micrOMEGAs 5:
Freeze-in

Based on
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IRN - Terascale 2017, Marseille

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Outline

- Introduction: evolution of a particle species
- Types of freeze-in
- Development of micrOMEGAs 5 and features
- Outlook

Evolution of a particle species

Consider a particle species χ in a FLRW Universe. The time/temperature evolution of its phase-space distribution can be described by a Boltzmann equation :

$$\boxed{\dot{n}_\chi + 3Hn_\chi = \sum_{A,B}(\xi_B - \xi_A)\mathcal{N}(A \to B)}$$

Evolution of a particle species

Consider a particle species χ in a FLRW Universe. The time/temperature evolution of its phase-space distribution can be described by a Boltzmann equation :

$$n_i = \frac{g_i}{(2\pi)^3} \int f_i(\vec{p}) d^3\vec{p}$$

General initial/final states containing $\xi_{A/B}$ particles of type χ

$$\boxed{\dot{n}_\chi + 3Hn_\chi = \sum_{A,B} (\xi_B - \xi_A) \mathcal{N}(A \to B)}$$

Integrated collision term for $A \to B$:

$$\mathcal{N}(in \to out) = \int \prod_{i=in} \left(\frac{d^3 p_i}{(2\pi)^3 2E_i} f_i \right) \prod_{j=out} \left(\frac{d^3 p_j}{(2\pi)^3 2E_j} (1 \mp f_j) \right)$$
$$\times (2\pi)^4 \delta^4 (\sum_{i=in} P_i - \sum_{j=out} P_j) C_{in} |\mathcal{M}|^2$$

Distribution functions

$$f_i = \frac{|\eta_i|}{e^{\frac{E_i}{T}} - \eta_i}$$

Matrix element

Freeze-in: general context

Freeze-in production of dark matter is based on two basic premises

arXiv:hep-ph/0106249
arXiv:0911.1120
arXiv:1706.07442...

- Dark matter interacts *very* weakly with the thermal bath.

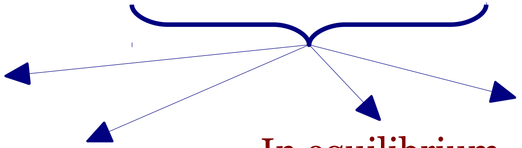
- It has a negligible initial abundance: $(1 \pm f_\chi) \sim 1$.

Assumption will be applied to all FIMPs

$$\boxed{\dot{n}_\chi + 3Hn_\chi = \sum_{A,B}(\xi_B - \xi_A)\mathcal{N}(A \to B)}$$

Then, we are allowed to only consider $\xi_A = 0$ terms in the Boltzmann equation, *i.e.* just DM production terms. In particular :

- Dark matter could be (pair-) produced from decays of a heavier particle.



Z2-even?

Z2-odd?

In equilibrium
with bath?

Not in equilibrium
with bath?

- Dark matter could be (pair-) produced from annihilations of bath particles.

Freeze-in vs freeze-out

· Naively, the freeze-in equation is simpler than the freeze-out one: quite similar but only including the DM production term.

· When working in full generality (i.e. for a general particle physics model), though, things can get much more involved:

$$\boxed{\dot{n}_\chi + 3Hn_\chi = \sum_{A,B}(\xi_B - \xi_A)\mathcal{N}(A \to B)}$$

- In FO equilibrium erases all memory: no dependence on the initial conditions.

Good or bad, depending on perspective!

- In FO no need to keep track of the heavier particles (modulo coannihilations).

Equilibrium is restored extremely fast

· If more than one particles in the spectrum are feebly coupled, need to write down Boltzmann equations for them as well.

Need to keep track of the evolution of all states
and the way they contribute to the DM abundance.

Digression: decay width in a medium

Consider the decay of a particle Y into two particles a, b in the early Universe. The number of decays per unit space-time volume is

$$\mathcal{N}(Y \to a, b) = \int \frac{d^3 p_Y}{(2\pi)^3 2E_Y} \frac{d^3 p_a}{(2\pi)^3 2E_a} \frac{d^3 p_b}{(2\pi)^3 2E_b} f_Y (1 \mp f_a)(1 \mp f_b) \\ \times (2\pi)^4 \delta(P_Y - P_a - P_b) C_{in} |\mathcal{M}|^2 \ .$$

Replacing $f_Y(p_Y) \longrightarrow (2\pi)^3 \delta^3(\vec{p} - \vec{p}_Y)/g_1$ we get :

$$\Gamma^{\rm bath}(Y \to a, b) = \frac{1}{2E_Y} \int \frac{d^3 p_a}{(2\pi)^3 2E_a} \frac{d^3 p_b}{(2\pi)^3 2E_b} (1 \mp f_a)(1 \mp f_b) \\ \times (2\pi)^4 \delta(P_Y - P_a - P_b) C_{in} \overline{|\mathcal{M}|}^2$$

Decay width of *Y* in the medium created by *a, b*

Defining : $$S\left(p/T, x_Y, x_a, x_b, \eta_a, \eta_b\right) = \frac{1}{2} \int_{-1}^{1} dc_\theta \frac{e^{E_Y^{\rm CF}/T}}{(e^{E_a^{\rm CF}/T} - \eta_a)(e^{E_b^{\rm CF}/T} - \eta_b)}$$

Calculable analytically

We obtain : $$\Gamma^{\rm bath}(Y \to a, b) = \frac{m_Y \Gamma_{Y \to a,b}}{E_Y^{\rm CF}} S\left(p/T, x_Y, x_a, x_b, \eta_a, \eta_b\right)$$

S contains all the stat. mech. information

Freeze-in through decays - 1

Dark matter could be produced from the decay of a heavier particle Y in equilibrium with the thermal bath: $Y \to X + \chi$ (only χ is a FIMP)

· Y in equilibrium → No need to write down dedicated Boltzmann equation.

$$\dot{n}_\chi + 3Hn_\chi = \int \frac{d^3p_Y}{(2\pi)^3 2E_Y}\frac{d^3p_X}{(2\pi)^3 2E_X}\frac{d^3p_\chi}{(2\pi)^3 2E_\chi} f_Y(1\mp f_X)(2\pi)^4\delta(P_Y - P_X - P_\chi)C_{in}|\mathcal{M}|^2$$

Eventually :

$$Y_\chi = \frac{g_Y\,|\eta_Y|}{2\pi^2}m_Y^2\left[\sum_X\left(\Gamma_{Y\to\chi,X}\int_{T_0}^{T_R}\frac{dT}{H'(T)s(T)}\tilde{K}_1(x_Y,0,x_X,\eta_Y,0,\eta_X)\right) + 2\Gamma_{Y\to\chi,\chi}\int_{T_0}^{T_R}\frac{dT}{H'(T)s(T)}\tilde{K}_1(x_Y,0,0,\eta_Y,0,0)\right]$$

Where :

$$\tilde{K}_1\left(x_1,x_2,x_3,\eta_1,\eta_2,\eta_3,\right) \equiv x_1\int_1^\infty \frac{dz\sqrt{z^2-1}e^{-x_1 z}}{1-\eta_1 e^{-x_1 z}} S\left(x_1\sqrt{z^2-1},x_1,x_2,x_3,\eta_2,\eta_3\right)$$

Reduces to Bessel function of the 1st kind for Maxwell-Boltzmann statistics.

Freeze-in through decays - 2

Dark matter could be produced from the decay of a heavier particle Y which is *not* in equilibrium with the thermal bath: $Y \to X + \chi$ (both χ and Y are FIMPs)

- Y not in equilibrium → Need to write down dedicated Boltzmann equation.

$$\dot{n}_\chi + 3Hn_\chi = \int \frac{d^3p_Y}{(2\pi)^3 2E_Y}\frac{d^3p_X}{(2\pi)^3 2E_X}\frac{d^3p_\chi}{(2\pi)^3 2E_\chi} f_Y(1\mp f_X)(2\pi)^4\delta(P_Y - P_X - P_\chi)C_{in}|\mathcal{M}|^2$$

$$\frac{dY_Y}{dt} = -\frac{1}{\gamma(x_Y,\eta_Y)}\Big(\underbrace{\Gamma^{\text{bath}}_{Y\to\text{all}}Y_Y}_{\text{Depletion term}} - \underbrace{\Gamma^{\text{bath}}_{Y\to\text{bath}}\bar{Y}_Y}_{\text{Production term}}\Big) \qquad \text{along with} \qquad \eta_Y = \frac{Y_Y}{Y_Y^{\text{eq}}}$$

- Here Y assumed to be in *kinetic* equilibrium with the bath.
- Opposite regime: Y is kinetically decoupled from the bath.

The two methods give very similar results, both options available to the user.

Basic types of freeze-in : scattering

Finally, dark matter could be produced directly from 2 → 2 processes, annihilations of Standard Model (or other bath) particles: $1 + 2 \rightarrow X + \chi$

$$\dot{n}_\chi + 3Hn_\chi = \int \prod_{i=1,2,X,\chi} \frac{d^3 |\vec{p}_i|}{(2\pi)^3 \, 2E_i} (2\pi)^4 \delta^4 (P_1 + P_2 - P_X - P_\chi) |\mathcal{M}|^2 f_1 f_2 (1 \mp f_X)$$

· In the special case of DM *pair*-production, we can get a fairly simple expression for the collision term :

$$\mathcal{N}(1, 2 \to \chi, \bar{\chi}) = \frac{g_1 g_2}{8\pi^4} T \, |\eta_1 \eta_2| \, C_{12} \int ds \, (p_{1,2}^{\rm CM})^2 \sqrt{s} \sigma(s) \tilde{K}_1 (\sqrt{s}/T, x_1, x_2, 0, \eta_1, \eta_2)$$

which we can promptly replace in the general yield expression.

· For DM production in association with a bath particle, we apply a correction factor to the integrand that leads to ***O***(2%) accuracy (compared against exact computation).

· For DM pair-production through a resonance, we correct the mediator width in order to match the result obtained from (potentially late) decays.

In practice

Important note: Freeze-out and freeze-in modules will be completely separated: when computing the relic abundance, you'll need to *assume* a production mechanism.

Basic steps (after writing down your Lagrangian) :

- `toFeebleList(particle_name)`: specify all feebly coupled particles in the spectrum.

All particles not in this list are assigned their equilibrium distributions

If you want to do things manually :

- `darkOmegaFiDecay(TR, Name, KE, plot)`: calculates the DM abundance from the decay of the particle `Name` into all odd feeble particles, assuming kinetic equilibrium (`KE` = 1) or not (`KE` = 0) with the Standard Model.

- `darkOmegaFi22(TR, Process, vegas, plot, &err)`: freeze-in through scattering for a given `Process`.

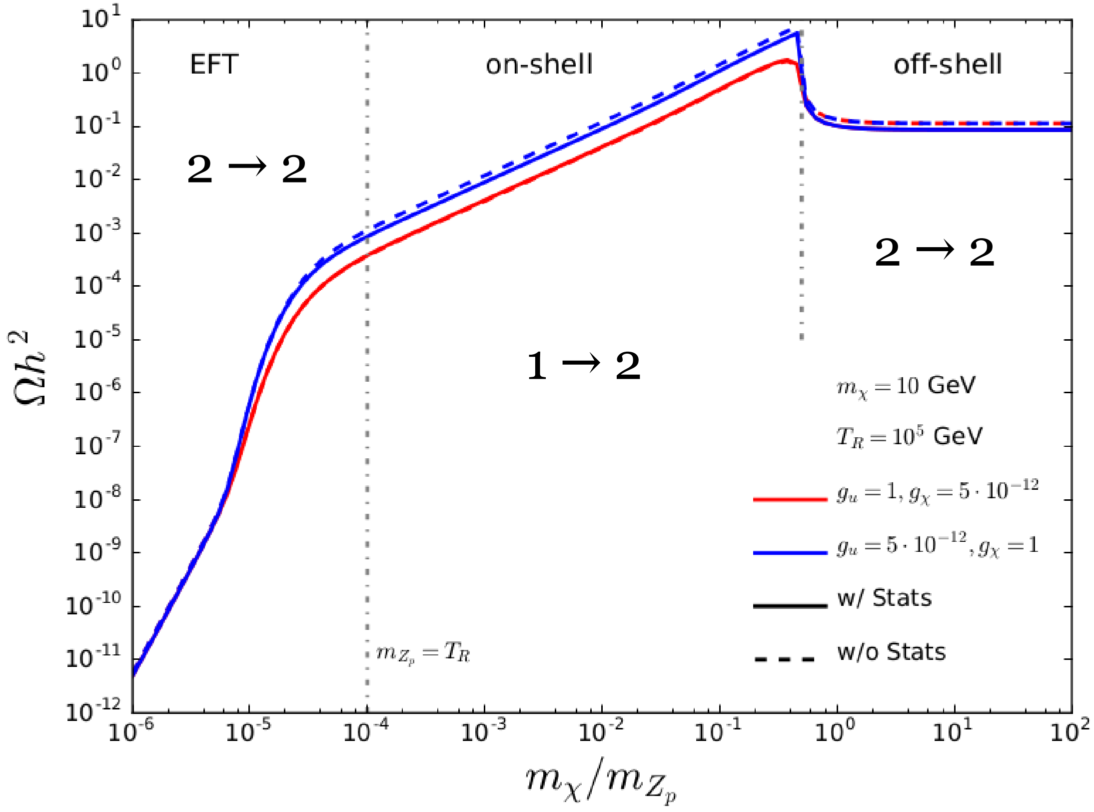
Or simply :

- `darkOmegaFi(TR)`: DM abundance after summing over all 2 → 2 processes involving particles in the bath in the initial state and at least one feeble particle in in the final state.

A simple example

Consider a simple vector portal model

$$\mathcal{L}_{\rm int} = -g_\chi Z'_\mu \bar{\chi}\gamma^\mu \chi - \sum_q g_q Z'_\mu \bar{q}\gamma^\mu q$$



Statistics *do* matter and are taken into account in micrOMEGAs. Can easily lead to factor 2 differences.

Outlook

· micrOMEGAs 5.0 will be able to handle feebly coupled dark matter candidates and compute their predicted abundance according to the freeze-in mechanism. Statistical distributions of bath particles are fully taken into account and are found to matter.

· Most major freeze-in scenarios will be covered.

· Our hope is that this will facilitate phenomenological studies (and model-building endeavours!) and help establish stronger connections between the early Universe phenomenology of FIMPs and their observational signatures.

· To appear (hopefully!) next week (so that we can take some vacation).

The ultimate goal (for the next versions): a unified treatment of all cases,with a smooth passage amongst the various regimes.

aka the most general form of the Boltzmann equation