Modified gravity theories and recent constraints

LPT

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Journée SFP: La Gravitation







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- Equivalence principle Locally a free-falling observer and an inertial observer are indistinguishable

• This means:

- -Gravity is a local condition of spacetime
- -Gravity sees all (including vacuum energy!)

-In Newtonnian gravity m_l and m_G happen to be the same, in GR it is a founding principle

• Theoretical consistency: In 4 dimensions, consider

 $\mathcal{L} = \mathcal{L}(\mathcal{M}, g, \nabla g, \nabla \nabla g)$. Then Lovelock's theorem in D = 4 states that GR with cosmological constant is the unique metric theory emerging from,

$$S_{(4)} = \int_{\mathcal{M}} d^4 x \sqrt{-g^{(4)}} \left[R - 2\Lambda \right]$$

giving,

- Equations of motion of 2^{nd} -order (Ostrogradski no-go theorem 1850!)
- given by a symmetric two-tensor, $G_{\mu
 u} + \Lambda g_{\mu
 u}$
- and admitting Bianchi identities.

Under these hypotheses GR is the unique massless-tensorial 4 dimensional theory of gravity!

Observational data

• Experimental consistency:

-Excellent agreement with solar system tests and strong gravity tests on binary pulsars

-Observational breakthrough GW170817: Non local, 40 Mpc and strong gravity test from binary neutron stars. $c_T=1\pm10^{-15}$



Q: What is the matter content of the Universe today?

Assuming homogeinity-isotropy and GR

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

cosmological and astrophysical observations dictate the matter content of the



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If we assume only ordinary sources of matter (DM included) there is disagreement between local, astrophysical and cosmological data.

Simplest way out: Assume a tiny cosmological constant $\rho_{\Lambda} = \frac{\Lambda_{obs}}{8\pi G} = (10^{-3} eV)^4$, ie modify Einstein's equation by,

 $G_{\mu\nu} + \Lambda_{obs} g_{\mu\nu} = 8\pi G T_{\mu\nu}$

- \bullet Cosmological constant introduces $\sqrt{\Lambda}$ and generates a cosmological horizon
- $\sqrt{\Lambda}$ is as tiny as the inverse size of the Universe today, $r_0 = H_0^{-1}$
- Note that $rac{\text{Solar system scales}}{ ext{Cosmological Scales}} \sim rac{10 \text{ A.U.}}{H_{c}^{-1}} = 10^{-14}$

Typical mass scale for neutrinos...

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- Vacuum energy fluctuations are at the UV cutoff of the QFT $\Lambda_{vac}/8\pi G\sim m_{Pl}^4...$
- Vacuum potential energy from spontaneous symmetry breaking $\Lambda_{EW} \sim (200 \, GeV)^4$
- Bare gravitational cosmological constant Abare

$\Lambda_{obs} \sim \Lambda_{vac} +$

Enormous Fine-tuning inbetween theoretical and observational value

• Why such a discrepancy between theory and observation? DIS

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Maybe $_{\Lambda_{obs}}$ is **not** a cosmological constant.

What if the need for exotic matter or cosmological constant is the sign for novel gravitational physics at very low energy scales or large distances.



-Same situation at the advent of GR.

-A next order correction with one additional parameter was enough to save Newton's laws (at the experimental precision of the time..)

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General issues to deal with

- Since GR is unique we need to introduce new and genuine gravitational degrees of freedom!
- They generically must not lead to higher derivative equations of motion. Additional degrees of freedom can lead to ghosts and vacuum is unstable (Ostrogradski theorem 1850 [Woodard 2006]). Since [Glayzes et al] we know that higher derivative EOM do not always lead to ghosts. What is essential is the number of propagating dof.
- Matter does not directly couple to novel gravity degrees of freedom. Matter sees only the metric and evolves in metric geodesics. As such EEP is preserved and space-time can be put locally in an inertial frame.
- Novel degrees of freedom need to be screened from local gravity experiments. Need a well defined GR local limit (Chameleon [Khoury 2013], Vainshtein [Babichev and Deffayet 2013]).
- Exact solutions essential in modified gravity in order to understand strong gravity regimes and novel characteristics. Need to deal with no hair paradigm, absence of Birkhoff theorem etc.
- A modified gravity theory should tell us something about the cosmological constant problem and in particular how to screen an a priori enormous cosmological constant. Self tuning and self acceleration.

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- $\bullet\,$ Weinberg's no go theorem states that we cannot have a Poincare invariant vacuum with $\Lambda \neq 0$
- Question: Can we break Poincare invariance for some additional field?
- Keep $g_{\mu\nu} = \eta_{\mu\nu}$ locally but allow for $\phi \neq constant$.
- Can we have a portion of flat spacetime whatever the value of the cosmological constant...
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Possible modified gravity theories

- Assume extra dimensions : Extension of GR to Lovelock theory with modified yet second order field equations. Braneworlds DGP model RS models, Kaluza-Klein compactification, String theory and holography.
- Graviton is not massless but massive! dRGT theory and bigravity theory.
- 4-dimensional modification of GR:
 - Scalar-tensor theories, f(R), Galileon/Hornedski theories \rightarrow Beyond Horndeski and DHOST theories.
 - Vector-tensor theories
- Lorentz breaking theories: Horava gravity, Einstein Aether theories
- Theories modifying geometry: inclusion of torsion, choice of geometric connection

Gravity in higher dimensions



- Inspired from string theory and holographic ideas
- small compact extra dimensions, the Kaluza-Klein paradigm relates higher dimensional metric theories to 4 dimensional modified gravity theories. Origin of Horndeski theory
- Braneworld idea and how a negative cosmological constant can accommodate large extra dimensions (RS and DGP models). Cutting and pasting portions of adS or flat spacetime using junction conditions.

Central idea

Matter lives on a distributional brane gravity lives in a higher dimensional space-time



Braneworld



Interesting phenomenology

DGP and RS models

- Introduction of self-acceleration idea [Deffayet]
- Understanding of FRW braneworld cosmology [Binetruy et al.]
- decoupling limit in DGP makes connection with Galileons in 4 dimensions [Lutty, Porrati, Rattazzi]

Braneworld



Self tuning

- In RS : Fine tuning of the cosmological constant; Fine tuning relieved with the presence of a scalar field in the bulk [Arkani-Hamed et al], [Kachru et al.]
- Keeping IR ads gives localised 4 dim graviton
- Keeping UV adS allows for self tuning
- Putting all of it together allows self tuning braneworlds [CC, Kiritsis, Nitti]

Scalar-tensor theories

- are the simplest modification of gravity with one additional degree of freedom
- Admit a uniqueness theorem due to Horndeski 1973 and extended to DHOST theories [Langlois et.al] [Crisostomi et.al.]
- contain or are limits of other modified gravity theories.
- (Can) have insightful screening mechanisms (Chameleon, Vainshtein)
- Include terms that can screen classically a big cosmological constant or give self accelerating solutions. Need a non trivial scalar field.
- Have non trivial hairy black hole solutions even around non trivial self accelerating vacua
- Theories are strongly constrained from gravity waves.

What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x \sqrt{-g} \left(L_2 + L_3 + L_4 + L_5 \right)$$

$$\begin{split} L_2 &= G_2(\phi, X), \\ L_3 &= G_3(\phi, X) \Box \phi, \\ L_4 &= G_4(\phi, X)R + G_{4X} \left[(\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right], \\ L_5 &= G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{G_{5X}}{6} \left[(\Box \phi)^3 - 3 \Box \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3 \right] \end{split}$$

the G_i are free functions of ϕ and $X \equiv -\frac{1}{2} \nabla^{\mu} \phi \nabla_{\mu} \phi$ and $G_{iX} \equiv \partial G_i / \partial X$.

 In fact same action as covariant Galileons [Deffayet, Esposito-Farese, Vikman]. Galileons are scalars with Galilean symmetry for flat spacetime.

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• Examples:
$$G_4 = 1 \longrightarrow R$$
.
 $G_4 = X \longrightarrow G^{\mu\nu} \nabla_{\mu} \phi \nabla_{\nu} \phi$.
 $G_3 = X \longrightarrow$ "DGP" term, $(\nabla \phi)^2 \Box \phi$
 $G_5 = lnX \longrightarrow$ gives GB term, $\hat{G} = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} - 4R^{\mu\nu} R_{\mu\nu} + R^2$
Action is unique modulo integration by parts.

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 Horndeski theory admits self accelerating vacua with a non trivial scalar field in de Sitter spacetime. A subset of Horndeski theory self tunes the cosmological constant.

Going beyond Horndeski [Gleyzes et.al], [Zumalacarregui et.al], [Deffayet et.al], [Langlois et.al],

[Crisostomi et.al]

What is the most general scalar-tensor theory with three propagating degrees of freedom?

It is beyond Horndeski but not quite DHOST yet...

$$S_{H} = \int d^{4}x \sqrt{-g} \left(L_{2} + L_{3} + L_{4} + L_{5} \right) ,$$

where

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where $XG_{5,X}F_4 = 3F_5 \left[G_4 - 2XG_{4,X} - (X/2)G_{5,\phi}\right]$. Beyond Horndeski acquires one extra function. BH has similar SA and ST solutions.

How are theories mapped under conformal and disformal transformations?

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\nabla_{\mu}\phi\nabla_{\nu}\phi$$

- Horndeski theory has G_2 , G_3 , G_4 , G_5 free functions.
- For $C(\phi)$ and $D(\phi)$ we remain within Horndeski.
- However if we take a disformal D(X) we jump to
- Beyond Horndeski (one more free function)
- Take a conformal C(X) and jump to
- DHOST Type | (one more free function) [Langlois, Noui], [Crisostomi, Koyama]

In other words DHOST type I are all related to some Horndeski theory. Remaining DHOST theories are pathological [Langleis, Noul, Vermizzi]

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- ST and SA solutions dictate time dependence for scalar for de Sitter or flat spacetimes.
- Scalar field is space and time dependent for static black holes...
- Birfurcating no hair theorem
- Example [Babichev, CC]:

$$S=\int d^4 \mathrm{x} \sqrt{-g} \left[\zeta R -2 \Lambda -\eta \left(\partial \phi
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- Solution: $f = h = 1 \frac{\mu}{r} + \frac{\eta}{3\beta}r^2$, $\phi = qt \pm \frac{q}{h}\sqrt{1-h}$ with $\Lambda_{\text{eff}} = -\zeta \eta/\beta$.
- Similar asymptotically flat solution : stealth Schwarzschild
- The effective cosmological constant is not the vacuum cosmological constant. In fact,
- $q^2\eta = \Lambda \Lambda_{eff} > 0$
- Hence for arbitrary $\Lambda > \Lambda_{eff}$ fixes q, integration constant.
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GW170817 constraints on scalar tensor theories [Creminelli, Vernizzi],

[Ezquiaga, Zumalacarregui]

- The combined observation of a gravity wave signal from a binary neutron star and its GRB counterpart constraints $c_T = 1$ to a 10^{-15} accuracy.
- For dark energy the scalar field (ST or SA) is non trivial at such distance scales (40Mpc) and generically mixes with the tensor metric perturbations modifying the light cone for gravity waves.
- For Horndeski the surviving theory has free $G_2(\phi, X)$, $G_3(\phi, X)$, $G_4(\phi)$ and $G_5 = 0$.
- For beyond Horndeski we have $G_5 = 0, F_5 = 0, 2G_{4,X} XF_4 = 0$ and theory,

$$\begin{split} L_{c_{T}=1} &= G_{2}(\phi,X) + G_{3}(\phi,X) \Box \phi + B_{4}(\phi,X)^{(4)}R \\ &- \frac{4}{X} B_{4,X}(\phi,X) (\phi^{\mu}\phi^{\nu}\phi_{\mu\nu}\Box\phi - \phi^{\mu}\phi_{\mu\nu}\phi_{\lambda}\phi^{\lambda\nu}) , \end{split}$$

 For DHOST we just make a conformal transformation of the above, G₂(φ, X)G₃(φ, X), B₄(φ, X), C(φ, X)

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What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x \sqrt{-g} \left(L_2 + L_3 + L_4 + L_5 \right)$$

$$\begin{split} L_2 &= G_2(\phi, X), \\ L_3 &= G_3(\phi, X) \Box \phi, \\ L_4 &= G_4(\phi, X) R + G_{4X} \left[(\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right], \\ L_5 &= G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{G_{5X}}{6} \left[(\Box \phi)^3 - 3 \Box \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3 \right] \end{split}$$

• Examples:
$$G_4 = 1 \longrightarrow R$$
.
 $G_4 = X \longrightarrow G^{\mu\nu} \nabla_{\mu} \phi \nabla_{\nu} \phi$.
 $G_3 = X \longrightarrow$ "DGP" term, $(\nabla \phi)^2 \Box \phi$
 $G_5 = lnX \longrightarrow$ gives GB term, $\hat{G} = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} - 4R^{\mu\nu} R_{\mu\nu} + R^2$

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Physical and disformed frames

Most general scalar tensor theory with $c_T = 1$ minimally coupled to matter parametrized by G_2, G_3, B_4, C

$$\begin{split} L_{c_{T}=1} &= G_{2} + G_{3} \Box \phi + B_{4} C^{(4)} R - \frac{4 B_{4,X} C}{X} \phi^{\mu} \phi^{\nu} \phi_{\mu\nu} \Box \phi \\ &+ \left(\frac{4 B_{4,X} C}{X} + \frac{6 B_{4} C_{,X}^{2}}{C} + 8 C_{,X} B_{4,X} \right) \phi^{\mu} \phi_{\mu\nu} \phi_{\lambda} \phi^{\lambda\nu} \\ &+ \frac{8 C_{,X} B_{4,X}}{X} (\phi_{\mu} \phi^{\mu\nu} \phi_{\nu})^{2} \, . \end{split}$$

Horndeski is related via a transformation

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\nabla_{\mu}\phi\nabla_{\nu}\phi$$

to the $L_{c_T=1}$ for given C and D.

- One can start with a $c_T \neq 1$ Horndeski theory and map it to a DHOST $c_T = 1$ theory for a specific function D.
- The former is what we could have called the Einstein frame respective to the latter, the Jordan frame...
- except that the metric is disformed in the procedure...
- The more the symmetry the better

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- Modifying GR is a difficult task but with countable possibilities. Even more so after the GW experiments. :)))
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- MG theories with higher order kinetic terms sensitive to higher order effects, $R_{NL} >> R_{Solar} >> R_{Sch}$.
- Where GR had a linear approximation MG does not!
- Vainshtein scale R_V >> R_S. A kind of Schw. scale for MG theories R_V = R_V(M, r_{Sch})
- Vainshtein mechanism gives GR as classical limit due to non-linear self-interaction.
- Restoring GR in massive gravity, in scalar-tensor models etc.

Review on the Vainshtein mechanism EB & C.Deffayet 2013