Precision of relic density calculations

Julia Harz

LPTHE / ILP, Paris

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What is the precision / uncertainty of your favorite DM tool?

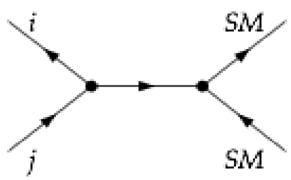
Prediction of the relic density

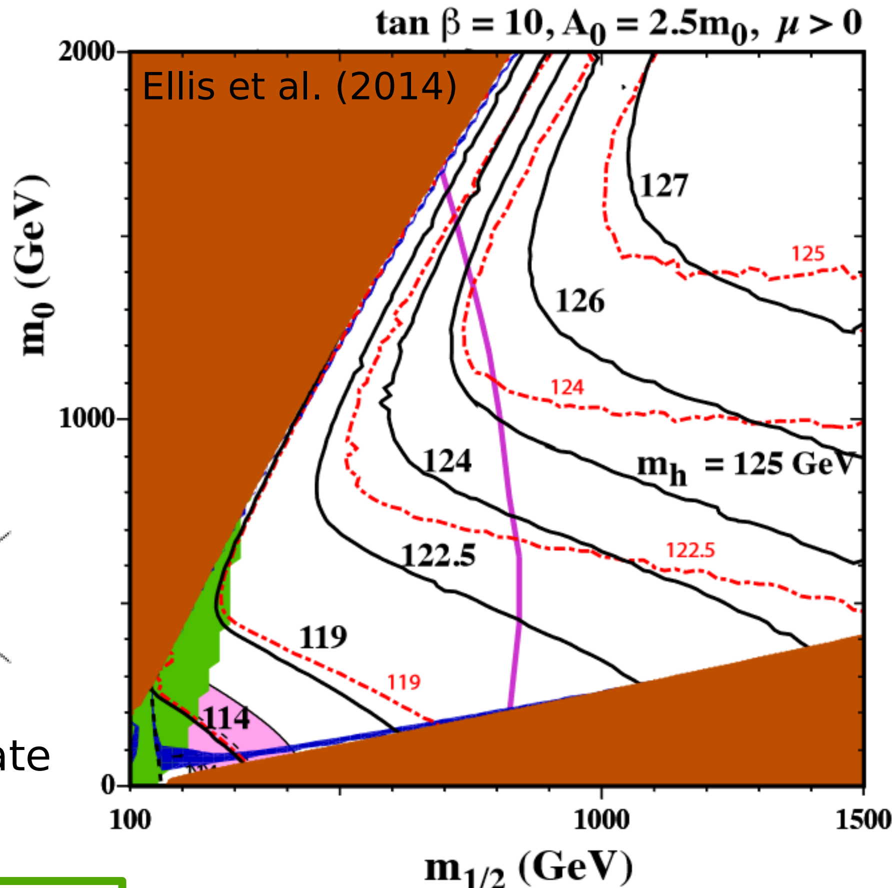
Boltzmann equations describes time evolution of relic density:

$$\dot{n} + 3Hn = -\langle \sigma_{\rm eff} v \rangle \left(n^2 - n_{\rm eq}^2\right)$$

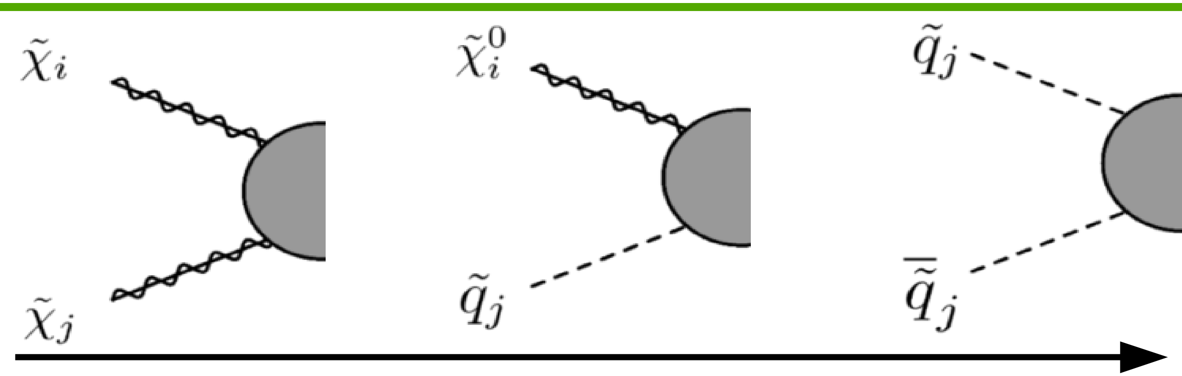
$$\langle \sigma_{\rm eff} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{\rm eq}}{n^{\rm eq}} \frac{n_j^{\rm eq}}{n^{\rm eq}}$$

$$\frac{n_i^{\rm eq}}{n^{\rm eq}} \propto \exp^{\frac{-(m_i - m_\chi)}{T}}$$





Particles with small mass differences can co-annihilate and dominate the contribution to the relic density



$$\Omega h^2 \propto \frac{1}{\langle \sigma_{\rm eff} v \rangle}$$

$$\Omega_{\rm CDM} h^2 = 0.1199 \pm 0.0022$$

DM density very precisely constrained by Planck satellite

Everything settled and straightforward?

October 1990

CfPA-TH-90-001
BA-90-79

UNIVERSITY OF CALIFORNIA, BERKELEY

CENTER FOR PARTICLE ASTROPHYSICS

Three Exceptions in the Calculation of Relic Abundances

KIM GRIEST

Center for Particle Astrophysics and Astronomy Department,
University of California, Berkeley,

and

DAVID SECKEL

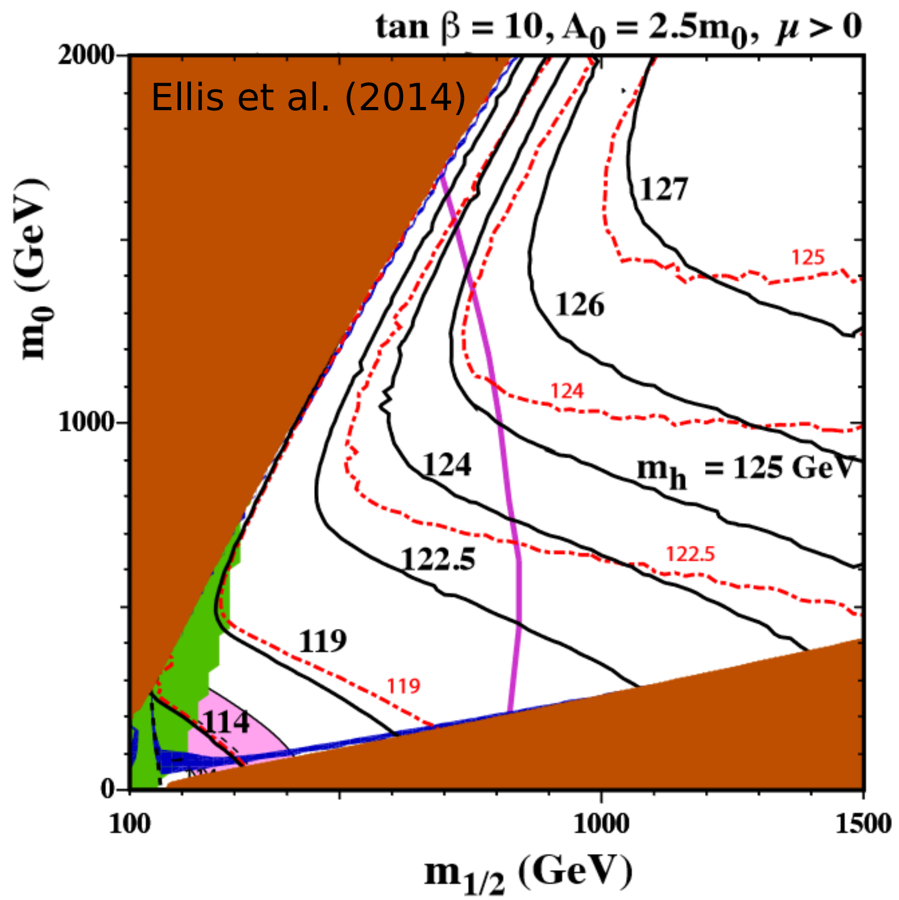
Bartol Research Institu
University of Delaware, Newark, 1

ABSTRACT

Calculation of relic abundances of elementary particles by following their annihilation and freeze-out in the early universe has become an important and standard tool in discussing particle dark matter candidates. We find three situations, all occurring in the literature, in which the standard methods of calculating relic abundances fail. The first situation occurs when another particle lies near in mass to the relic particle and shares a quantum number with it. An example is a light squark with neutralino dark matter. The additional particle must be included in the reaction network, since its annihilation can control the relic abundance. The second situation occurs when the relic particle lies near a mass threshold. Previously, annihilation into particles heavier than the relic particle was considered kinematically forbidden, but we show that if the mass difference is $\sim 5\% - 15\%$, these "forbidden" channels can dominate the cross section and determine the relic abundance. The third situation occurs when the annihilation takes place near a pole in the cross section. Proper treatment of the thermal averaging and the annihilation after freeze-out shows that the dip in relic abundance caused by a pole is not nearly as sharp or deep as previously thought.

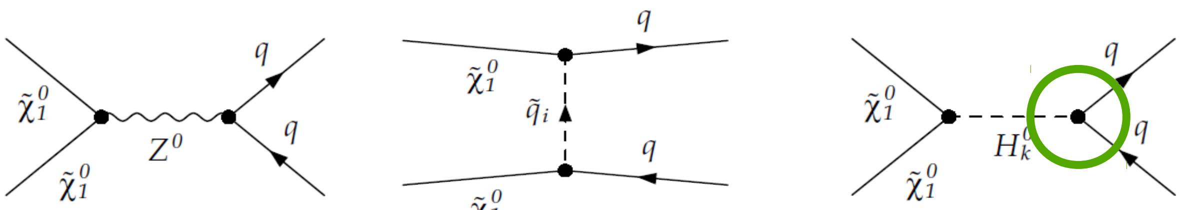
Everything settled and straightforward?

- No higher order corrections
- No proper theoretical uncertainties
- No Sommerfeld effect
- No bound-state formation

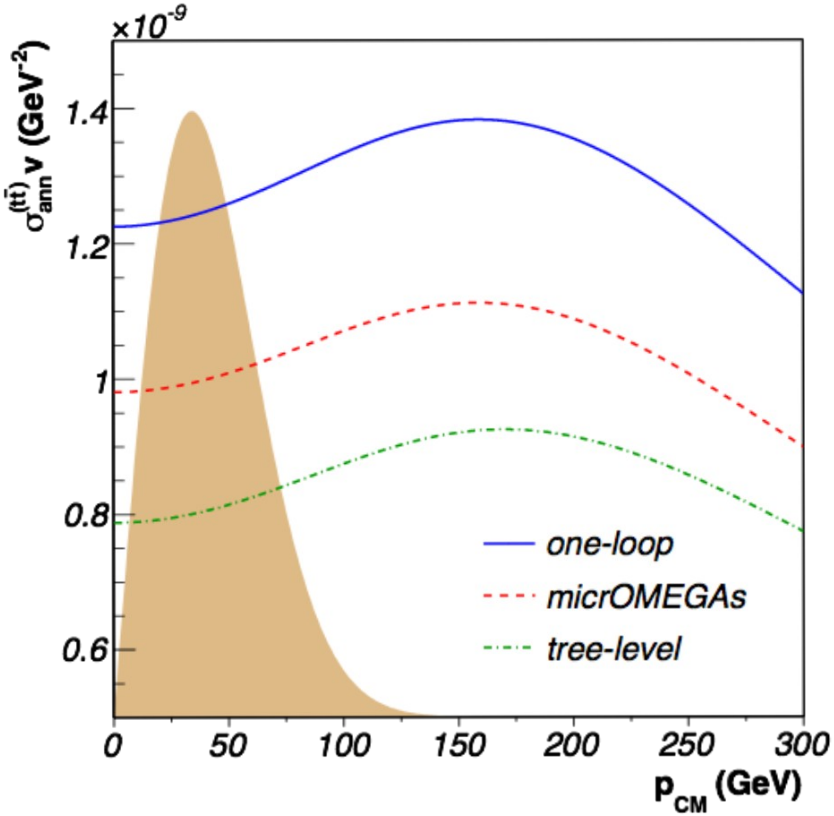


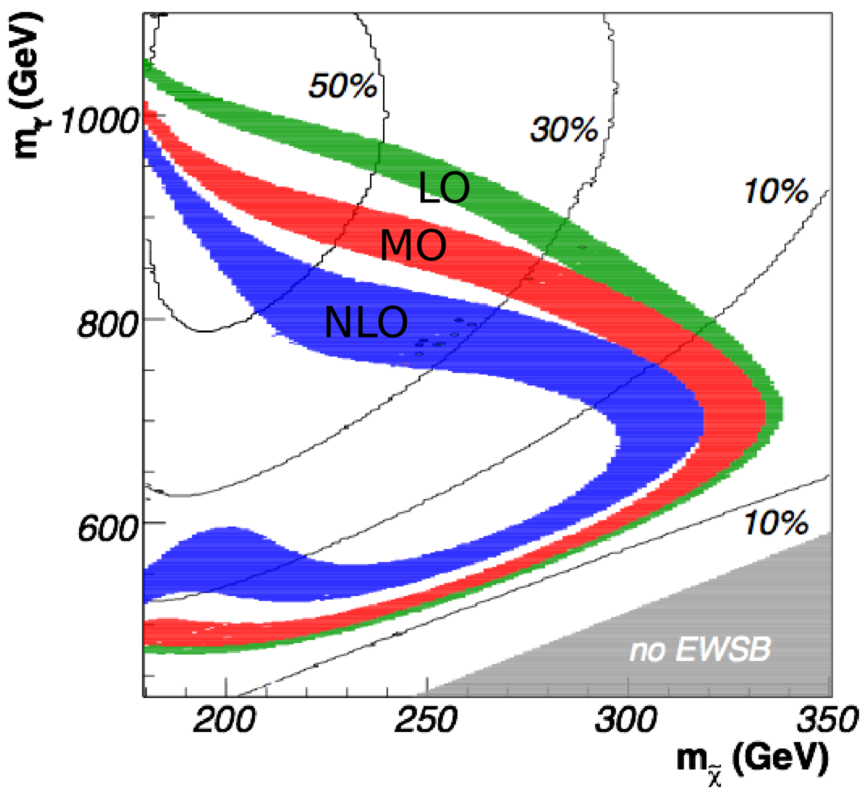
GAMBIT, e.g. uses a 5% uncertainty on the DM density for the whole parameter space

(1) Higher order corrections to $\sigma_{\rm eff}$



leading sub-leading





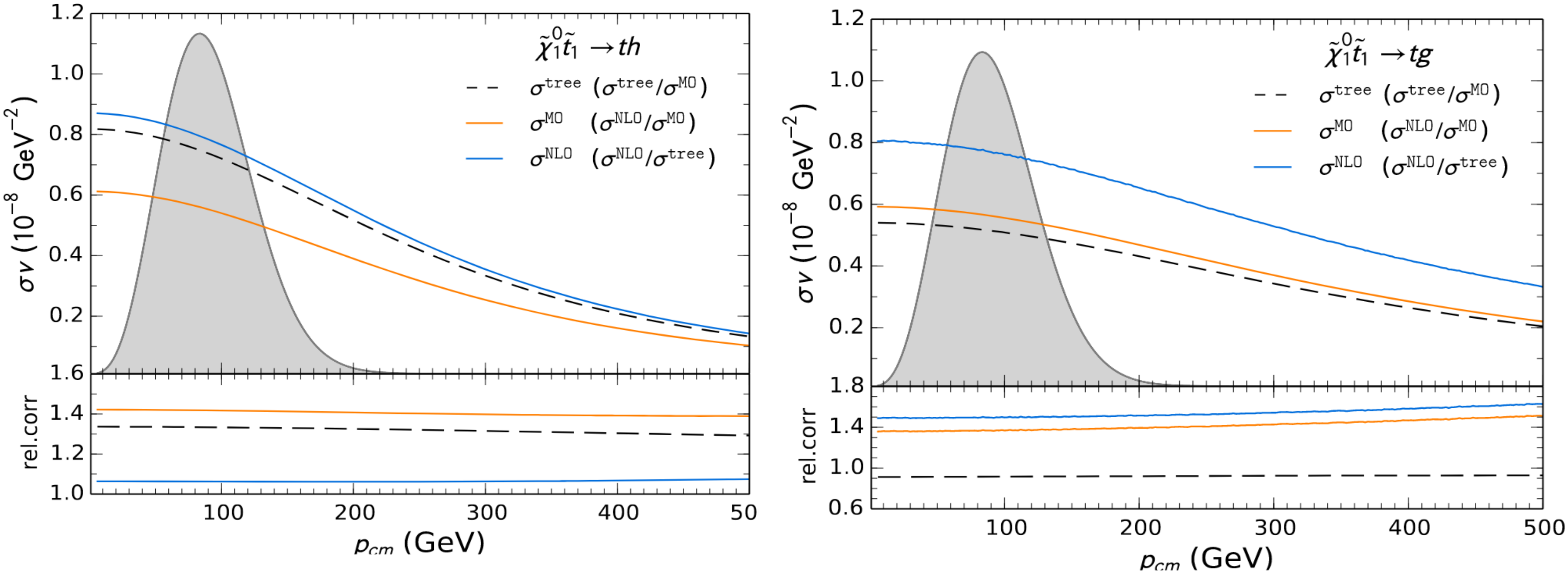
While effective couplings are a good approximation around Higgs-resonances, they fail when other channels are important, as e.g. at a Z-resonance → 20% correction to MO

→ corrections of ~ 100 GeV in the physical mass plane

B. Herrmann, M. Klasen and K. Kovařík, Phys. Rev. D 79: 061701 (2009)
B. Herrmann, M. Klasen and K. Kovařík, Phys. Rev. D 80: 085025 (2009)

(1) Higher order corrections to $\sigma_{\rm eff}$

Coannihilation scenario on cross section level

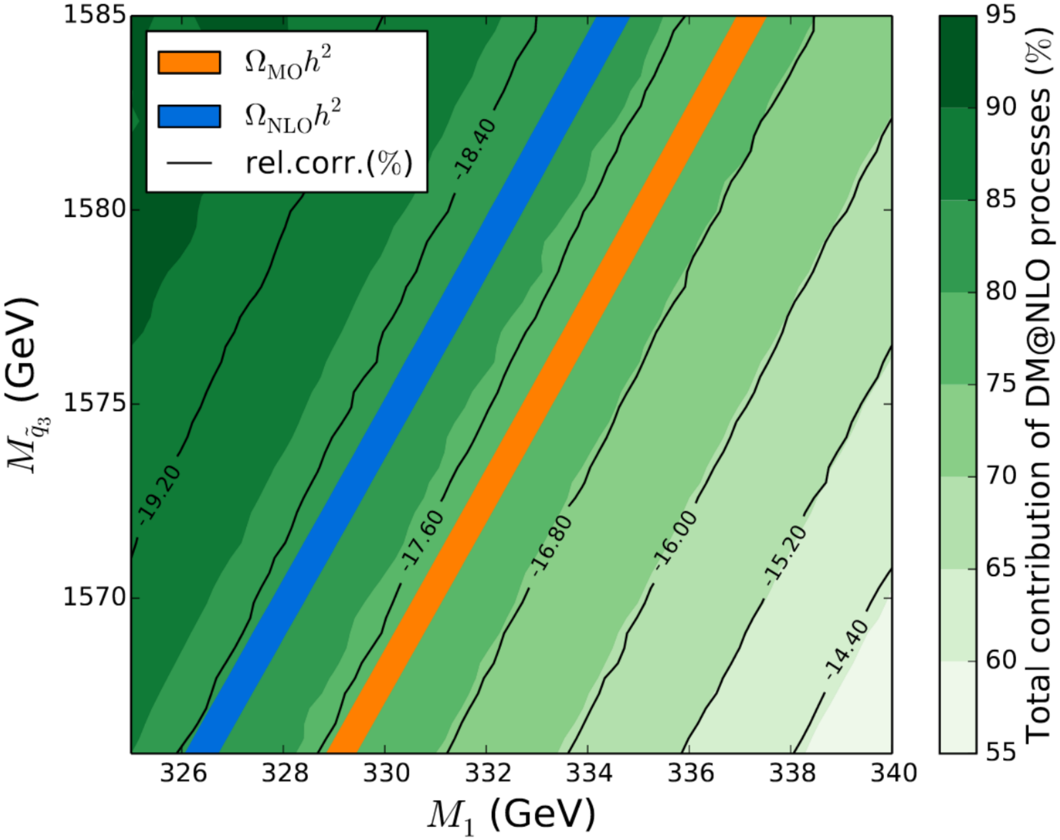


→ Corrections of around 40% to the MicrOMEGAs cross section only from loop corrections

J. Harz, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h, Phys. Rev. D 87, 054031 (2013)
J. Harz, B. Herrmann, M. Klasen, and K. Kovařík, Phys. Rev. D 91, 034028 (2015)

(1) Higher order corrections to $\sigma_{\rm eff}$

Coannihilation scenario on relic density level



→ corrections of around 20% on the relic density

This concerns only QCD corrections
→ EW corrections have to be added up

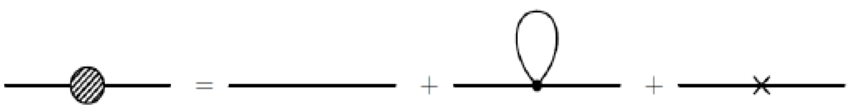
J. Harz, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h, Phys. Rev. D 87, 054031 (2013)
J. Harz, B. Herrmann, M. Klasen, and K. Kovařík, Phys. Rev. D 91, 034028 (2015)

(2) Theoretical uncertainty of σ_{eff}

- No real estimation of theoretical uncertainty of the cross section calculation so far!

  → scale variation $\mu_R/2 < \mu < 2\mu_R$ gives an estimate

UV- divergences → Renormalisation schemes



$$\frac{i}{p^2-m^2} + \frac{i}{p^2-m^2}\left(-i\Sigma(p^2)\right)\frac{i}{p^2-m^2} = \frac{i}{p^2-m^2}\left(1+\frac{\Sigma(p^2)}{p^2-m^2}\right)$$

$$-i\Sigma(p^2) = -i\frac{\lambda}{2}\frac{1}{16\pi^2}A_0(m^2) + i\left[-m^2\delta Z_m\right]$$

$$-i\Sigma(p^2) = -i\frac{\lambda}{2}\frac{m^2}{16\pi^2}\left[\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi - \ln\left(\frac{m^2-i\epsilon}{\mu^2}+1\right)\right] - im^2\delta Z_m.$$

OS - scheme

$$\text{Re}\,\Sigma(p^2)\Big|_{p^2=m^2} = 0$$

$$\delta Z_m^{OS} = -\frac{\lambda}{32\pi^2}\left[\Delta + 1 - \ln\left(\frac{m^2-i\epsilon}{\mu^2}\right)\right]$$

$$\Sigma(p^2) = 0.$$

renormalised parameter fixed

DRbar - scheme

$$\delta Z_m^{\overline{\text{DR}}} = -\frac{\lambda}{32\pi^2}\Delta$$

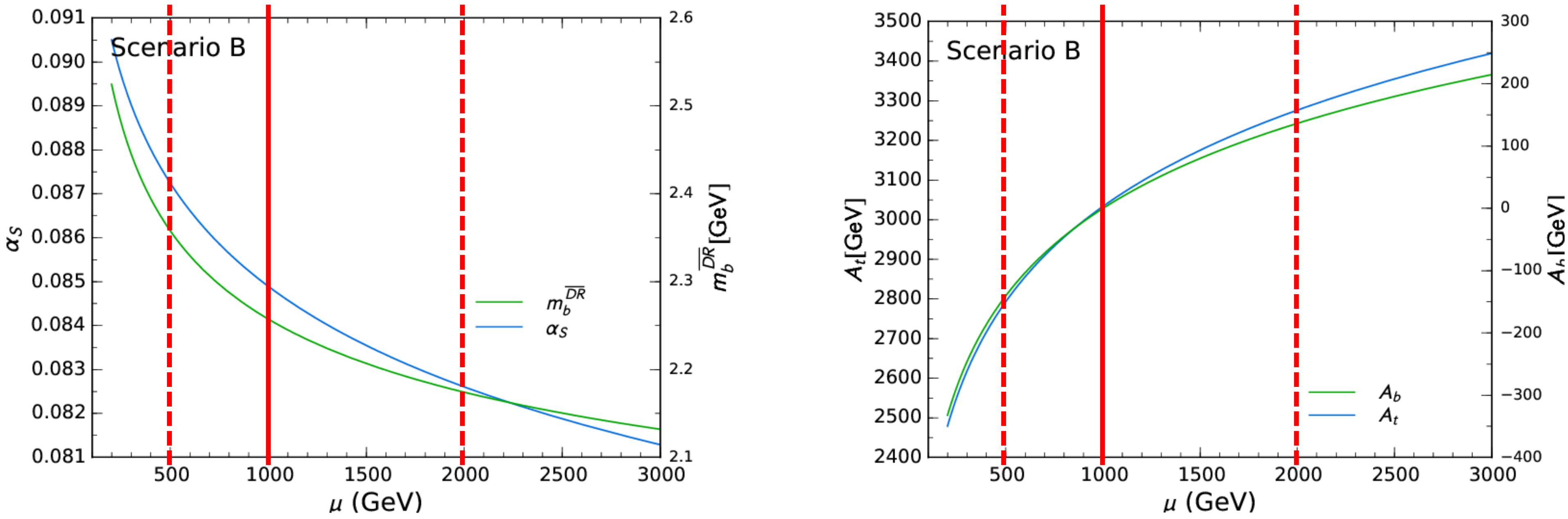
$$\Sigma(p^2) = \frac{\lambda m^2}{32\pi^2}\left[1 - \ln\left(\frac{m^2-i\epsilon}{\mu^2}\right)\right]$$

renormalised parameter scale dependent

(2) Theoretical uncertainty of $\sigma_{\rm eff}$

- No real estimation of theoretical uncertainty of the cross section calculation so far!

→ scale variation $\mu_R/2 < \mu < 2\mu_R$ gives an estimate

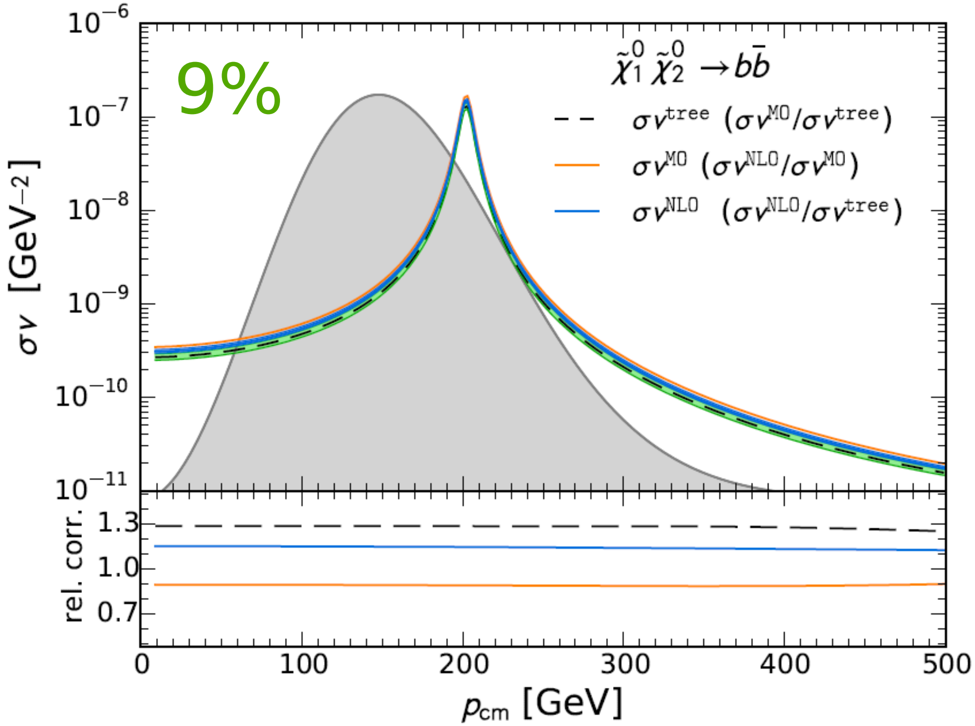


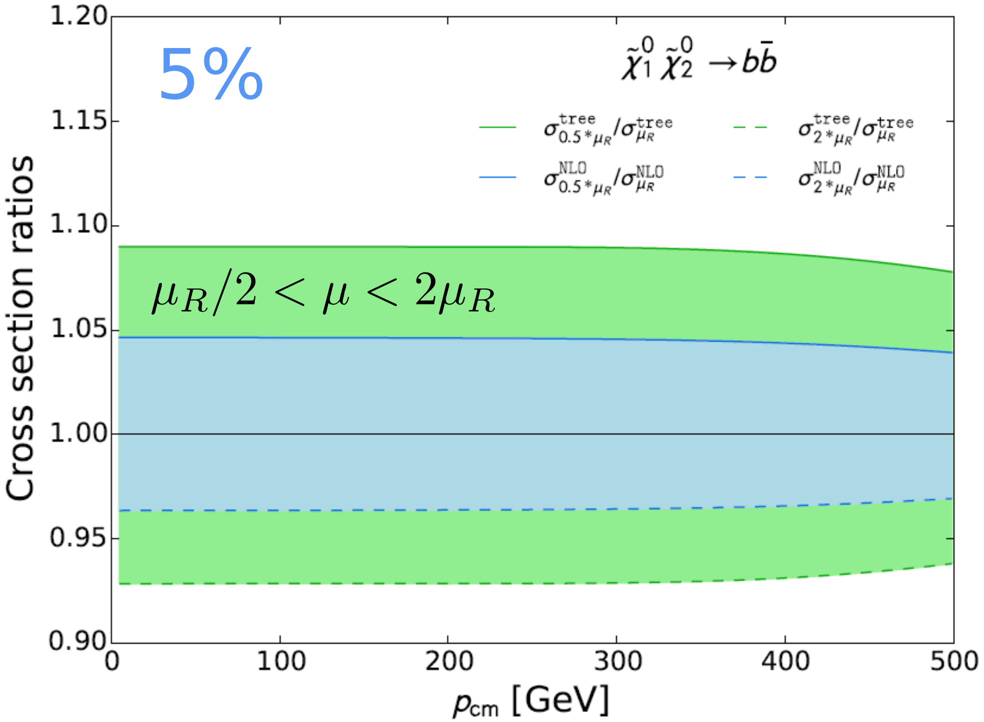
- Calculation contains explicit uncancelled logs of the renormalisation scale
- Depends implicitly on scale dependent parameters, such as $\alpha_s, \theta_{\tilde{t}}, \theta_{\tilde{b}} A_t, A_b, m_b, m_{\tilde{t}_2}$

→ first study of this kind in the context of DM in the literature!

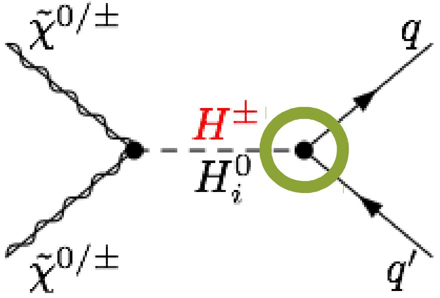
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D93, 114023 (2016)

(2a) renormalisation scale variation



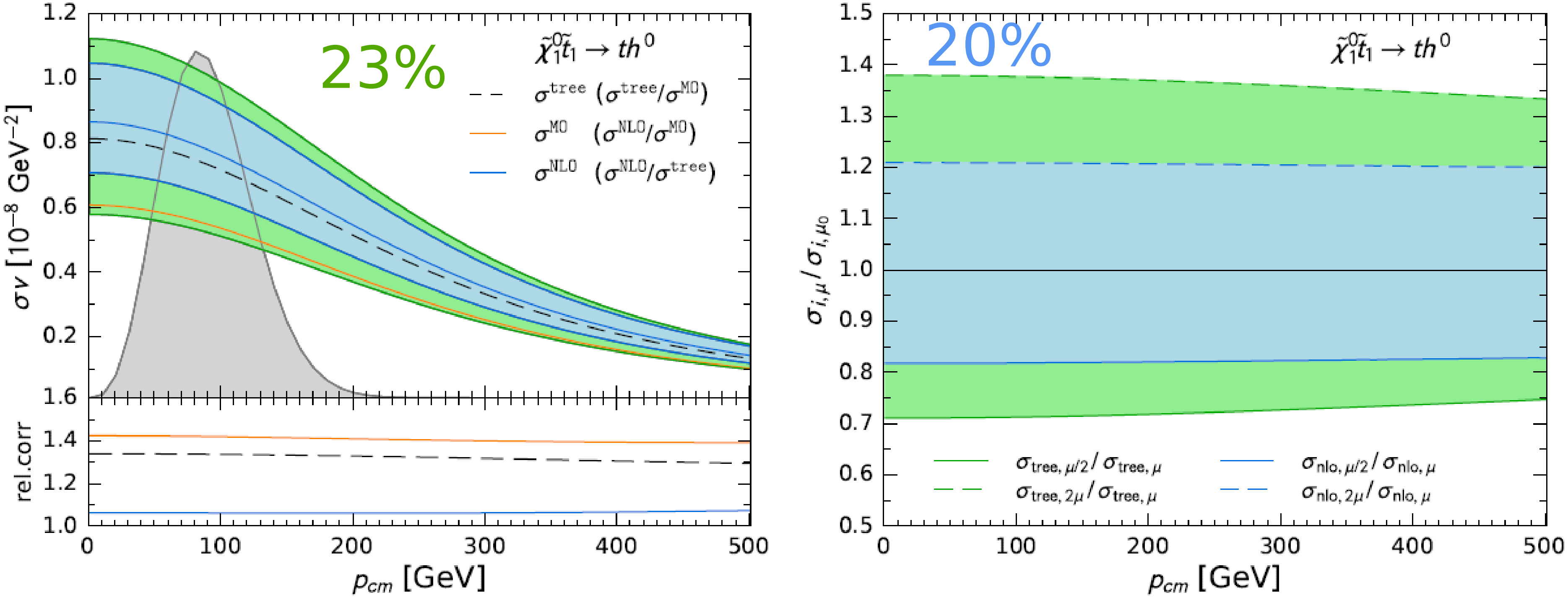


- we find 10-15% correction to the micrOMEGAs value
- error 2 times larger than naively expected

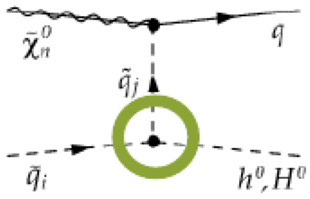


J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D93, 114023 (2016)

(2a) renormalisation scale variation



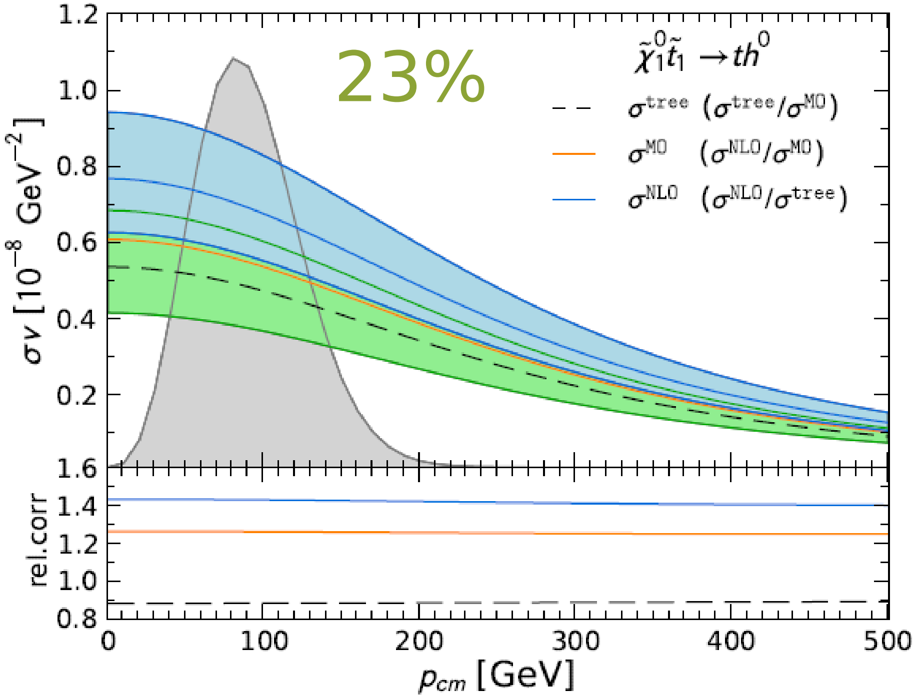
- we find 45% correction to the micrOMEGAs value
- error 20 times larger than naively expected

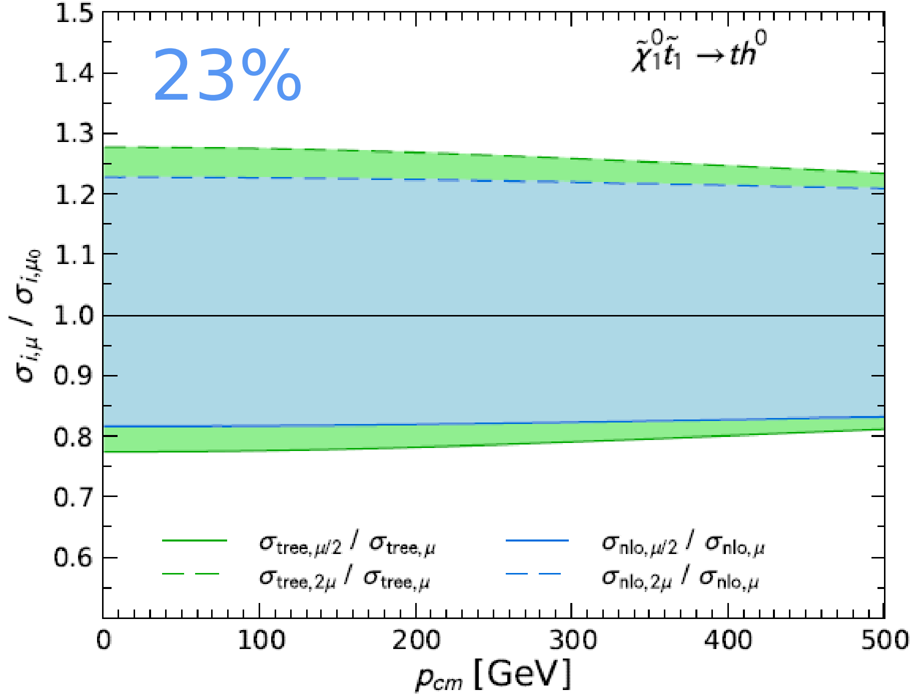


J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D93, 114023 (2016)

(2b) renormalisation scheme variation

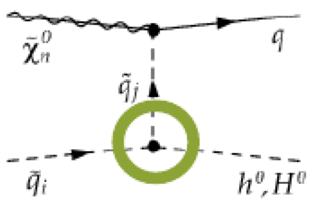
Analysis – using $m_t^{\overline{\rm DR}}$ instead of $m_t^{\rm OS}$





- Leads to a large K-factor of 1.4 instead of 1.05

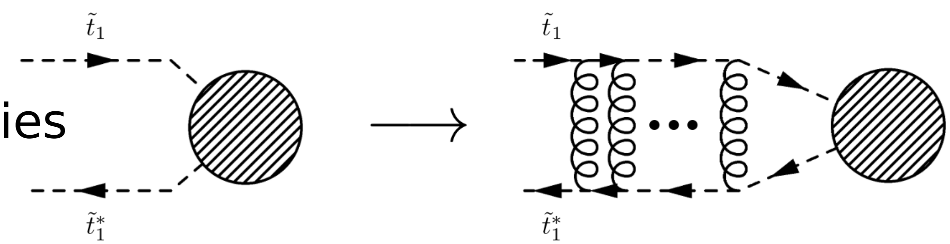
  → confirms our choice

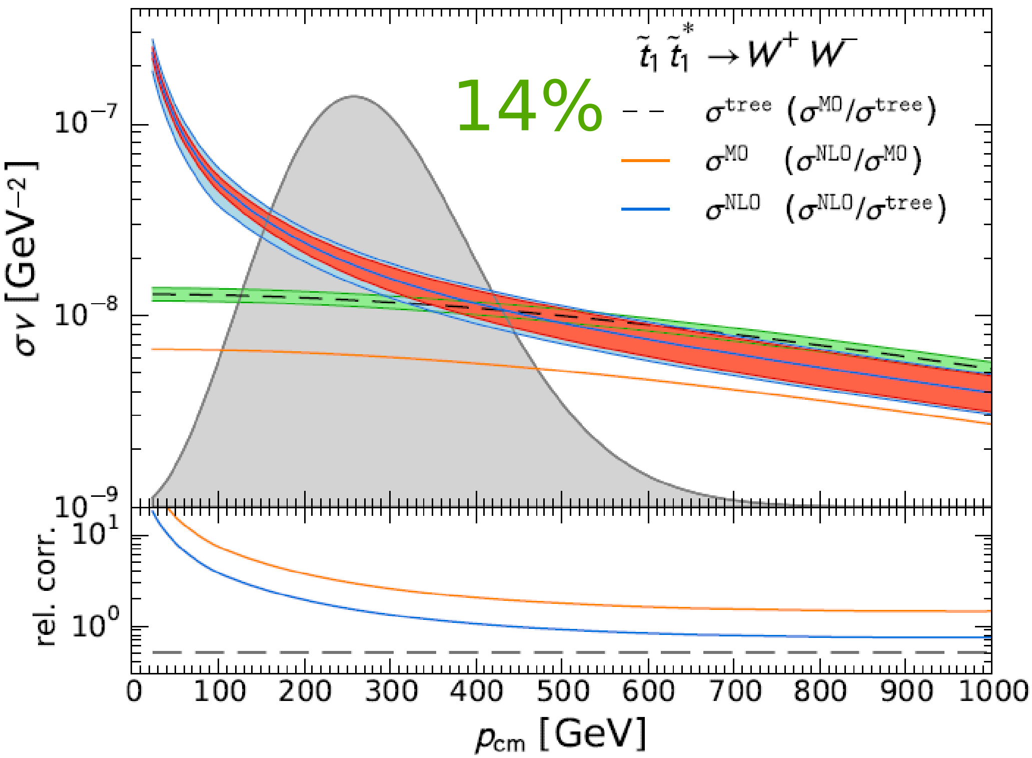


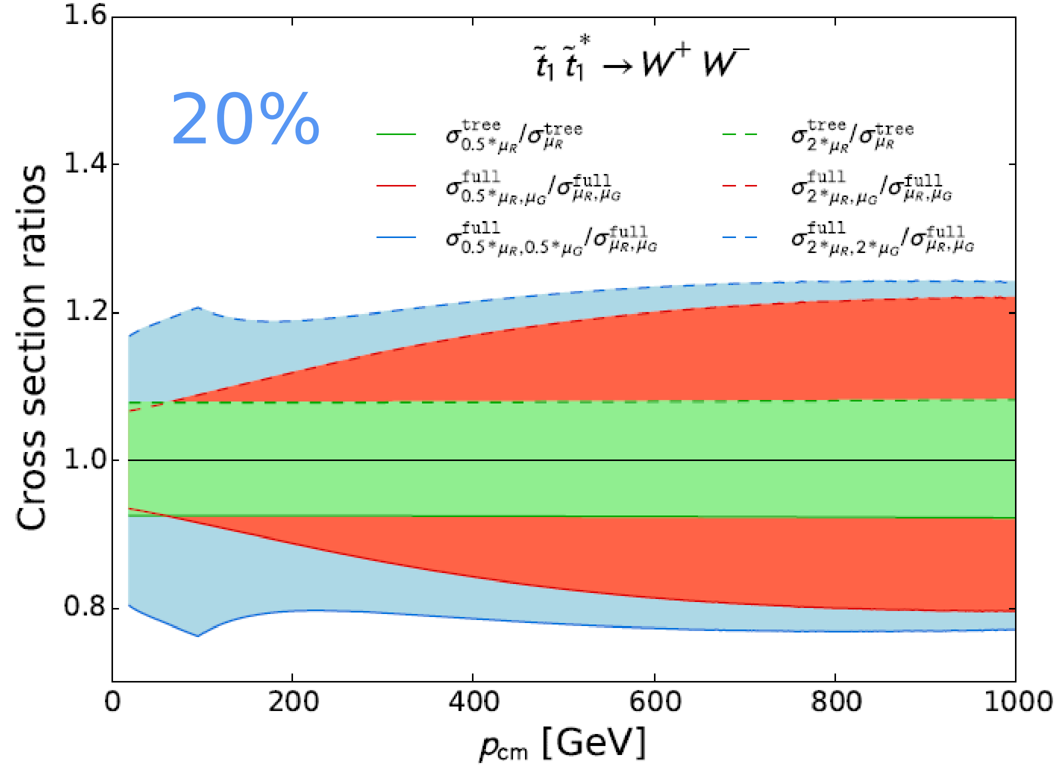
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D93, 114023 (2016)

(3) Sommerfeld enhancement

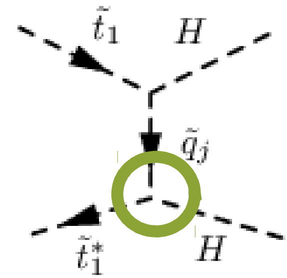
Non-perturbative effects for small velocities





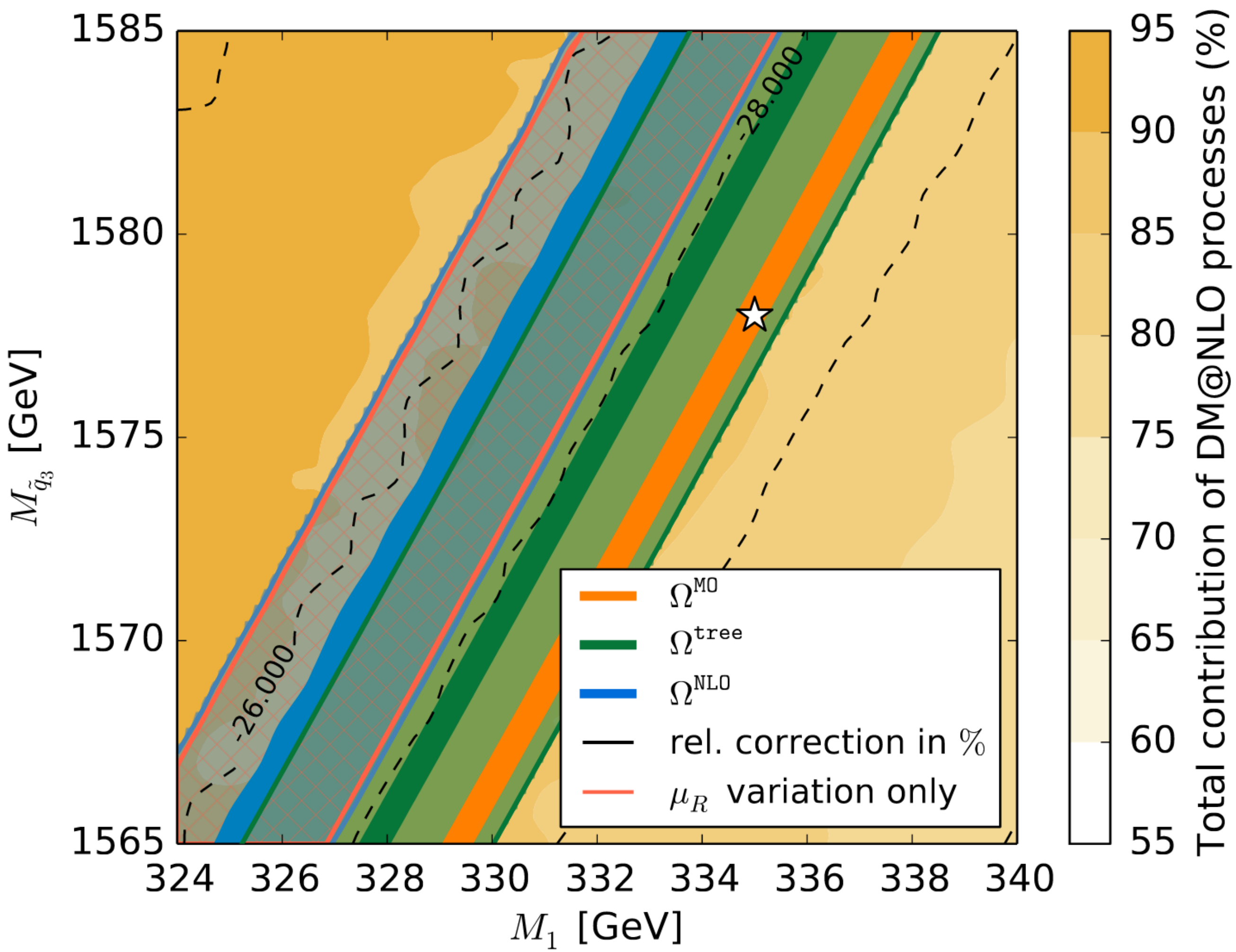


- Large K-factor in relevant region K=1-9
- Theoretical uncertainty of 20%



J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D93, 114023 (2016)

(1-3) Summary



		C
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow$	$t\bar{t}$	16%
$\tilde{\chi}_1^0\tilde{t}_1 \rightarrow$	th^0	23%
	tg	23%
	tZ^0	5%
	bW^+	11%
$\tilde{t}_1\tilde{t}_1^* \rightarrow$	h^0h^0	5%
	Z^0Z^0	2%
	W^+W^-	3%
Total		88%

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D93, 114023 (2016)

(4) Higgs enhancement

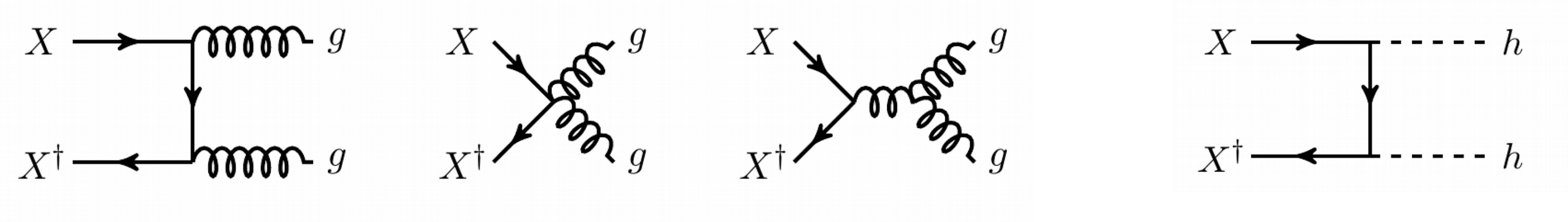
- **Simplified model:**

  DM Majorana fermion χ

  co-annihilating with complex scalar X charged under SU(3)$_c$

$$\delta\mathcal{L} = (D_{\mu,ij}X_j)^\dagger\,(D^\mu_{ij'}X_{j'}) - m_X^2\,X_j^\dagger X_j + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{1}{2}m_h^2 h^2 + g_h m_X\;h\,X_j^\dagger X_j$$

- **Annihilation processes:**

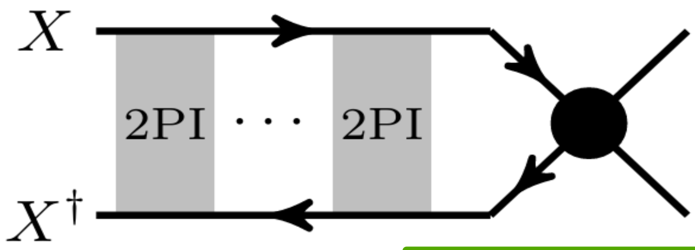


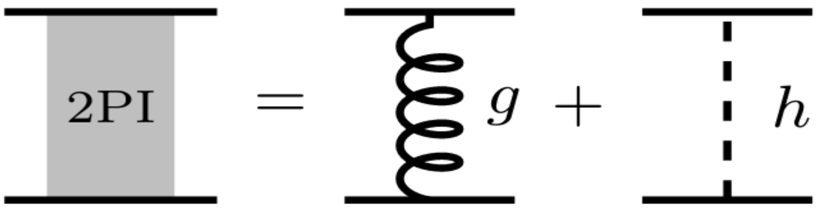
we neglect p-wave suppressed contributions $X\bar{X} \to q\bar{q}, X\bar{X} \to gh$

$$(\sigma v_{\rm rel})_{XX^\dagger\to gg} = \frac{14}{27}\frac{\pi\alpha_s^2}{m_X^2}\times\left(\frac{2}{7}\,S_0^{[\mathbf{1}]} + \frac{5}{7}\,S_0^{[\mathbf{8}]}\right) \qquad (\sigma v_{\rm rel})_{XX^\dagger\to hh} = \frac{4\pi\alpha_h^2}{3m_X^2}\,\frac{(1-m_h^2/m_X^2)^{1/2}}{[1-m_h^2/(2m_X^2)]^2}\times S_0^{[\mathbf{1}]}$$

(4) Higgs enhancement

- Higgs as mediator of long-range interactions





$$V(r) = -\frac{\alpha_g}{r} - \frac{\alpha_h}{r}\, e^{-m_h r}$$

Characteristic parameters, comparing physical scales:

$$\zeta_{g,h} \equiv \frac{\mu\alpha_{g,h}}{\mu v_{\rm rel}} = \frac{\alpha_{g,h}}{v_{\rm rel}}$$

$$d_h \equiv \frac{\mu\alpha_h}{m_h}$$

$$\left\{\nabla^2_{\mathbf{z}} + 1 + \frac{2}{z}\left[\zeta_g + \zeta_h \exp\left(-\frac{\zeta_h z}{d_h}\right)\right]\right\}\phi_{\mathbf{k}} = 0$$

$$\Rightarrow \quad S_0(\zeta_g, \zeta_h, d_h) \equiv |\phi_{\mathbf{k}}(0)|^2$$

$$S_0^{[\mathbf{1}]} = S_0\left[\frac{4\alpha_s}{3v_{\rm rel}}, \frac{\alpha_h}{v_{\rm rel}}, \frac{m_X\alpha_h}{2m_h}\right]$$

$$S_0^{[\mathbf{8}]} = S_0\left[-\frac{\alpha_s}{6v_{\rm rel}}, \frac{\alpha_h}{v_{\rm rel}}, \frac{m_X\alpha_h}{2m_h}\right]$$

Note the different scales of $\alpha_s(Q)$

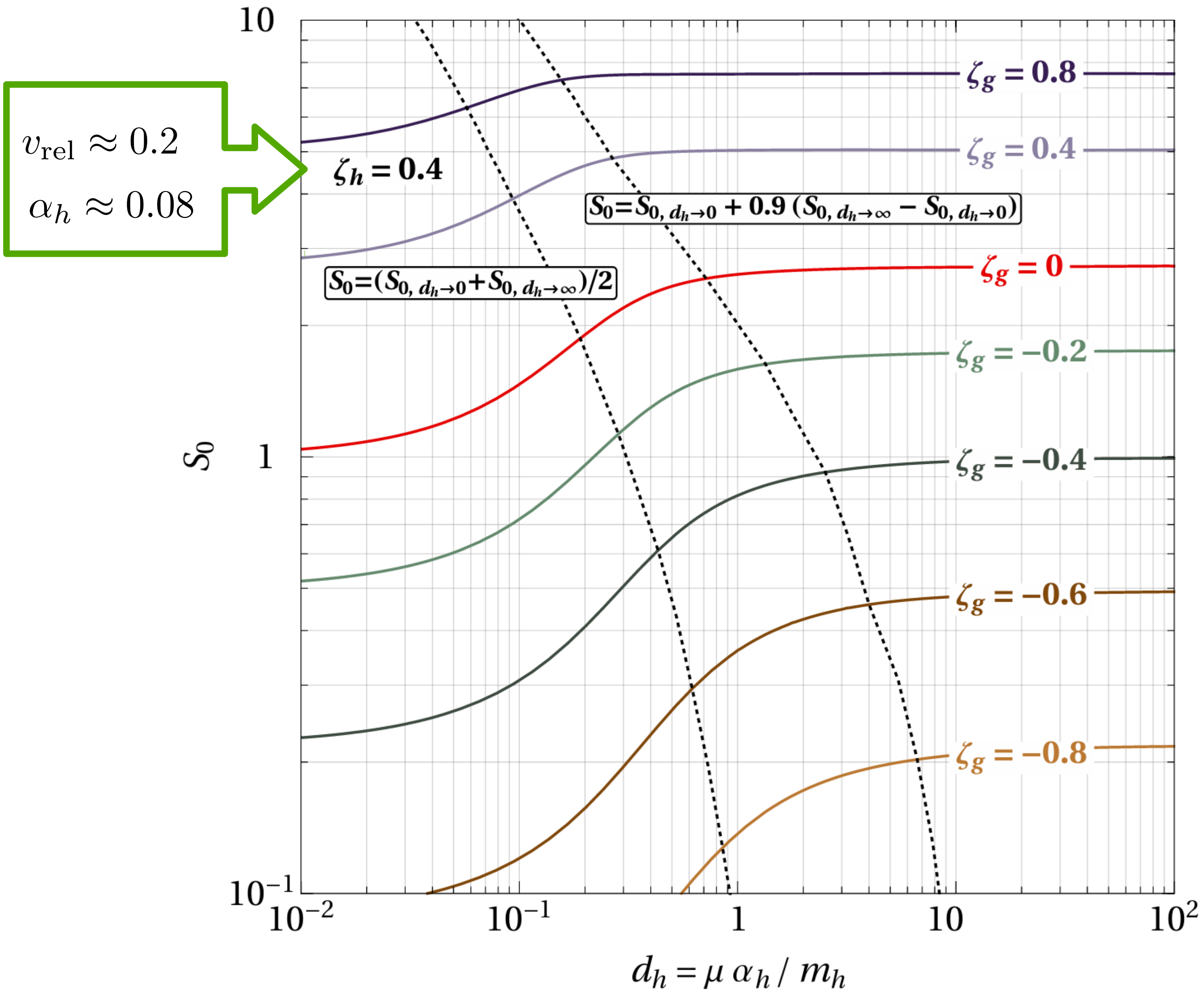
$Q = m_X$ annihilation

$Q = \mu v_{\rm rel}$ Sommerfeld

$$(\sigma v_{\rm rel})_{XX^\dagger\to gg} = \frac{14}{27}\frac{\pi\alpha_s^2}{m_X^2} \times \left(\frac{2}{7}\,S_0^{[\mathbf{1}]} + \frac{5}{7}\,S_0^{[\mathbf{8}]}\right)$$

$$(\sigma v_{\rm rel})_{XX^\dagger\to hh} = \frac{4\pi\alpha_h^2}{3m_X^2}\,\frac{(1-m_h^2/m_X^2)^{1/2}}{[1-m_h^2/(2m_X^2)]^2} \times S_0^{[\mathbf{1}]}$$

(4) Higgs enhancement

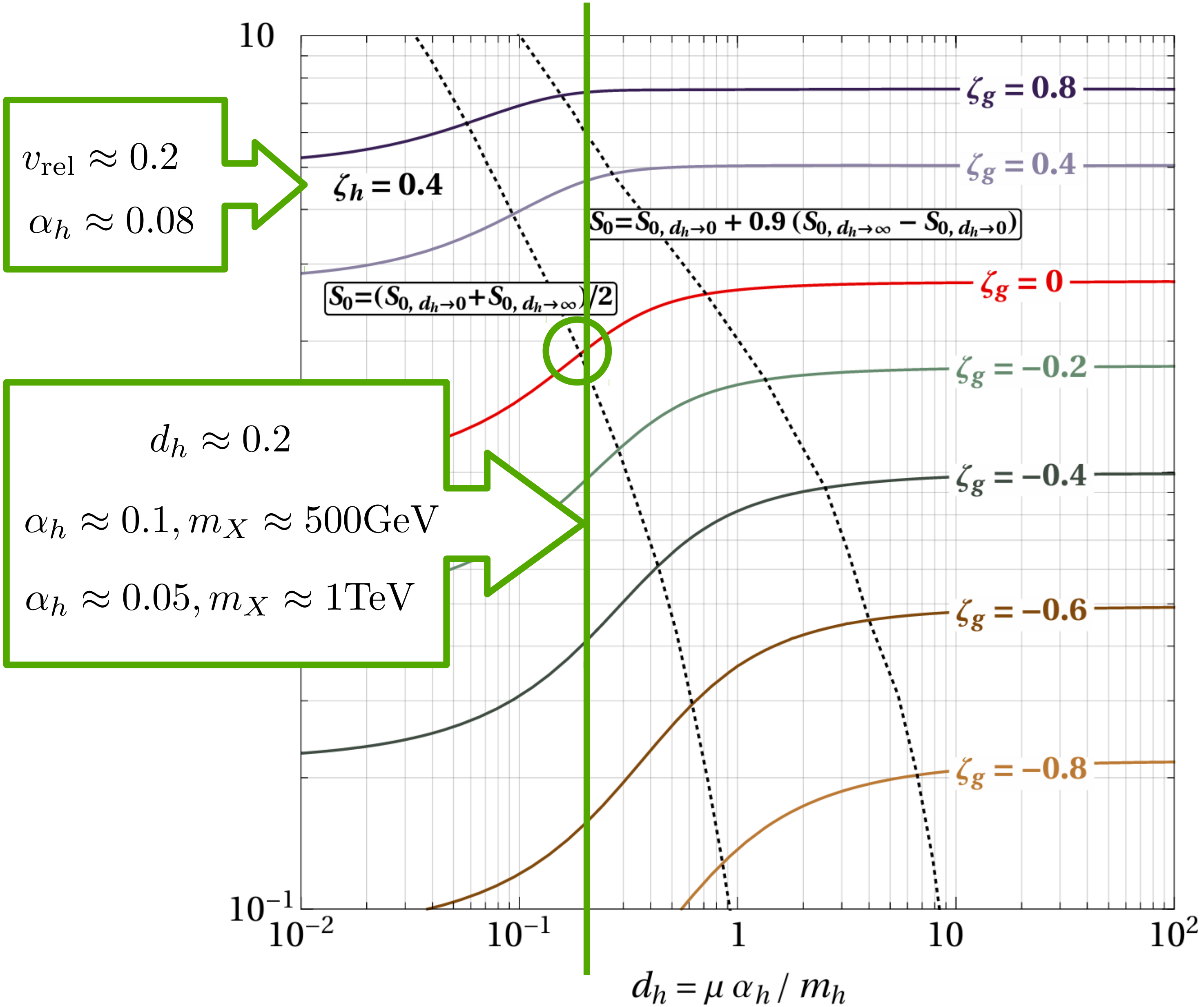


$$\zeta_{g,h} \equiv \frac{\mu \alpha_{g,h}}{\mu v_{\rm rel}} = \frac{\alpha_{g,h}}{v_{\rm rel}}$$

$$d_h \equiv \frac{\mu \alpha_h}{m_h}$$

JH, K. Petraki, arXiv:1711.03552

(4) Higgs enhancement



$$\zeta_{g,h} \equiv \frac{\mu\alpha_{g,h}}{\mu v_{\rm rel}} = \frac{\alpha_{g,h}}{v_{\rm rel}};$$

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JH, K. Petraki, arXiv:1711.03552

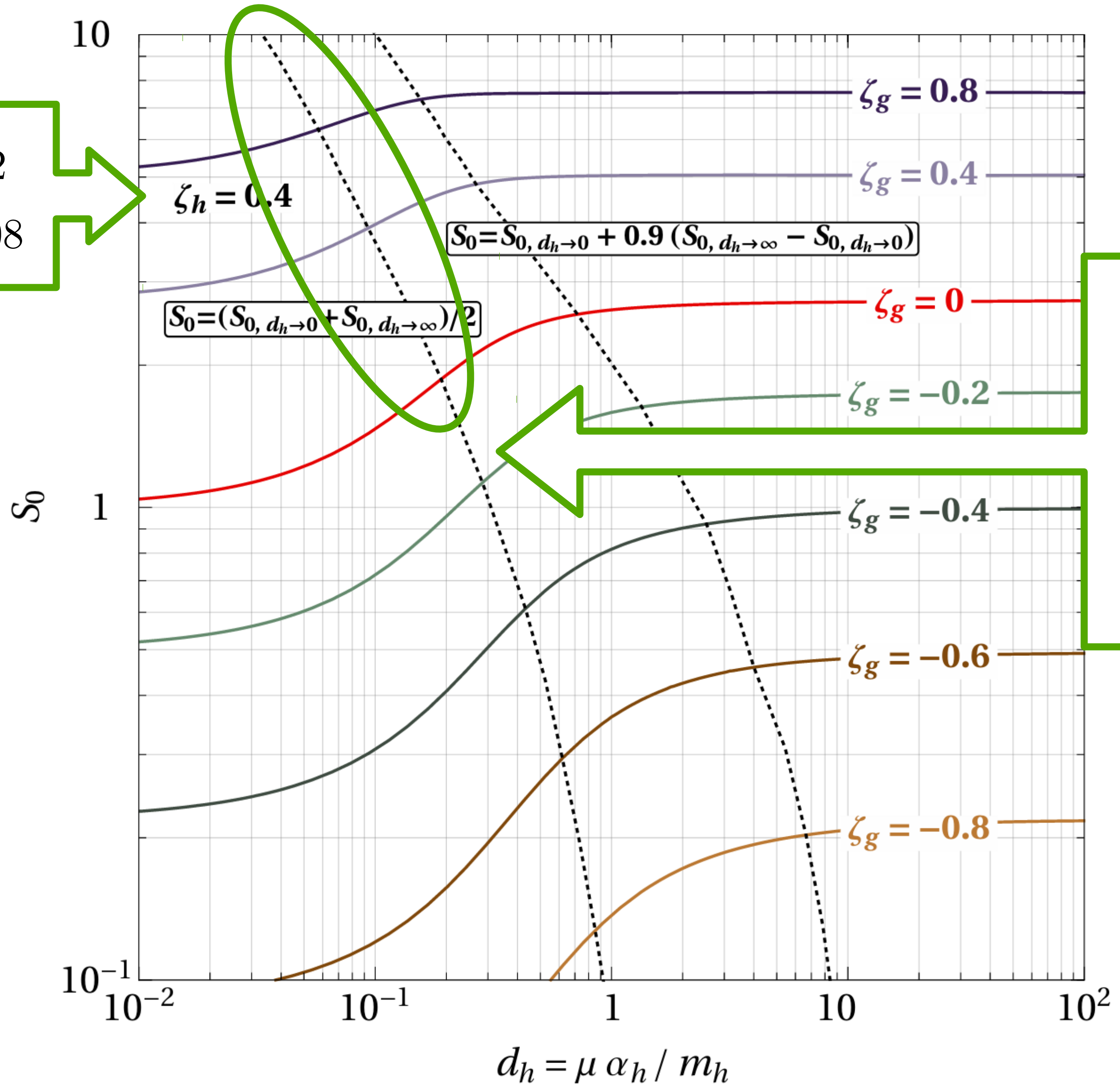
(4) Higgs enhancement

$$\zeta_{g,h} \equiv \frac{\mu \alpha_{g,h}}{\mu v_{\rm rel}} = \frac{\alpha_{g,h}}{v_{\rm rel}};$$

$$d_h \equiv \frac{\mu \alpha_h}{m_h}$$

$v_{\rm rel} \approx 0.2$

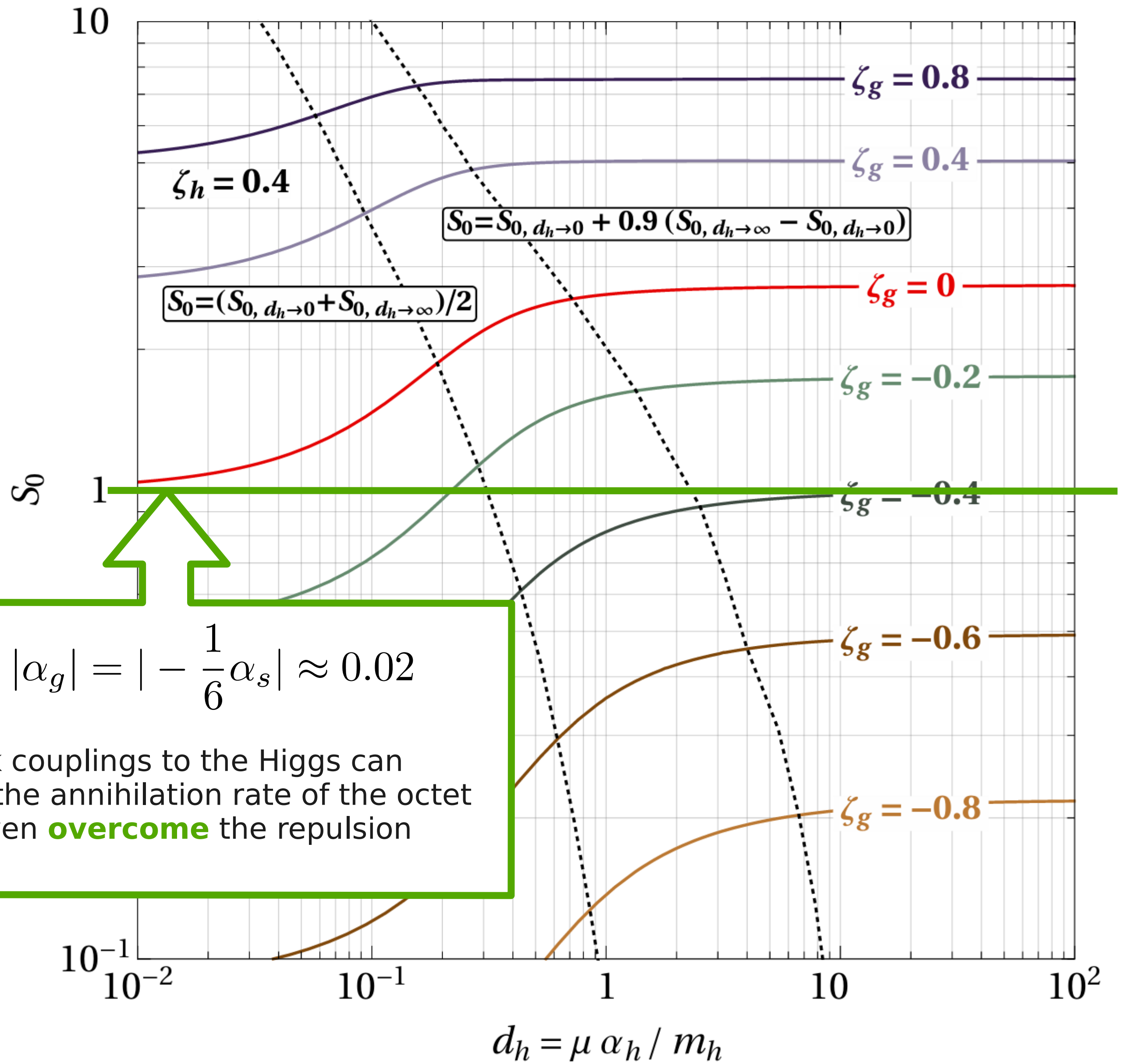
$\alpha_h \approx 0.08$



Yukawa interaction manifests as long-range for an even smaller hierarchy of scales than anticipated

JH, K. Petraki, arXiv:1711.03552

(4) Higgs enhancement



$$\zeta_{g,h} \equiv \frac{\mu \alpha_{g,h}}{\mu v_{\rm rel}} = \frac{\alpha_{g,h}}{v_{\rm rel}};$$

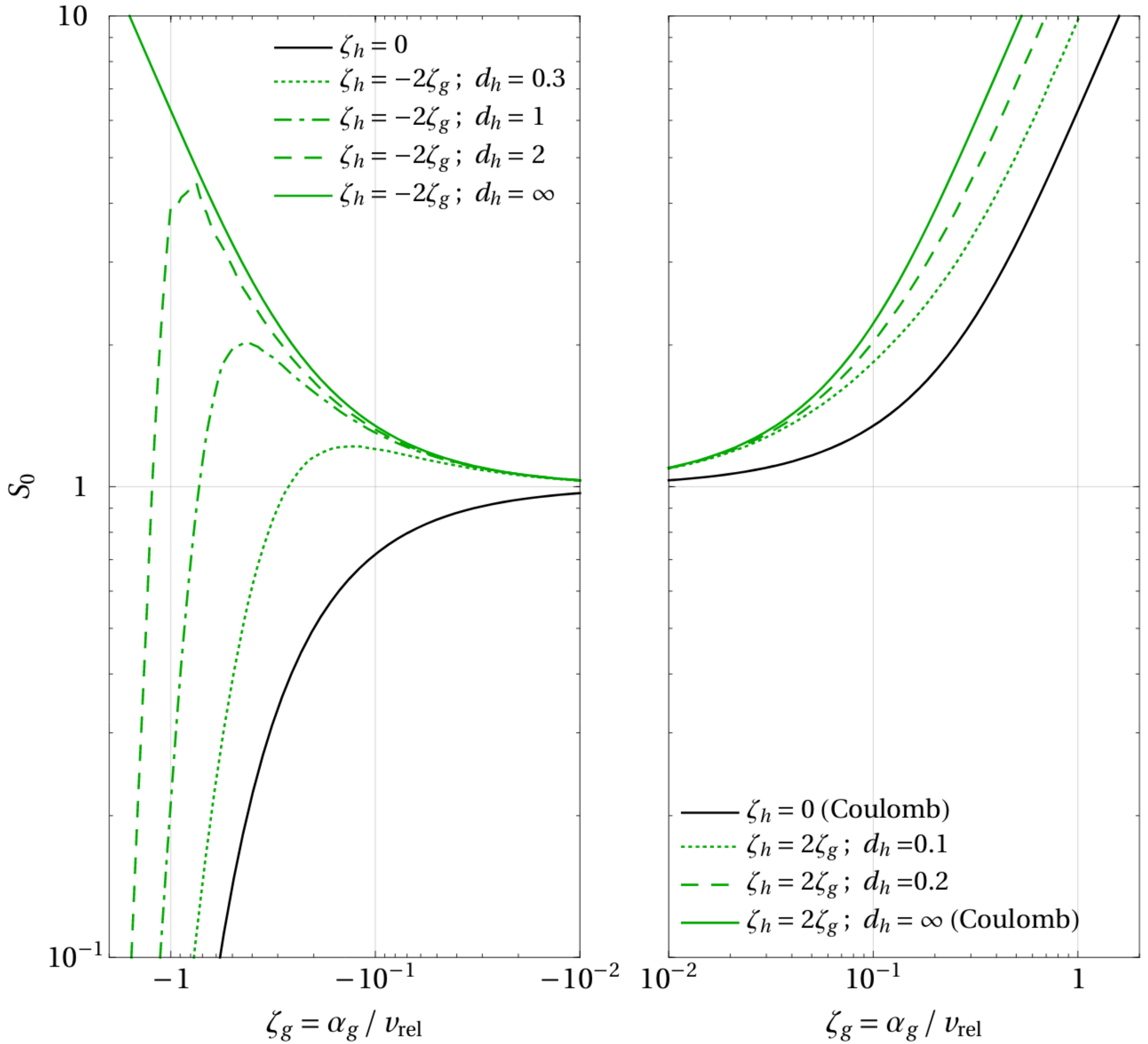
$$d_h \equiv \frac{\mu \alpha_h}{m_h}$$

$$\alpha_h \approx |\alpha_g| = |-\frac{1}{6}\alpha_s| \approx 0.02$$

Even weak couplings to the Higgs can **enhance** the annihilation rate of the octet state or even **overcome** the repulsion

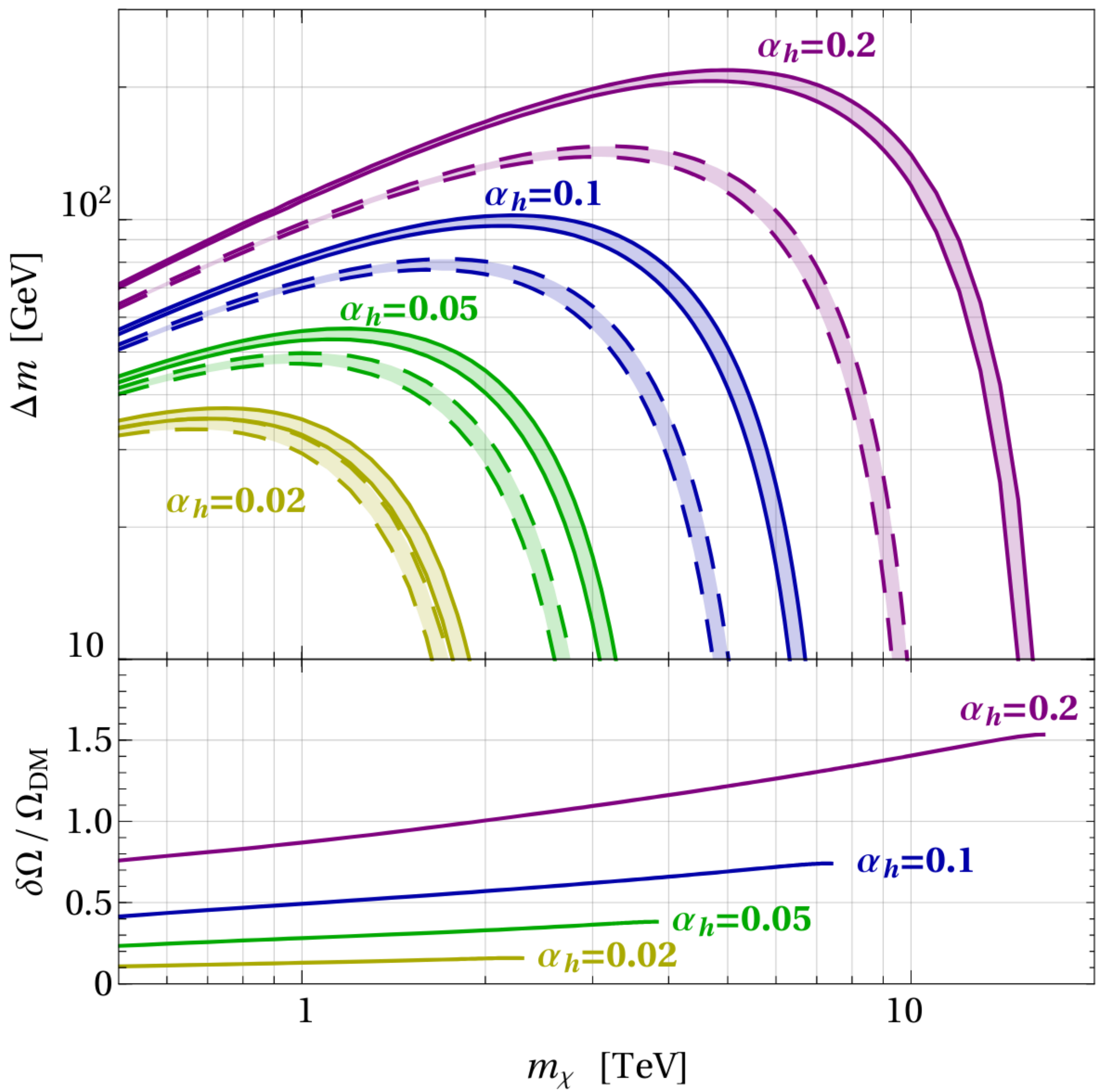
JH, K. Petraki, arXiv:1711.03552

(4) Higgs enhancement



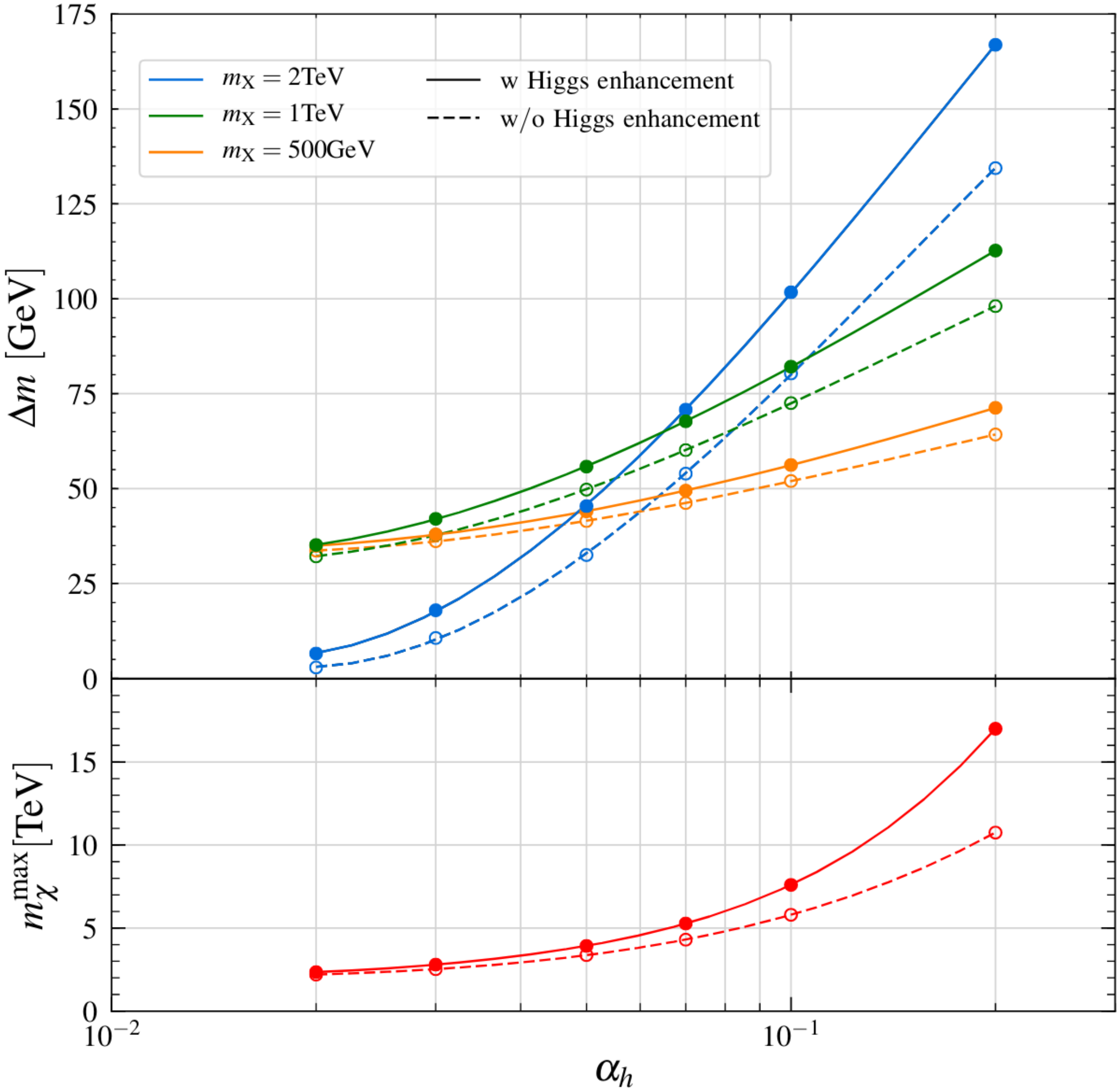
JH, K. Petraki, arXiv:1711.03552

(4) Higgs enhancement



JH, K. Petraki, arXiv:1711.03552

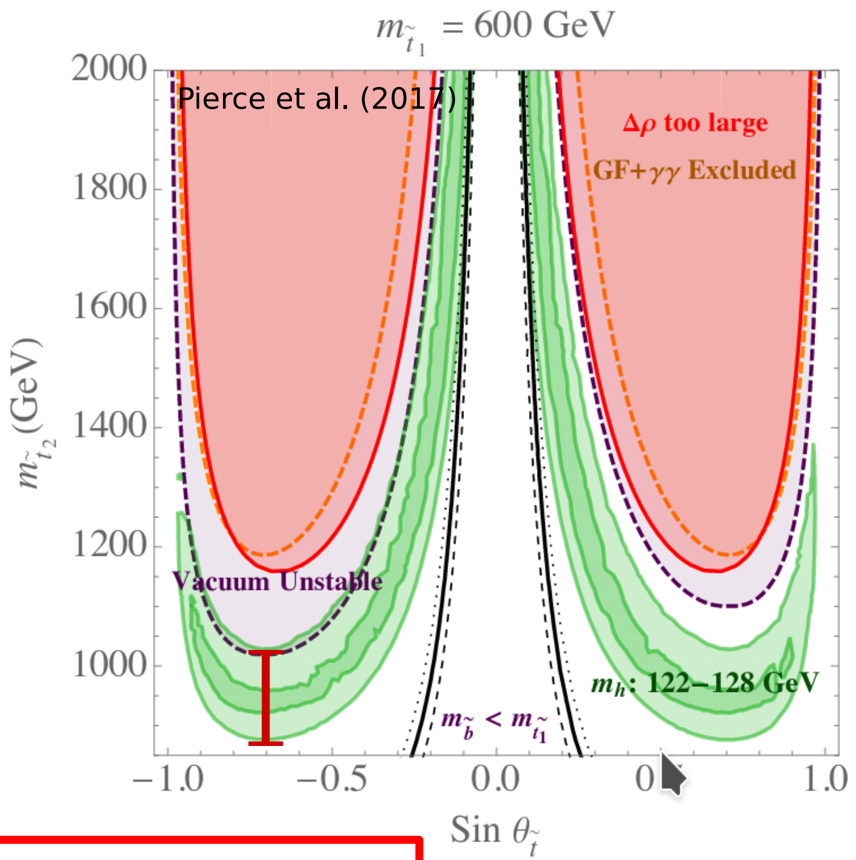
(4) Higgs enhancement

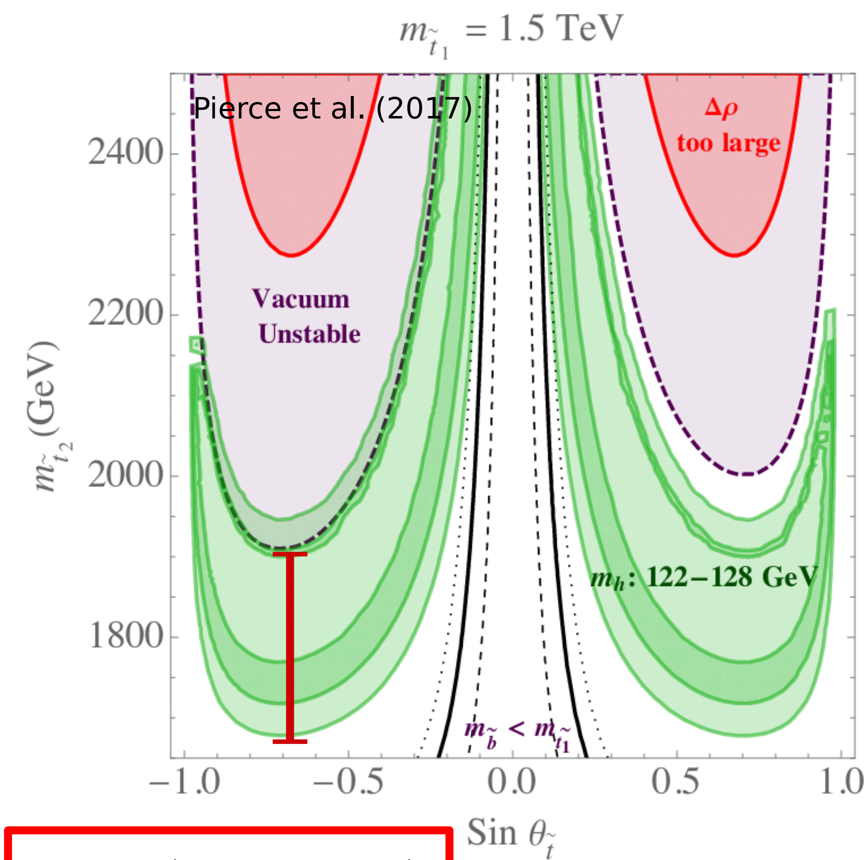


JH, K. Petraki, arXiv:1711.03552

(4) Higgs enhancement

Let's have a look at the MSSM:





$\alpha_h \approx (0.02 - 0.07)$

$\alpha_h \approx (0.01 - 0.05)$

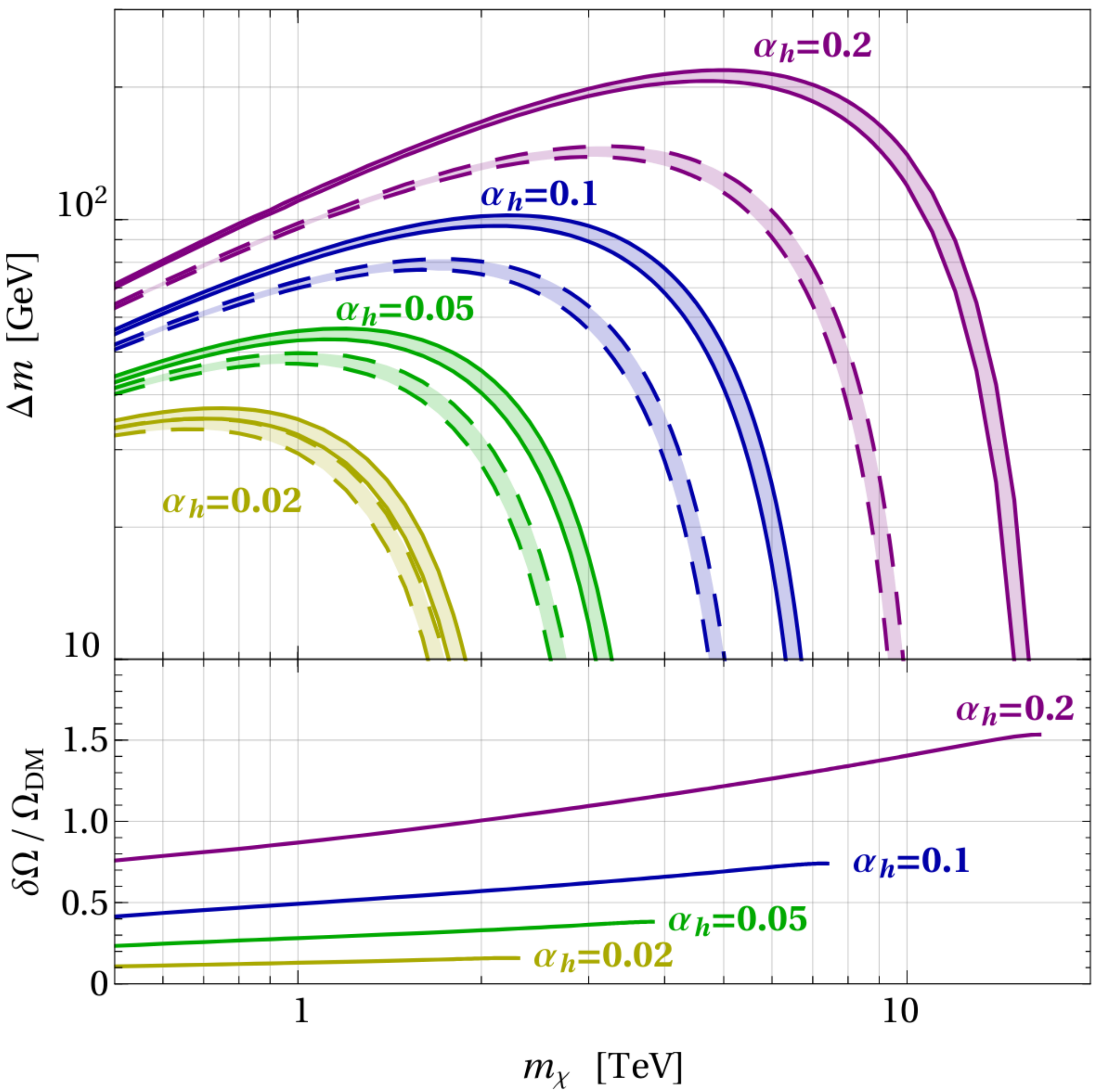
We found a scenario within the MSSM with $\alpha_h \approx 0.15$

that was checked with vevacious

$m_{\tilde{\chi}_1^0} = 982.5$ GeV

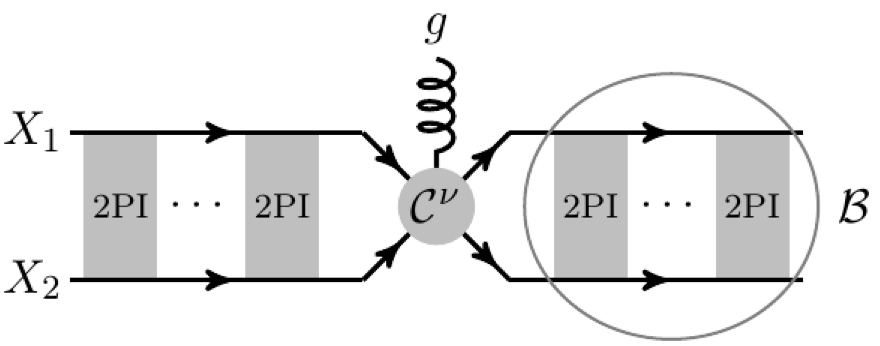
$m_{\tilde{t}_1} = 1066.1$ GeV

(4) Higgs enhancement

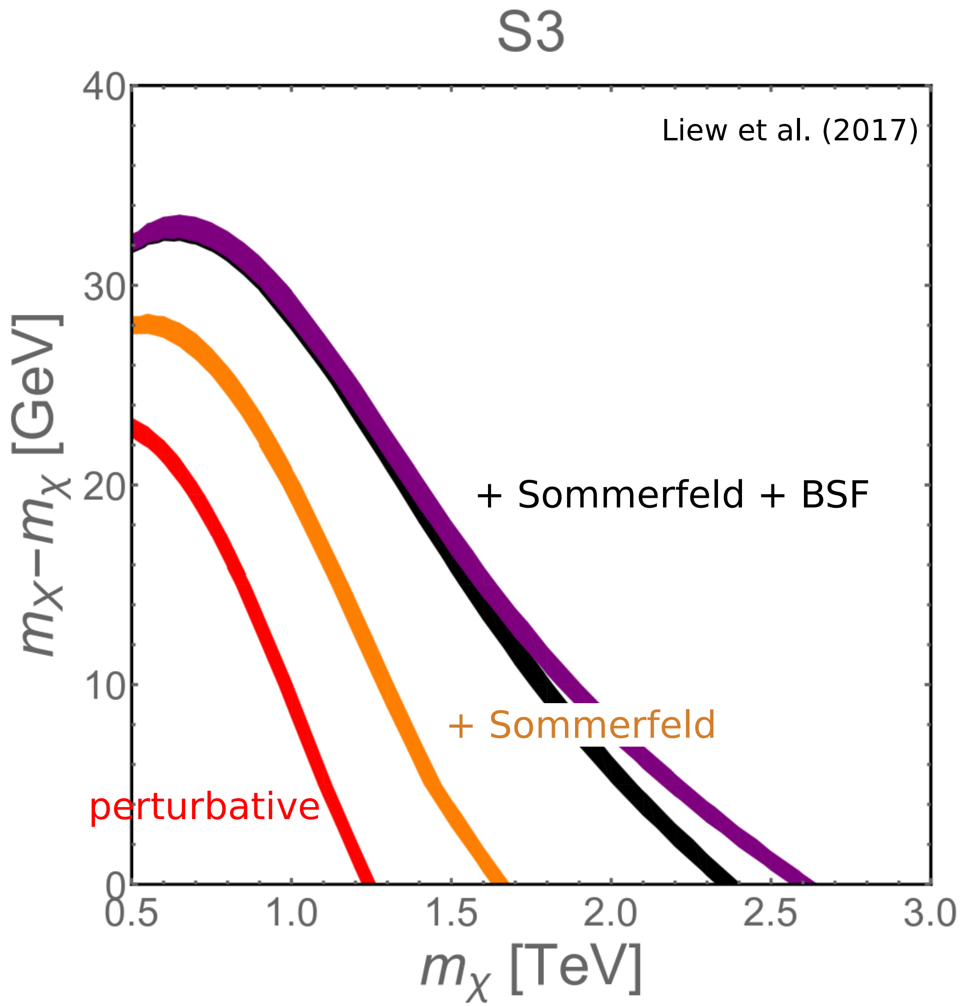


JH, K. Petraki, arXiv:1711.03552

(5) bound-state formation



$$\mathcal{U}_{[\mathbf{8}]}(X+X^\dagger) \to \mathcal{B}_{[\mathbf{1}]}(XX^\dagger) + g_{[\mathbf{8}]}$$
$$\mathcal{U}_{[\mathbf{1}]}(X+X^\dagger) \to \{\mathcal{B}_{[\mathbf{8}]}(XX^\dagger) + g_{[\mathbf{8}]}\}_{[\mathbf{1}]}$$
$$\mathcal{U}_{[\mathbf{8}]}(X+X^\dagger) \to \{\mathcal{B}_{[\mathbf{8}]}(XX^\dagger) + g_{[\mathbf{8}]}\}_{[\mathbf{8}]}$$



bound-state formation with Higgs as an additional mediator
work in progress (JH and K. Petraki)

(6) DM tool related uncertainties

Gambit (2017)

Model	Ωh^2 DarkSUSY	Ωh^2 micrOMEGAs
1	0.1396	0.08322
2	0.01479	0.007722
3	0.05507	0.03119
4	0.002217	0.001421
5	0.1103	0.1093
6	0.02212	0.02272
7	0.004588	0.003339
8	0.005815	0.004286
9	0.003000	0.003148

1	Resonant annihilation via A^0 gaugino-like neutralino
2	Resonant annihilation via A^0, mixed neutralino
3	Resonant annihilation via A^0, Higgsino-like neutralino
4	Resonant annihilation via h
5	$\tilde{\tau}$ coannihilations
6	$\tilde{t}$ coannihilations, gaugino-like neutralino
7	$\tilde{t}$ coannihilations, mixed neutralino
8	$\tilde{t}$ coannihilations, Higgsino-like neutralino
9	Chargino coannihilations

MicrOMEGAs xsec at freeze-out consistently larger than the one of DarkSUSY

→ comprehensive study of uncertainties

work in progress (T. Bringmann, J. Edsjö, JH, B. Herrmann, C. Niblaeus, P. Scott in the context of GAMBIT)

Conclusions

Be aware that out-of-the-box calculations neglect

→ higher order corrections

→ theoretical uncertainty

→ Sommerfeld enhancement

→ bound-state formation

→ Higgs enhancement

of which each can lead to sizeable effects by its own!