

PHENOMENOLOGICAL POLYAKOV-LOOP POTENTIALS

RAINER STIELE

In collaboration with [Pedro Costa \(CFisUC, Coimbra\)](#), [Hubert Hansen \(IPNL, Lyon\)](#) and [Urko Reinosa \(CPHT, Palaiseau\)](#)

Outline

- 1 Introduction**
- 2 Phenomenological Polyakov-loop potential revisited: adjustment to thermodynamics**
- 3 Phenomenological Polyakov-loop potential revisited: adjustment to first-principle continuum potential**
- 4 Conclusions**

Phenomenological vs. First Principle

$$\mathcal{U}_{\text{poly}}(\Phi, \bar{\Phi}, t) / T^4 = p_2(t) \Phi \bar{\Phi} + p_3 (\Phi^3 + \bar{\Phi}^3) + p_4 (\Phi \bar{\Phi})^2$$

O. Scavenius, A. Dumitru and J. Lenaghan, *Phys. Rev. D* 66 (2002)

C. Ratti, M. A. Thaler and W. Weise, *Phys. Rev. D* 73 (2006)

$$\mathcal{U}_{\text{log}}(\Phi, \bar{\Phi}, t) / T^4 = p_2(t) \Phi \bar{\Phi} + l(t) \ln \left[1 - 6\Phi \bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi \bar{\Phi})^2 \right]$$

S. Roessner, T. Hell, C. Ratti and W. Weise, *Nucl. Phys. A* 814 (2008)

VS.

$$V[A_0] = V_{\Lambda}[A_0] + \frac{1}{\beta V} \int_{\Lambda} dt \partial_t \Gamma_k[A_0],$$

L. Fister and J. M. Pawłowski,

Phys. Rev. D 88 (2013)

$$V^{(1)}(T, r) = \frac{3}{2} \mathcal{F}_m(T, r) - \frac{1}{2} \mathcal{F}_0(T, r),$$

$$\begin{aligned} V^{(2)}(T, r) &= m^2 C_G \sum_{\kappa} J_{\kappa}^*(1n) \\ &+ \frac{3g^2}{8} \sum_{\kappa, \lambda, \tau} C_{\kappa\lambda\tau} \left[\frac{5}{2} U^{\kappa} V^{\lambda} - \frac{7}{m^2} \bar{U}^{\kappa} \bar{V}^{\lambda} \right] \\ &+ \frac{g^2 m^2}{16} \sum_{\kappa, \lambda, \tau} C_{\kappa\lambda\tau} [33 S_{mmm}^{\kappa\lambda\tau}(2n) + S_{m00}^{\kappa\lambda\tau}(2n)]. \end{aligned}$$

U. Reinosa, J. Serreau, M. Tissier, and N. Wschebor, *Phys. Rev. D* 93 (2016)

$$e(a, L) = \sum_{\sigma} \int d^3 p [\omega_G(\mathbf{p}_{\sigma}) - \chi_{\text{IR}}(\mathbf{p}_{\sigma})]. \quad (195)$$

Subtracting the value at $a = 0$ we have

$$\bar{e}(a, L) = e(a, L) - e(a = 0, L) = \bar{e}_G^{\omega}(a, L) - \bar{e}_{\text{IR}}^{\omega}(a, L),$$

H. Reinhardt and J. Heffner, *Phys. Rev. D* 88 (2013)

$$\beta^4 V_{\text{eff}}(x) = \beta^4 W(x) + \frac{6}{\pi^2} \sum_{m=1}^{\infty} \frac{1 - \cos(2\pi m x)}{m^4} h(\beta m)$$

M. Quandt and H. Reinhardt, *Phys. Rev. D* 94 (2016)

Application of a Phenomenological Potential

... for a simple model of QCD

$$\begin{aligned} \mathcal{L}_{\text{PQM}} = & \bar{q} \left[i \gamma_{\mu} (\partial^{\mu} - i A^{\mu} \delta_{\mu 0} + \mu_f \delta^{\mu 0}) - g \frac{\lambda_a}{2} (\sigma_a + i \gamma_5 \pi_a) \right] q \\ & + \frac{1}{2} (\partial_{\mu} \sigma_a \partial^{\mu} \sigma_a + \partial_{\mu} \pi_a \partial^{\mu} \pi_a) - U(\sigma_a, \pi_a) - \mathcal{U}(\Phi[A_0], \bar{\Phi}[A_0]; T) \end{aligned}$$

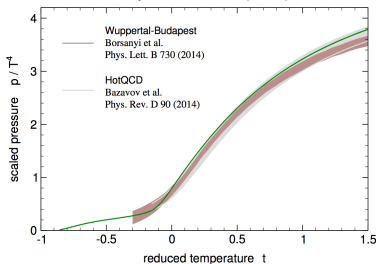
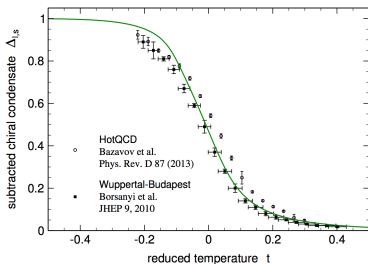
$$\begin{aligned} \mathcal{L}_{\text{PNJL}} = & \bar{q} \left[i \gamma_{\mu} (\partial^{\mu} - i A^{\mu} \delta_{\mu 0} + \mu_f \delta^{\mu 0}) - m_0 \right] q + G \left[(\bar{q} q)^2 + (\bar{q} i \gamma_5 \vec{\tau} q)^2 \right] \\ & - \mathcal{U}(\Phi[A_0], \bar{\Phi}[A_0]; T) \end{aligned}$$

Application of a Phenomenological Potential

$$\begin{aligned} \mathcal{L}_{\text{PQM}} &= \bar{q} \left[i\gamma_\mu (\partial^\mu - iA^\mu \delta_{\mu 0} + \mu_f \delta^{\mu 0}) - g \frac{\lambda_a}{2} (\sigma_a + i\gamma_5 \pi_a) \right] q \\ &\quad + \frac{1}{2} (\partial_\mu \sigma_a \partial^\mu \sigma_a + \partial_\mu \pi_a \partial^\mu \pi_a) - U(\sigma_a, \pi_a) - \mathcal{U}(\Phi[A_0], \bar{\Phi}[A_0]; T) \\ \mathcal{L}_{\text{PNJL}} &= \bar{q} [i\gamma_\mu (\partial^\mu - iA^\mu \delta_{\mu 0} + \mu_f \delta^{\mu 0}) - m_0] q + G [(\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2] \\ &\quad - \mathcal{U}(\Phi[A_0], \bar{\Phi}[A_0]; T) \end{aligned}$$

L. Haas, RS, J. Braun, J. Pawłowski and J. Schaffner-Bielich, Phys. Rev. D 87 (2013)

T. K. Herbst, M. Mitter, J. Pawłowski, B.-J. Schaefer and RS, Phys. Lett. B 731 (2014)



Captures the relevant properties

Application of a Phenomenological Potential

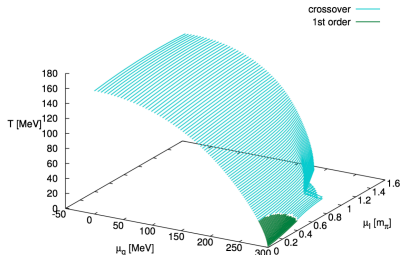
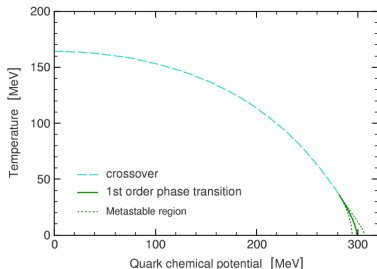
$$\mathcal{L}_{\text{PQM}} = \bar{q} \left[i \gamma_{\mu} (\partial^{\mu} - i A^{\mu} \delta_{\mu 0} + \mu_f \delta^{\mu 0}) - g \frac{\lambda_a}{2} (\sigma_a + i \gamma_5 \pi_a) \right] q$$

$$+ \frac{1}{2} (\partial_{\mu} \sigma_a \partial^{\mu} \sigma_a + \partial_{\mu} \pi_a \partial^{\mu} \pi_a) - U(\sigma_a, \pi_a) - \mathcal{U}(\Phi[A_0], \bar{\Phi}[A_0]; T)$$

$$\mathcal{L}_{\text{PNJL}} = \bar{q} [i \gamma_{\mu} (\partial^{\mu} - i A^{\mu} \delta_{\mu 0} + \mu_f \delta^{\mu 0}) - m_0] q + G [(\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2]$$

$$- \mathcal{U}(\Phi[A_0], \bar{\Phi}[A_0]; T)$$

... Sketch of the phase diagram and isospin dependence



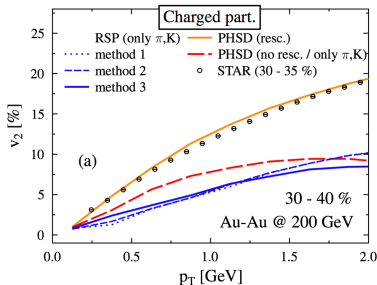
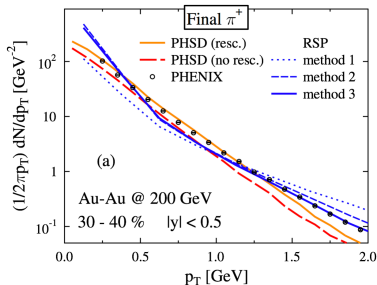
RS and J. Schaffner-Bielich, *Phys. Rev. D* 93 (2016)

RS, E. S. Fraga and J. Schaffner-Bielich, *arXiv:1307.2851v1*

Application of a Phenomenological Potential

$$\begin{aligned} \mathcal{L}_{\text{PQM}} &= \bar{q} \left[i\gamma_\mu (\partial^\mu - iA^\mu \delta_{\mu 0} + \mu_f \delta^{\mu 0}) - g \frac{\lambda_a}{2} (\sigma_a + i\gamma_5 \pi_a) \right] q \\ &\quad + \frac{1}{2} (\partial_\mu \sigma_a \partial^\mu \sigma_a + \partial_\mu \pi_a \partial^\mu \pi_a) - U(\sigma_a, \pi_a) - \mathcal{U}(\Phi[A_0], \bar{\Phi}[A_0]; T) \\ \mathcal{L}_{\text{PNJL}} &= \bar{q} [i\gamma_\mu (\partial^\mu - iA^\mu \delta_{\mu 0} + \mu_f \delta^{\mu 0}) - m_0] q + G [(\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2] \\ &\quad - \mathcal{U}(\Phi[A_0], \bar{\Phi}[A_0]; T) \end{aligned}$$

... Input to calculate transport properties in RHICs



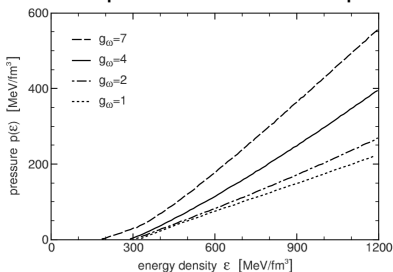
R. Marty, E. Bratkovskaya, W. Cassing and J. Aichelin, *Phys. Rev. C* 92 (2015)

Application of a Phenomenological Potential

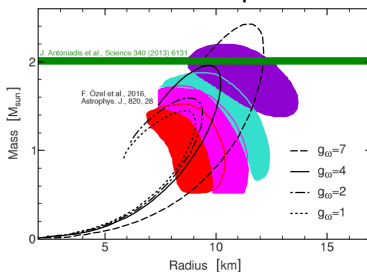
$$\mathcal{L}_{\text{PQM}} = \bar{q} \left[i \gamma_{\mu} (\partial^{\mu} - i A^{\mu} \delta_{\mu 0} + \mu_f \delta^{\mu 0}) - g \frac{\lambda_a}{2} (\sigma_a + i \gamma_5 \pi_a) \right] q \\ + \frac{1}{2} (\partial_{\mu} \sigma_a \partial^{\mu} \sigma_a + \partial_{\mu} \pi_a \partial^{\mu} \pi_a) - U(\sigma_a, \pi_a) - \mathcal{U}(\Phi[A_0], \bar{\Phi}[A_0]; T)$$

$$\mathcal{L}_{\text{PNJL}} = \bar{q} [i \gamma_{\mu} (\partial^{\mu} - i A^{\mu} \delta_{\mu 0} + \mu_f \delta^{\mu 0}) - m_0] q + G [(\bar{q}q)^2 + (\bar{q}i\gamma_5\bar{\tau}q)^2] \\ - \mathcal{U}(\Phi[A_0], \bar{\Phi}[A_0]; T)$$

... equation of state of quark matter inside compact stars

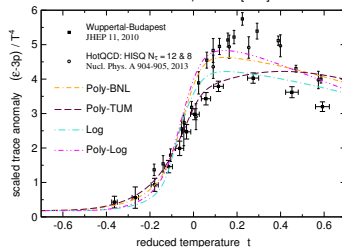
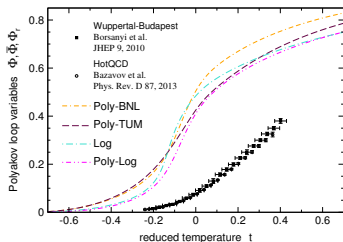
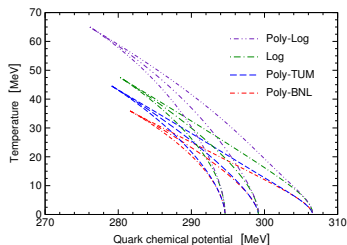
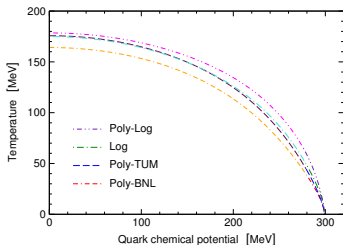


A. Zacchi, RS, J. Schaffner-Bielich, *Phys. Rev. D* 92 (2015)



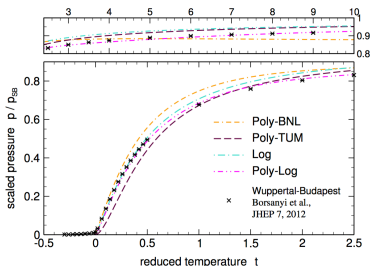
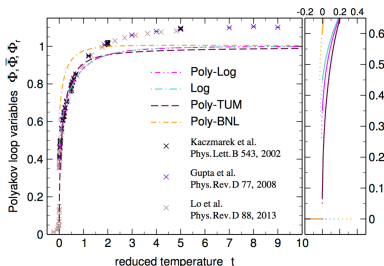
Problem with existing parameterisations

Results with different parameterisations ... differ



RS and J. Schaffner-Bielich, *Phys. Rev. D* 93 (2016)

Different parameterisations ... differ already in YM

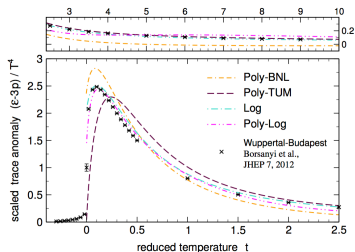


L. Haas, RS, J. Braun, J. M. Pawłowski and J. Schaffner-Bielich, *Phys. Rev. D* 87 (2013)

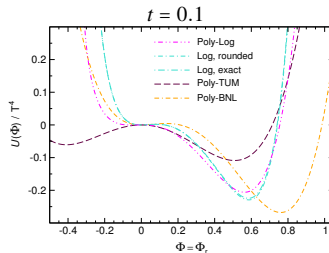
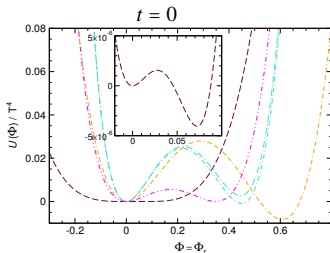
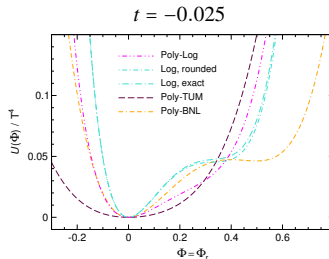
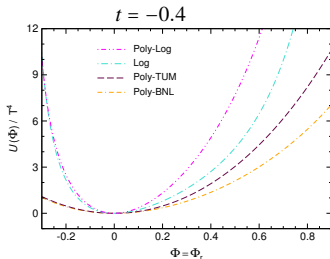
$$\mathcal{U}_{\text{poly}}(\Phi, \bar{\Phi}, t) / T^4 = p_2(t) \Phi \bar{\Phi} + p_3 (\Phi^3 + \bar{\Phi}^3) + p_4 (\Phi \bar{\Phi})^2$$

$$\mathcal{U}_{\text{log}}(\Phi, \bar{\Phi}, t) / T^4 = p_2(t) \Phi \bar{\Phi} + l(t) \ln [1 - 6\Phi \bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi \bar{\Phi})^2]$$

$$\mathcal{U}_{\text{poly+log}}(\Phi, \bar{\Phi}, t) / T^4 = p_2(t) \Phi \bar{\Phi} + p_3(t) (\Phi^3 + \bar{\Phi}^3) + p_4(t) (\Phi \bar{\Phi})^2 + l(t) \ln [1 - 6\Phi \bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi \bar{\Phi})^2]$$



Different parametrisations ... differ already in YM



RS, PhD thesis, 2014, Universität Heidelberg, <http://www.ub.uni-heidelberg.de/archiv/16887>

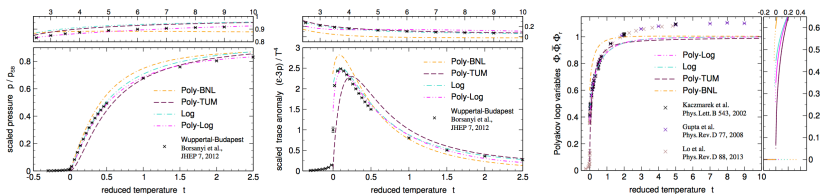
Determining Phenomenological Potentials ?

T-dependent parameters fitted to T-dependence of YM thermodynamics and Polyakov loop

$$\mathcal{U}_{\text{poly}}(\bar{\Phi}, \bar{\Phi}, t)/T^4 = p_2(t) \bar{\Phi} \bar{\Phi} + p_3(\bar{\Phi}^3 + \bar{\Phi}^3) + p_4(\bar{\Phi} \bar{\Phi})^2$$

$$\mathcal{U}_{\text{log}}(\bar{\Phi}, \bar{\Phi}, t)/T^4 = p_2(t) \bar{\Phi} \bar{\Phi} + l(t) \ln[1 - 6\bar{\Phi} \bar{\Phi} + 4(\bar{\Phi}^3 + \bar{\Phi}^3) - 3(\bar{\Phi} \bar{\Phi})^2]$$

$$\mathcal{U}_{\text{poly+log}}(\bar{\Phi}, \bar{\Phi}, t)/T^4 = p_2(t) \bar{\Phi} \bar{\Phi} + p_3(t)(\bar{\Phi}^3 + \bar{\Phi}^3) + p_4(t)(\bar{\Phi} \bar{\Phi})^2 + l(t) \ln[1 - 6\bar{\Phi} \bar{\Phi} + 4(\bar{\Phi}^3 + \bar{\Phi}^3) - 3(\bar{\Phi} \bar{\Phi})^2]$$



Poly-BNL: *O. Scavenius, A. Dumitru and J. Lenaghan, Phys. Rev. D 66 (2002)*

Poly-TUM: *C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D 73 (2006)*

Log: *S. Roessner, T. Hell, C. Ratti and W. Weise, Nucl. Phys. A 814 (2008)*

Poly+Log: *P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich, C. Sasaki, Phys. Rev. D 88 (2013)*

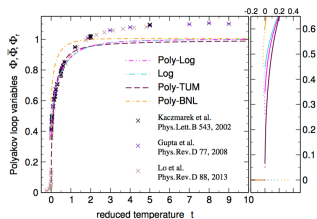
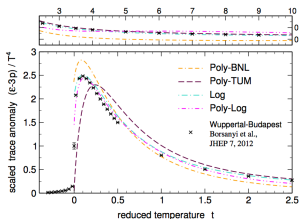
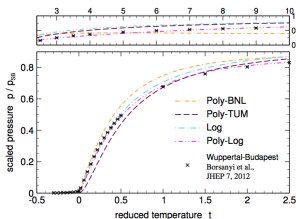
Existing Phenomenological Parameterisations

T-dependent parameters fitted to T-dependence of YM thermodynamics and Polyakov loop

$$\mathcal{U}_{\text{poly}}(\Phi, \bar{\Phi}, t)/T^4 = p_2(t) \Phi \bar{\Phi} + p_3(\Phi^3 + \bar{\Phi}^3) + p_4(\Phi \bar{\Phi})^2$$

$$\mathcal{U}_{\text{log}}(\Phi, \bar{\Phi}, t)/T^4 = p_2(t) \Phi \bar{\Phi} + l(t) \ln[1 - 6\Phi \bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi \bar{\Phi})^2]$$

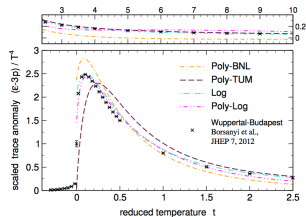
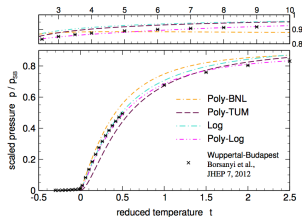
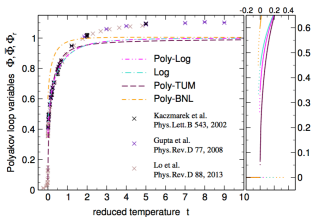
$$\mathcal{U}_{\text{poly+log}}(\Phi, \bar{\Phi}, t)/T^4 = p_2(t) \Phi \bar{\Phi} + p_3(t)(\Phi^3 + \bar{\Phi}^3) + p_4(t)(\Phi \bar{\Phi})^2 + l(t) \ln[1 - 6\Phi \bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi \bar{\Phi})^2]$$



→ not 'well' done

Existing Phenomenological Parameterisations

T-dependent parameters fitted to T-dependence of
YM thermodynamics and Polyakov loop



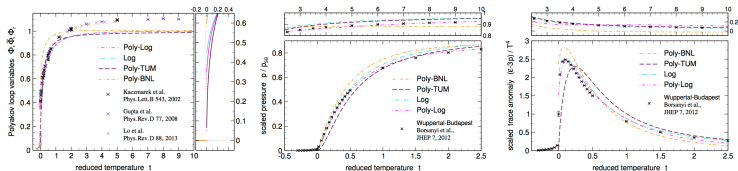
→ NO constraint from $T < T_c$: $\Phi = 0$, $p = -\mathcal{U} = 0$

$$\mathcal{U}_{\text{poly}}(\Phi = 0, t)/T^4 = 0, \quad \mathcal{U}_{\text{log}}(\Phi = 0, t)/T^4 = 0, \quad \mathcal{U}_{\text{poly+log}}(\Phi = 0, t)/T^4 = 0$$

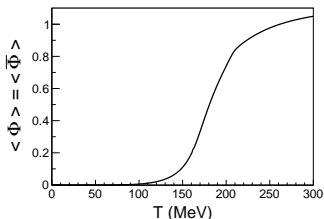
but at $T < T_c$ is transition in QCD: $\lesssim 160 \text{ MeV}$

Existing Phenomenological Parameterisations

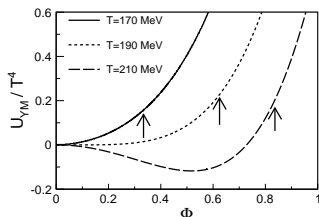
T-dependent parameters fitted to T-dependence of YM thermodynamics and Polyakov loop



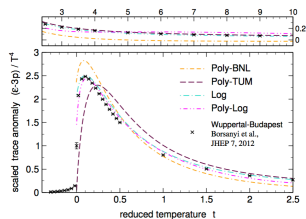
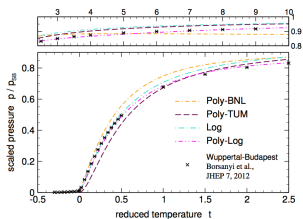
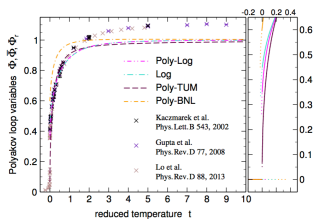
→ from $T \geq T_c$ only information from one single point: $(\Phi_{\min}, U/T^4(\Phi_{\min}))$, which is not the point which determines Φ in QCD!



J. M. Torres-Rincon and J. Aichelin, Phys. Rev. C 96 (2017)



Existing Phenomenological Parameterisations



→ not 'well' done

→ NO constraint from $T < T_c$: $\Phi = 0$, $p = -\mathcal{U} = 0$

→ from $T \geq T_c$ only information from one single point: $(\Phi_{\min}, \mathcal{U}/T^4(\Phi_{\min}))$, which is not the point which determines Φ in QCD!

⇒ Revise and improve phenomenological Polyakov-loop potentials

Revise phenomenological Polyakov-loop potentials

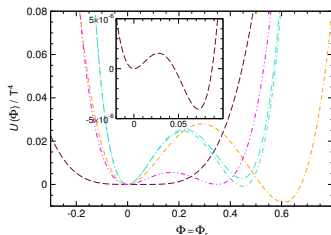
To obtain consistency between different parameterisations
 ... apply theoretical constraints

$$\lim_{T \rightarrow \infty} \Phi \rightarrow 1$$

$$\lim_{T \rightarrow \infty} p/T^4 = -\mathcal{U}/T^4 \rightarrow p_{SB}/T^4$$

$$\mathcal{U}/T^4 (\Phi_{\min} \neq 0, T = T_c) = \mathcal{U}/T^4 (\Phi = 0, T = T_c)$$

	$\Phi_{T \rightarrow \infty}$	$(-\mathcal{U}/p_{SB})_{T \rightarrow \infty}$
Poly-BNL	1.0	0.859
Poly-TUM	1.0	0.997
Log, exact	1.0	1.0
Log, rounded	1.0	1.0
Poly+Log	0.998	0.933

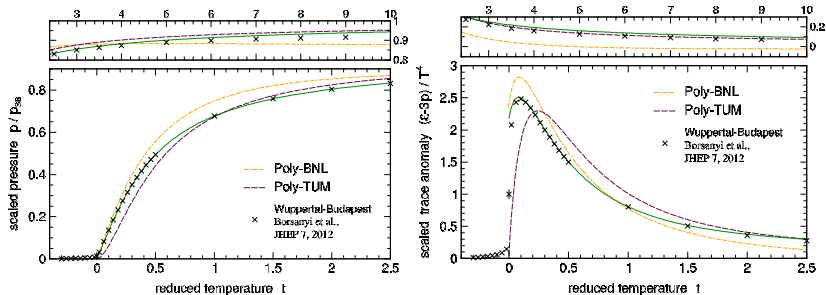


New fit to latest lattice data

To obtain consistency between different parameterisations
 ... apply theoretical constraints:

$$\lim_{T \rightarrow \infty} \Phi \rightarrow 1, \quad \lim_{T \rightarrow \infty} -\mathcal{U}/p_{\text{SB}} \rightarrow 1, \quad \mathcal{U}/T_c^4 (\Phi_{\text{min}} \neq 0) = \mathcal{U}/T_c^4 (\Phi = 0)$$

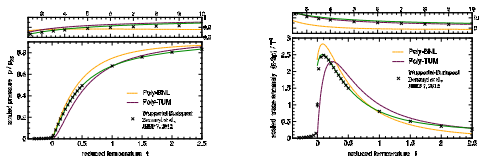
... do a fit to latest lattice data



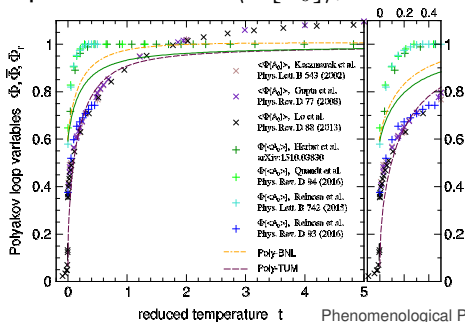
→ much better results for thermodynamics possible

New fit to latest lattice data

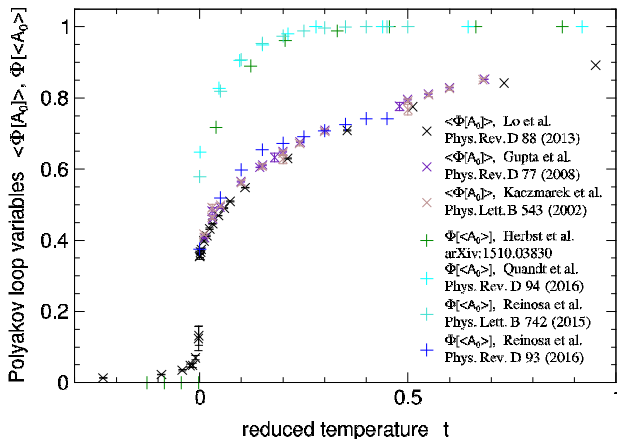
Reproducing thermodynamics ...



... Polyakov-loop neither lattice $\langle \Phi [A_0] \rangle$, nor continuum $\Phi [\langle A_0 \rangle]$



Lattice $\langle \Phi [A_0] \rangle$ vs. continuum $\Phi [\langle A_0 \rangle]$

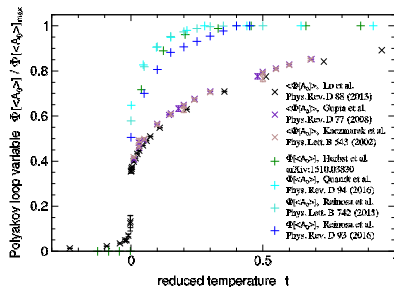
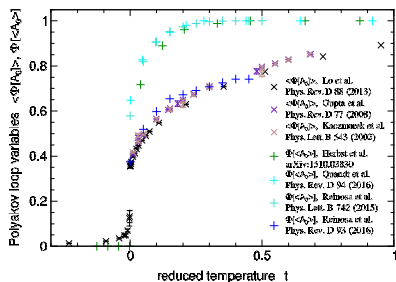


$$\Phi [\langle A_0 \rangle] \geq \langle \Phi [A_0] \rangle$$

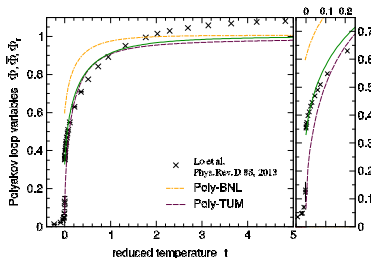
J. Braun, H. Gies and J. Pawłowski, Phys. Lett. B 684, 262-267, 2010

Continuum Polyakov loop $\Phi [\langle A_0 \rangle]$

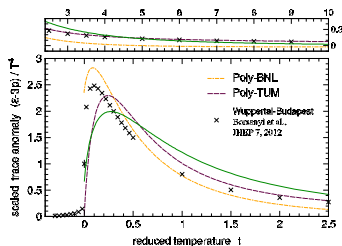
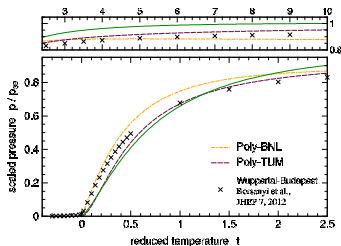
renormalisation factor of $\Phi [\langle A_0 \rangle]$?



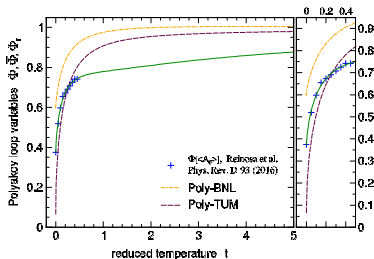
Fitting lattice $\langle \Phi [A_0] \rangle \dots$



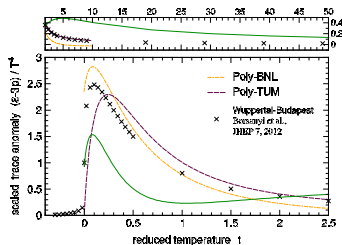
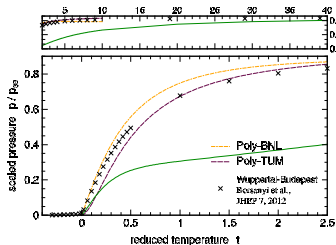
... no agreement to thermodynamics



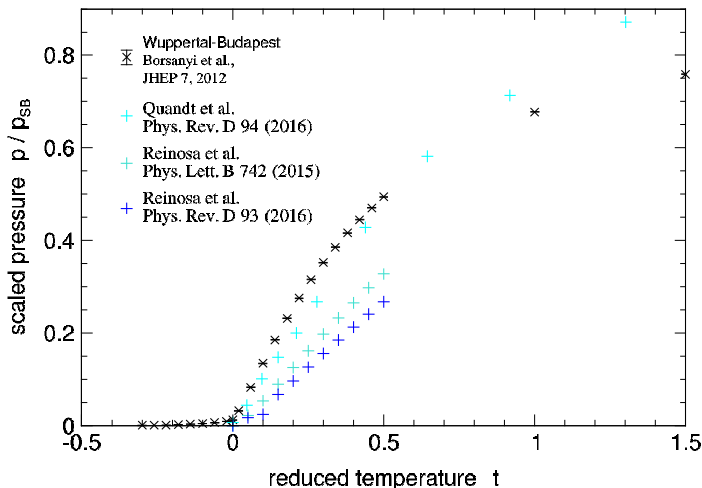
Fitting continuum perturbative NLO $\Phi[\langle A_0 \rangle] \dots$



... no agreement to thermodynamics

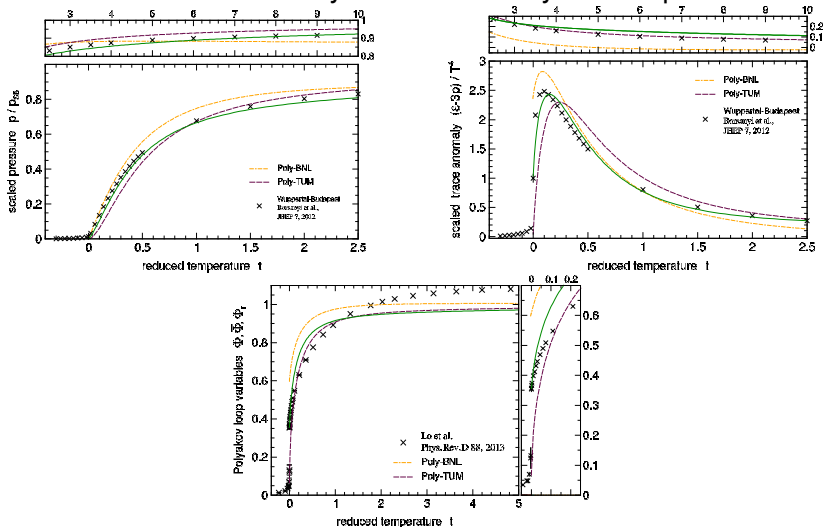


Thermodynamics in first-principle continuum



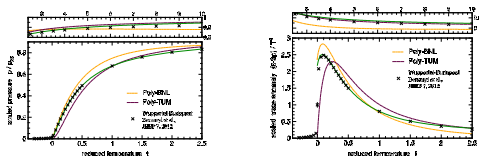
New fit to latest lattice data

... to thermodynamics and Polyakov loop

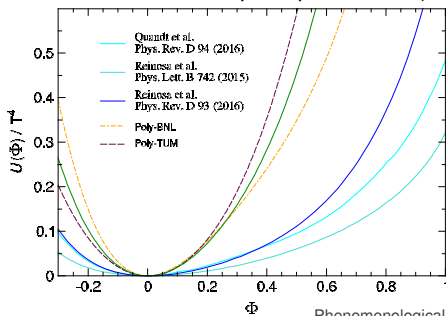


New fit to latest lattice data

Reproducing thermodynamics ...



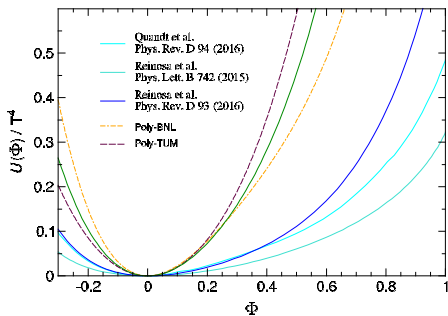
... shape of potential different from first-principle ones (e.g. @ $t = -0.15$)



New fit to latest lattice data

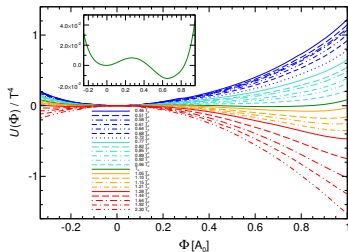
Reproducing thermodynamics ...

... shape of potential different from first-principle ones (e.g. @ $t = -0.15$)



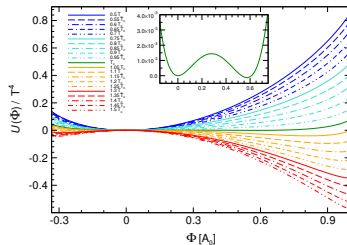
→ far steeper \Rightarrow different Φ in QCD

Polyakov-loop Potential in first-principle continuum



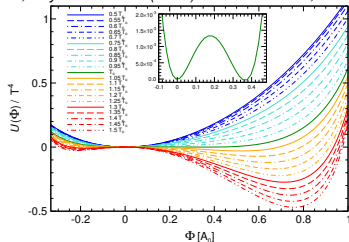
covariant approach

M. Quandt and H. Reinhardt, *Phys. Rev. D* 94 (2016)



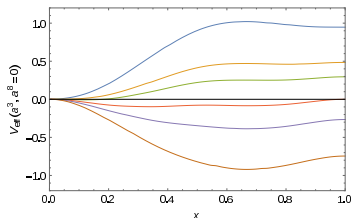
LO pert. theory

U. Reinosa, et al., *Phys. Lett. B* 742 (2015)



NLO pert. theory: U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, *Phys. Rev. D* 93 (2016)

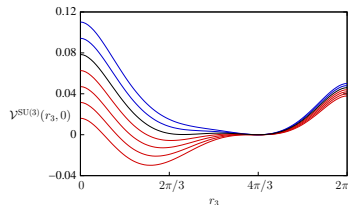
Gauge-field Potential in first-principle continuum



covariant approach

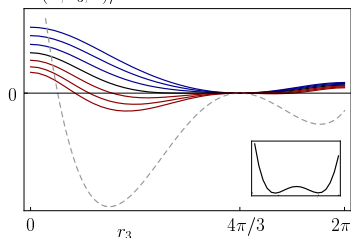
M. Quandt and H. Reinhardt, *Phys. Rev. D* 94 (2016)

$$V(T, r_3, 0)/T^4$$



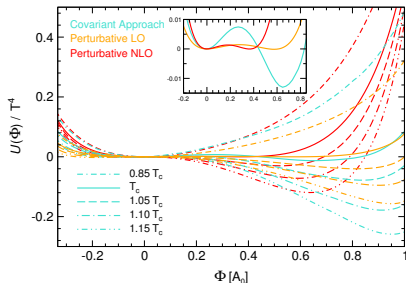
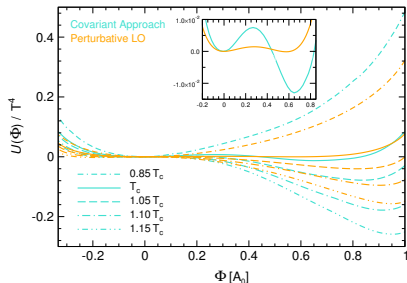
LO pert. theory

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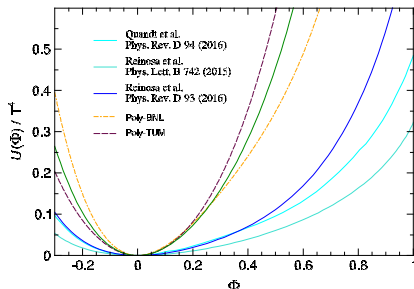
Polyakov-loop Potential in first-principle continuum



Revise phenomenological Polyakov-loop potentials

Reproducing thermodynamics ...

... shape of potential different from first-principle ones



→ adjusting to first-principle potentials includes information away from the minimum

Revise phenomenological Polyakov-loop potentials

Reproducing thermodynamics and first-principle continuum potential . . .

- include information of $T > T_c$ (thermodynamics) and $T \leq T_c$ (shape of potential)
- include information of global shape of potential

Revise phenomenological Polyakov-loop potentials

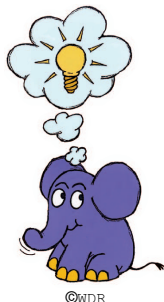
Reproducing thermodynamics and first-principle continuum potential . . .

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Conclusions

- Effective Polyakov-loop potential for phenomenological investigation of QCD
 - Existing parameterisations differ between each other and from latest lattice results =
 - Are only adjusted to deconfined phase → no information from temperature range relevant for QCD transition
 - Only one single point of potential is adjusted, the minimum
→ Potential differs from calculated ones in first-principle, continuum calculations
- ⇒ Try to reproduce total shape of potential of first-principle, continuum calculations



Thank You for your attention!

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