Testing gravitation theories with the Event Horizon Telescope and GRAVITY

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based on a collaboration with

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Outline

1. Observing the black hole at the Galactic center
2. The theoretical framework and the no-hair theorem
3. Examples : boson stars and hairy black holes

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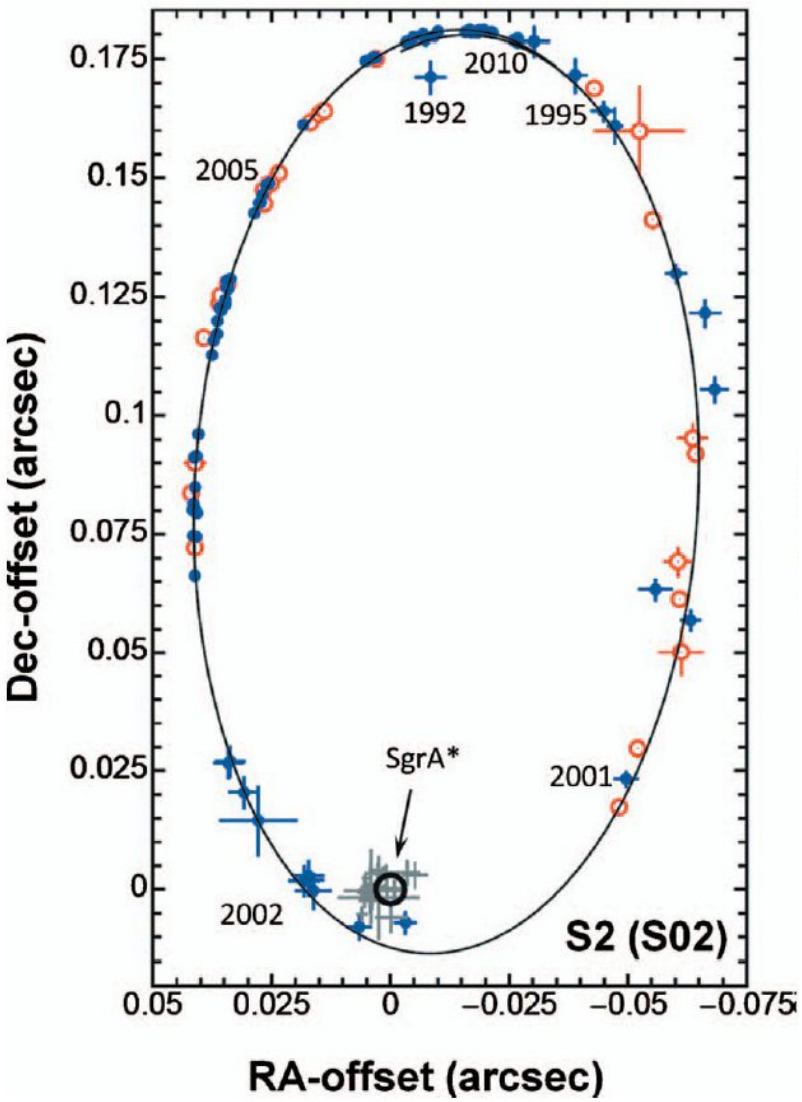
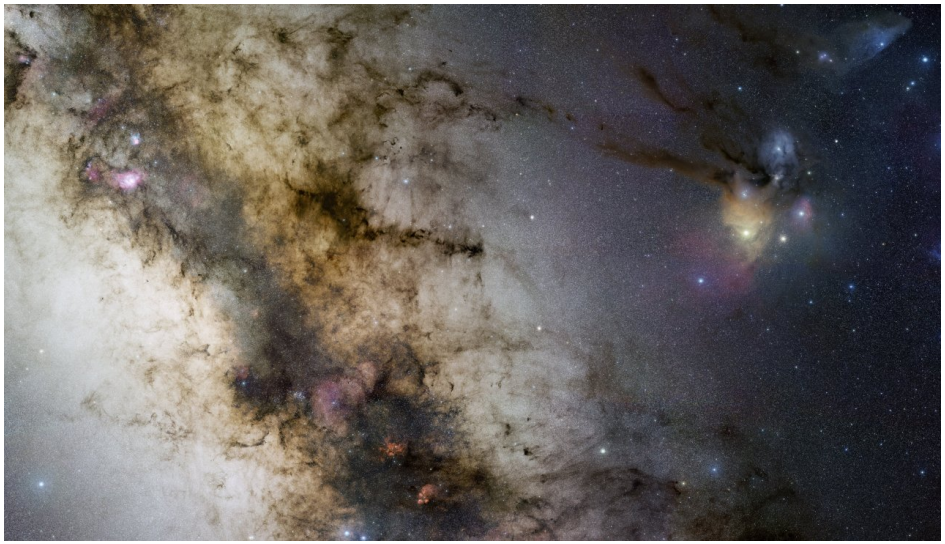
The black hole at the centre of our galaxy : Sgr A*





[ESO (2009)]

Mass of Sgr A* black hole deduced from stellar dynamics :

$$M_{\rm BH} = 4.3 \times 10^6 \, M_\odot$$

← Orbit of the star S2 around Sgr A*

$P = 16\,\mathrm{yr}, \quad r_{\rm per} = 120\,\mathrm{UA} = 1400\,R_{\rm S},$
$V_{\rm per} = 0.02\,c$

[Genzel, Eisenhauer & Gillessen, RMP **82**, 3121 (2010)]

Next periastron passage : mid 2018

Can we see it from the Earth ?

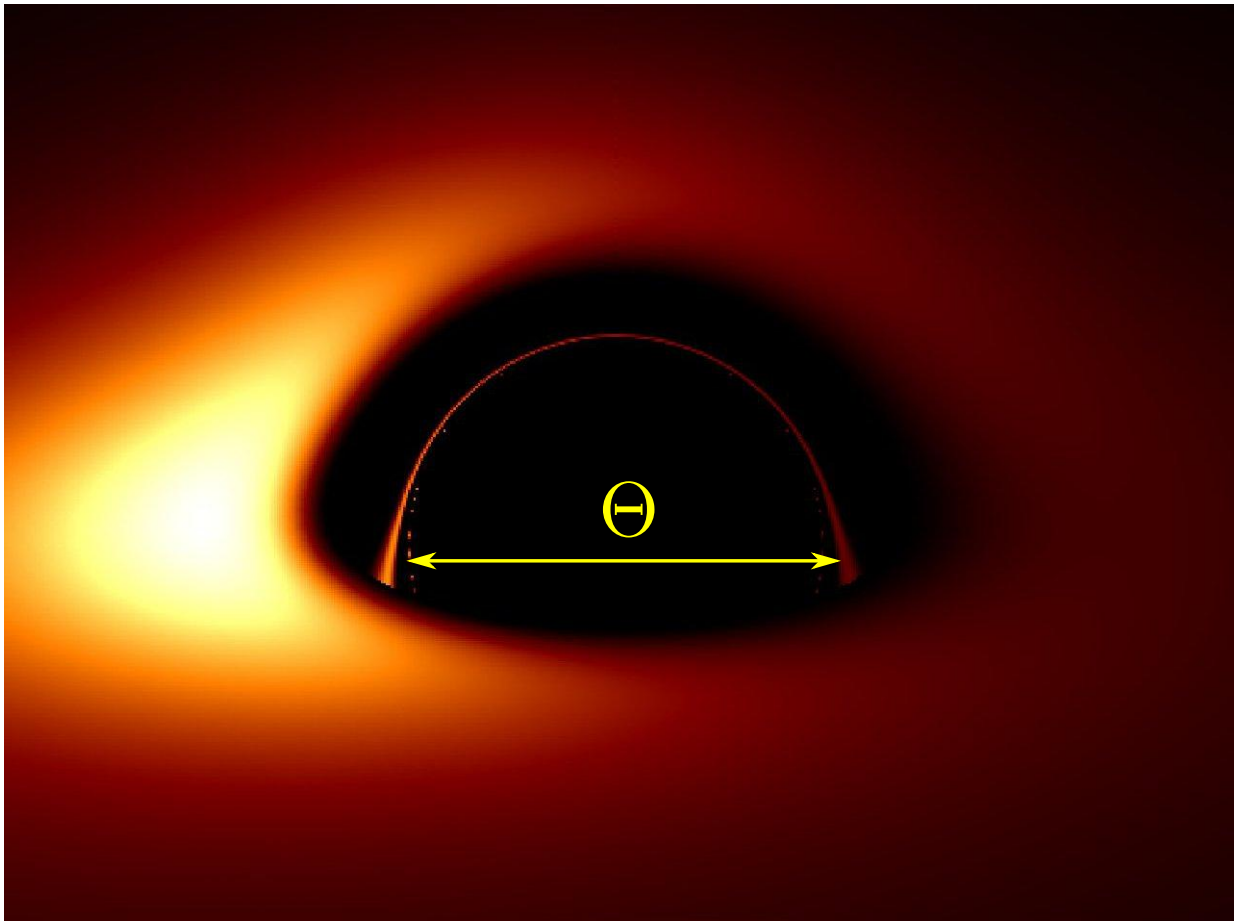


Image of a thin accretion disk around a Schwarzschild BH

[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Angular diameter of the silhouette of a Schwarzschild BH of mass M seen from a distance d :

$$\Theta = 6\sqrt{3}\,\frac{GM}{c^2 d} \simeq 2.60\,\frac{2R_{\rm S}}{d}$$

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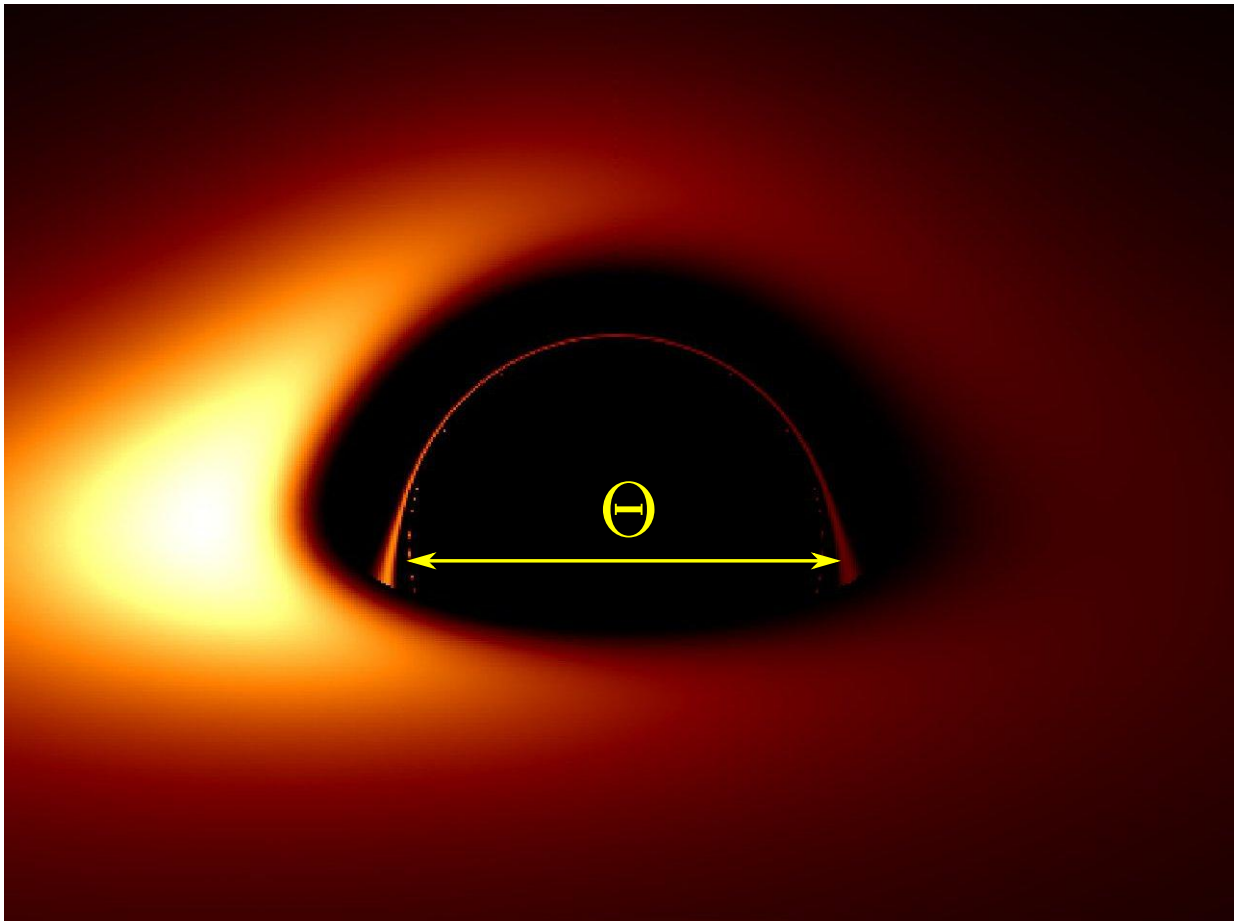

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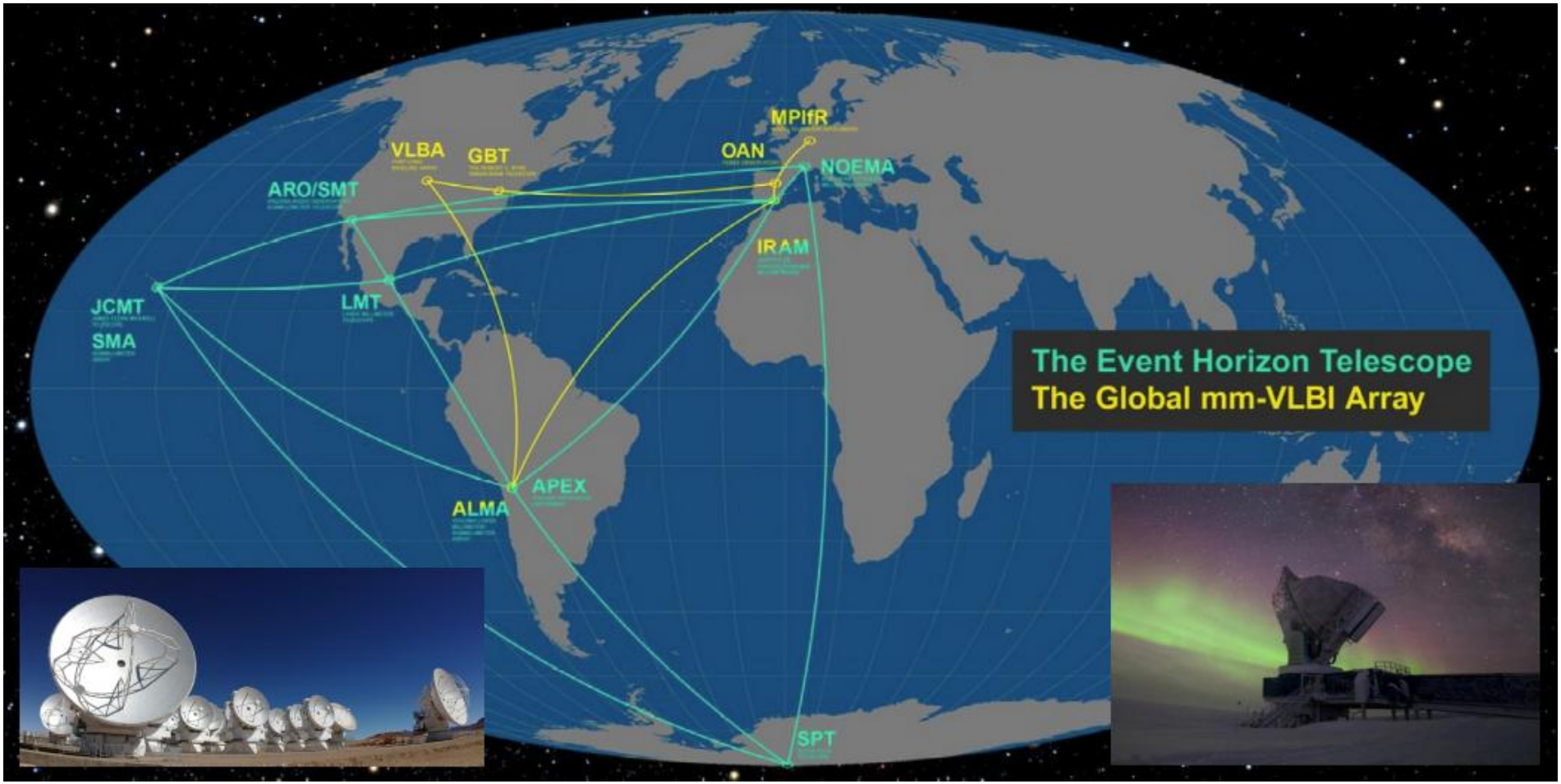
Largest black holes in the Earth's sky :

Sgr A* : $\Theta = 53\ \mu$as
M87 : $\Theta = 21\ \mu$as
M31 : $\Theta = 20\ \mu$as

Remark : black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$

Reaching μas resolution : the Event Horizon Telescope



http://eventhorizontelescope.org/

Very Large Baseline Interferometry (VLBI) at $\lambda = 1.3$ mm

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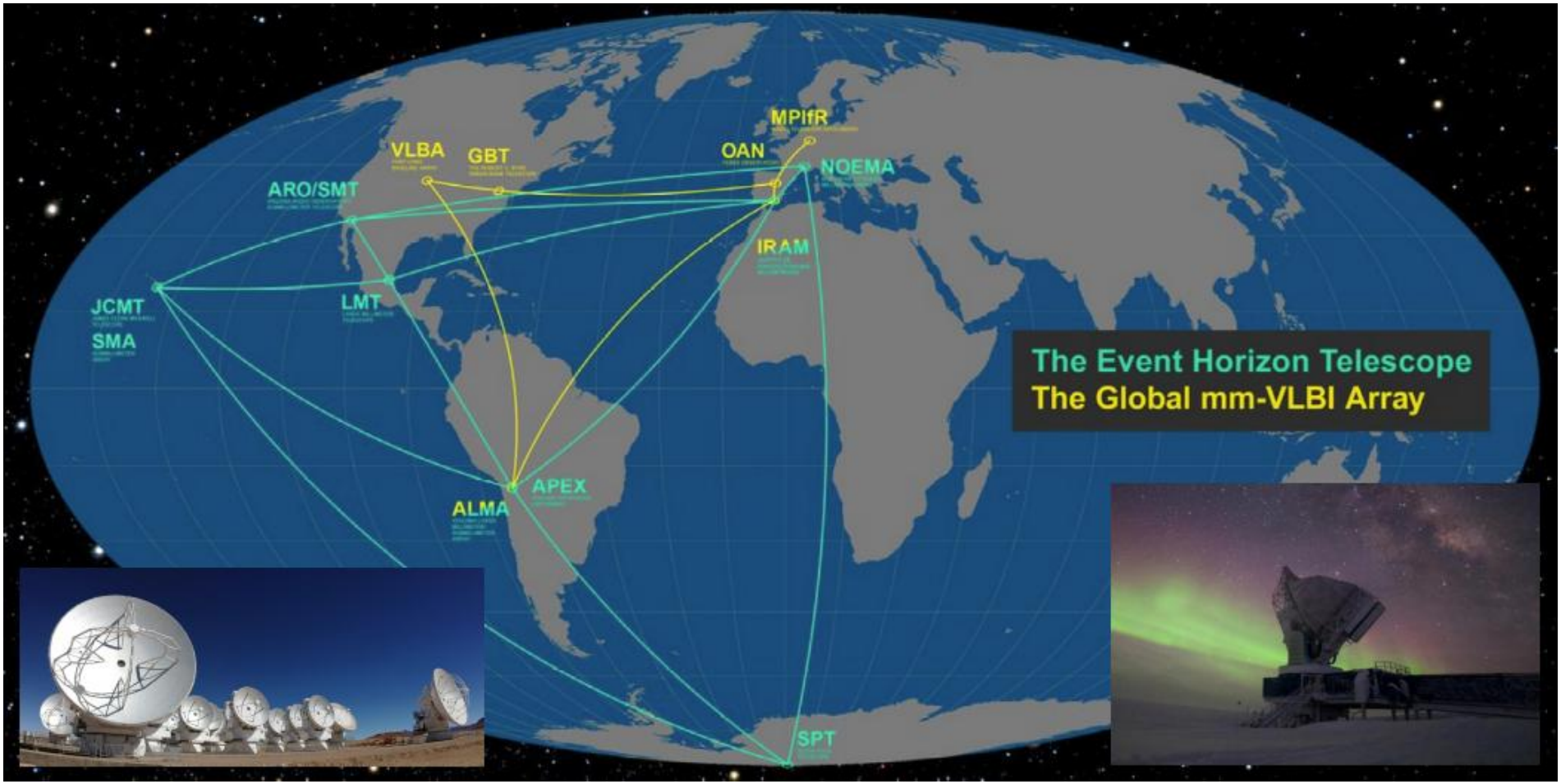

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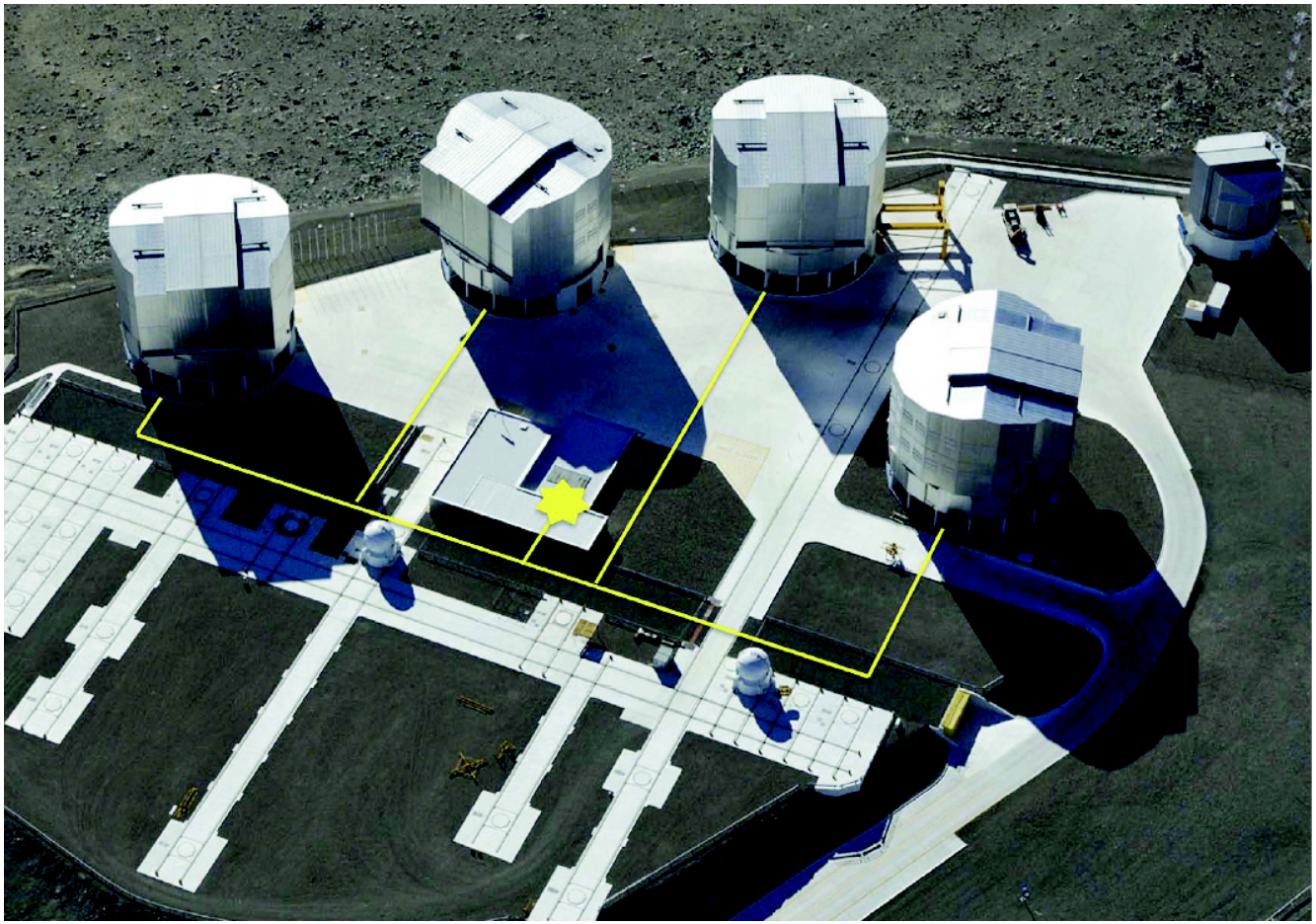
Very Large Baseline Interferometry (VLBI) at $\lambda = 1.3$ mm

April 2017 : large observation campaign $\Longrightarrow$ first image soon ?

Near-infrared optical interferometry : GRAVITY



[Gillessen et al. 2010]

GRAVITY instrument at VLTI (start : 2016)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes)

astrometric precision on orbits : $10\ \mu\mathrm{as}$

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The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

Within 4-dimensional general relativity, a *stationary* black hole in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 numbers :

- the total mass M
- the total specific angular momentum $a = J/(Mc)$
- the total electric charge Q

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

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Astrophysical black holes have to be electrically neutral :

- $Q = 0$: Kerr solution (1963)

Other special cases :

- $a = 0$: Reissner-Nordström solution (1916, 1918)
- $a = 0$ and $Q = 0$: Schwarzschild solution (1916)
- $a = 0$, $Q = 0$ and $M = 0$: Minkowski metric (1907)

The Kerr metric is specific to black holes

Spherically symmetric (non-rotating) case :

Birkhoff theorem

Within 4-dimensional general relativity, the spacetime outside any spherically symmetric body is described by Schwarzschild metric

$\Longrightarrow$ No possibility to distinguish a non-rotating black hole from a non-rotating dark star by monitoring orbital motion or fitting accretion disk spectra

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Rotating axisymmetric case :
No Birkhoff theorem
Moreover, no "reasonable" matter source has ever been found for the Kerr metric (the only known source consists of two counter-rotating thin disks of collisionless particles [Bicak & Ledvinka, PRL **71**, 1669 (1993)])

$\Longrightarrow$ The Kerr metric is specific to rotating black holes (in 4-dimensional general relativity)

Lowest order no-hair theorem : quadrupole moment

Asymptotic expansion (large r) of the metric in terms of multipole moments $(\mathcal{M}_k, \mathcal{J}_k)_{k\in\mathbb{N}}$ [Geroch (1970), Hansen (1974)] :

- $\mathcal{M}_k$: mass 2^k-pole moment
- $\mathcal{J}_k$: angular momentum 2^k-pole moment

$\Longrightarrow$ For the Kerr metric, all the multipole moments are determined by (M, a) :

- $\mathcal{M}_0 = M$
- $\mathcal{J}_1 = aM = J/c$
- $\boxed{\mathcal{M}_2 = -a^2 M = -\dfrac{J^2}{c^2 M}}$ (1) $\leftarrow$ mass quadrupole moment
- $\mathcal{J}_3 = -a^3 M$
- $\mathcal{M}_4 = a^4 M$
- $\cdots$

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Measuring the three quantities M, J, $\mathcal{M}_2$ provides a compatibility test w.r.t. the Kerr metric, by checking (1)

Theoretical alternatives to the Kerr black hole

Within general relativity

The compact object is not a black hole but

- boson stars
- gravastar
- dark stars
- ...

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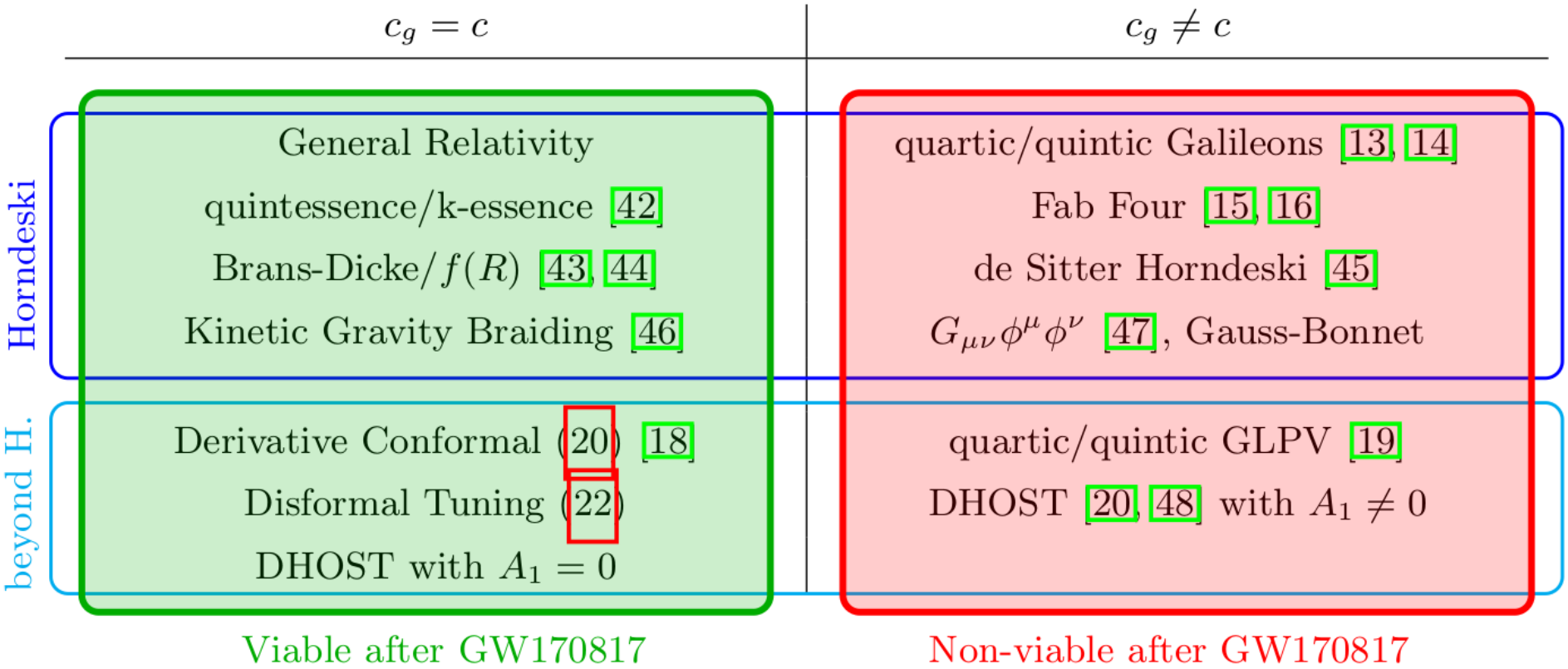
Beyond general relativity

The compact object is a black hole but in a theory that differs from 4-dimensional GR :

- Horndeski theories
- Chern-Simons gravity
- Hořava-Lifshitz gravity
- Higher-dimensional GR
- ...

Viable scalar-tensor theories after GW170817



[Ezquiaga & Zumalacárregui, arXiv:1710.05901]

→ *see talks by Ed Porter and Christos Charmousis*

Testing the Kerr black hole hypothesis

Observational tests

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)

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  - ring-down phase of binary black hole mergers (LIGO, Virgo, LISA)
  - EMRI : extreme-mass-ratio binary inspirals (LISA)

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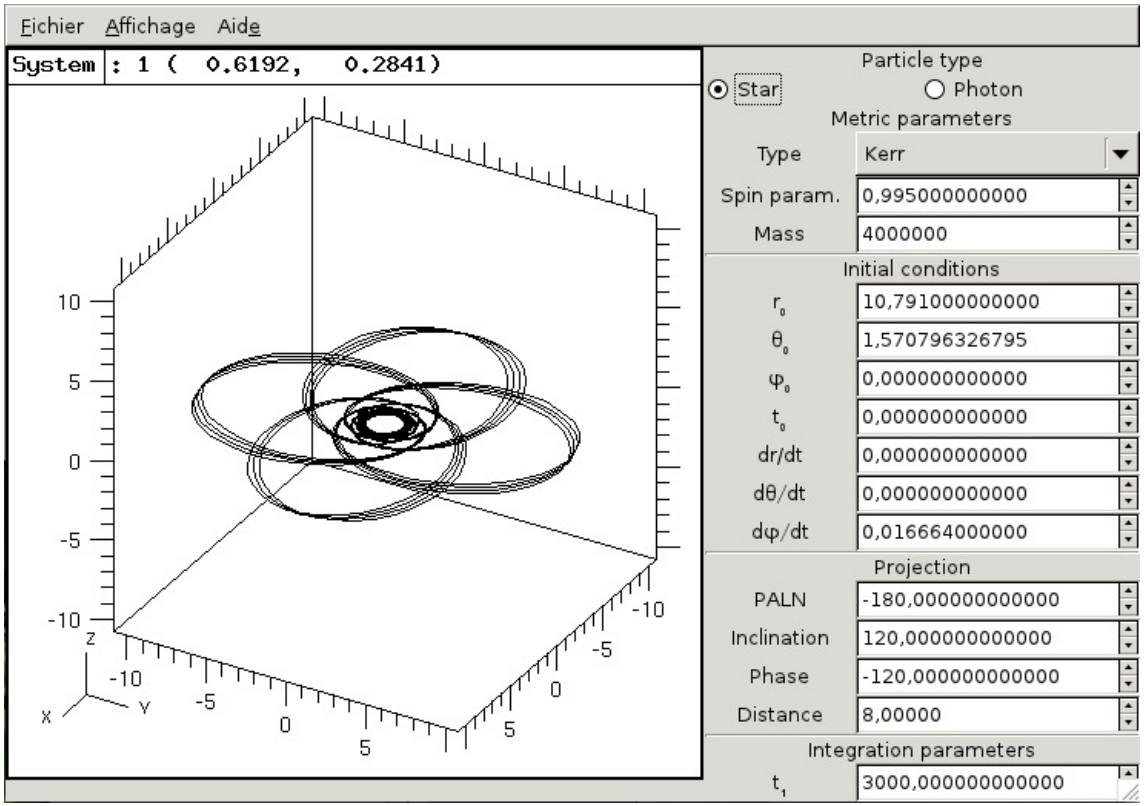
Need for a good and versatile **geodesic integrator**
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric

Gyoto code

Main developers : T. Paumard & F. Vincent



- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick and Python interfaces
- Free software (GPL) : http://gyoto.obspm.fr/

[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

[Vincent, Gourgoulhon & Novak, CQG **29**, 245005 (2012)]

Geodesics with the free computer algebra system SageMath

SageMath : Python-based open-source mathematical software

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Timelike geodesic in Schwarzschild spacetime

This Jupyter/SageMath worksheet presents the numerical computation of a timelike geodesic in Schwarzschild spacetime, given an initial point and tangent vector. It uses the `integrated_geodesic` functionality introduced by Karim Van Aelst in **SageMath 8.1**, in the framework of the SageManifolds project.

A version of SageMath at least equal to 8.1 is required to run this worksheet:

In [1]:
```
version()
```
Out[1]: 'SageMath version 8.1.rc0, Release Date: 2017-11-08'

In [2]:
```
%display latex  # LaTeX rendering turned on
```

We define first the spacetime manifold M and the standard Schwarzschild-Droste coordinates on it:

In [3]:
```
M = Manifold(4, 'M')
X.<t,r,th,ph> = M.chart(r't r:(0,+oo) th:(0,pi):\theta ph:\phi')
X
```
Out[3]: $(M, (t, r, \theta, \phi))$

http://nbviewer.jupyter.org/github/sagemanifolds/SageManifolds/blob/master/Worksheets/v1.1/SM_simple_geod_Schwarz.ipynb

Geodesics with the free computer algebra system SageMath

In [3]:
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X.<t,r,th,ph> = M.chart(r't r:(0,+oo) th:(0,pi):\theta ph:\phi')
X
```

Out[3]: $(M, (t, r, \theta, \phi))$

For graphical purposes, we introduce $\mathbb{R}^3$ and some coordinate map $M \to \mathbb{R}^3$:

In [4]:
```
R3 = Manifold(3, 'R^3', latex_name=r'\mathbb{R}^3')
X3.<x,y,z> = R3.chart()
to_R3 = M.diff_map(R3, {(X, X3): [r*sin(th)*cos(ph),
                                  r*sin(th)*sin(ph), r*cos(th)]})
to_R3.display()
```

Out[4]:
$$\begin{aligned} M &\longrightarrow \mathbb{R}^3 \\ (t, r, \theta, \phi) &\longmapsto (x, y, z) = (r\cos(\phi)\sin(\theta), r\sin(\phi)\sin(\theta), r\cos(\theta)) \end{aligned}$$

Then, we define the Schwarzschild metric:

In [5]:
```
g = M.lorentzian_metric('g')
m = var('m'); assume(m >= 0)
g[0,0], g[1,1] = -(1-2*m/r), 1/(1-2*m/r)
g[2,2], g[3,3] = r^2, (r*sin(th))^2
g.display()
```

Out[5]:
$$g = \left(\frac{2\,m}{r} - 1\right) \mathrm{d}t \otimes \mathrm{d}t + \left(-\frac{1}{\frac{2\,m}{r} - 1}\right) \mathrm{d}r \otimes \mathrm{d}r + r^2 \mathrm{d}\theta \otimes \mathrm{d}\theta + r^2 \sin(\theta)^2 \mathrm{d}\phi \otimes \mathrm{d}\phi$$

We pick an initial point and an initial tangent vector:

In [6]:
```
p0 = M.point((0, 8*m, pi/2, 1e-12), name='p_0')
v0 = M.tangent_space(p0)((1.297513, 0, 0, 0.0640625/m), name='v_0')
v0.display()
```

Out[6]:
$$v_0 = 1.29751300000000\,\frac{\partial}{\partial t} + \frac{0.0640625000000000}{m}\,\frac{\partial}{\partial \phi}$$

Geodesics with the free computer algebra system SageMath

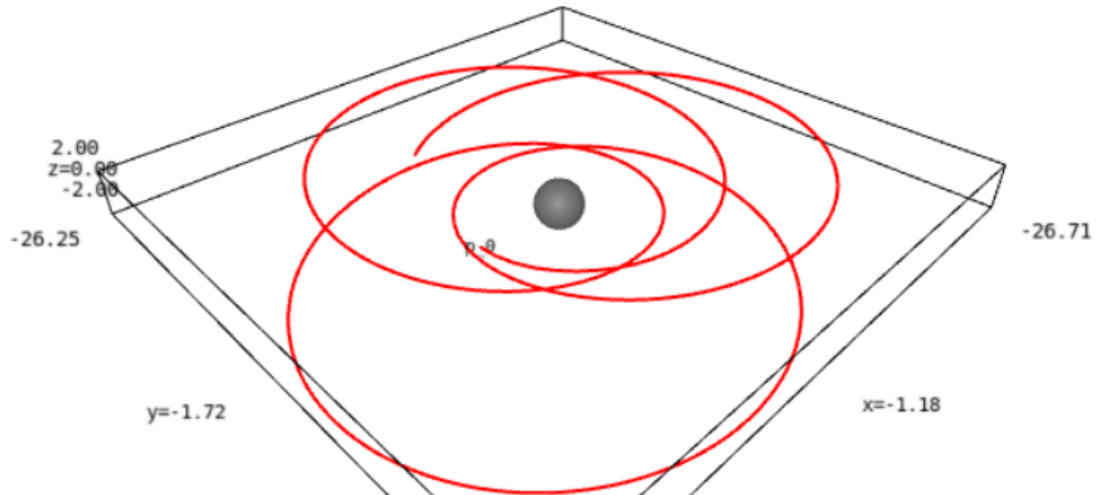
We declare a geodesic with such initial conditions, denoting by s the affine parameter (proper time), with $(s_{\min}, s_{\max}) = (0, 1500\,m)$:

In [7]:
```
s = var('s')
geod = M.integrated_geodesic(g, (s, 0, 1500), v0); geod
```

Out[7]: Integrated geodesic in the 4-dimensional differentiable manifold M

We ask for the numerical integration of the geodesic, providing some numerical value for the parameter m, and then plot it in terms of the Cartesian chart X3 of $\mathbb{R}^3$:

In [8]:
```
sol = geod.solve(parameters_values={m: 1})  # numerical integration
interp = geod.interpolate()                 # interpolation of the solution for the plot
graph = geod.plot_integrated(chart=X3, mapping=to_R3, plot_points=500,
                             thickness=2, label_axes=False)          # the geodesic
graph += p0.plot(chart=X3, mapping=to_R3, size=4, parameters={m: 1}) # the starting point
graph += sphere(size=2, color='grey')                                # the event horizon
show(graph, viewer='threejs', online=True)
```



Geodesics with the free computer algebra system SageMath

In [11]:
```
g.christoffel_symbols_display()
```

Out[11]:

$$\Gamma^{t}_{\,t\,r} = -\frac{m}{2\,mr-r^2}$$
$$\Gamma^{r}_{\,t\,t} = -\frac{2\,m^2-mr}{r^3}$$
$$\Gamma^{r}_{\,r\,r} = \frac{m}{2\,mr-r^2}$$
$$\Gamma^{r}_{\,\theta\,\theta} = 2\,m-r$$
$$\Gamma^{r}_{\,\phi\,\phi} = (2\,m-r)\sin(\theta)^2$$
$$\Gamma^{\theta}_{\,r\,\theta} = \frac{1}{r}$$
$$\Gamma^{\theta}_{\,\phi\,\phi} = -\cos(\theta)\sin(\theta)$$
$$\Gamma^{\phi}_{\,r\,\phi} = \frac{1}{r}$$
$$\Gamma^{\phi}_{\,\theta\,\phi} = \frac{\cos(\theta)}{\sin(\theta)}$$

In [12]:
```
g.riemann().display_comp()
```

Out[12]:

$$\mathrm{Riem}(g)^{t}_{\,r\,t\,r} = -\frac{2\,m}{2\,mr^2-r^3}$$
$$\mathrm{Riem}(g)^{t}_{\,r\,r\,t} = \frac{2\,m}{2\,mr^2-r^3}$$
$$\mathrm{Riem}(g)^{t}_{\,\theta\,t\,\theta} = -\frac{m}{r}$$
$$\mathrm{Riem}(g)^{t}_{\,\theta\,\theta\,t} = \frac{m}{r}$$
$$\mathrm{Riem}(g)^{t}_{\,\phi\,t\,\phi} = -\frac{m\sin(\theta)^2}{r}$$
$$\mathrm{Riem}(g)^{t}_{\,\phi\,\phi\,t} = \frac{m\sin(\theta)^2}{r}$$

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Boson stars

Boson star = localized configurations of a self-gravitating complex scalar field Φ
$\equiv$ "Klein-Gordon geons" [Bonazzola & Pacini (1966), Kaup (1968), Ruffini & Bonazzola (1969)]

- Minimally coupled scalar field : $\mathcal{L} = \dfrac{1}{16\pi} R - \dfrac{1}{2}\left[\nabla_\mu \bar{\Phi} \nabla^\mu \Phi + V(|\Phi|^2)\right]$
- Scalar field equation : $\nabla_\mu \nabla^\mu \Phi = V'(|\Phi|^2)\, \Phi$
- Einstein equation : $R_{\alpha\beta} - \dfrac{1}{2} R g_{\alpha\beta} = 8\pi T_{\alpha\beta}$

  with $T_{\alpha\beta} = \nabla_{(\alpha} \bar{\Phi} \nabla_{\beta)} \Phi - \dfrac{1}{2}\left[\nabla_\mu \bar{\Phi} \nabla^\mu \Phi + V(|\Phi|^2)\right] g_{\alpha\beta}$

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Examples :

- free field : $V(|\Phi|^2) = \dfrac{m^2}{\hbar^2} |\Phi|^2$, $\quad m$: boson mass

  $\Longrightarrow$ field equation = Klein-Gordon equation : $\nabla_\mu \nabla^\mu \Phi = \dfrac{m^2}{\hbar^2} \Phi$

- a standard self-interacting field : $V(|\Phi|^2) = \dfrac{m^2}{\hbar^2} |\Phi|^2 + \lambda |\Phi|^4$

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Boson stars as black-hole mimickers

Boson stars can be very compact and are the less exotic alternative to black holes : they require only a scalar field and since 2012 we know that at least one fundamental scalar field exists in Nature : the Higgs boson !

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Maximum mass

- Free field : $M_{\rm max} = \alpha \dfrac{\hbar}{m} = \alpha \dfrac{m_{\rm P}^2}{m}$, with $\alpha \sim 1$
- Self-interacting field : $M_{\rm max} \sim \left(\dfrac{\lambda}{4\pi} \right)^{1/2} \dfrac{m_{\rm P}^2}{m} \times \dfrac{m_{\rm P}}{m}$

$m_{\rm P} = \sqrt{\hbar} = \sqrt{\hbar c / G} = 2.18\ 10^{-8}\ {\rm kg}$: Planck mass

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Maximum mass

- Free field : $M_{\rm max} = \alpha \dfrac{\hbar}{m} = \alpha \dfrac{m_{\rm P}^2}{m}$, with $\alpha \sim 1$
- Self-interacting field : $M_{\rm max} \sim \left(\dfrac{\lambda}{4\pi} \right)^{1/2} \dfrac{m_{\rm P}^2}{m} \times \dfrac{m_{\rm P}}{m}$

$m_{\rm P} = \sqrt{\hbar} = \sqrt{\hbar c / G} = 2.18\ 10^{-8}$ kg : Planck mass

m	$M_{\rm max}$ (free field)	$M_{\rm max}$ (self-interacting field, $\lambda = 1$)
125 GeV (Higgs)	$2\ 10^{9}$ kg	$2\ 10^{26}$ kg
1 GeV	$3\ 10^{11}$ kg	$2\, M_\odot$
0.5 MeV	$3\ 10^{14}$ kg	$5\ 10^{6}\, M_\odot$

Rotating boson stars

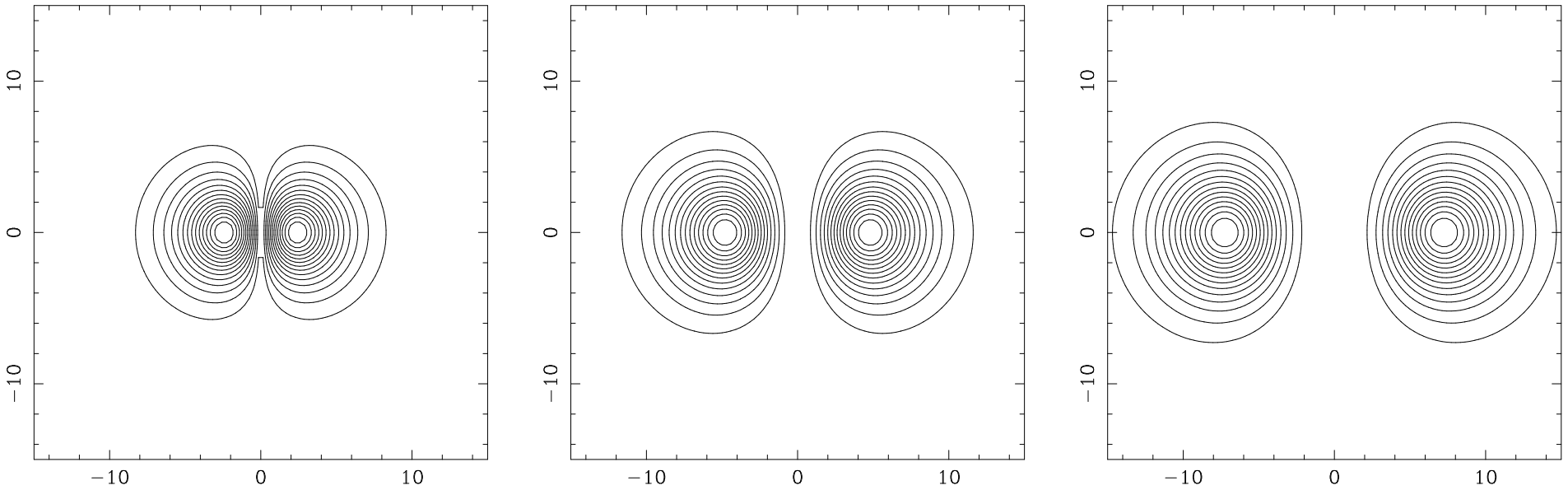
Solutions computed by means of Kadath [Grandclément, JCP **229**, 3334 (2010)]

http://kadath.obspm.fr/

→ *see Philippe Grandclément's talk*

Isocontours of $\Phi_0(r,\theta)$ in the plane $\varphi = 0$ for $\omega = 0.8\dfrac{m}{\hbar}$:

$k = 1$ $k = 2$ $k = 3$



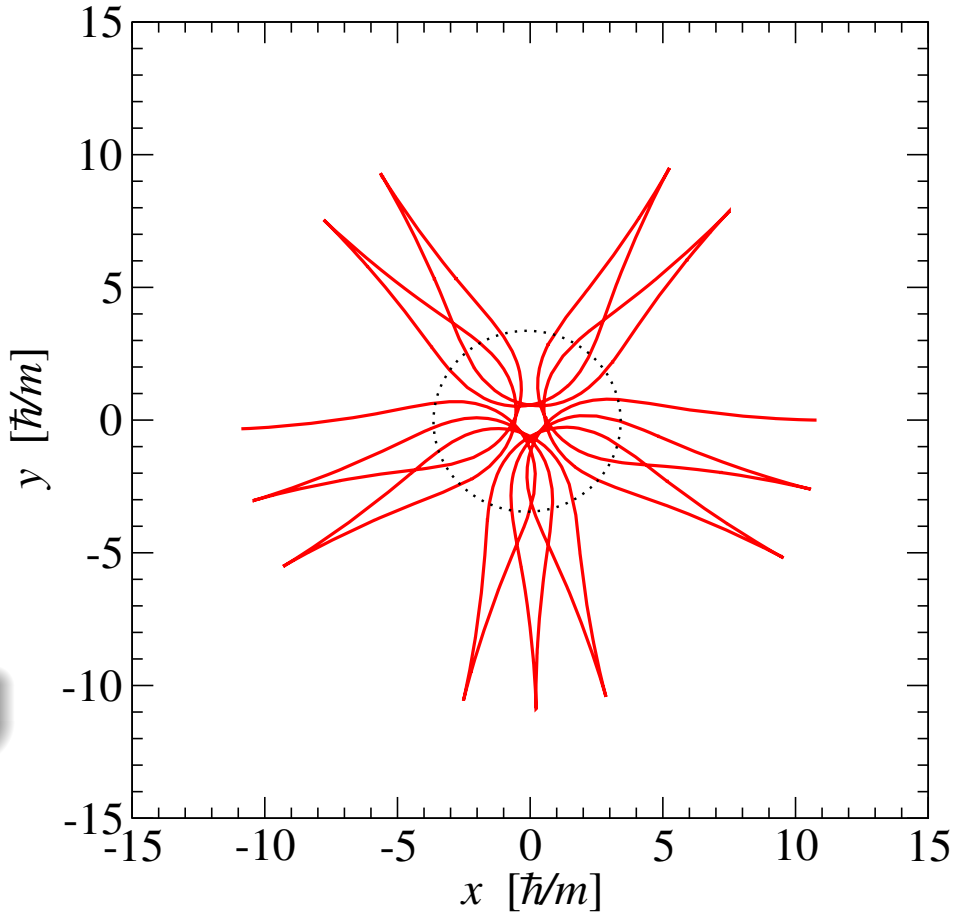
[Granclément, Somé & Gourgoulhon, PRD **90**, 024068 (2014)]

Initially-at-rest orbits around rotating boson stars

Orbit with a rest point around a rotating boson star based on the scalar field
$\Phi = \Phi_0(r,\theta)e^{i(\omega t + k\varphi)}$
with $k = 2$ and $\omega = 0.75\, m/\hbar$

Orbit = timelike geodesic computed by means of `Gyoto`

$\Longrightarrow$ strong Lense-Thirring effect



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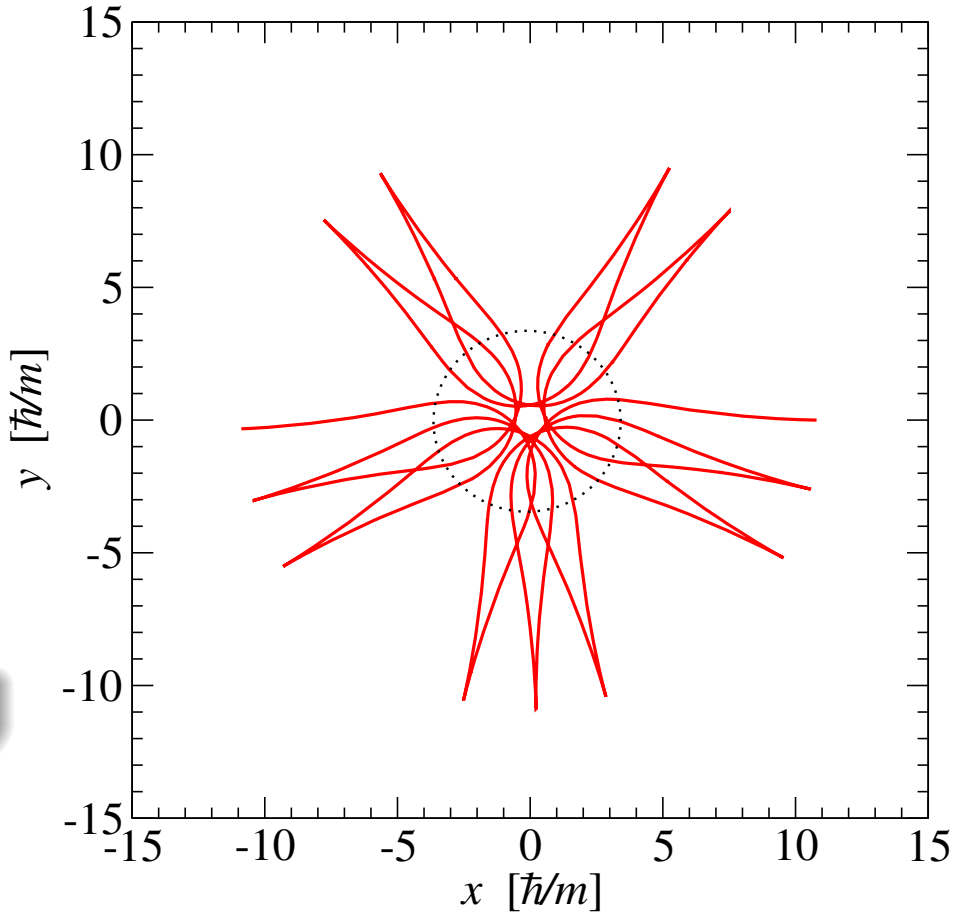
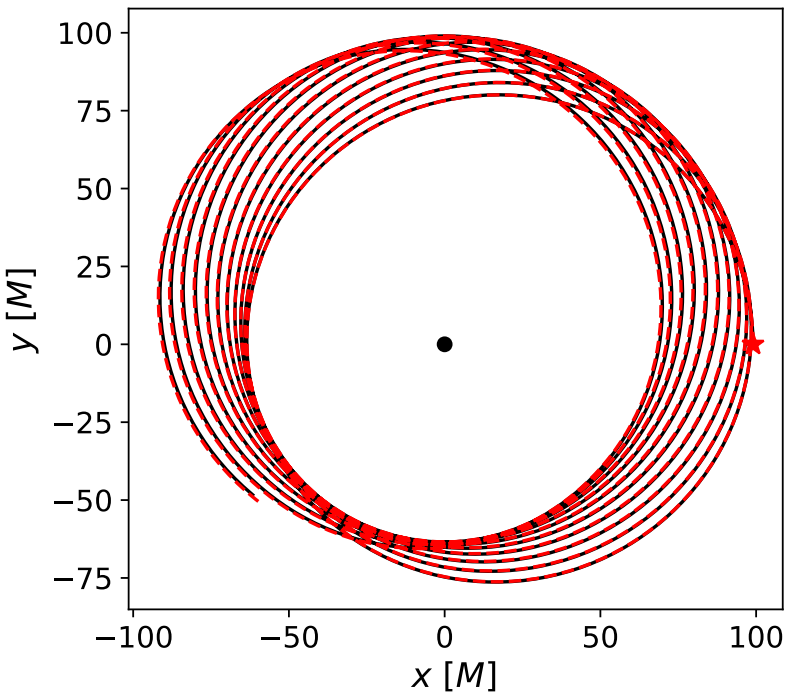


[Granclément, Somé & Gourgoulhon, PRD **90**, 024068 (2014)]

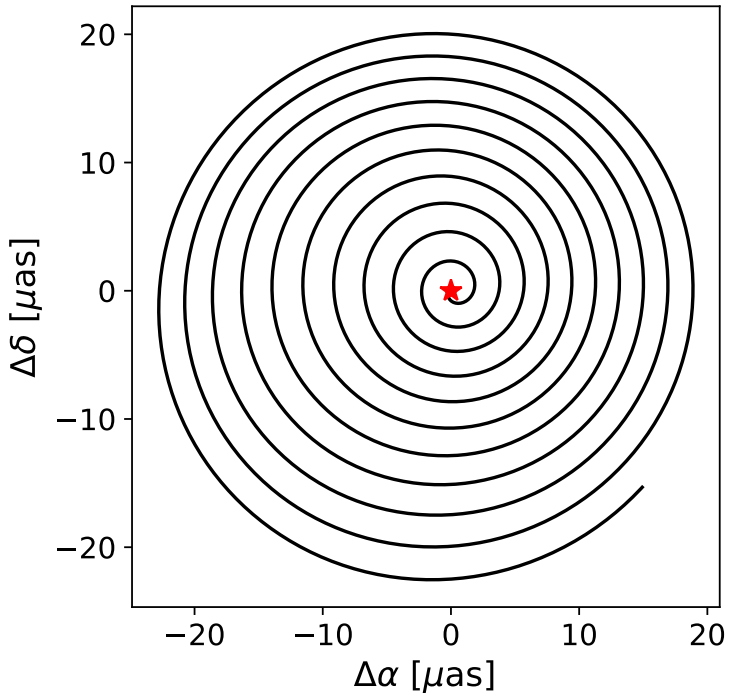
No equivalent in Kerr spacetime

Comparing orbits with a Kerr BH

Same reduced spin for the boson star and the Kerr BH : $a = 0.802\,M$
Boson star (BS) : $k = 1$ and $\omega = 0.8\,m/\hbar$
Orbit with pericenter of $60\,M$ and apocenter of $100\,M$



The two orbits

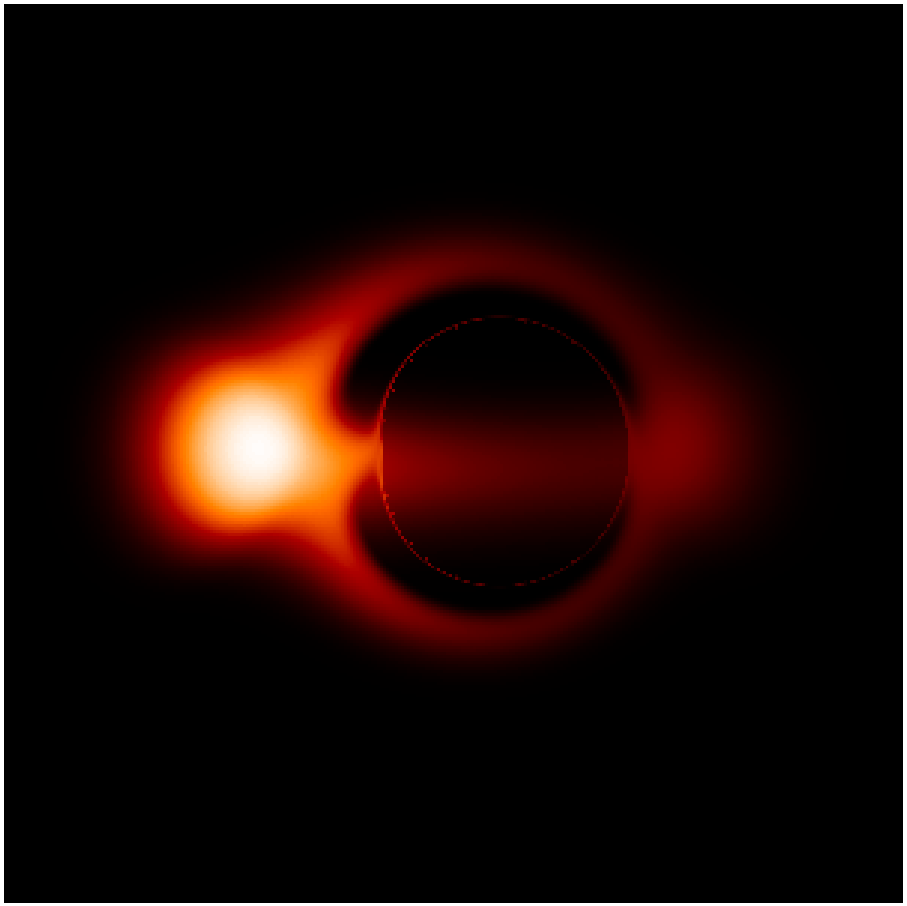


Difference between the BS orbit and the BH one for Sgr A*

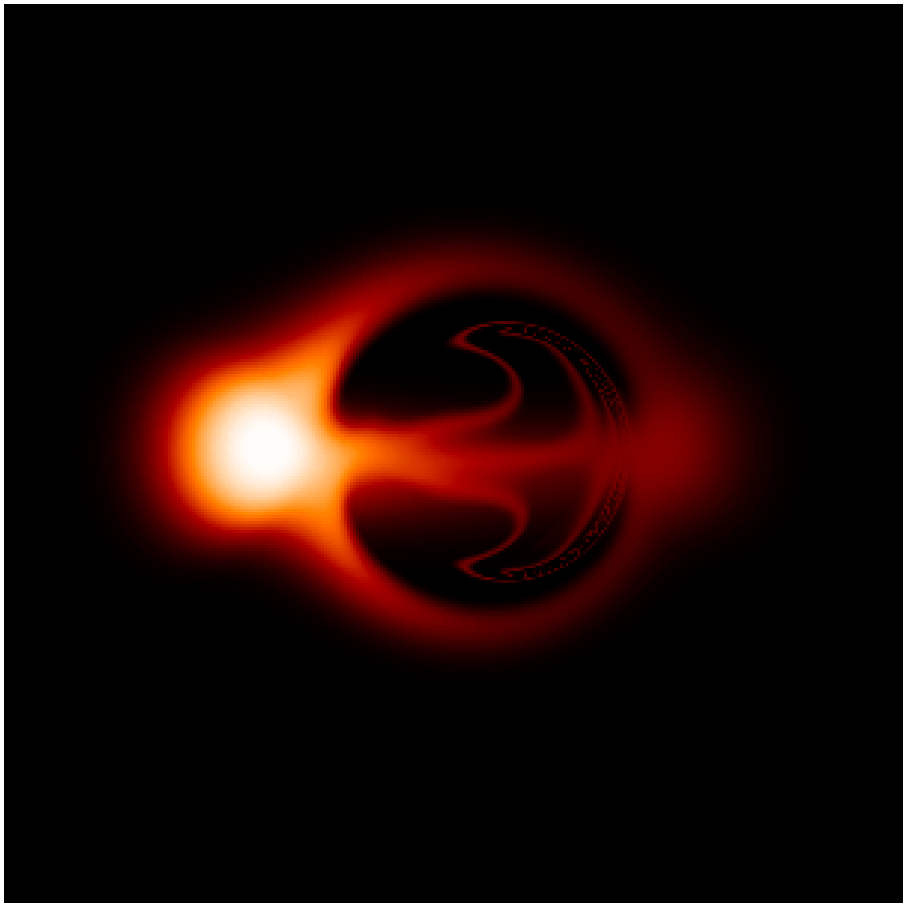
[Grould, Meliani, Vincent, Grandclément & Gourgoulhon, CQG **34**, 215007 (2017)]

Image of an accretion torus : comparing with a Kerr BH

Kerr BH $a/M = 0.9$



Boson star $k = 1$, $\omega = 0.70\, m/\hbar$



[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, CQG **33**, 105015 (2016)]

Hairy black holes

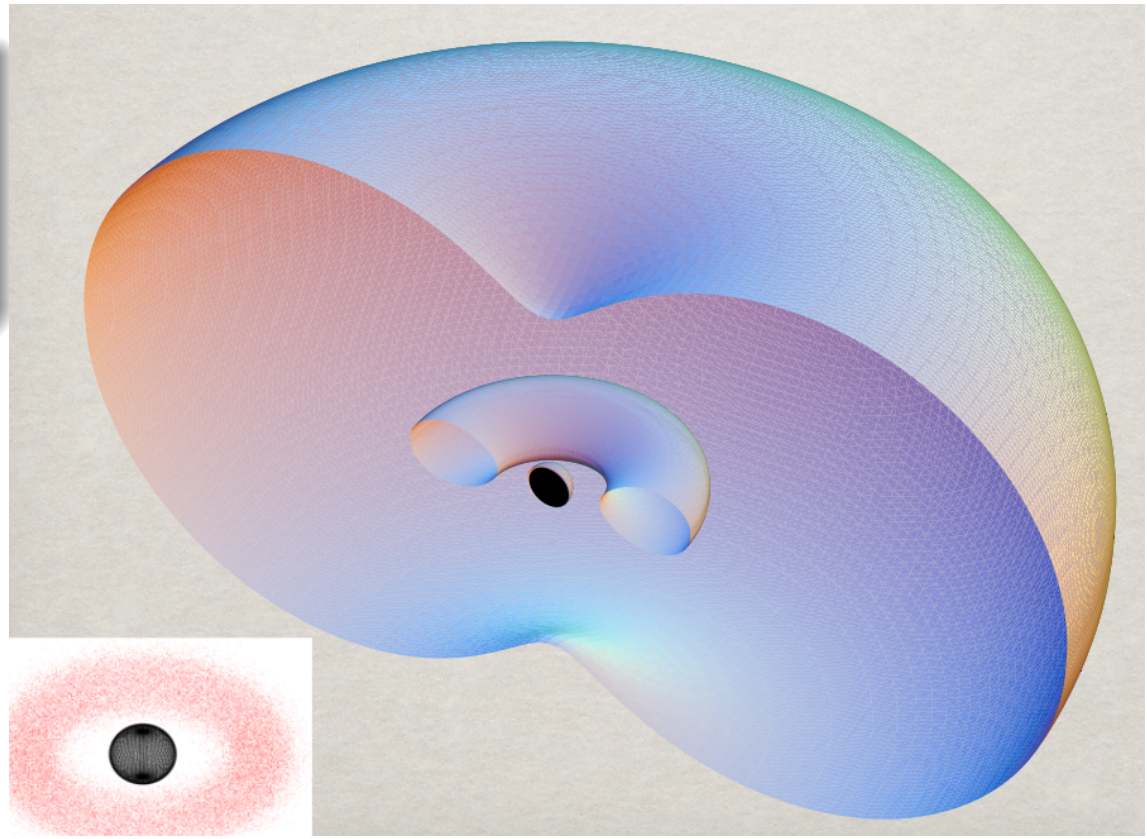
Herdeiro & Radu discovery (2014)

A black hole can have a complex scalar hair

Stationary axisymmetric configuration with a self-gravitating massive complex scalar field Φ and an event horizon

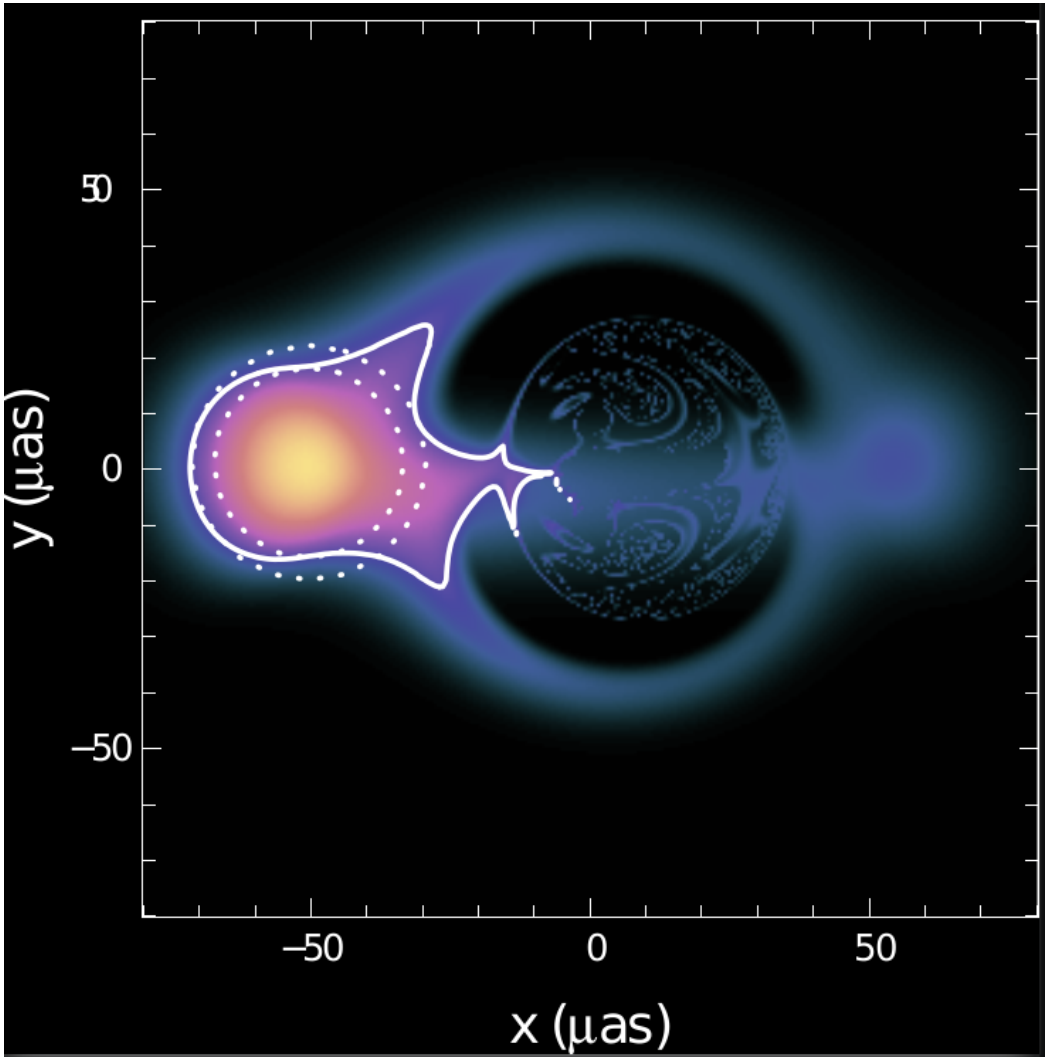
$$\Phi(t, r, \theta, \varphi) = \Phi_0(r, \theta) e^{i(\omega t + k\varphi)}$$

$$\omega = k\Omega_{\mathrm{H}}, \quad k \in \mathbb{N}$$



[Herdeiro & Radu, PRL **112**, 221101 (2014)]

Hairy black hole



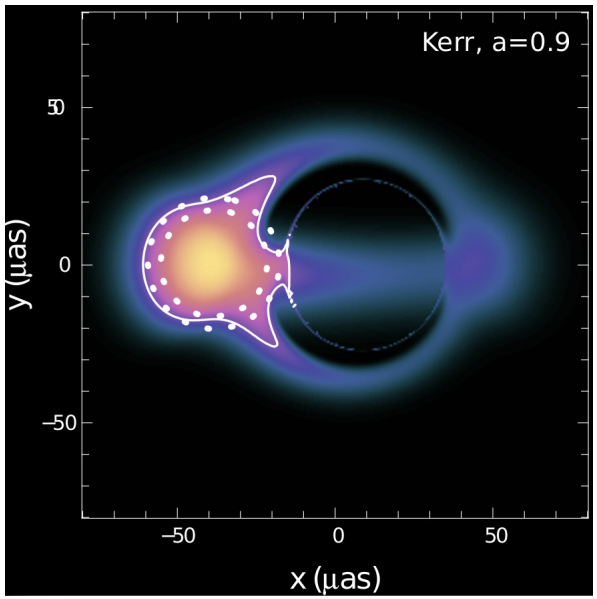
Accretion torus around a scalar-field-hairy rotating black hole

[Vincent, Gourgoulhon, Herdeiro & Radu, Phys. Rev. D **94**, 084045 (2016)]

Alternatives to the Kerr black hole

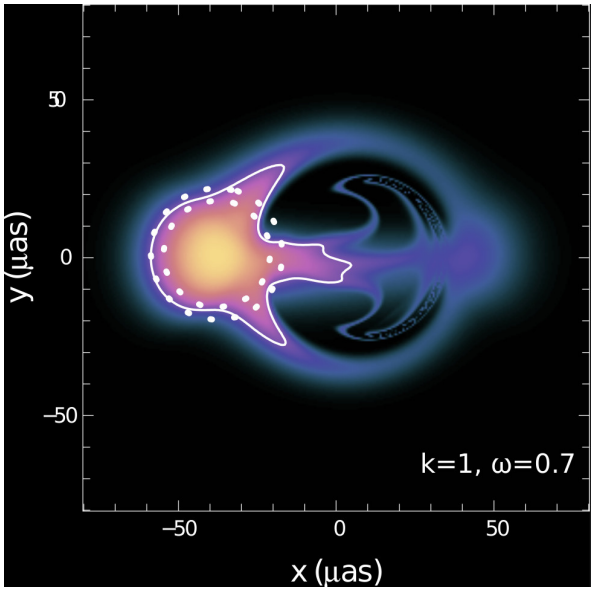
Kerr black hole

$a/M = 0.9$



boson star [[1]]

$k=1,\ \omega=0.7\,m/\hbar$

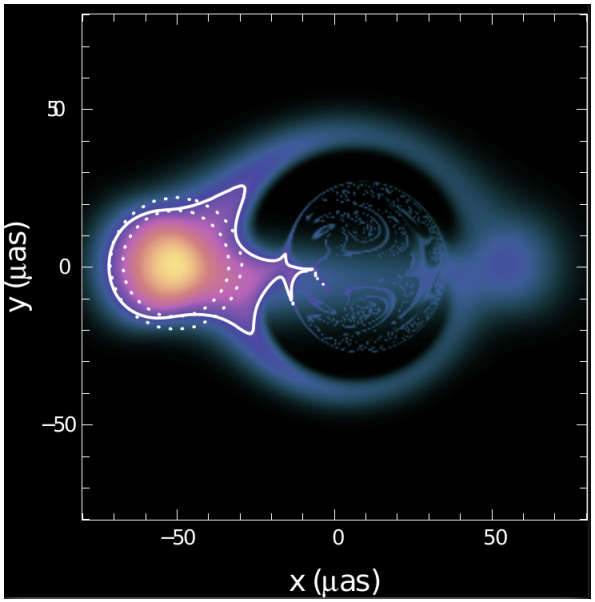


Kadath → metric

Gyoto → ray-tracing

hairy black hole [[2]]

$a/M = 0.9$



HR code → metric

(via Lorene)

Gyoto → ray-tracing

[[1] Vincent, Meliani, Grandclément, Gourgoulhon & Straub, Class. Quantum Grav. **33**, 105015 (2016)]

[[2] Vincent, Gourgoulhon, Herdeiro & Radu, Phys. Rev. D **94**, 084045 (2016)]

Conclusions and perspectives

Black hole physics is entering a new observational era, with the advent of high-angular-resolution telescopes and gravitational wave detectors, which provide unique opportunities to test general relativity in the strong field regime, notably by searching for some violation of the *no-hair theorem*.

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Work in progress

- rotating *regular* black holes [F. Lamy et al.]
- rotating black holes in cubic Galileon gravity [K. Van Aelst et al.]