

# The $\Delta(1232)$ and nuclear saturation

Andreas Ekström



# Acknowledgements

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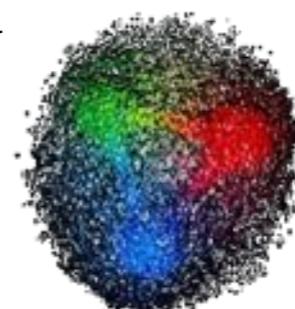
Gustav Jansen, ORNL

# Overview

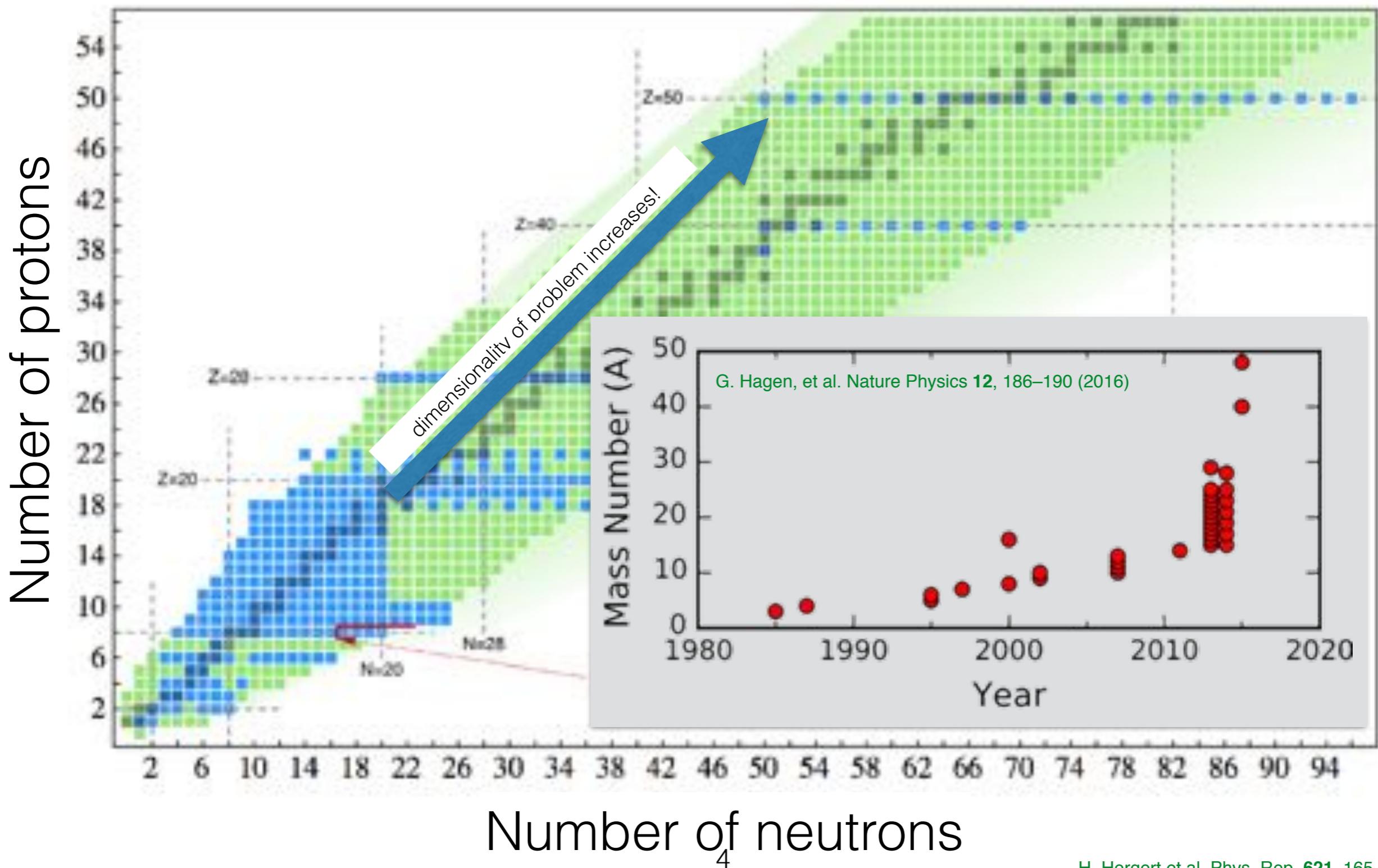
Nuclear physics spans a broad scientific scope. We would like to understand the origin, stability, and evolution of subatomic matter; how it organizes itself and what phenomena emerge.

Question: How does the nuclear chart emerge from QCD?

$$\sum_{i=1}^A \frac{p_i^2}{2m_i} + \underbrace{\sum_{i < j = 1}^A V_{ij} + \sum_{i < j < k = 1}^A W_{ijk}}_{\text{chiral effective field theory}} |\Psi_A\rangle = E |\Psi_A\rangle$$

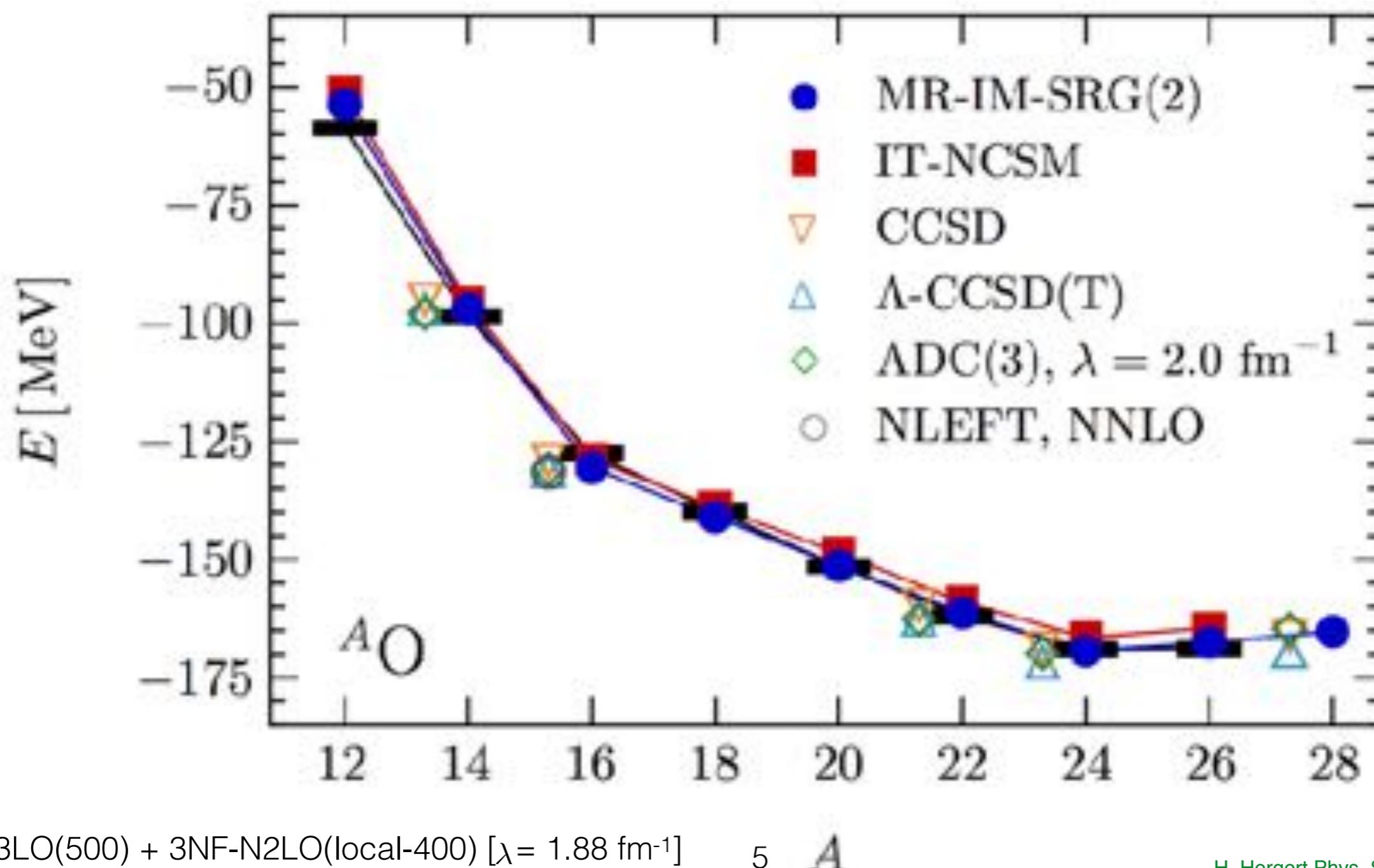


# Progress in ab initio calculations



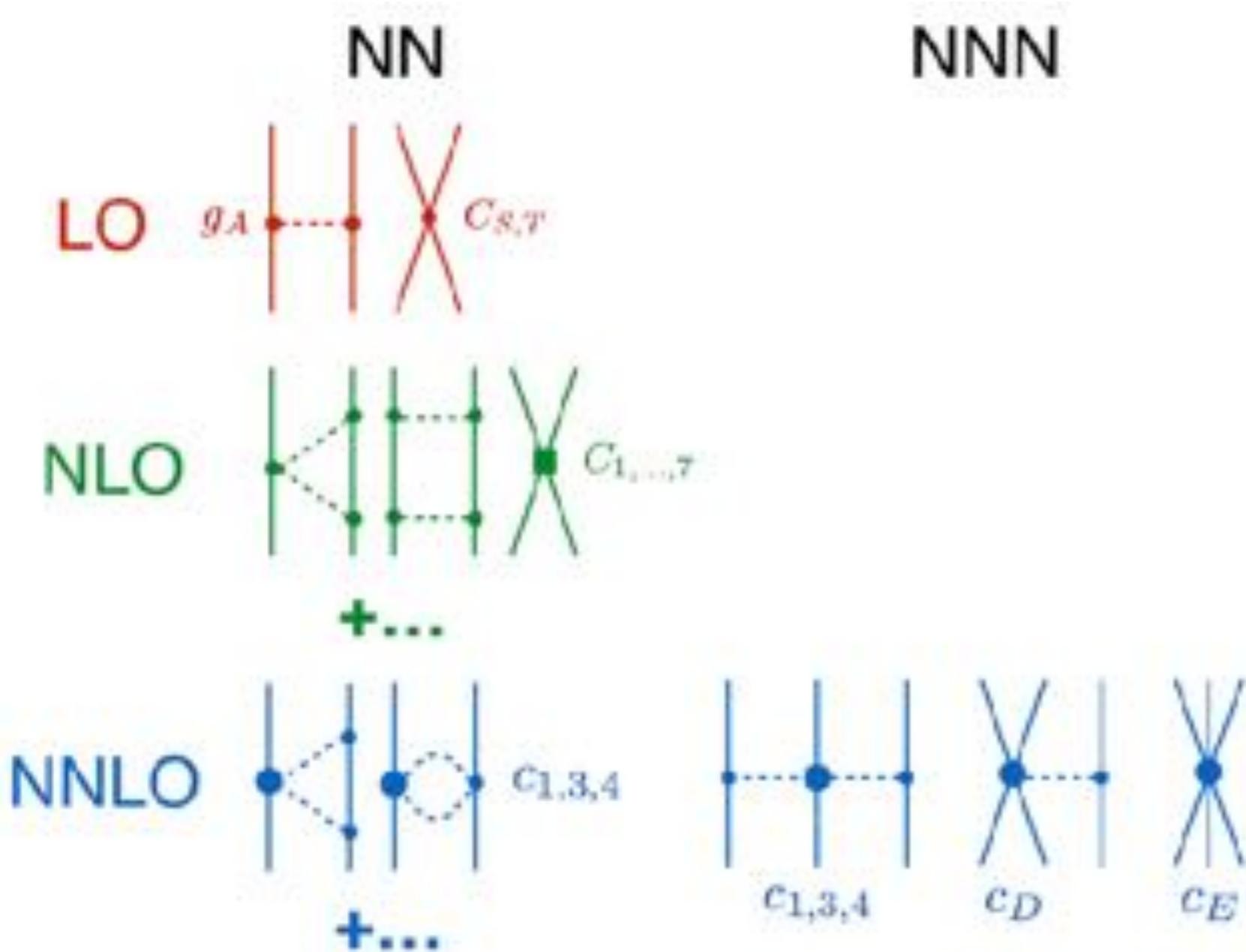
# Complementary methods agree

**Same** chiral interaction, but **different** many-body methods



# Chiral effective field theory

*Nucleons interact via a potential built from perturbative pion contributions, and **indirectly** everything else.*

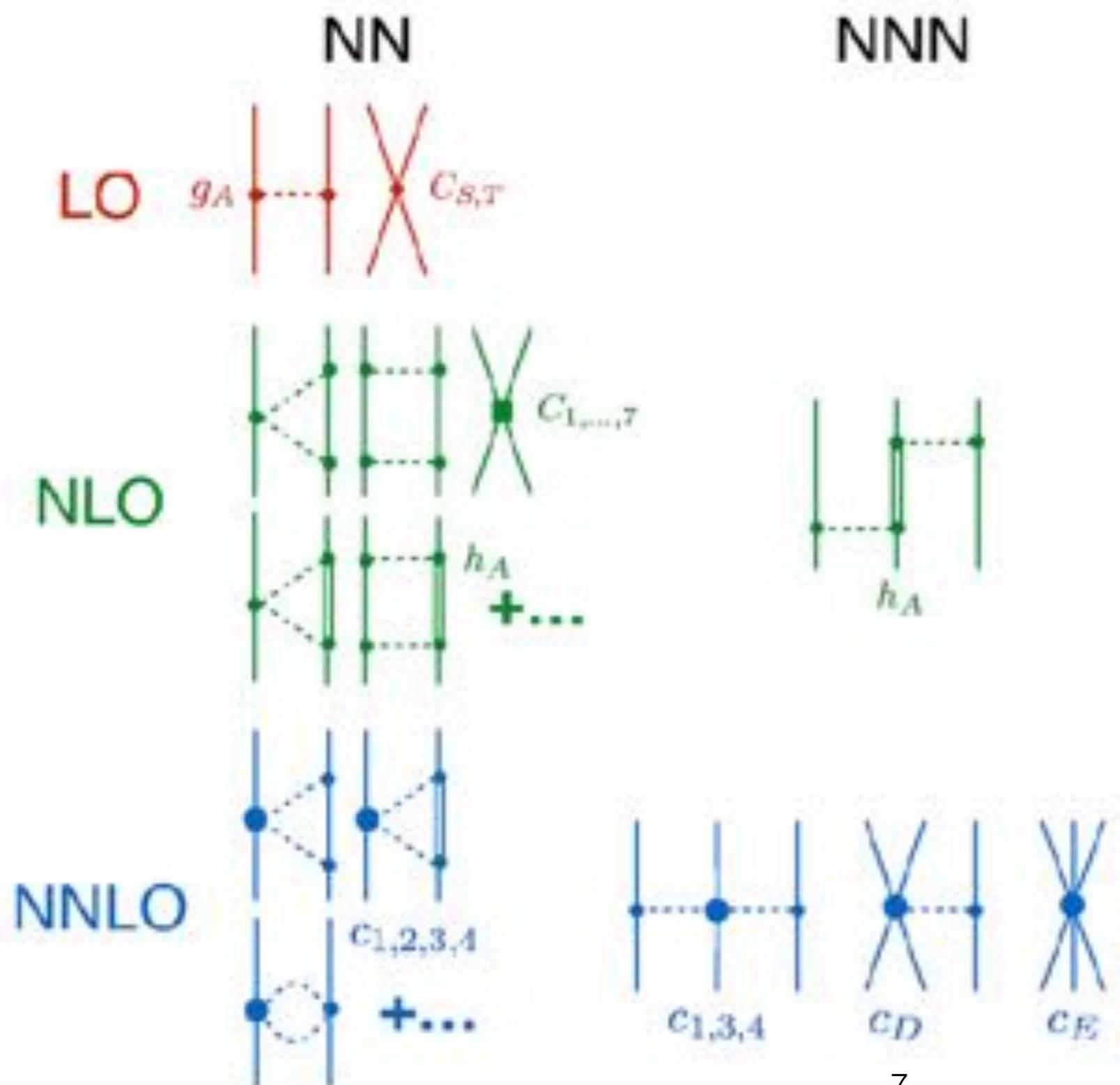


- Chiral symmetry dictates long-ranged physics (pion-exchange).
- 2N,3N,4N,... - forces
- Coupling constants fit from data **once**
- On-going work: power counting, optimization strategies, uncertainty quantification.

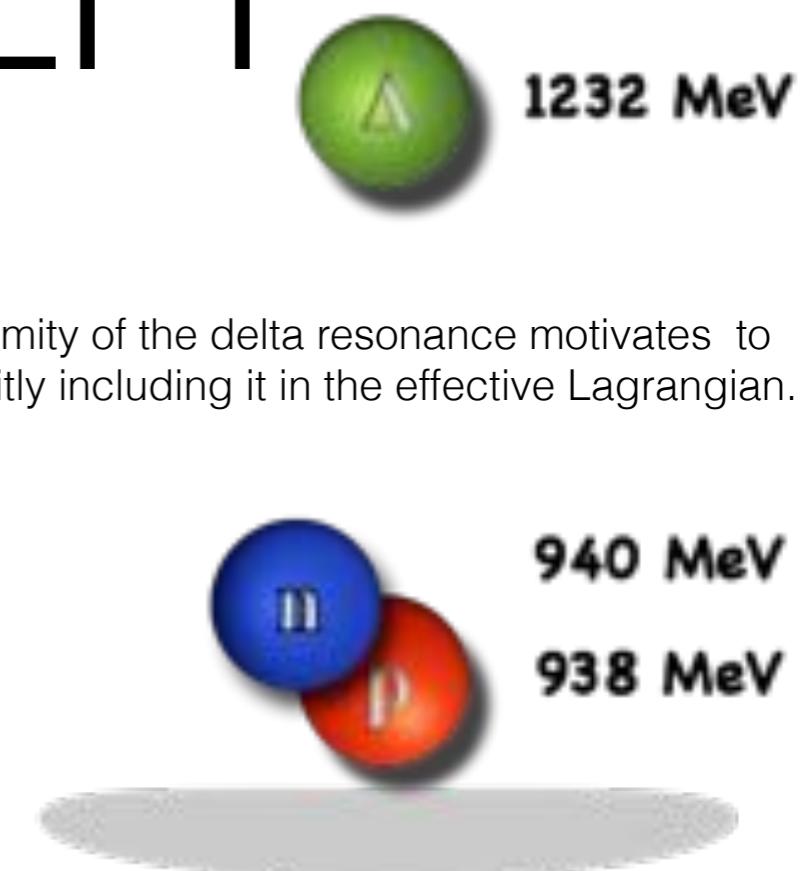


Weinberg, van Kolck,  
Meissner, Epelbaum,  
Kaiser, Machleidt,  
Kaplan, Savage, Bernard, ...

# $\Delta$ -full Chiral EFT



Proximity of the delta resonance motivates to explicitly including it in the effective Lagrangian.



$$\Delta \equiv m_\Delta - m_N = 293 \text{ MeV} \approx 2.1 m_\pi$$

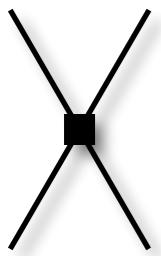
More *natural* values for the LECs!

But also more LECs and digrams (extensive N3LO)

Ordoñez, Ray, van Kolck 1994  
Hemmert, Holstein, Kambor 1998  
Kaiser, Gerstendorfer, Weise 1998  
Krebs, Epelbaum, Meissner 2007

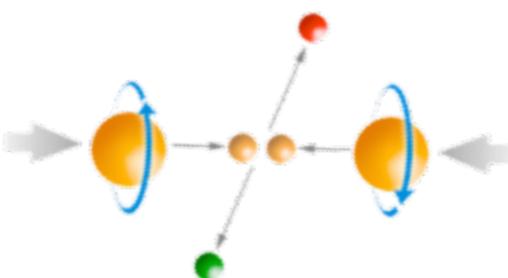
# Optimizing the LECs

Levenberg-Marquardt  
w. AD derivatives

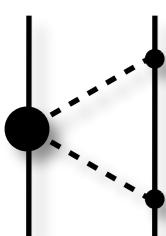


$$x_{\star} = \underset{x}{\operatorname{argmin}} \chi^2(x)$$

B. D. Carlsson et al. Phys. Rev. X **6**, 011019 (2016)  
A. Eksröm et al. Phys. Rev. Lett. **110**, 192502 (2013)

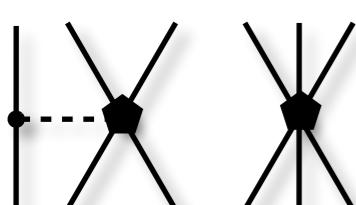


partial-wave NN scattering **phase shifts** of the Granada group up to 200 MeV scattering energy in the laboratory system. [R. Navarro-Perez et al, Phys. Rev. C \*\*88\*\*, 064002 \(2013\).](#)



Sub-leading pi-N LECs precisely determined in recent **Roy-Steiner analysis**

[D. Siemens et al. Physics Letters B \*\*770\*\* \(2017\) 27–34](#)

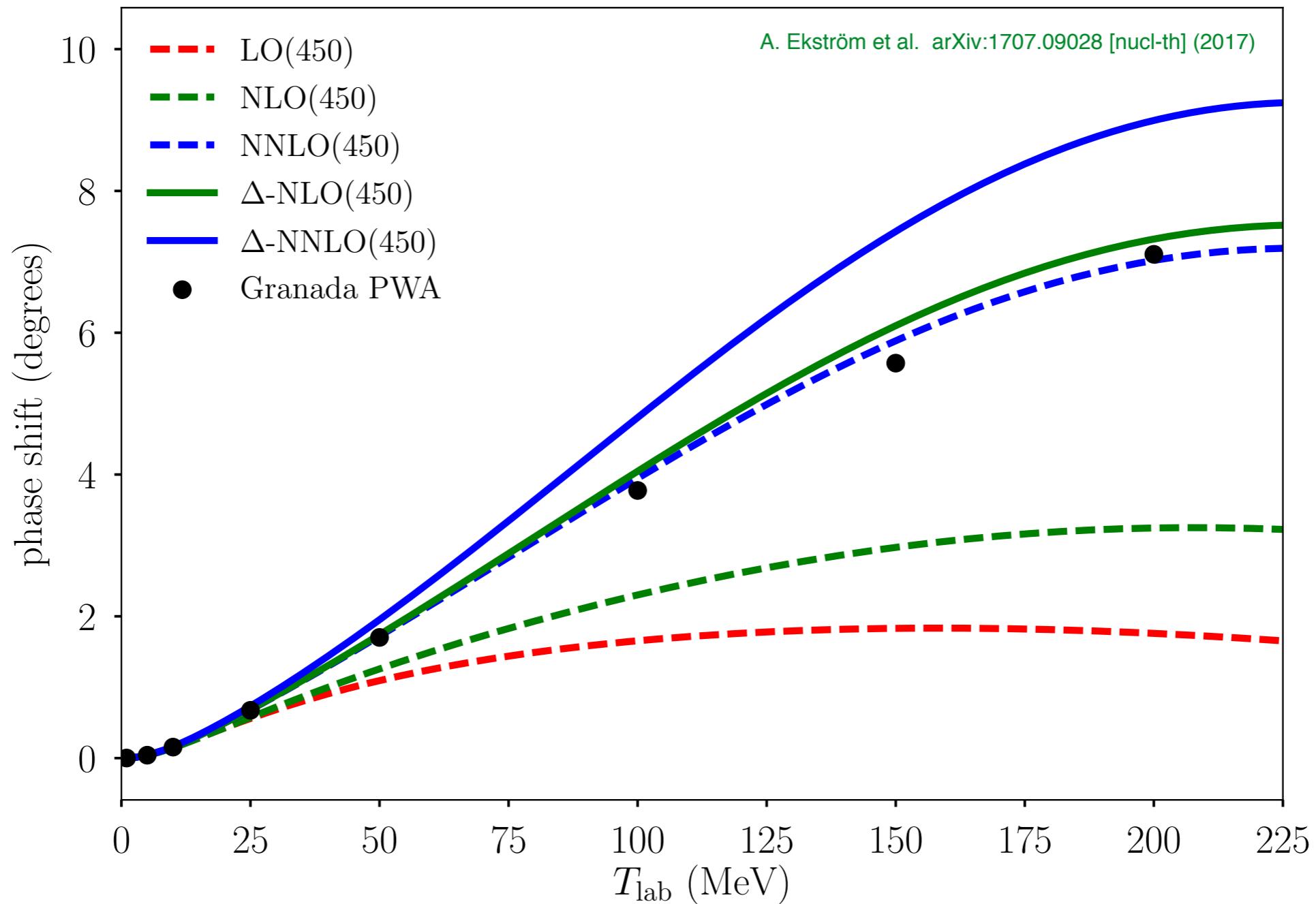


Determined from **NCSM**  $E_{\text{gs}}(^4\text{He})$  &&  $R_{\text{pt-p}}(^4\text{He})$   
n.b. only relevant at NNLO

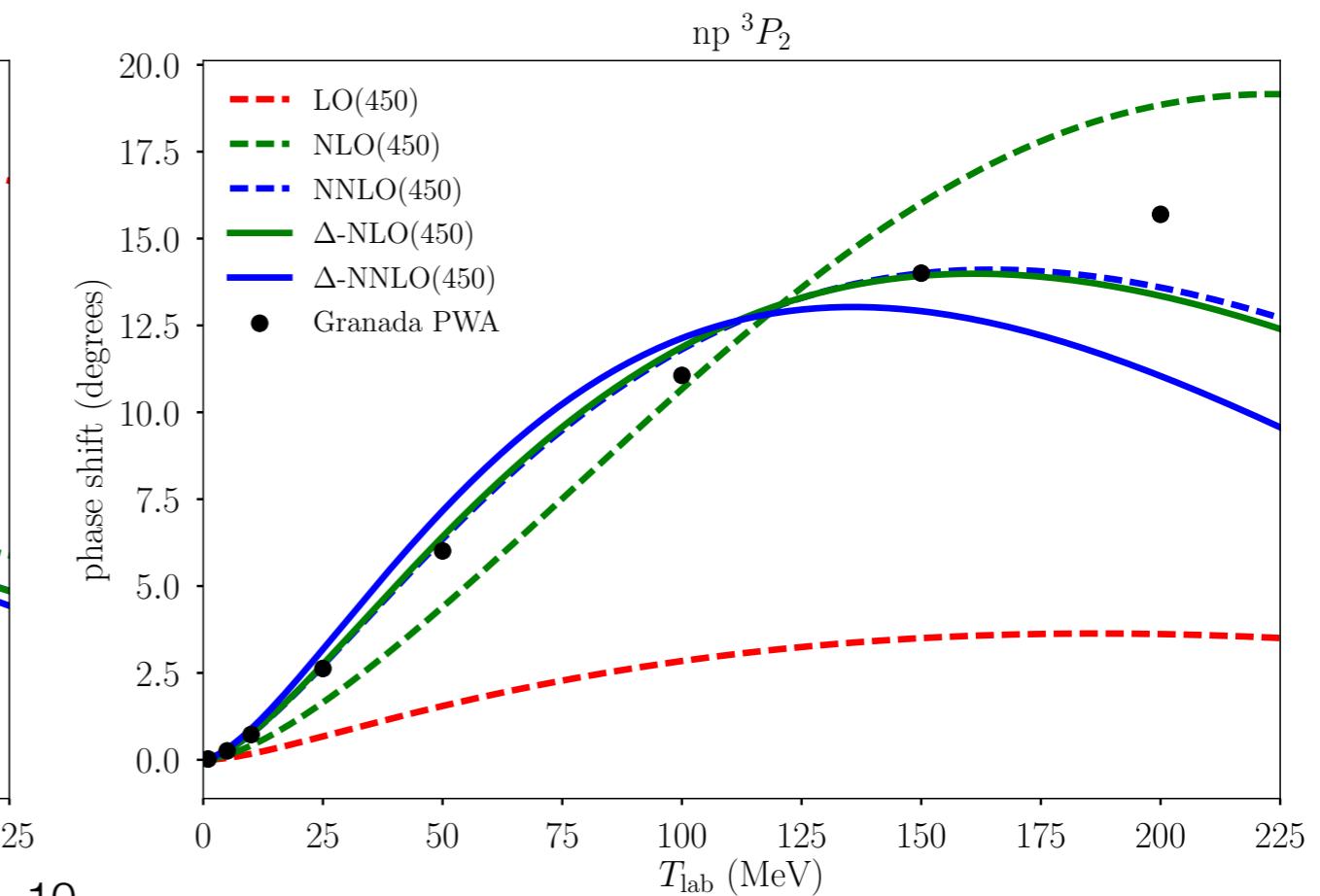
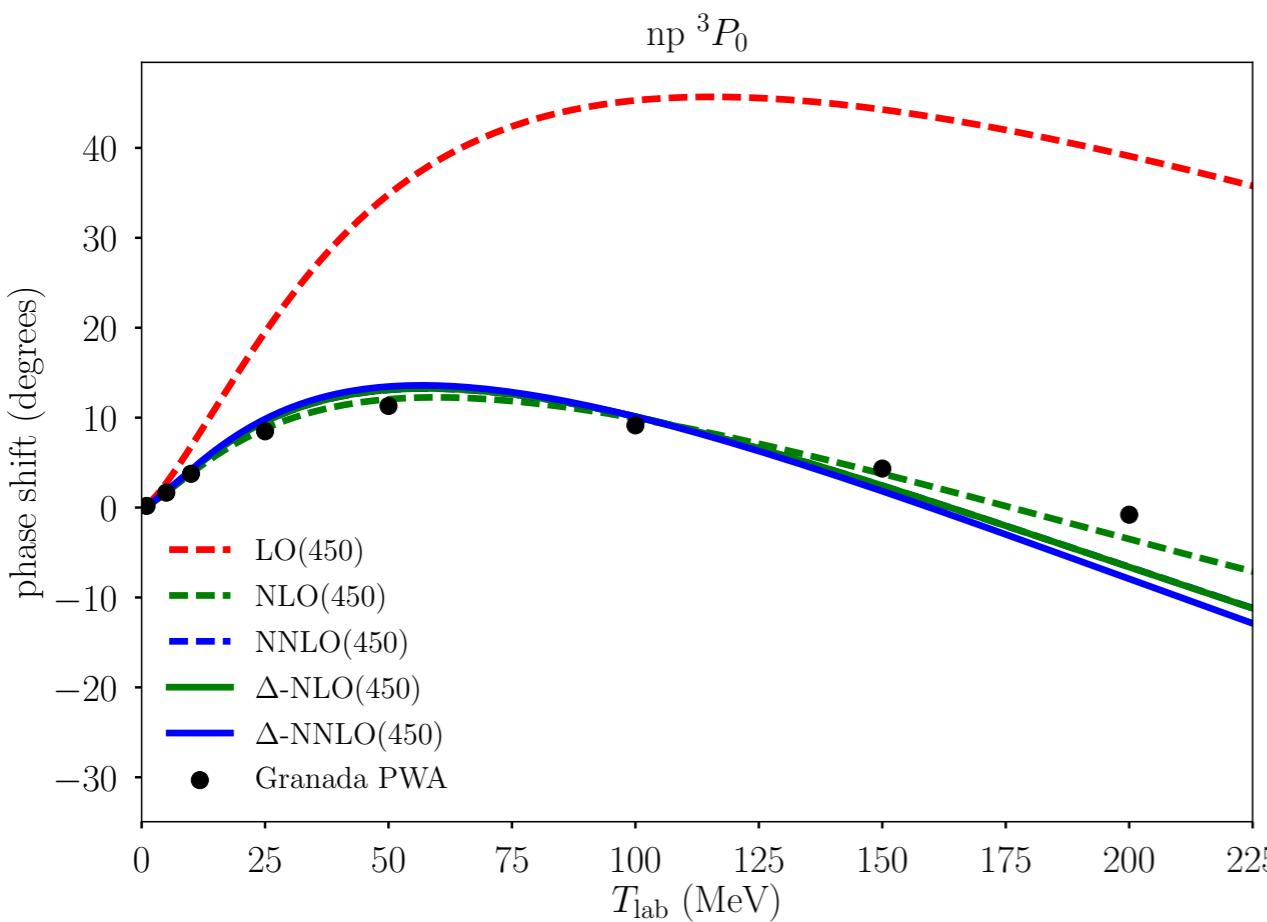
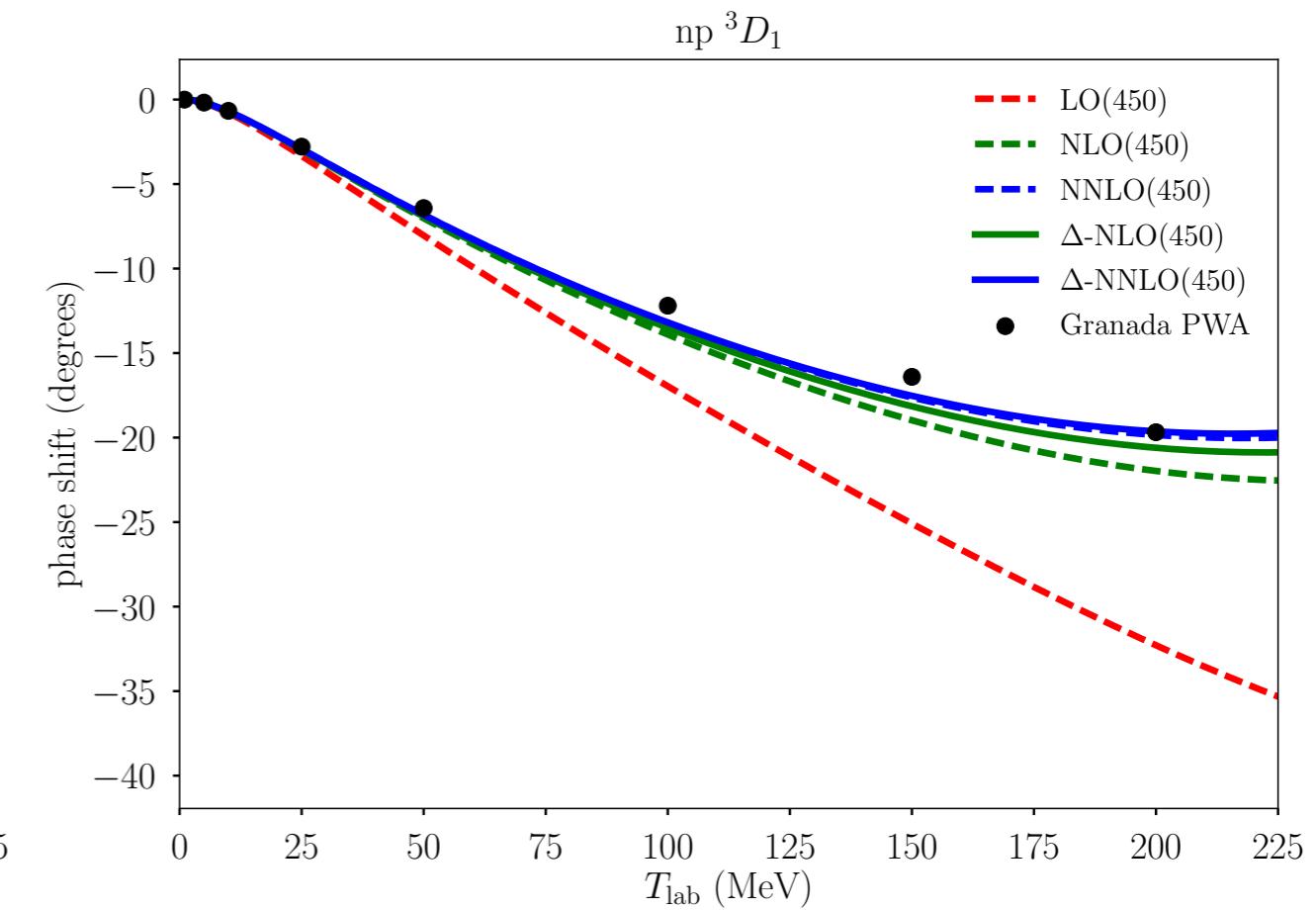
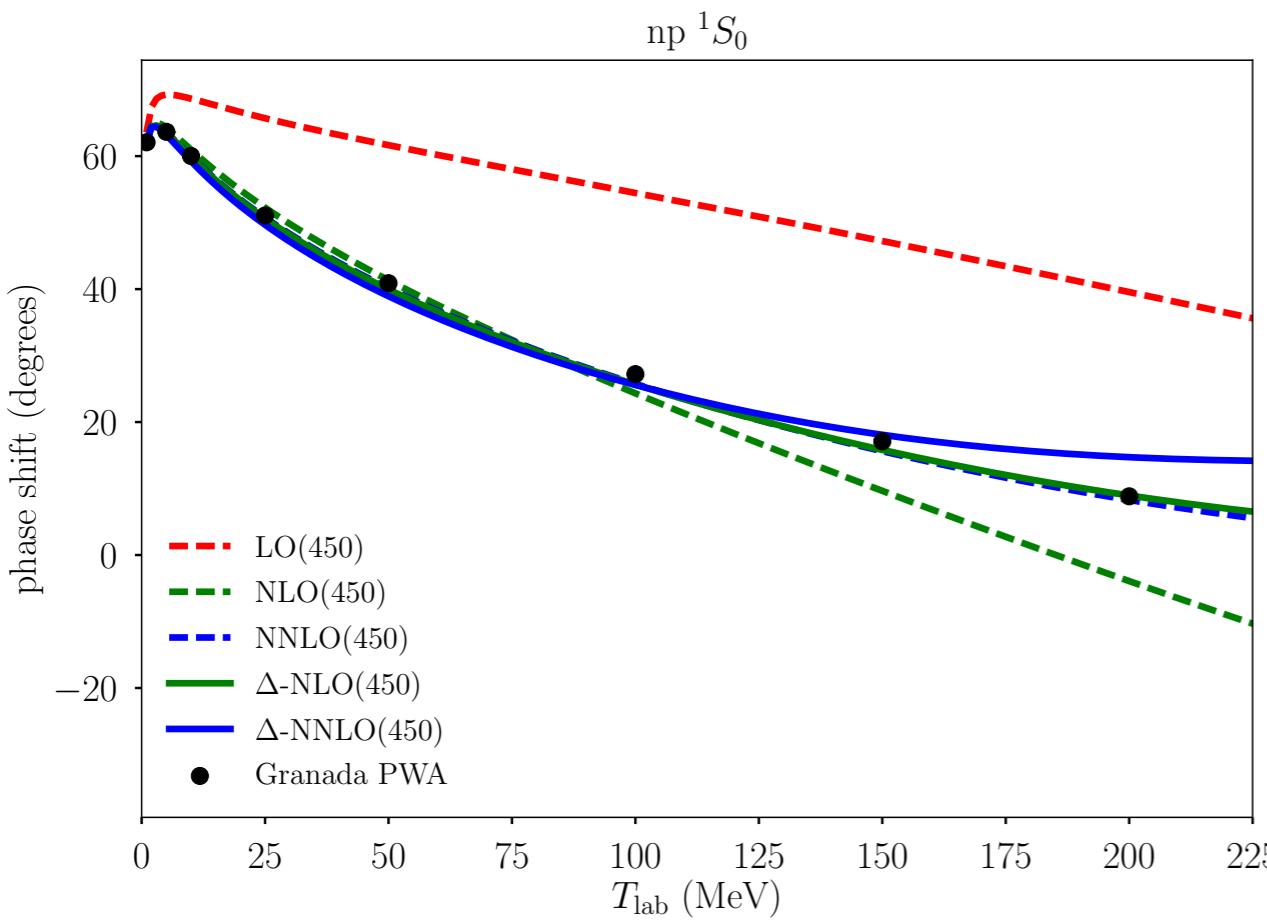
$\Delta$ -full
$c_1 = -0.74(2)$
$c_2 = -0.49(17)$
$c_3 = -0.65(22)$
$c_4 = +0.96(11)$
$h_A = 1.40 \pm 0.05$
.....
$\Delta$ -less
$c_1 = -0.74(2)$
$c_2 = +1.81(3)$
$c_3 = -3.61(3)$
$c_4 = +2.44(3)$

# Peripheral NN-phase shifts

np  $^1D_2$

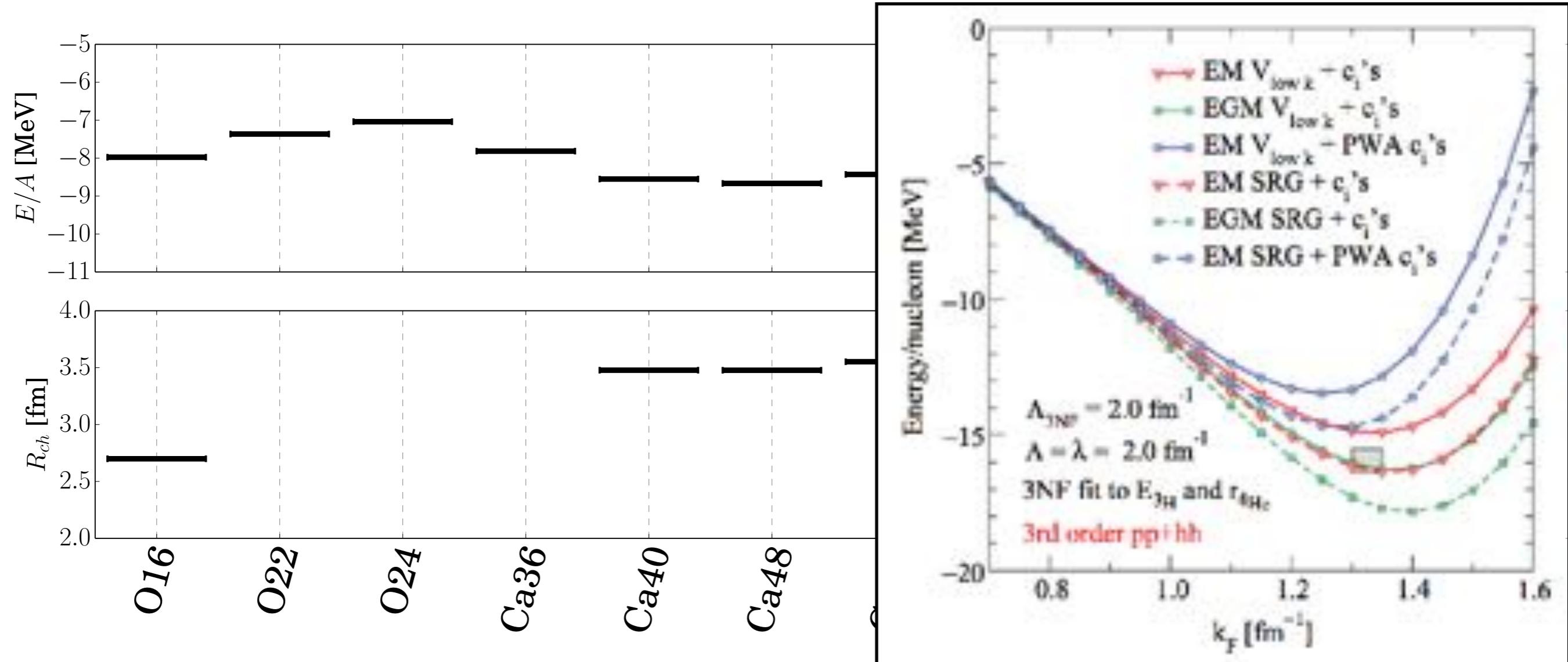


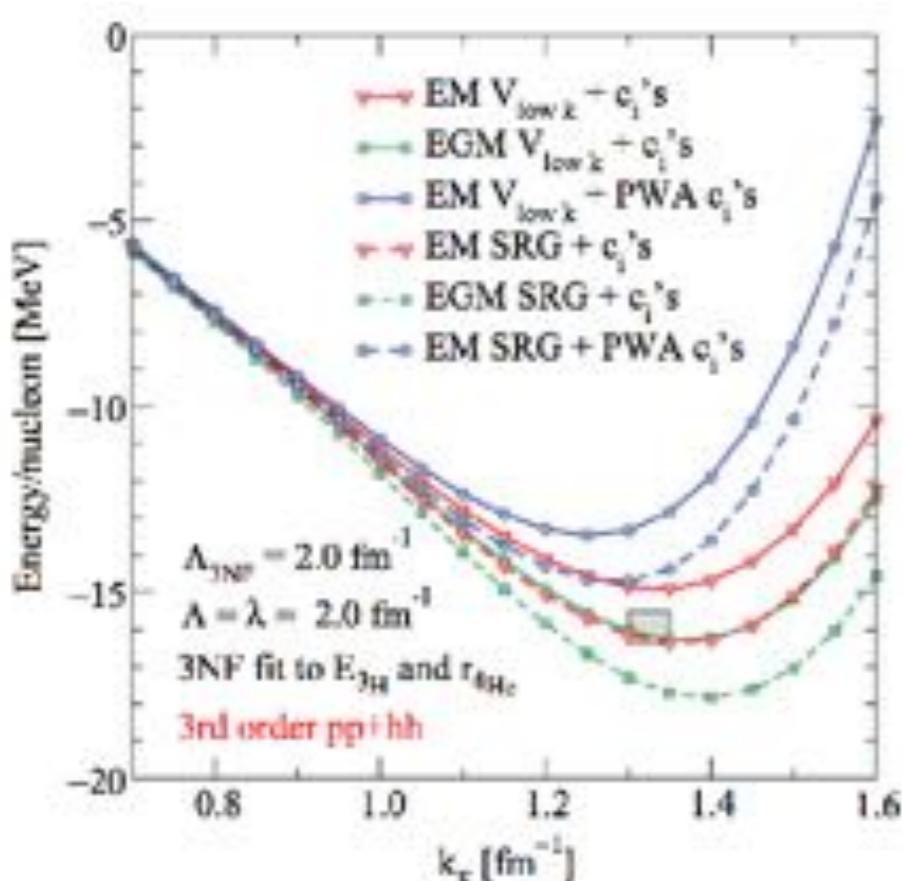
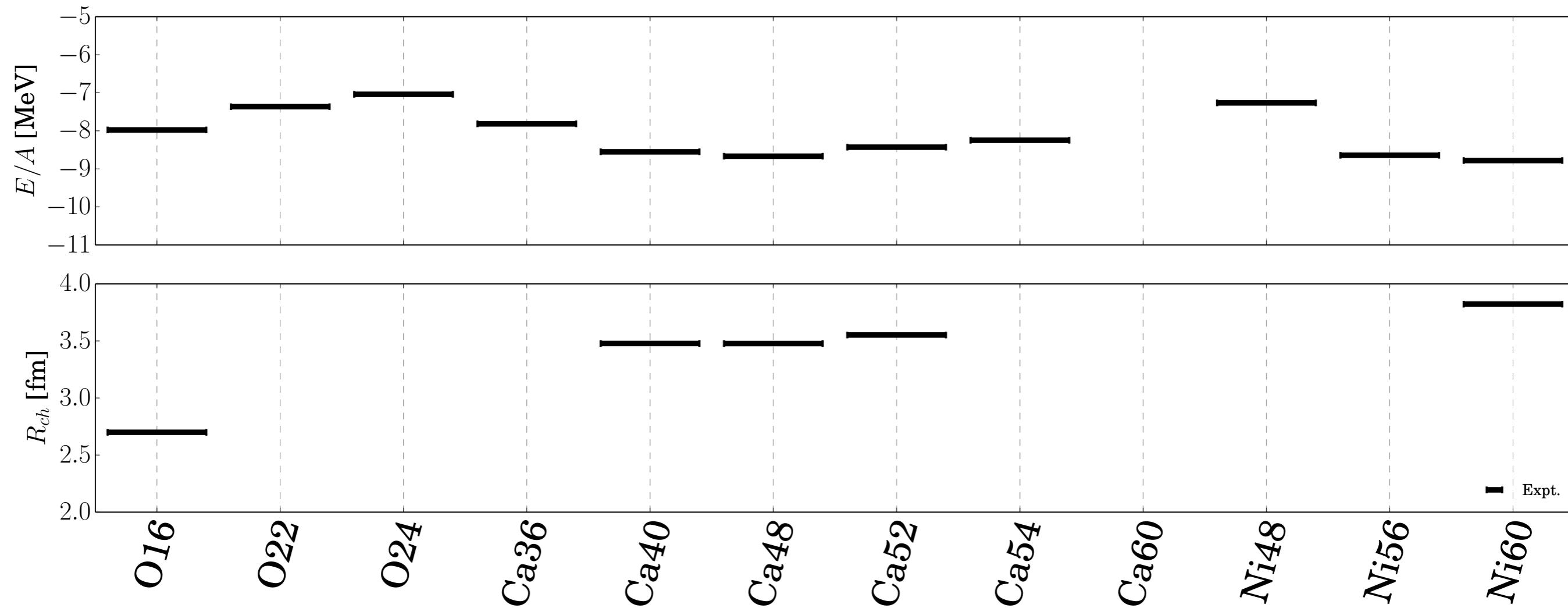
$\Delta$ -NLO  $\approx$  NNLO

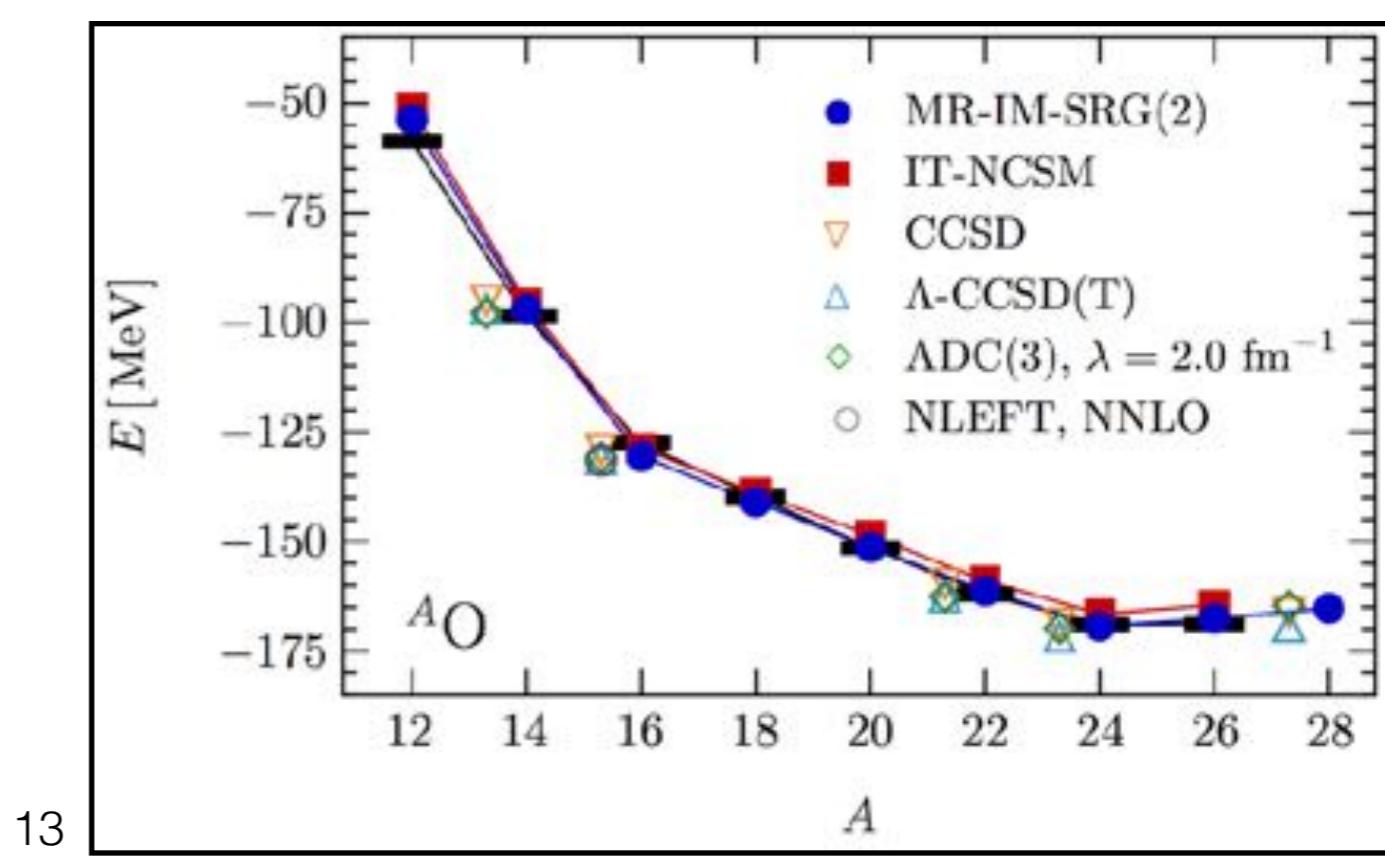
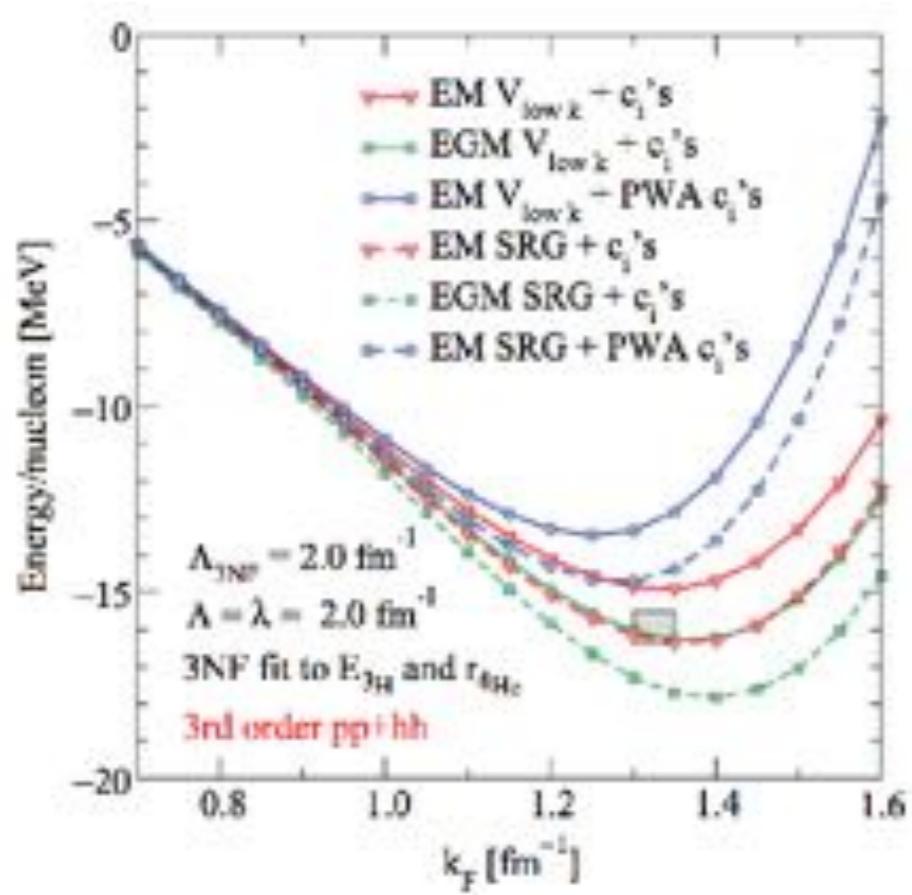
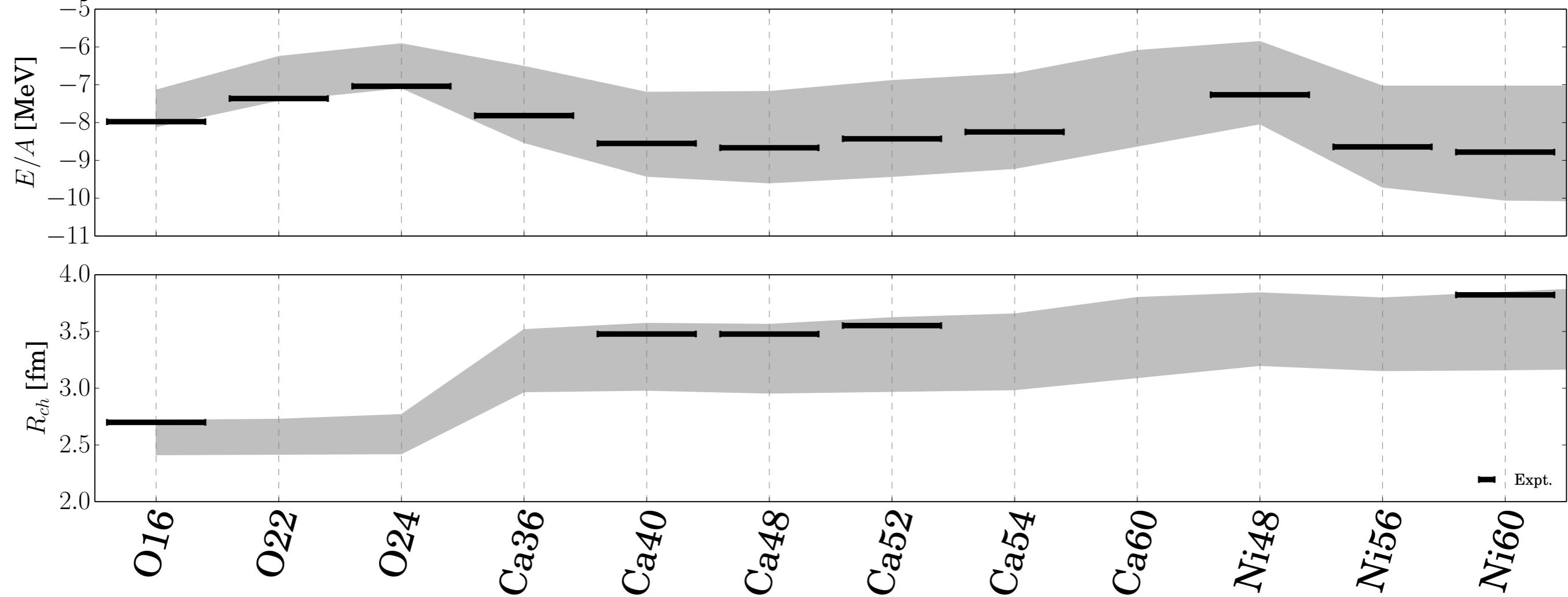


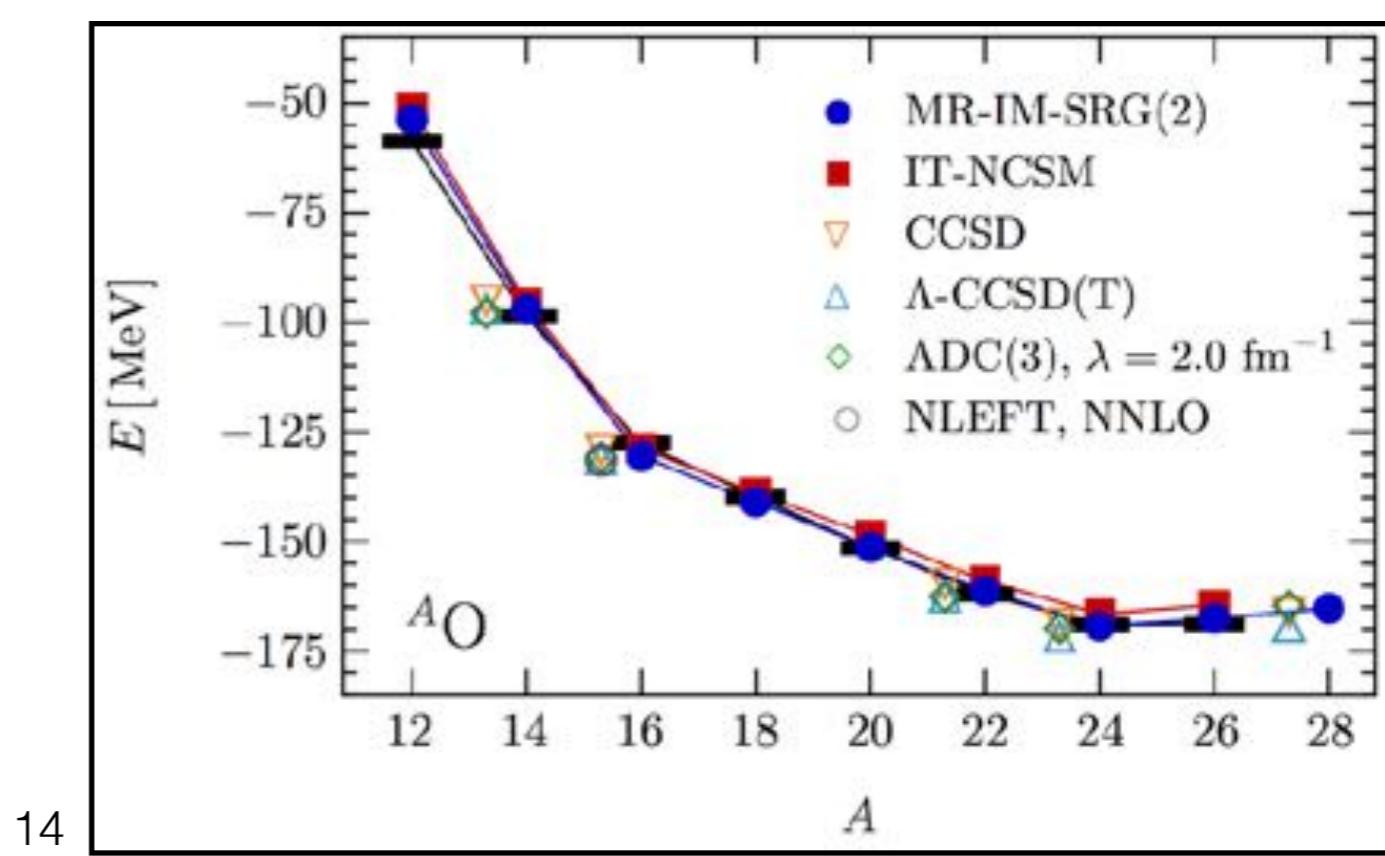
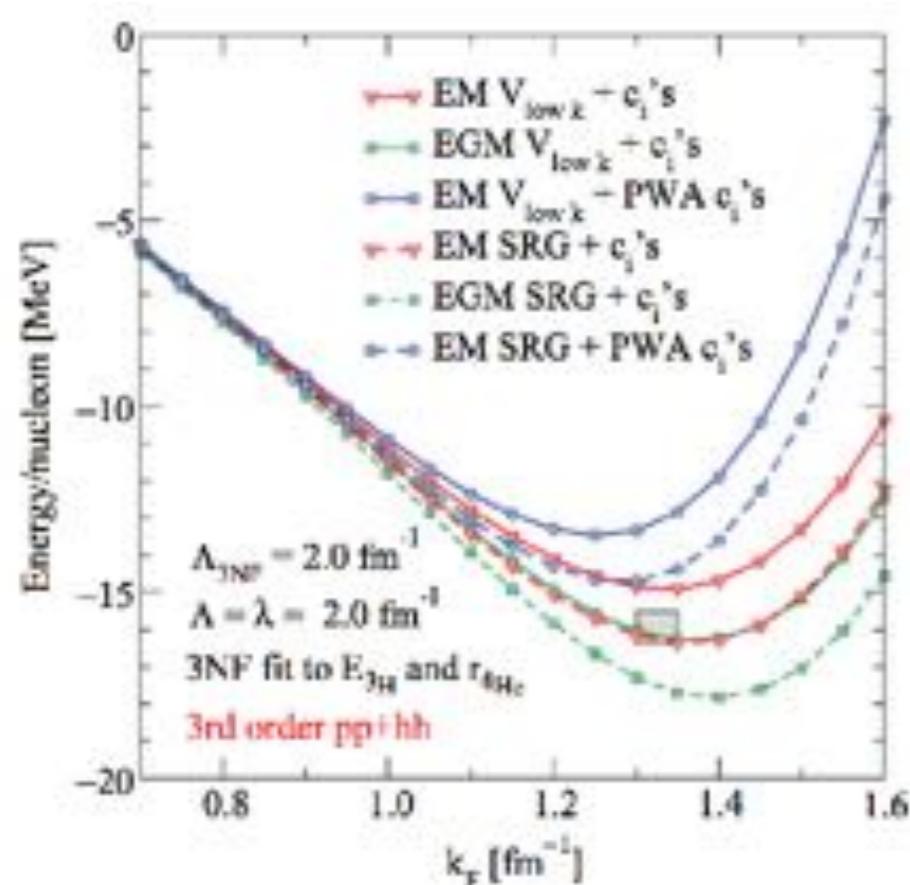
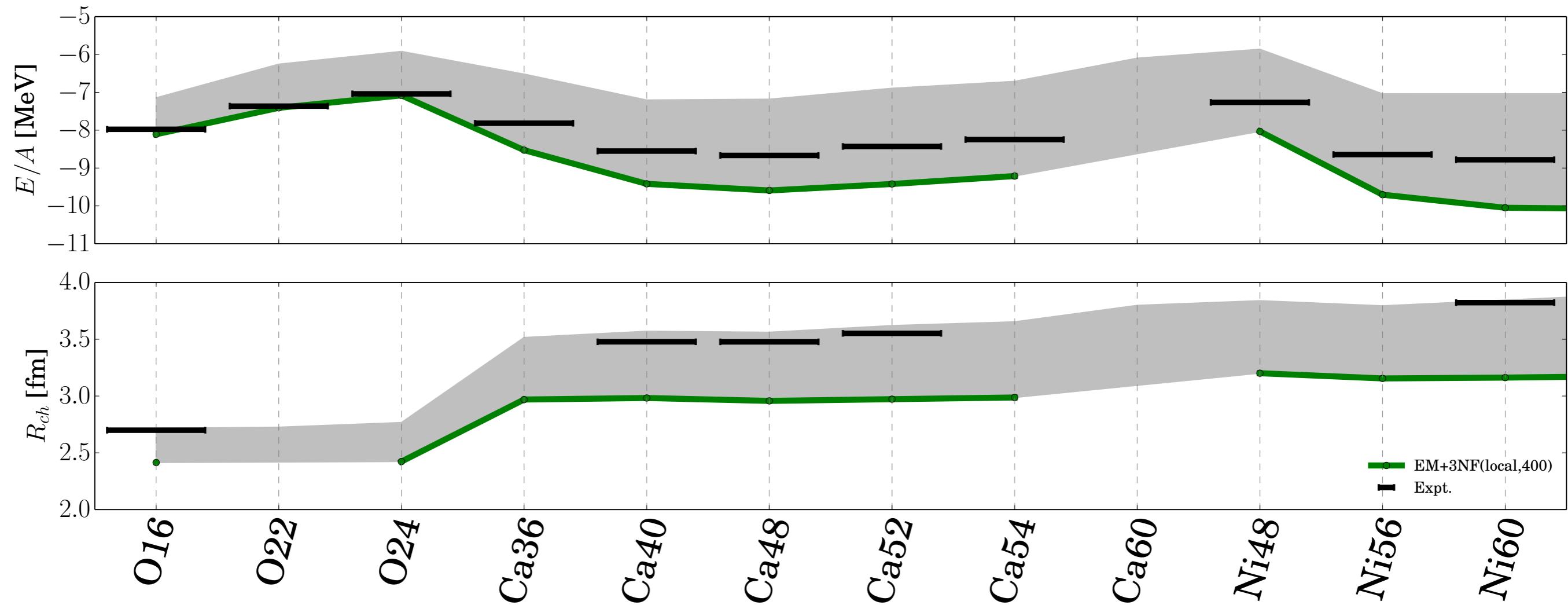
# Energy & charge radius

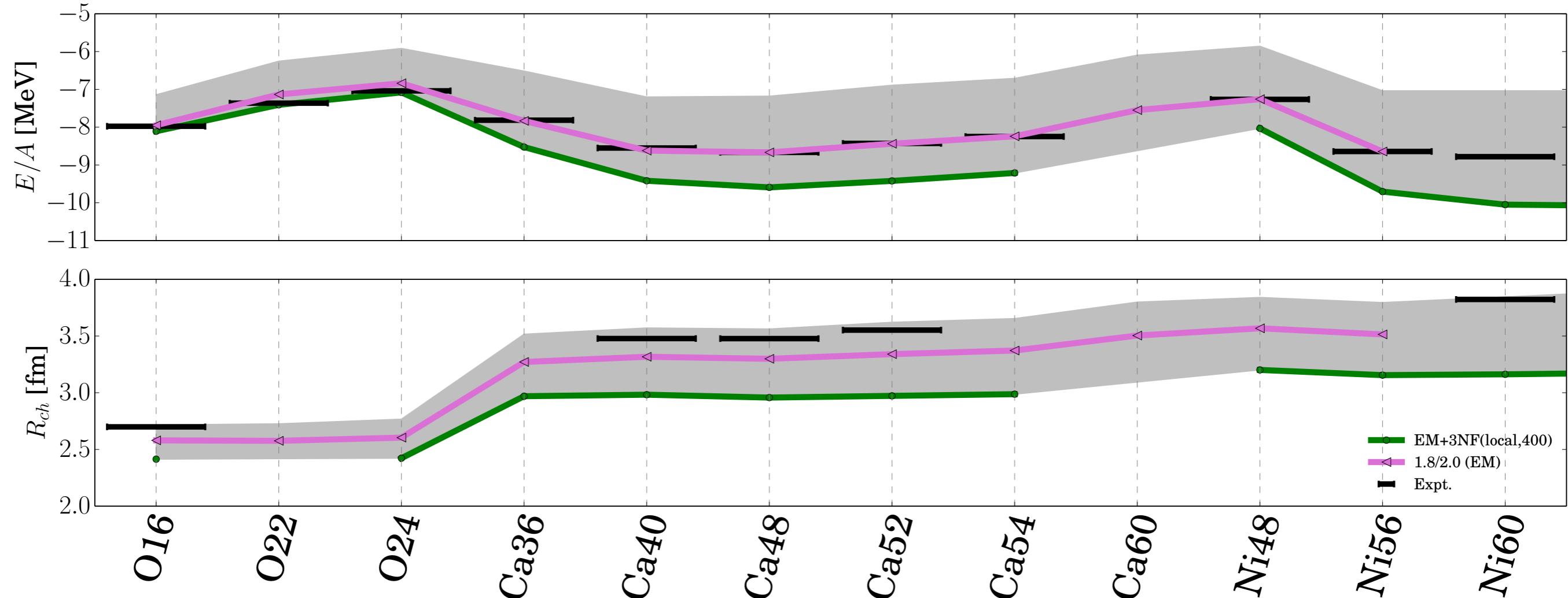
K. Hebeler, et al. PRC **83**, 031301(R) (2011)









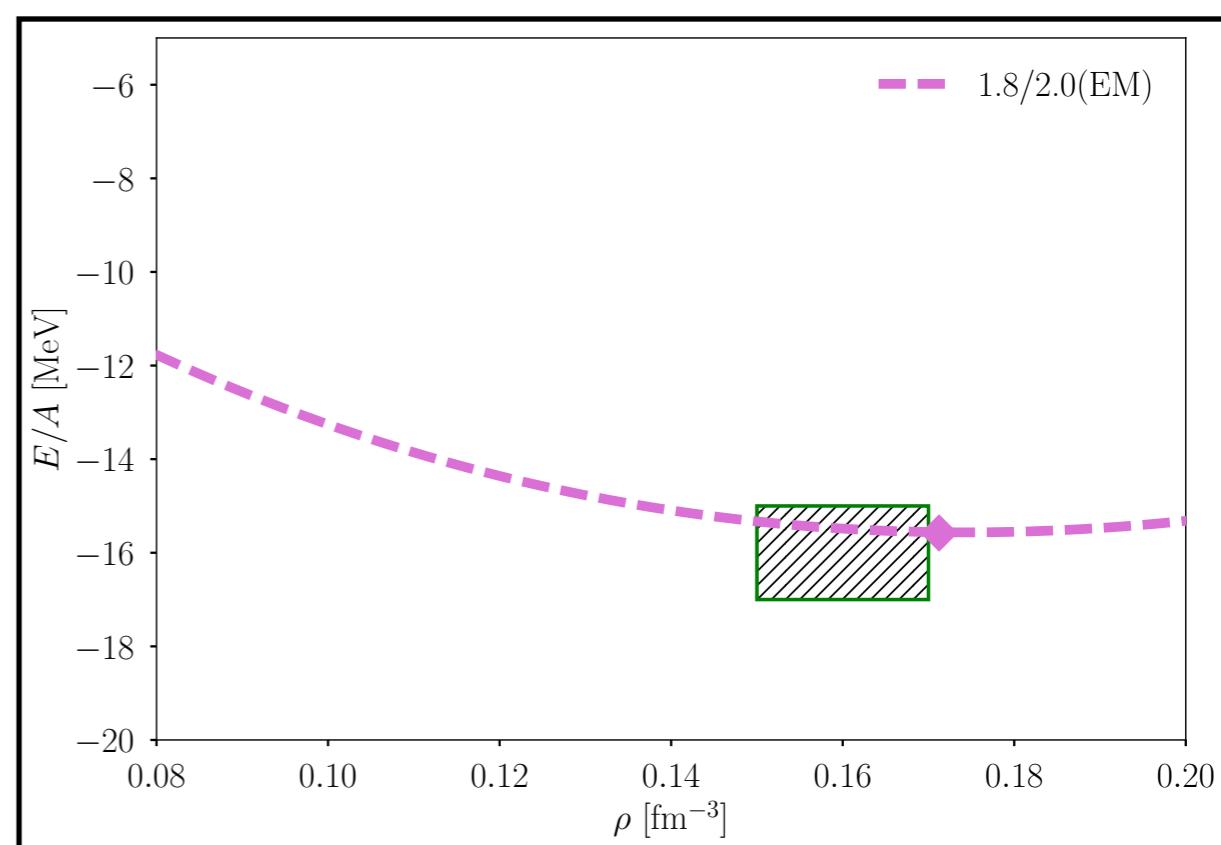


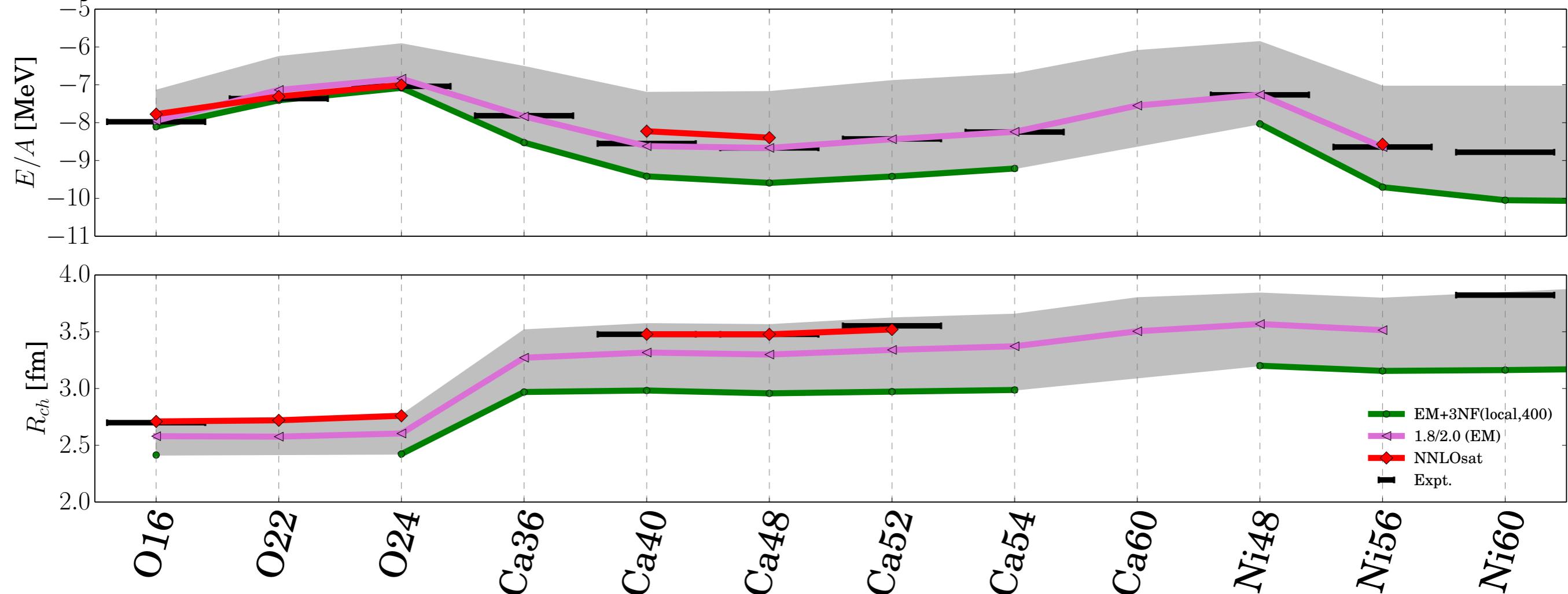
The **SRG-evolved (1.8/2.0)**  
EM-N3LO(500) + N2LO(2.0)  
reproduces energies very well!

K. Hebeler, et al. PRC **83**, 031301(R) (2011)  
J. Simonis, et al. PRC **96**, 014303 (2017)

CC calculations in nuclear matter

G. Hagen, et al. PRC **89**, 014319 (2014)



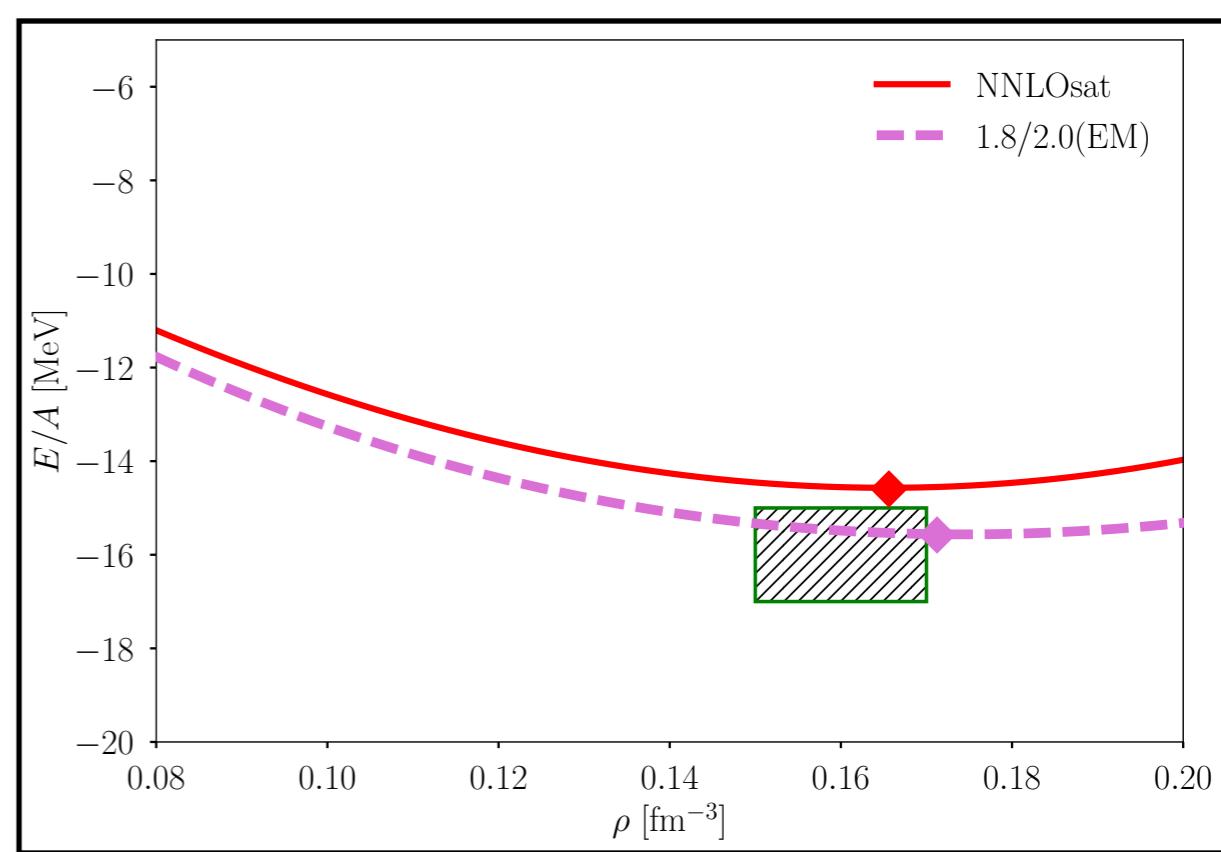


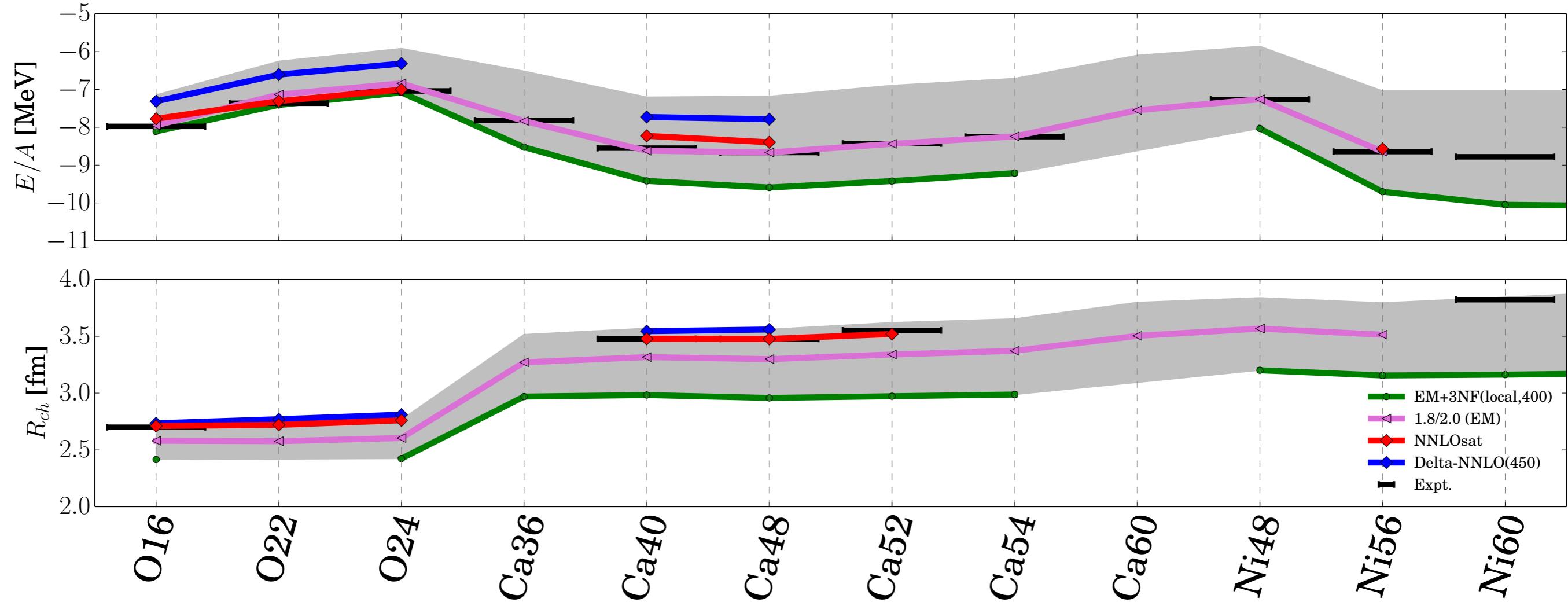
**NNLOsat** is designed to reproduce energies and radii of medium-mass nuclei very well!

A. Ekström, et al. PRC **91**, 051301(R) (2015)

CC calculations in nuclear matter

G. Hagen, et al. PRC **89**, 014319 (2014)

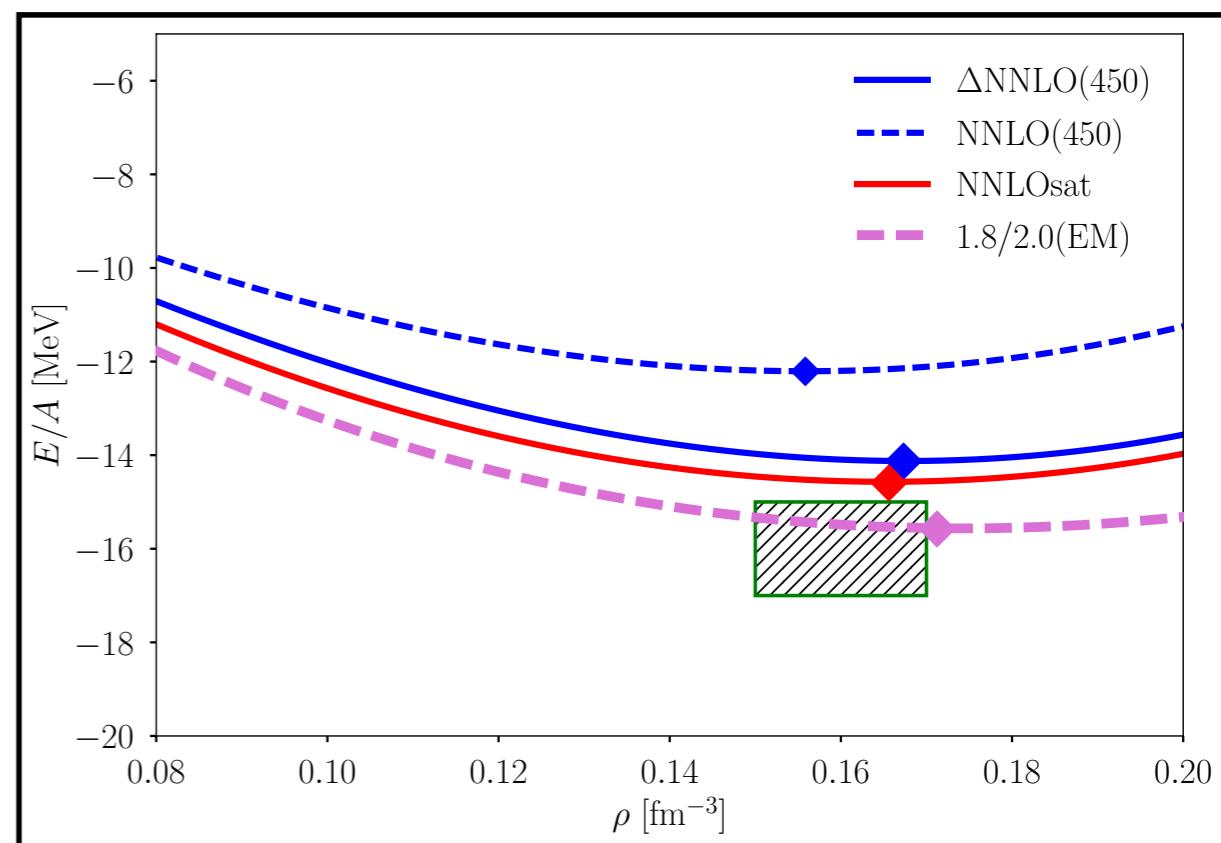




**ΔNNLO(450) predicts**  
energies and radii of medium-mass  
nuclei rather well!

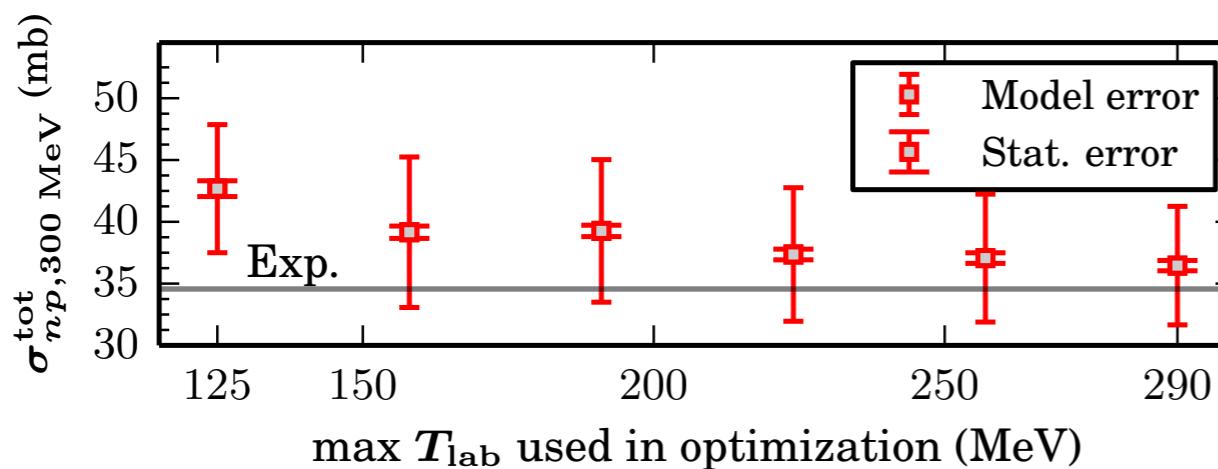
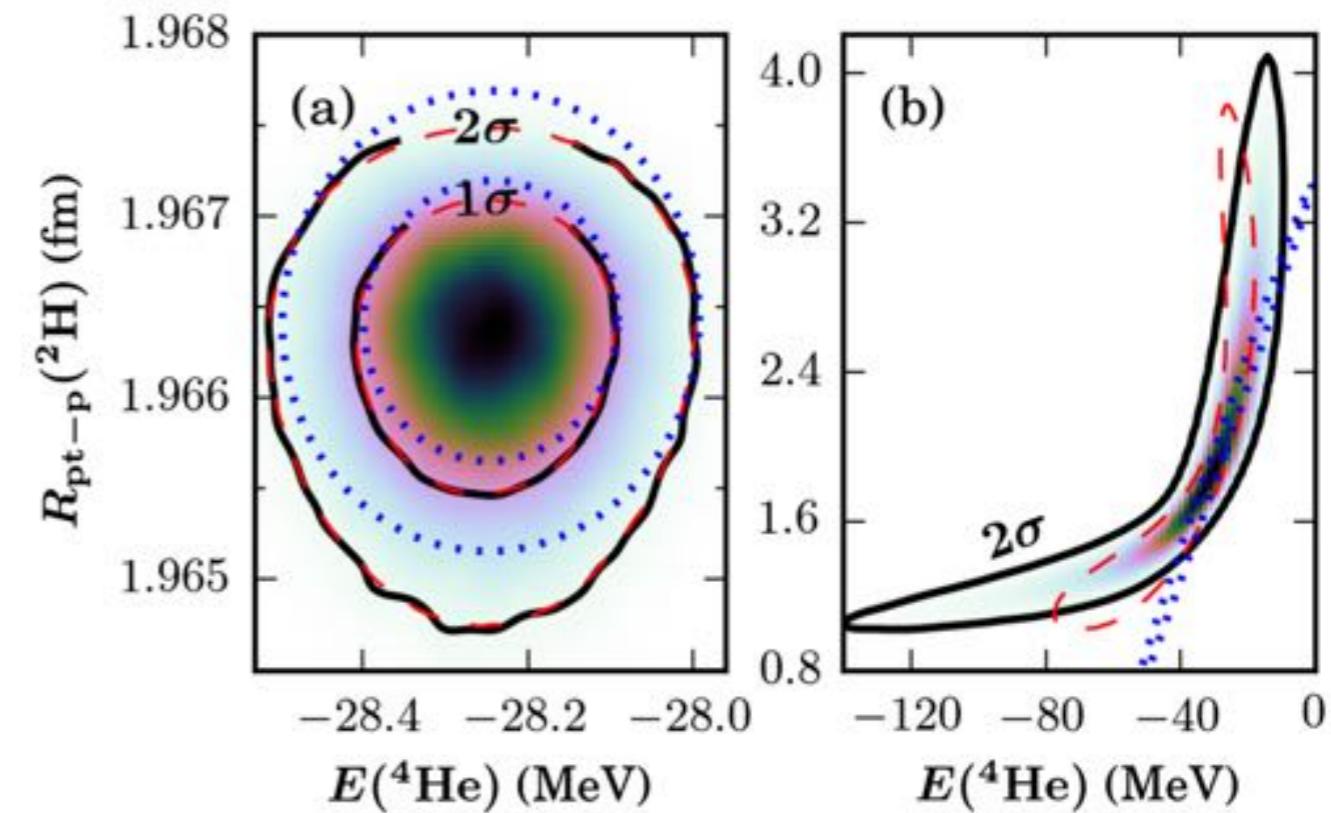
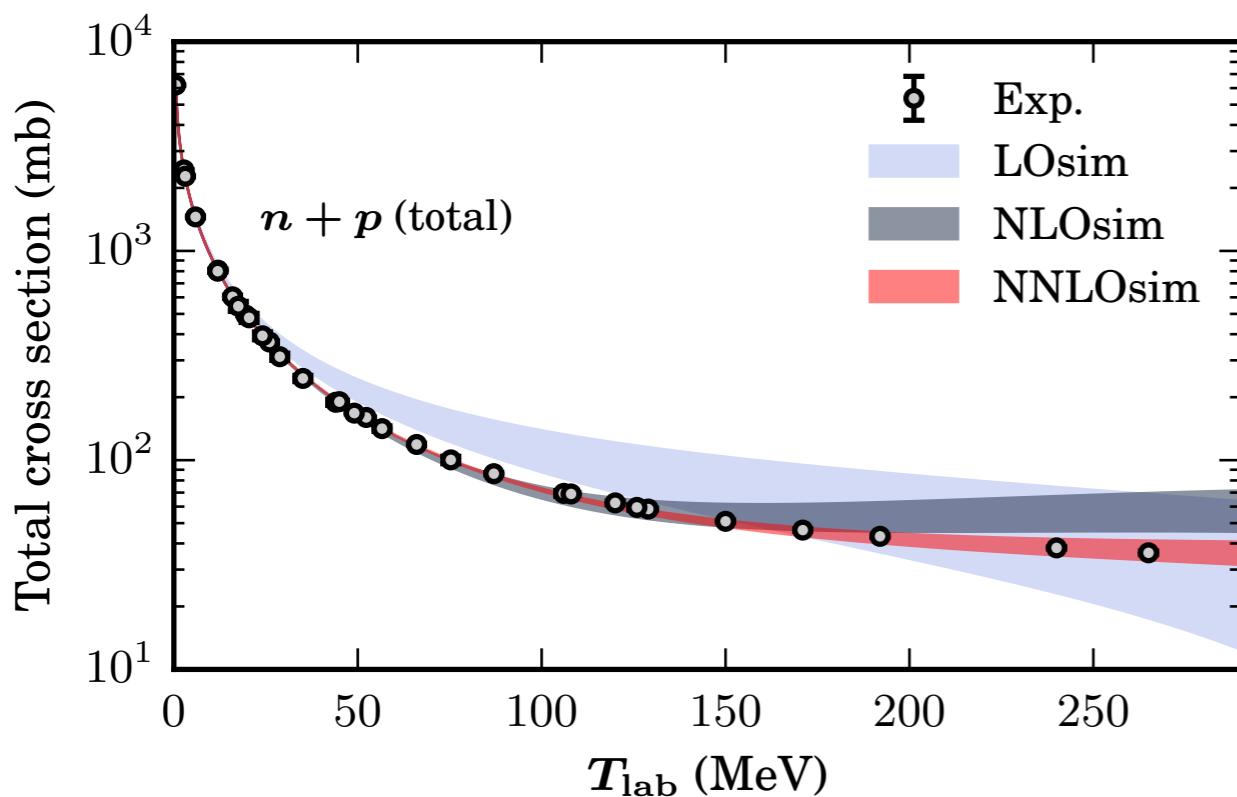
A. Ekström et al. arXiv:1707.09028 [nucl-th] (2017)

The  $\Delta$  is significant



# Uncertainty in the interaction

*“Calculations are only as good as their input”*



$$\chi^2(\boldsymbol{\alpha}) \equiv \sum_{i \in M} \left( \frac{\mathcal{O}_i^{\text{theo}}(\boldsymbol{\alpha}) - \mathcal{O}_i^{\text{exp}}}{\sigma_i} \right)^2 \equiv \sum_{i \in M} r_i^2(\boldsymbol{\alpha}).$$

$$\begin{aligned} \sigma^2 &= \sigma_{\text{exp}}^2 + \sigma_{\text{theo}}^2 \\ &= \sigma_{\text{exp}}^2 + \sigma_{\text{numerical}}^2 + \sigma_{\text{method}}^2 + \sigma_{\text{model}}^2 \end{aligned}$$

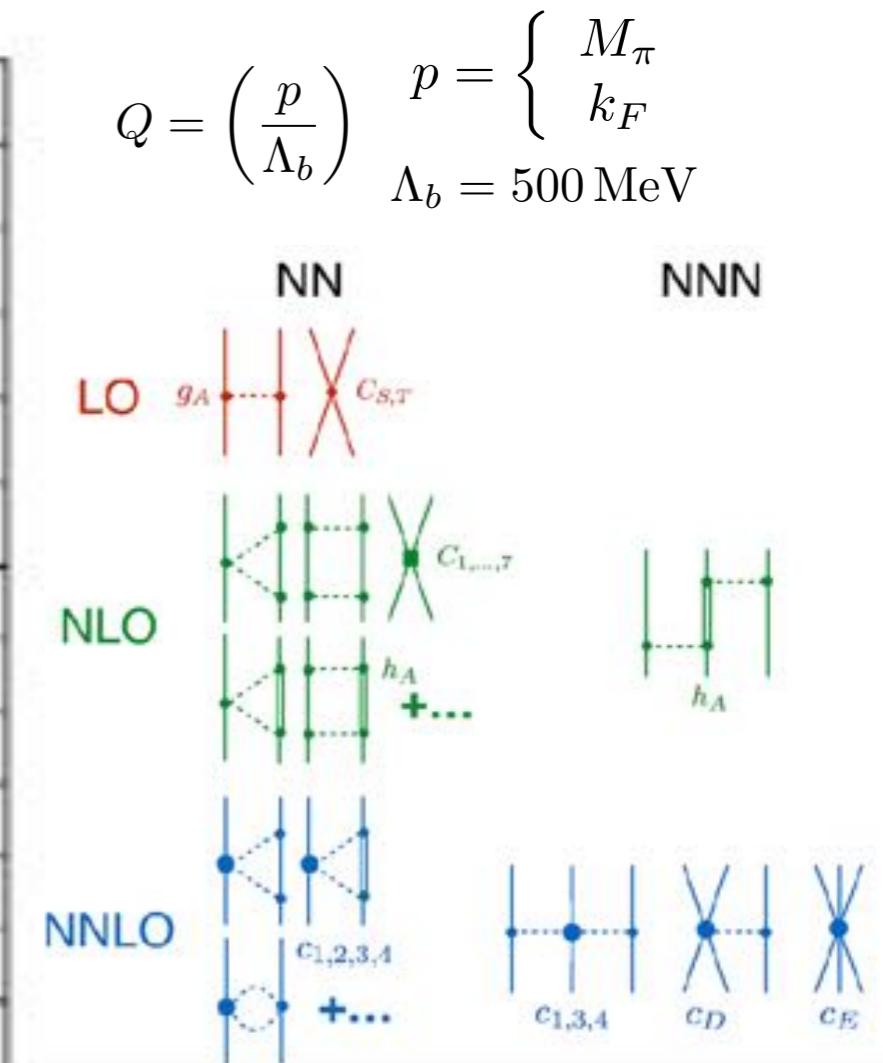
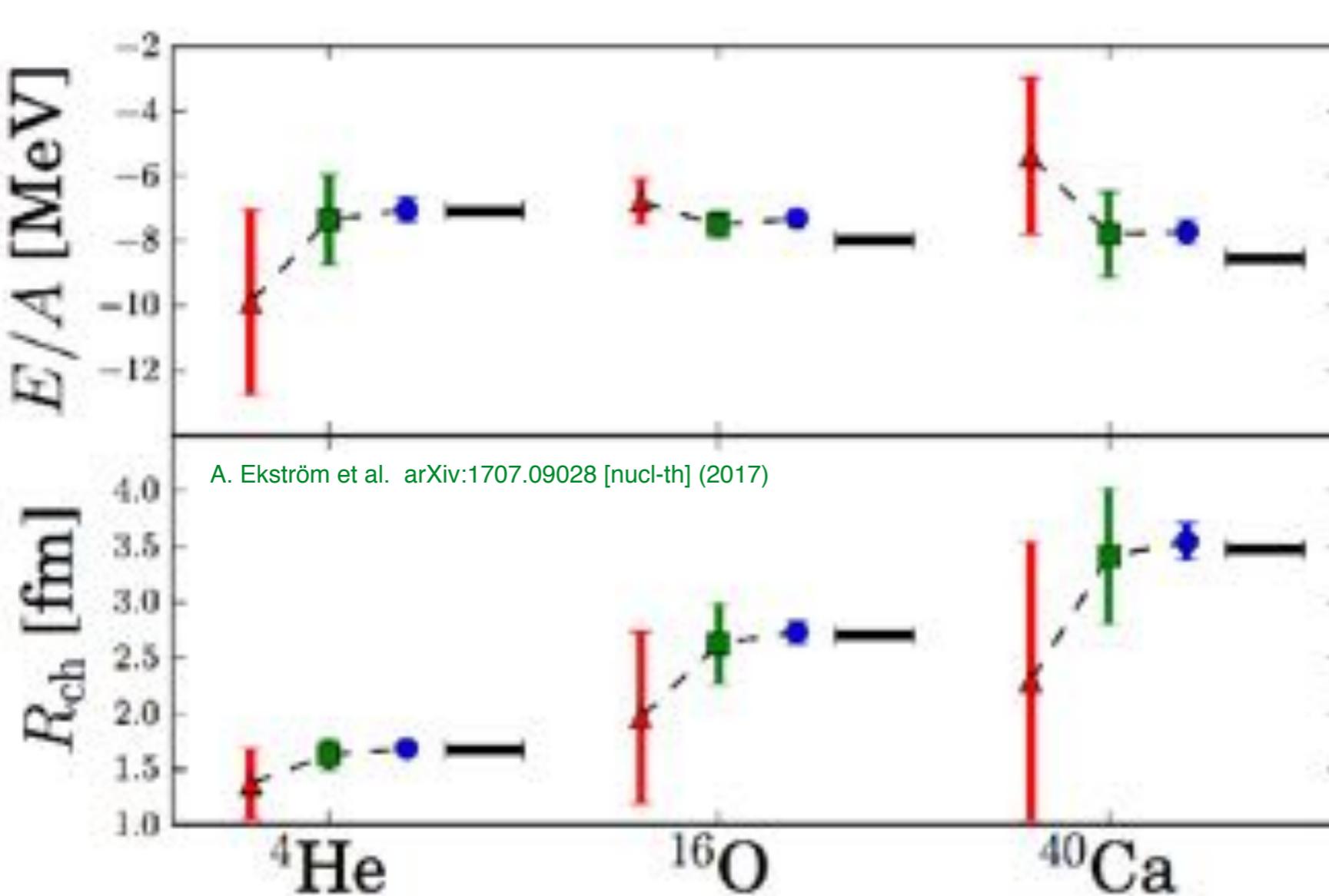
$$\sigma_{\text{model,x}}^{(\text{amp})} = C_x \left( \frac{Q}{\Lambda_\chi} \right)^{\nu_x + 1}, \quad x \in \{NN, \pi N\}$$

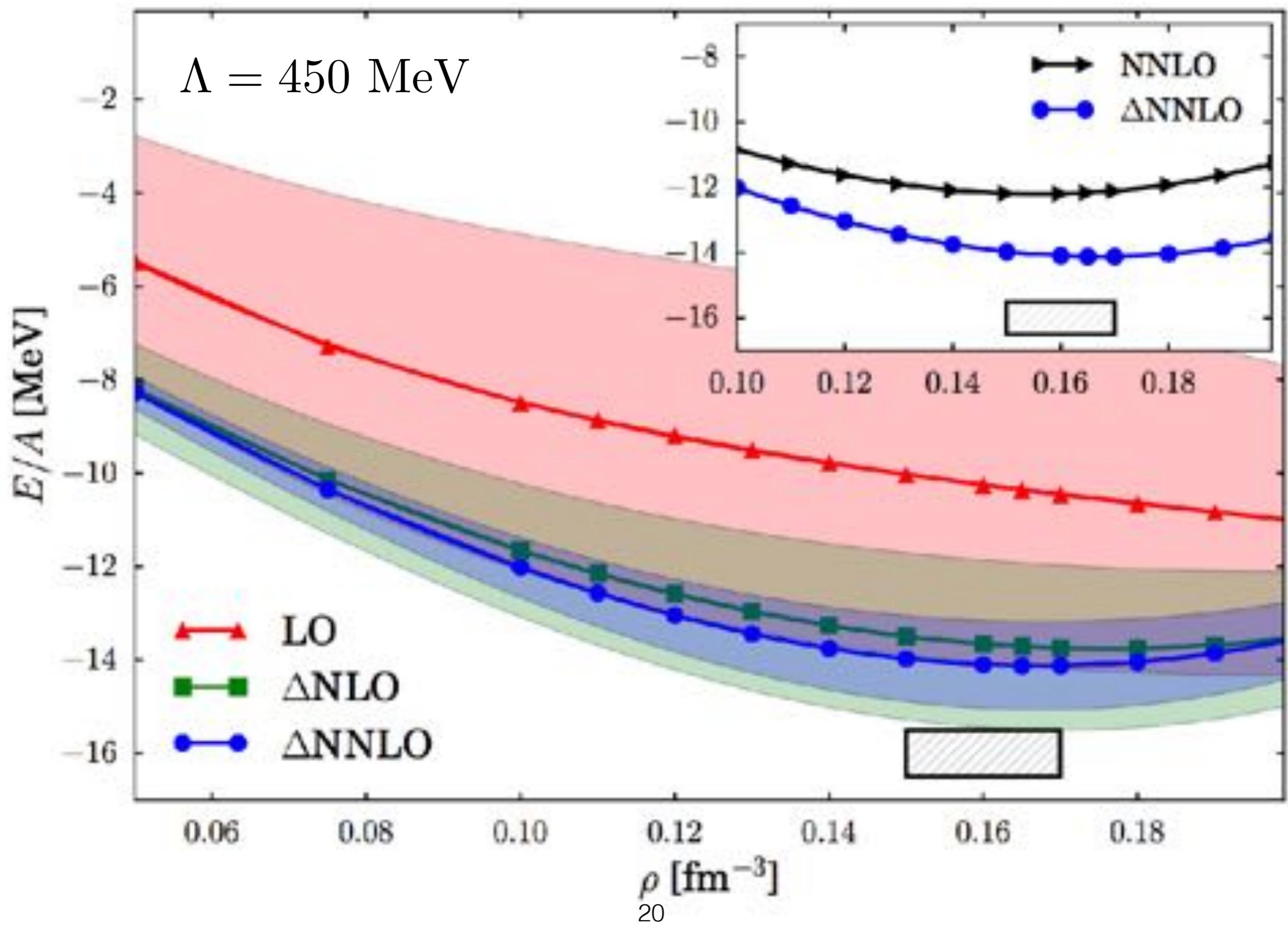
Up to factors of order unity, we can estimate the truncation error

$$X = X_0 \sum_{n=0}^{\infty} c_n Q^n$$

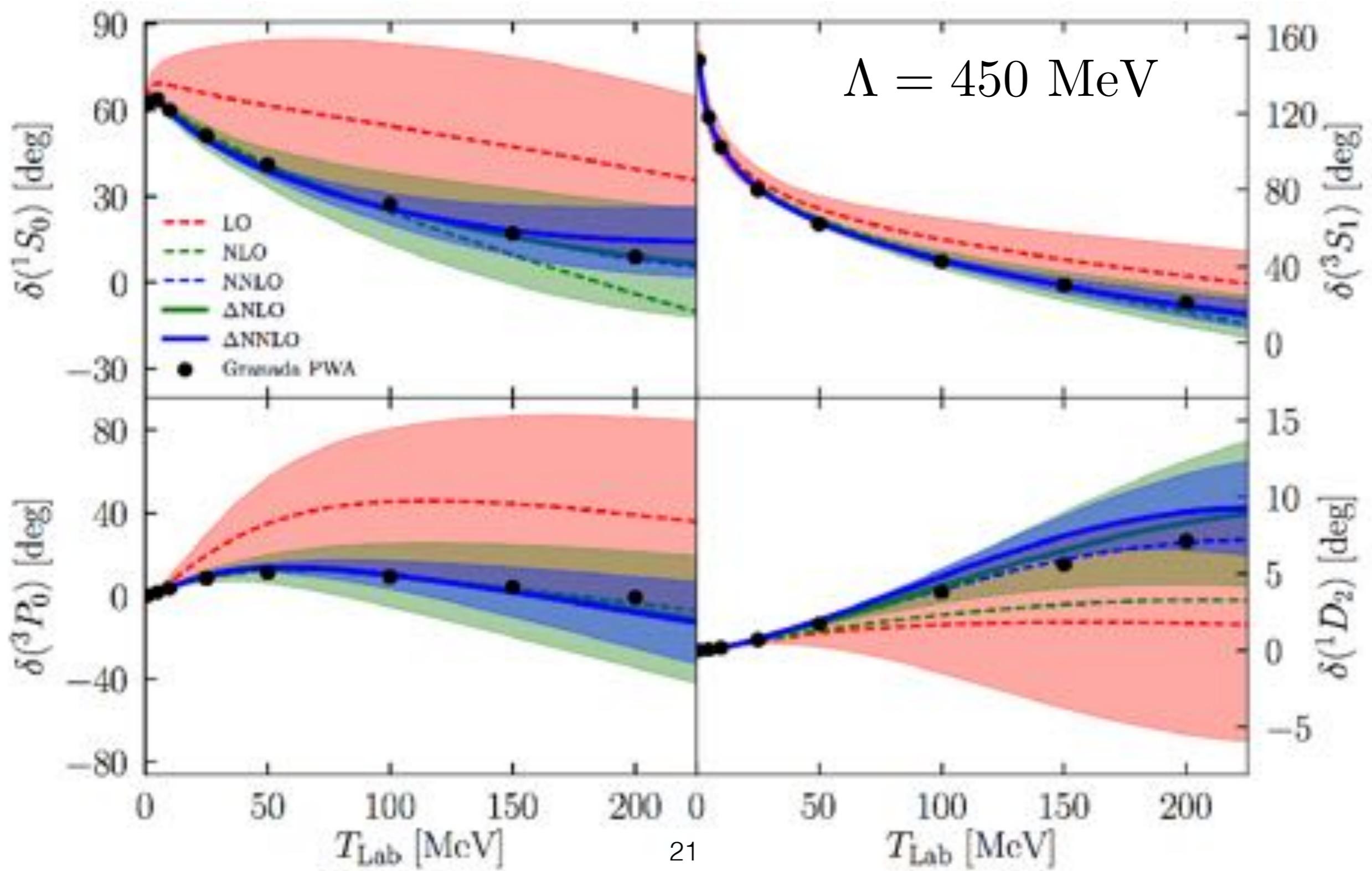
$$\sigma_X(\text{N}j\text{LO}) = X_0 Q^{j+2} \max(|c_0|, |c_1|, \dots, |c_{j+1}|)$$

E. Epelbaum et al Phys. Rev. Lett. **115**, 122301 (2015)  
 R. J. Furnstahl et al, Phys. Rev. C **92**, 024005 (2015)



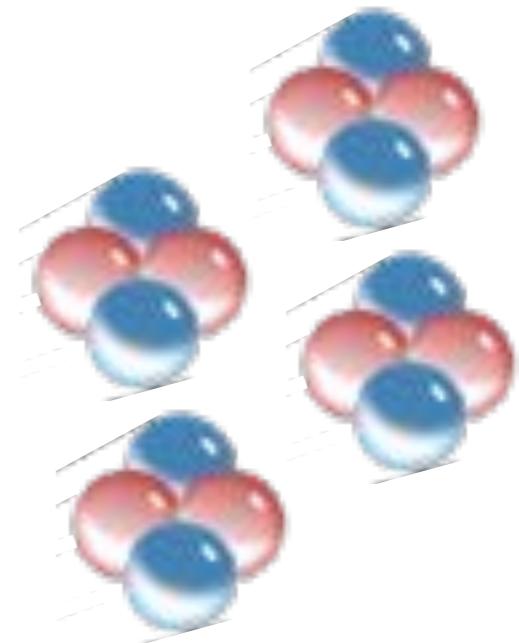


# Phase shifts, truncation errors



# Stability with respect to alpha breakup

Several recent calculations observe that,  
contrary to well-established experiments,  
 $^{16}\text{O}$  is not stable against decay into  $4\alpha$  particles



- Lattice EFT calculations (improved LO interaction “A”)  
(observed for  $^8\text{Be}$ ,  $^{12}\text{C}$ ,  $^{16}\text{O}$ ,  $^{20}\text{Ne}$ ) S. Elhatisari et al. Phys. Rev. Lett. **117**, 132501 (2016)
- Pionless EFT calculations at LO L. Contessit et al. arXiv:1701.06516 [nucl-th] (2017)
- Chiral EFT calculations using optimized NNLO<sub>sim</sub> B. D. Carlsson et al. Phys. Rev. X **6**, 011019 (2016)
- $\Delta$ -less chiral EFT calculations LO, NLO, NNLO A. Ekström et al. arXiv:1707.09028 [nucl-th] (2017)

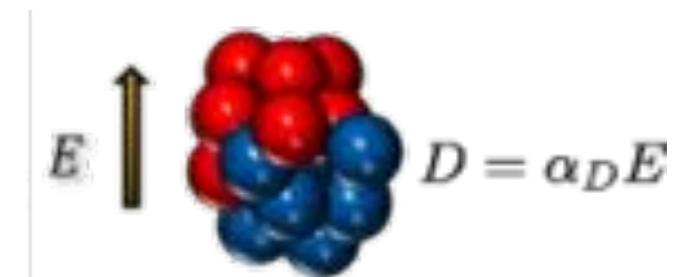
The  $\Delta$ -full NLO, NNLO interactions yield  $^{16}\text{O}$  (and  $^{40}\text{Ca}$ )  
stable with respect to  $\alpha$  breakup A. Ekström et al. arXiv:1707.09028 [nucl-th] (2017)

# Coupled-cluster calculations in the Lambda-CCSD(T) formulation, $hw=16$ , $E3\text{max}=16hw$ , 3NF-NO2b HF

TABLE II. Binding energies ( $E$ ) (in MeV), charge radii (in fm), proton point radii (in fm), neutron point radii (in fm), and neutron skin (in fm) for  $^8\text{He}$ ,  $^{16,22,24}\text{O}$ , and  $^{40,48}\text{Ca}$  at  $\Delta\text{NLO}$  and  $\Delta\text{NNLO}$ , and compared to experiment.

	$E$			$R_{ch}$			$R_p$		$R_n$		$R_{skin}$	
	$\Delta\text{NLO}$	$\Delta\text{NNLO}$	Exp. [65]	$\Delta\text{NLO}$	$\Delta\text{NNLO}$	Exp. [51]	$\Delta\text{NLO}$	$\Delta\text{NNLO}$	$\Delta\text{NLO}$	$\Delta\text{NNLO}$	$\Delta\text{NLO}$	$\Delta\text{NNLO}$
$^8\text{He}$	27.5	27.0	31.40	1.90	1.97	1.924(31)	1.77	1.85	2.63	2.70	0.85	0.85
$^{16}\text{O}$	120.3	117.0	127.62	2.63	2.73	2.699(5)	2.49	2.61	2.47	2.58	-0.02	-0.03
$^{22}\text{O}$	146.2	145.4	162.04	2.66	2.77		2.54	2.66	2.88	3.00	0.34	0.34
$^{24}\text{O}$	152.2	151.6	168.96	2.70	2.81		2.59	2.71	3.11	3.22	0.52	0.51
$^{40}\text{Ca}$	312.2	309.1	342.05	3.41	3.55	3.478(2)	3.31	3.45	3.26	3.40	-0.05	-0.05
$^{48}\text{Ca}$	373.4	373.8	416.00	3.45	3.56	3.477(2)	3.36	3.47	3.51	3.62	0.15	0.15

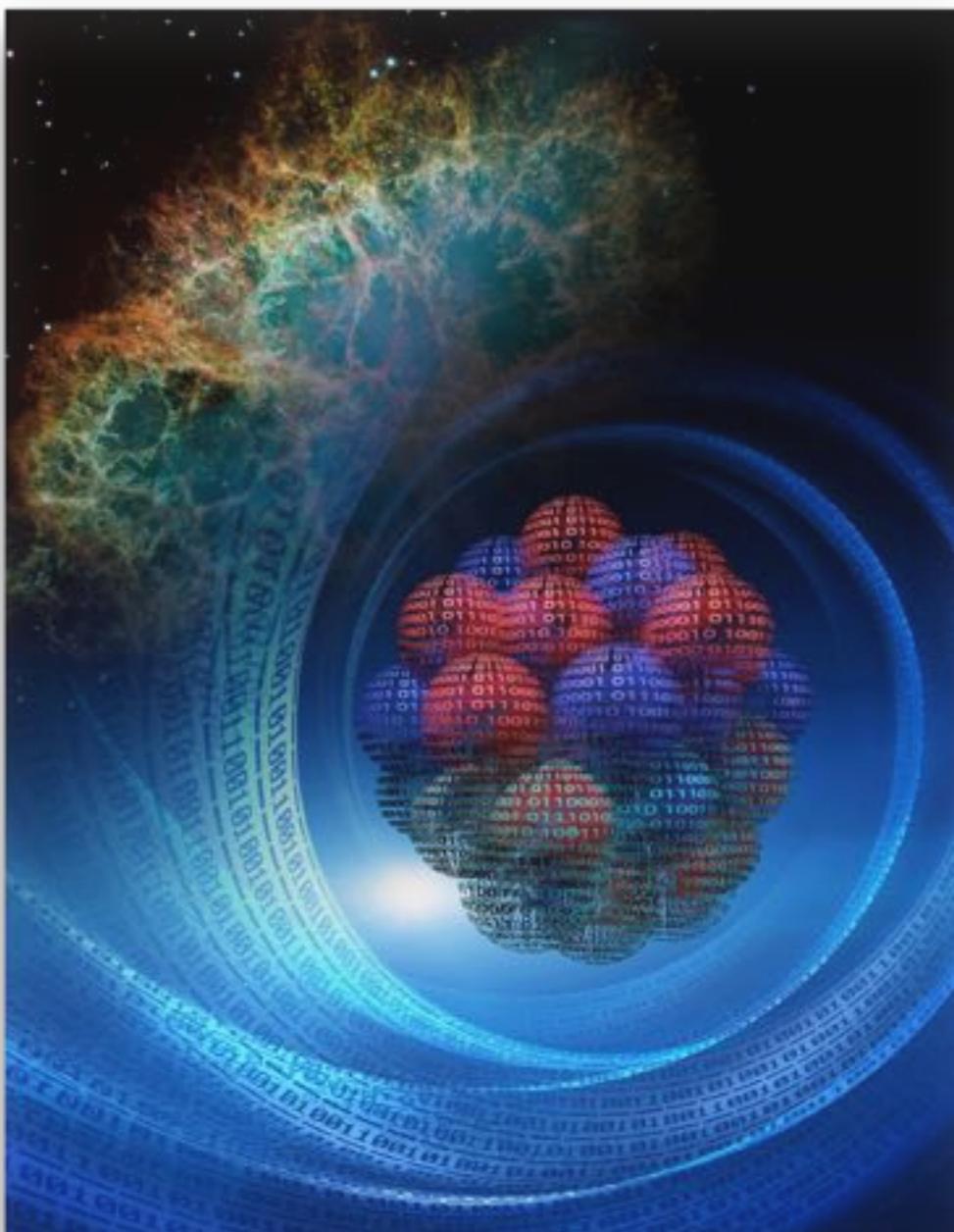
- 😊 •  $\Delta\text{NNLO}$  predicts  $E$  and  $R_{ch}$  rather well.
- 😊 • Neutron skin in  $^{48}\text{Ca}$  consistent with estimated ranges:  
0.14–0.20 fm from E-dipole polarizability & ab initio predictions 0.12–0.15 fm
- 😊 • Low-lying states in  $^{17}\text{O}$  in good agreement with data.
- 😢 •  $^{25}\text{O}$  is bound at  $\Delta\text{NNLO}$  with respect to  $^{24}\text{O}$  by about 0.5 MeV
- 😢 •  $2^+$  state in  $^{24}\text{O}$  is too low compared to experiment.



$$\alpha_D = 2\alpha \int \frac{R(w)}{\omega} d\omega$$

$$R_{\text{skin}} = R_n - R_p$$

# Conclusions



- ✓ The inclusion of  $\Delta$ -isobars in the nuclear interaction addresses some long-standing problems regarding nuclear saturation.
- ✓  $\Delta$ -full NNLO interactions provide  $^{16}\text{O}$  and  $^{40}\text{Ca}$  that are stable with respect to alpha-breakup
- ✓  $\Delta$ -full interactions up to NNLO using Roy-Steiner LECs.
- ✓ How much room for improvement hidden in the uncertainties of the pion-nucleon couplings?
- ✓ Next step(s): exploit information from 3N scattering, decay probabilities, bulk properties of heavier nuclei ?