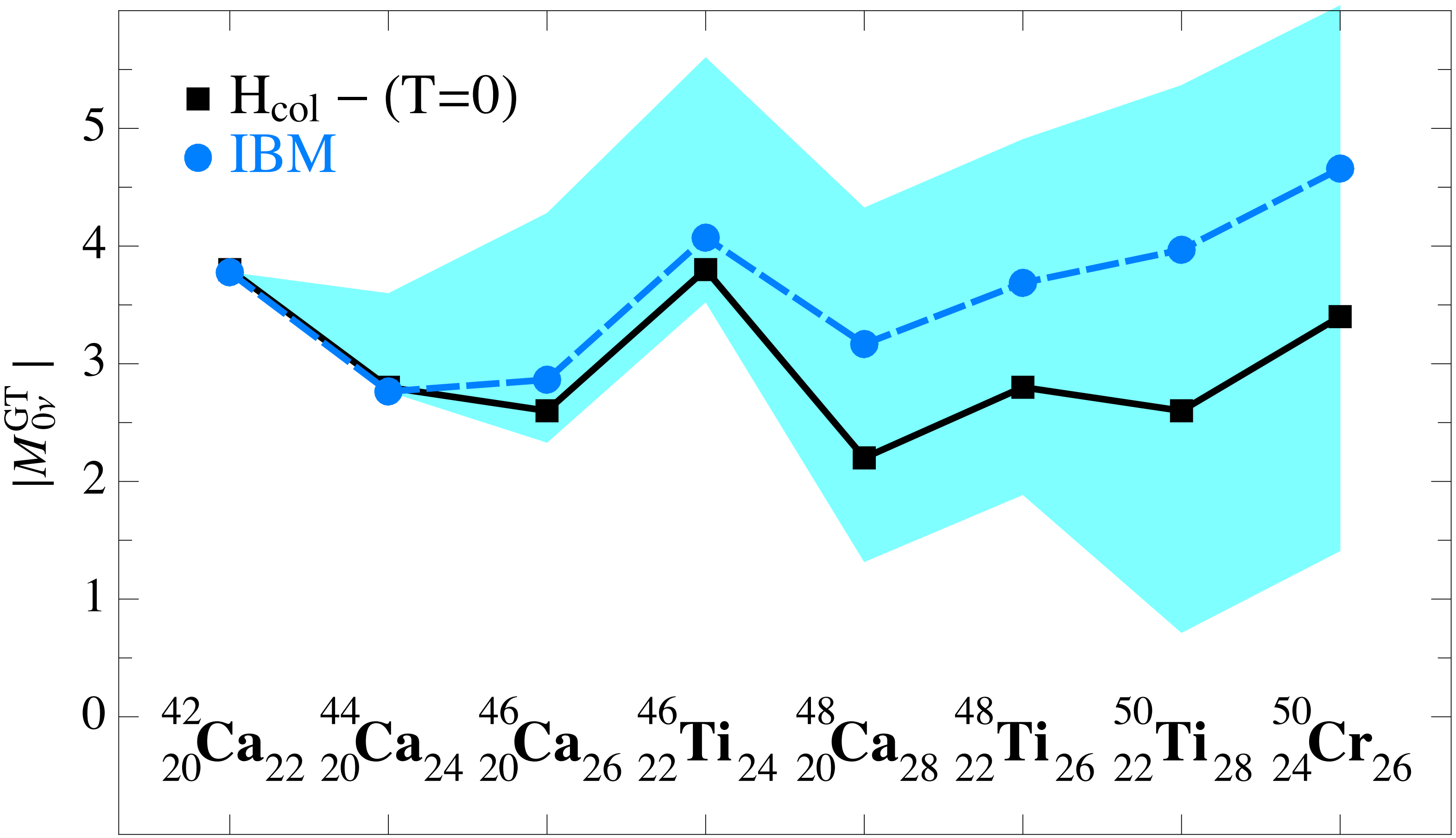
Gamow-Teller $\beta\beta$ in *sd*-IBM



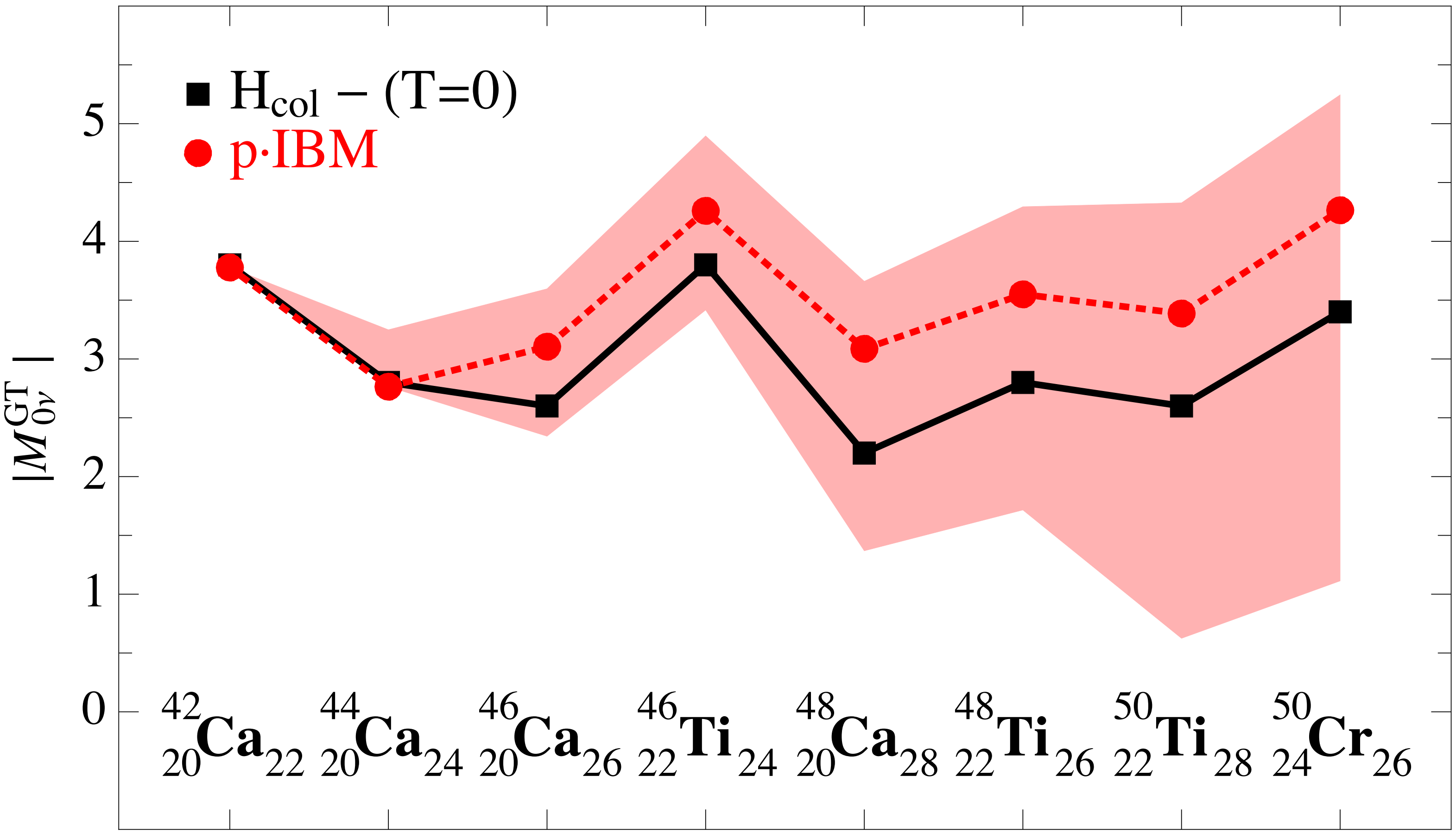
Gamow-Teller $\beta\beta$ in *sdp*-IBM



Gamow-Teller $\beta\beta$ in *sd*-IBM



Gamow-Teller $\beta\beta$ in *sdp*-IBM



Conclusions and open problems

Lee-Suzuki transformation is used to define an effective collective Hamiltonian.

In light nuclei (^{48}Ca, ^{76}Ge, ^{82}Se):

Isospin invariance must be included in the IBM.

Isoscalar-pair bosons are not needed for energy spectra but they are important for $\beta\beta$ decay.

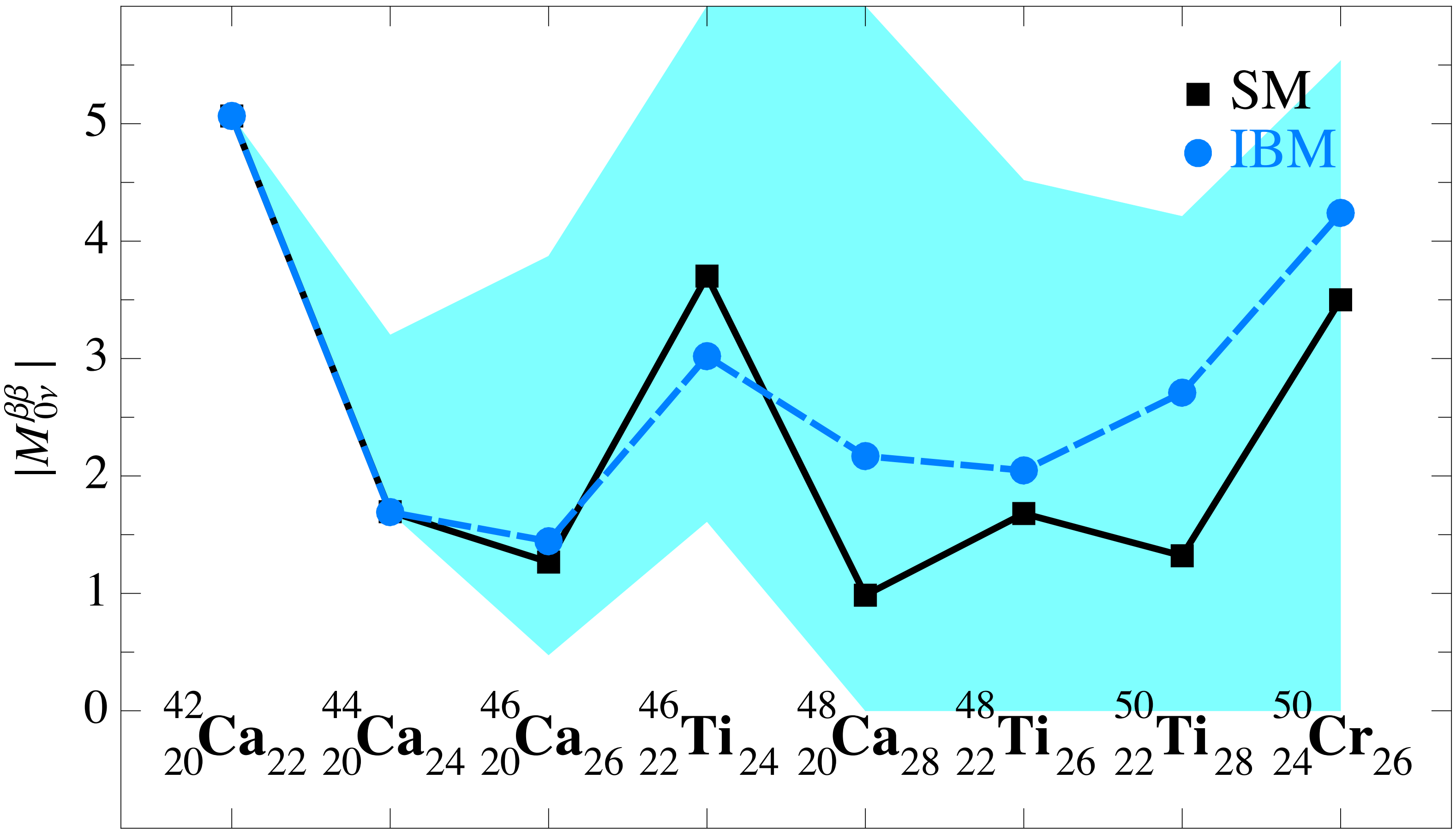
Open problems:

What is the boson-number dependence of the Hamiltonian?

How to couple this study to phenomenology?

P. Van Isacker, J. Engel & K. Nomura, arXiv:1708.05925

Total $\beta\beta$ in *sd*-IBM



Total $\beta\beta$ in *sdp*-IBM