Unification of gauge and Yukawa couplings

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Summary

- Gauge-Higgs idea, naturalness, unification
- Group theory, $\sin\theta_W$, Yukawa couplings
- Renormalisation group evolutions
- Conclusion

Natural Higgs boson

- Naturalness implies the 125 GeV Higgs mass is not due to a tuning of the parameters, typically $\delta m \sim \Lambda_{NP}$ is a problem…

- Only special types of scalars may satisfy this requirement:

  - supersymmetry (scalar $\leftrightarrow$ fermion)

  - compositeness (condensate, goldstones)

  - gauge-higgs (scalar $\leftrightarrow$ gauge boson)

A scalar from extra dimensions

A 5D field (with a compact 5th dimension is a Kaluza-Klein tower when seen from our 4D perspective (Fourier decomposition)

$$\phi(x^\mu, y) = \sum_n \phi_n(x) e^{iny/R}$$

For the equations of motion

$$(\partial_\mu \partial^\mu - \partial_y^2)\phi(x^\mu, y) = \sum_n e^{iny/R}(\partial_\mu \partial^\mu - \frac{n^2}{R^2})\phi_n(x^\mu) = 0$$

Orbifold projections (parity)

$$\phi(x^\mu, y) = \sum_{n=0}^{\infty} \cos(\frac{n}{R}y)\phi_n^+ + \sum_{n=1}^{\infty} \sin(\frac{n}{R}y)\phi_n^-$$

zero mode (no KK mass scale)

A gauge scalar from extra dimensions

$$A_M = (A_\mu, A_5)$$

Orbifold parity projection

$$A_M^+ = (A_\mu^+, A_5^-)$$

Contains a zero mode vector

$$A_M^- = (A_\mu^-, A_5^+)$$

Contains a zero mode scalar

5D gauge symmetry broken by parity projection (but 4D preserved)

Higgs scalar "protected" by the gauge symmetry (Hosotani mechanism), vev "geometrisation"

Hosotani mechanism

EWSB giving a vev to the Higgs is a gauge transformation

$$\Omega(y) = e^{i\alpha T_h y/R} \qquad h \to h - \partial_y \left(\frac{\alpha \; y}{R} \right) = h - \frac{\alpha}{R}$$

Spectrum is shifted and the zero mode gets a mass

$$M_0 = \frac{\alpha}{R} \qquad\qquad M_n = \frac{n+\alpha}{R}$$

Yukawa couplings are related to gauge too

$$\bar{\psi} D_5 \gamma^5 \psi \to \frac{g}{\sqrt{2}} \bar{\psi}_L h \psi_R + h.c. \to y_f = \frac{g}{\sqrt{2}}$$

A SU(3) toy model

SU(2) U(1)

$$A_M^+ : \frac{1}{\sqrt{2}} \begin{bmatrix} W_3/2 & W^+/\sqrt{2} & 0 \\ W^-/\sqrt{2} & -W_3/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \frac{1}{2\sqrt{3}} \begin{bmatrix} B & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & -2B \end{bmatrix}$$

Higgs

$$A_M^- : \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & 0 \end{bmatrix}$$

Weinberg angle, top mass

$$A_M^- : \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & 0 \end{bmatrix}$$

Higgs doublet, but U(1) charge

$$\frac{1}{2\sqrt{3}}(1-(-2)) = \frac{\sqrt{3}}{2}$$

Therefore the group theory value for the Weinberg angle is not the correct one

$$g' = \sqrt{3}\, g \to \sin^2\theta_W = \frac{3}{4}$$

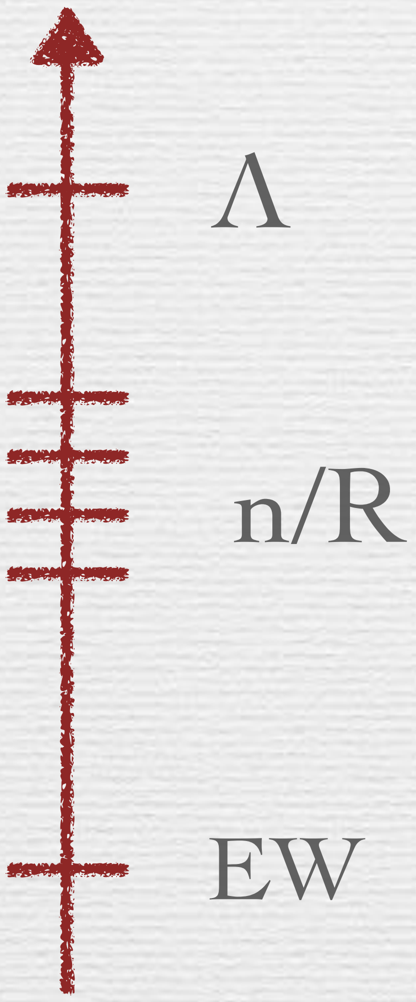
Fermion mass related to W mass

$$m_f = m_W$$

Improving the model…

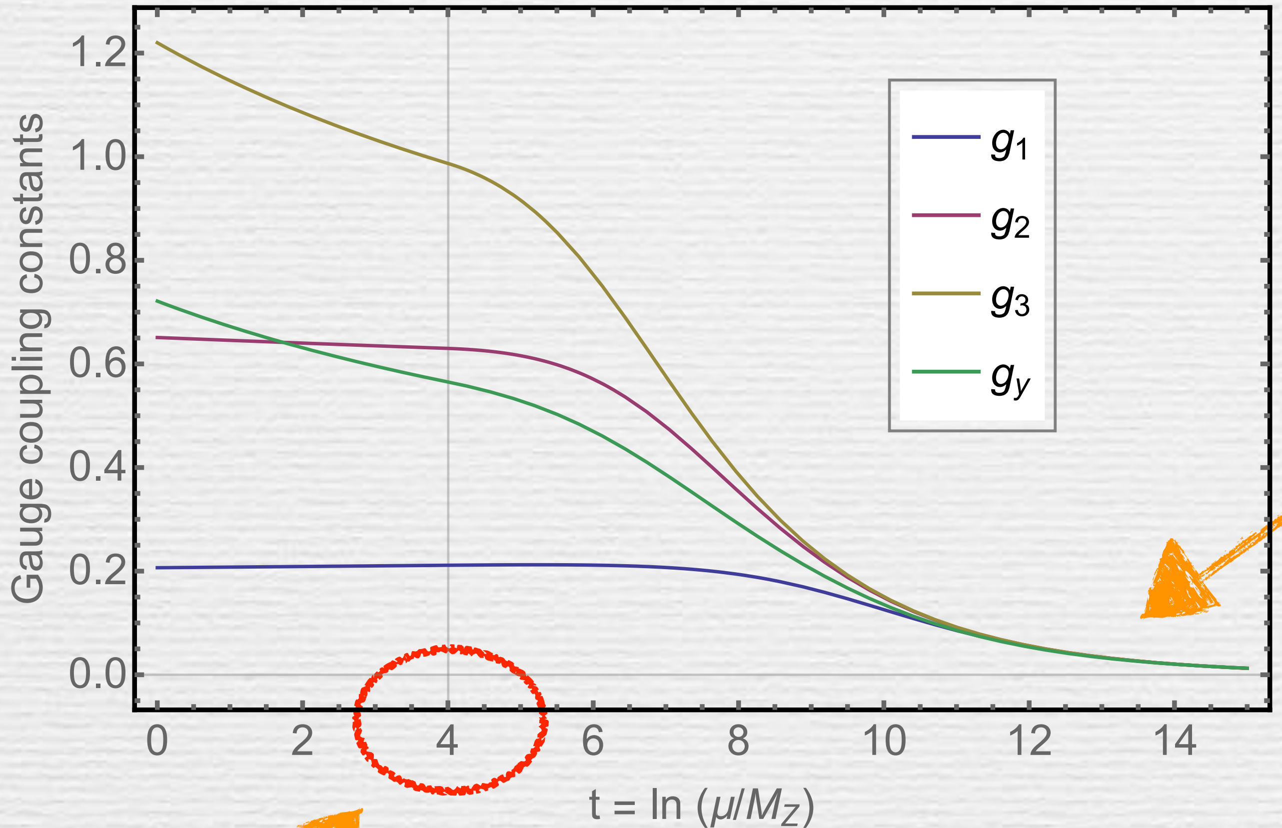
- Adding extra $U(1)_X$: changes the U(1) charge, but ad hoc
- Use different group for embedding S(2)xU(1), example G_2 is closer to the Weinberg angle value, but still incompatible
- Embed the top quark in a higher representation
- Modify the geometry
- Add localised fields and couplings
- …or keep the model and work harder!

Renormalisation group evolution



- Group theory prediction at unification, need to run down at the EW scale
- Λ and EW not that far, but running in Xdim is fast, linear effect, not log
- Here Yukawa coupling is also gauge

Reconsidering SU(3)



SU(2) and U(1) unify asymptotically

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1st KK mode kicks in the running

$$\{g_1, g_2, g_3, g_y\} = \left\{\frac{g'}{\sqrt{3}}, g, g_s, \sqrt{2}\ y\right\}$$

	$SU(2)_L$ g	$U(1)_Y$ g'	Yuk. y	$SU(3)_c$ g_s
SU(3) GHU	$g_{\rm GHU}$	$\sqrt{3}\ g_{\rm GHU}$	$g_{\rm GHU}/\sqrt{2}$	-
SM	0.66	0.35	1.0	1.2

Running of the gauge couplings

$$16\pi^2 \frac{dg_i}{dt} = b_i^{\rm SM}\; g_i^3 + (S(t)-1)\, b_i^{\rm GHU}\; g_i^3$$

$$S(t) = \begin{cases} \mu\, R = M_Z\, R\, e^t & \text{for} \quad \mu > 1/R\,, \\ 1 \quad \text{for} \quad M_Z < \mu < 1/R, \end{cases}$$

S(t) encodes the sum of the KK contributions to the running

$$b_i^{SM} = \left[\frac{41}{10}, -\frac{19}{6}, -7\right], \quad b_i^{SU(3)} = \left[-\frac{17}{6}, -\frac{17}{2}, -\frac{17}{2}\right]$$

Running of the Yukawa coupling

$$16\pi^2 \, \frac{d\, y_t}{d\, t} = \beta_t^{SM} + \Big(S(t) - 1\Big)\, \beta_t^{\mathrm{GHU}}$$

$$\beta_t = y_t \left[c_t \; y_t^2 + \sum_i d_i \; g_i^2 \right]$$

$$c_t^{\mathrm{SM}} = \frac{9}{2}\,, \quad c_t^{SU(3)} = \frac{21}{2} \qquad d_i^{SM} = \left[-\frac{5}{12}, -\frac{9}{4}, -8\right], \quad d_i^{SU(3)} = \left[-\frac{35}{24}, -\frac{39}{8}, -4\right]$$

Conclusions

- Gauge-Higgs models should be reconsidered
- Renormalisation group running is important to bring the unification scale results to the EW one
- SU(3) is still a toy model but interesting features emerge from the running
- Work towards more realistic models in progress