The massive Landau-DeWitt gauge at finite temperature and density

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Introduction

The Landau gauge is hampered by the Gribov ambiguity
$\Rightarrow$ Faddeev-Popov action needs to be modified a priori.

One possible model is

$$S = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{c}^a (D_\mu c)^a + i h^a \partial_\mu A^a_\mu + \frac{1}{2} m^2 A^a_\mu A^a_\mu \right\}$$

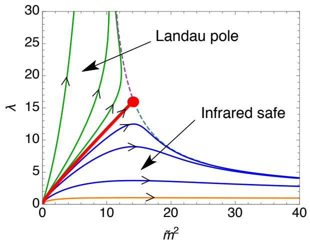
$\Rightarrow$ CF model for $\xi = 0$ or “massive extension of the Landau gauge”.

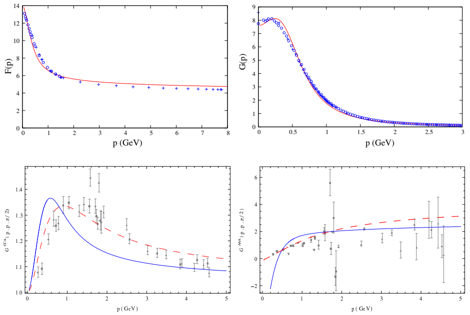
Introduction

$$S = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{c}^a (D_\mu c)^a + i h^a \partial_\mu A^a_\mu + \frac{1}{2} m^2 A^a_\mu A^a_\mu \right\}$$

The model seems to capture many properties of vacuum Landau-gauge correlation functions, from an IR-safe perturbative approach.

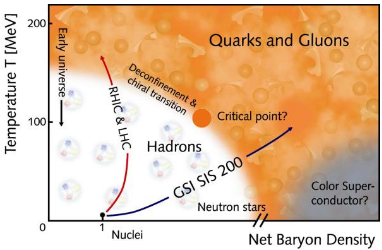
[→ talk by Matthieu Tissier]

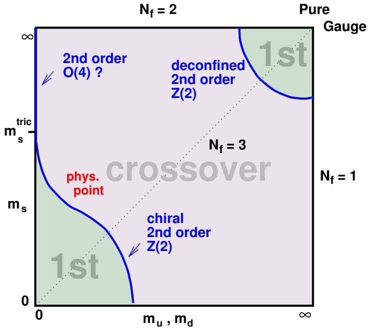




Introduction

What has this approach to tell us about properties at finite temperature and density? Confinement and χSB?





This talks deals with the confinement/deconfinement transition.

[χSB aspects $\rightarrow$ talk by Marcela Peláez]

Outline

I. The massive extension at finite temperature and density.

II. Review of results.

III. Some open issues.

I. The massive Landau gauge at finite temperature and density

I. The massive Landau-DeWitt gauge at finite temperature and density

Deconfinement and Center symmetry breaking

In the pure Yang-Mills theory, deconfinement is understood as the spontaneous breaking of center symmetry.

What is center symmetry? The YM action is invariant under

$$(A^U)^a_\mu(\tau,\vec{x})t^a \equiv U(\tau,\vec{x})A^a_\mu(\tau,\vec{x})t^a U^\dagger(\tau,\vec{x}) + \frac{i}{g}U(\tau,\vec{x})\partial_\mu U^\dagger(\tau,\vec{x})$$

At finite $T \equiv 1/\beta$, the periodicity of the gauge field implies

$$\underbrace{U(\tau+\beta,\vec{x}) = Z\,U(\tau,\vec{x})}_{\text{twisted gauge transformations}} \quad \text{with} \quad Z \in \underbrace{\{\mathbb{1}, e^{i2\pi/3}\mathbb{1}, e^{i4\pi/3}\mathbb{1}\}}_{\text{center of SU(3)}} \simeq \mathbb{Z}_3$$

- $Z = \mathbb{1}$ (periodic): usual, unphysical, gauge transformations.
- $\mathcal{G}_{\text{twisted}}/\mathcal{G}_{\text{periodic}} \simeq \mathbb{Z}_3$: physical center transformations.

Deconfinement and Center symmetry breaking

In the pure Yang-Mills theory, deconfinement is understood as the spontaneous breaking of center symmetry.

What has this to do with the deconfinement transition? The free energy cost ΔF_q for bringing in a quark is an order parameter for center symmetry:

1) $$e^{-\beta\Delta F_q} = \frac{1}{N_c}\left\langle \underbrace{\mathrm{tr}\, P\, e^{ig\int_0^\beta d\tau\, A_0^a(\tau,\vec{x})t^a}}_{\equiv L_q} \right\rangle \equiv \ell_q$$ [a.k.a. Polyakov loop]

2) Under a center transformation $L_q \to Z L_q$. Then:
- if center symmetry is not broken, $\ell_q = 0$, $\Delta F_q = \infty$
- if center symmetry is broken, $\ell_q \neq 0$, $\Delta F_q < \infty$

Deconfinement $\Leftrightarrow$ Center symmetry breaking.

Center symmetry and Gauge-fixing

Can one study center symmetry breaking within a continuum setting?

Problems:

1) Continuum approaches require gauge-fixing.
   In such a context, center symmetry is usually not explicit.

2) The Polyakov loop is an intricate object.
   Can one define equivalent but simpler order parameters?

Solution: Generalize the gauge-fixing by including a background.

[J. Braun, H. Gies, J.M. Pawlowski, Phys.Lett. B684 (2010) 262-267]

The Landau-DeWitt (LDW) gauge

Idea: replace the Landau gauge-fixing condition $0 = \partial_\mu A_\mu^a$ by a covariant version of it

$$0 = \bar{D}_\mu^{ab}(A_\mu^b - \bar{A}_\mu^b) \equiv (\partial_\mu \delta^{ab} + g f^{acb} \bar{A}_\mu^c)(A_\mu^b - \bar{A}_\mu^b)$$

with $\bar{A}_\mu^a$ some given field configuration (background).

The gauge-fixed action reads

$$S_{\bar{A}}[A] = \int d^4x \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{D}_\mu \bar{c}^a (D_\mu c)^a + i h^a \bar{D}_\mu (A - \bar{A})_\mu^a \right\}$$

Center symmetry is manifest in the sense that: $S_{\bar{A}^U}[A^U] = S_{\bar{A}}[A]$.

This property is inherited by the quantum action: $\Gamma_{\bar{A}^U}[A^U] = \Gamma_{\bar{A}}[A]$.

⚠ Not enough however to grant a good handle on center symmetry.

States and Symmetries

Discussing the breaking of a physical symmetry $\mathcal{S}$ requires one to first identify the invariant states under this symmetry.

Problem: A priori, it is not easy to do so in terms of $\Gamma_{\bar{A}}[A]$:

1) the state of the system is obtained by minimizing $\Gamma_{\bar{A}}[A]$ with respect to A at fixed $\bar{A}$. Let us denote it by $A_{\rm min}(\bar{A})$;

2) if $\mathcal{S}$ is such that $\Gamma_{\bar{A}^{\mathcal{S}}}[A^{\mathcal{S}}] = \Gamma_{\bar{A}}[A]$, then

$$A^{\mathcal{S}}_{\rm min}(\bar{A}) = A_{\rm min}(\bar{A}^{\mathcal{S}^{-1}})$$

$\Rightarrow$ the transformation $\mathcal{S}$ connects states described in different gauges ($\neq \bar{A}$'s). This makes it difficult to identify the invariant states.

States and Symmetries

Discussing the breaking of a physical symmetry $\mathcal{S}$ requires one to first identify the invariant states under this symmetry.

Solution: work not with arbitrary backgrounds but with backgrounds $\bar{A}_s$ obeying the self-consistent condition $\bar{A}_s = \langle A \rangle_{\bar{A}_s}$. Now,

1) the $\bar{A}_s$ are the absolute minima of $\tilde{\Gamma}[\bar{A}] \equiv \Gamma_{\bar{A}}[A = \bar{A}]$;

2) the functional $\tilde{\Gamma}[\bar{A}]$ is symmetric under $\mathcal{S}$, $\tilde{\Gamma}[\bar{A}^{\mathcal{S}}] = \tilde{\Gamma}[\bar{A}]$.

$\Rightarrow$ a minimum of $\tilde{\Gamma}[\bar{A}]$ (a state) is transformed by $\mathcal{S}$ into another minimum (another state) of the same functional.

States and Symmetries

Discussing the breaking of a physical symmetry $\mathcal{S}$ requires one to first identify the invariant states under this symmetry.

There is a final subtlety however: $\tilde{\Gamma}[\bar{A}]$ is invariant under unphysical transformations, the periodic gauge transformations $(Z = \mathbb{1})$.

It follows that the state of the system is not described by a single minimum $\bar{A}_s$ of $\tilde{\Gamma}[\bar{A}]$ but rather by the whole orbit $\{\bar{A}_s^U | U \in \mathcal{G}_{Z=\mathbb{1}}\}$.

Therefore, the invariant states correspond to the invariant orbits. In other words, a given $\bar{A}_s$ represents an invariant state if it is invariant but only modulo a periodic gauge tranformation: $\bar{A}_s^{\mathcal{S}} = \bar{A}_s^U$ for some $U \in \mathcal{G}_{Z=\mathbb{1}}$.

Example: Finding the center-symmetric state

We restrict to homogeneous and isotropic backgrounds:

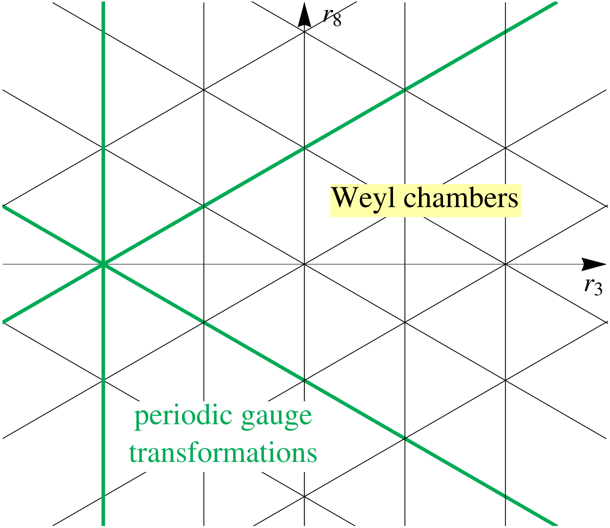
$$\bar{A}_\mu(\tau, \vec{x}) = \delta_{\mu 0} \bar{A}$$

Without loss of generality, we can take $\bar{A}$ in the Cartan subalgebra:

$$g\bar{A} = T \left(r_3 \frac{\lambda_3}{2} + r_8 \frac{\lambda_8}{2} \right)$$

The evaluation of $\tilde{\Gamma}[\bar{A}]$ boils down to that of a potential $V(r_3, r_8)$.

Example: Finding the center-symmetric state



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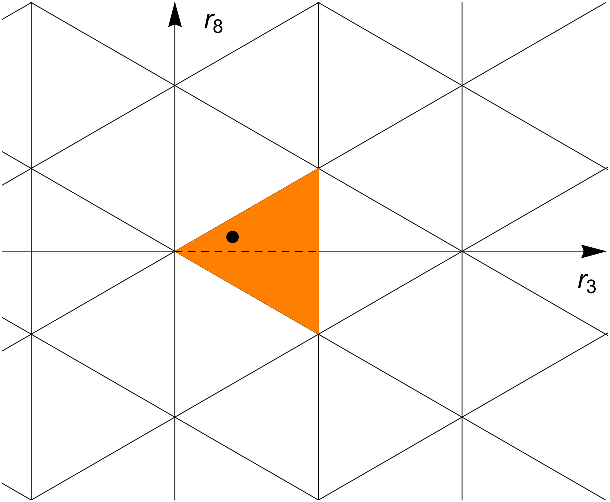

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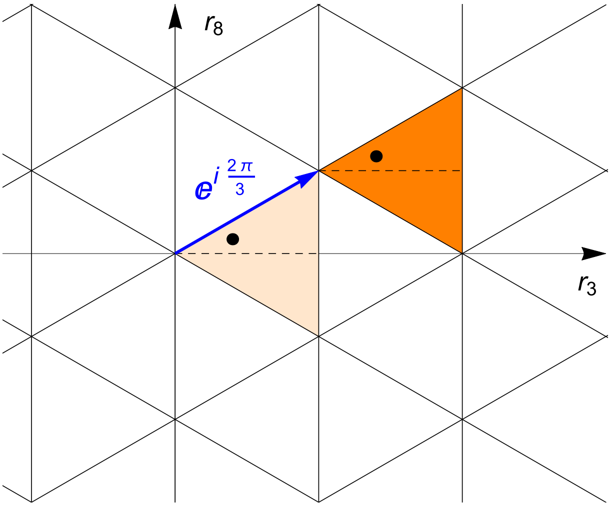

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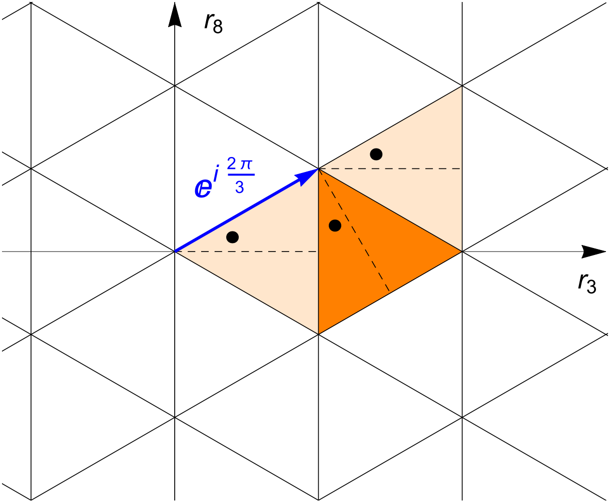
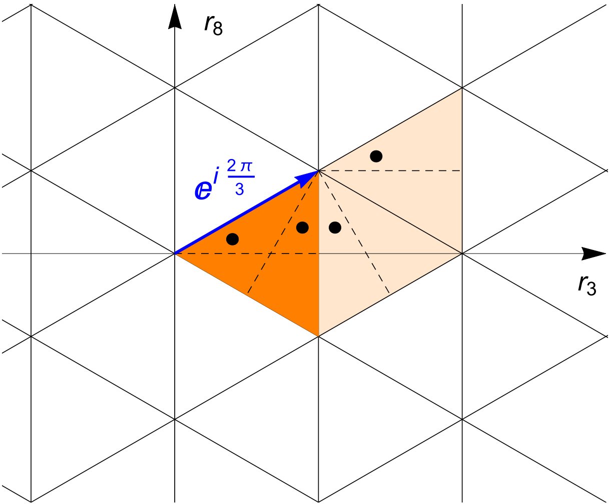


Example: Finding the center-symmetric state



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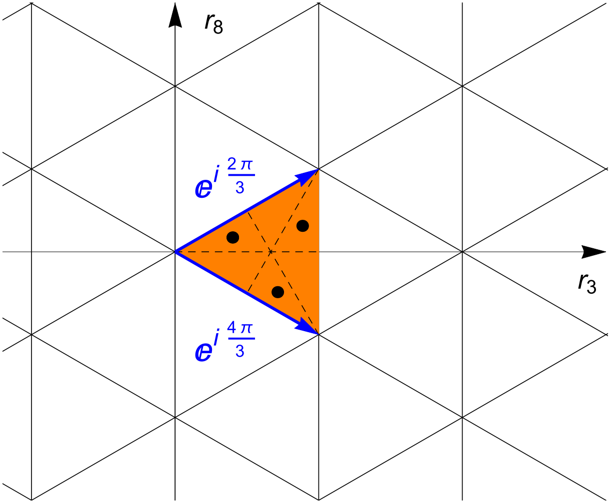

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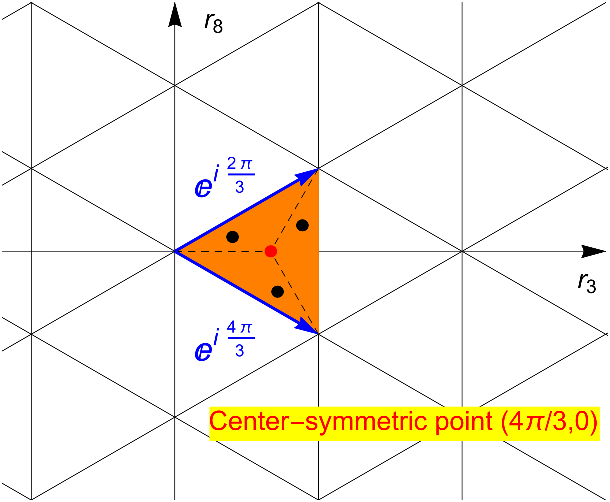

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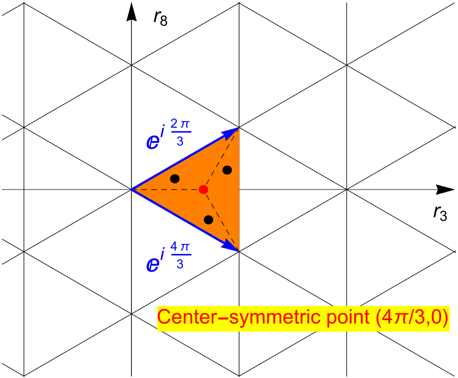
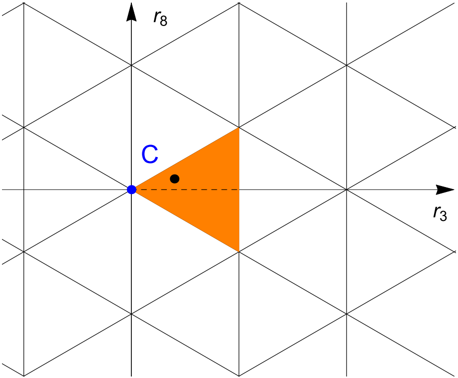


Conclusion: the self-consistent backgrounds play the role of order parameters for center symmetry and thus for the deconfinement transition.

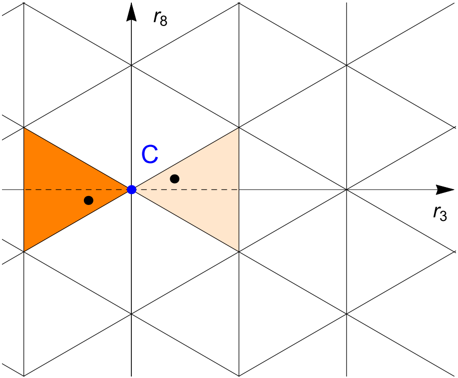
Finding the C-symmetric states



$$\bar{A}^C = -A^{\mathrm{t}} = -A^*$$

Similar considerations apply to any other physical symmetry, such as C.

Finding the C-symmetric states



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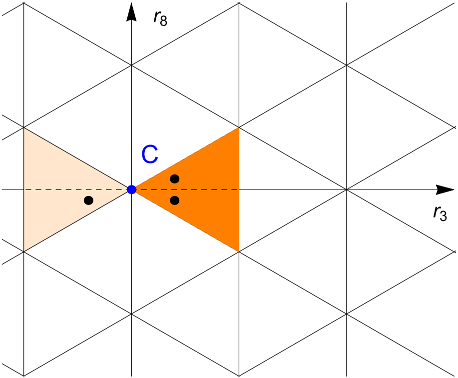

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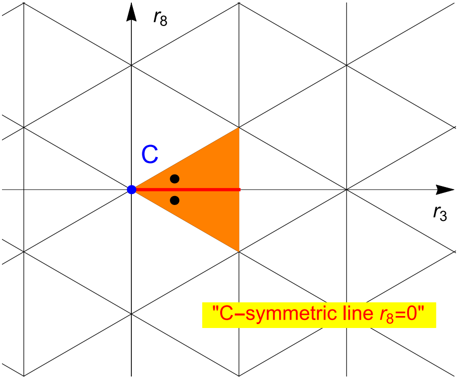


$$\bar{A}^C = -A^{\mathrm{t}} = -A^*$$

The red segment corresponds to the C-invariant states.

Massive extension of the LDW gauge

In view of the success of the massive extension of the Landau gauge in the vacuum ...

$$\mathcal{L} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \partial_\mu \bar{c}^a (D_\mu c)^a + ih^a \partial_\mu A^a_\mu + \frac{1}{2}m^2 A^a_\mu A^a_\mu$$

... we try the same type of extension with the LDW gauge at finite temperature:

$$\mathcal{L} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{D}_\mu \bar{c}^a (D_\mu c)^a + ih^a \bar{D}_\mu (A - \bar{A})^a_\mu + \frac{1}{2}m^2 (A - \bar{A})^a_\mu (A - \bar{A})^a_\mu$$

The mass term is introduced in such a way that the invariance properties $S_{\bar{A}^U}[A^U] = S_{\bar{A}}[A]$ and $\Gamma_{\bar{A}^U}[A^U] = \Gamma_{\bar{A}}[A]$ remain true.

I now review the perturbative predictions of this model at finite T.

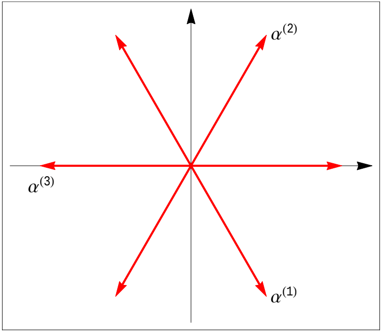
II. Review of results

Pure YM: massive LDW gauge at one-loop

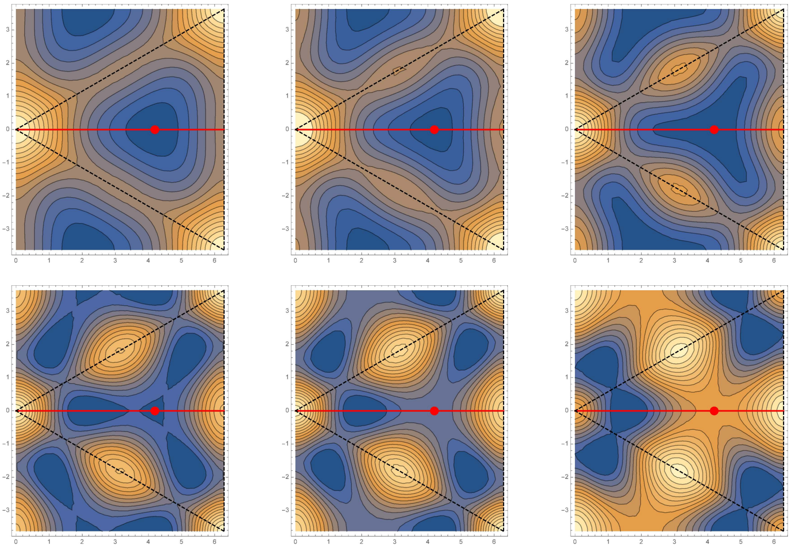
$$V(r) = \frac{3}{2}\sum_{\kappa} T \sum_{n\in\mathbb{N}} \int \frac{d^3q}{(2\pi)^3} \ln\left[(\omega_n + T\kappa\cdot r)^2 + q^2 + m^2\right] \leftarrow \text{gluons}$$

$$- \frac{1}{2}\sum_{\kappa} T \sum_{n\in\mathbb{N}} \int \frac{d^3q}{(2\pi)^3} \ln\left[(\omega_n + T\kappa\cdot r)^2 + q^2\right] \leftarrow \text{ghosts}$$

with $r \equiv (r_3, r_8)$ and $\kappa \in \{0, 0, -\alpha^{(1)}, \alpha^{(1)}, -\alpha^{(2)}, \alpha^{(2)}, -\alpha^{(3)}, \alpha^{(3)}\}$:

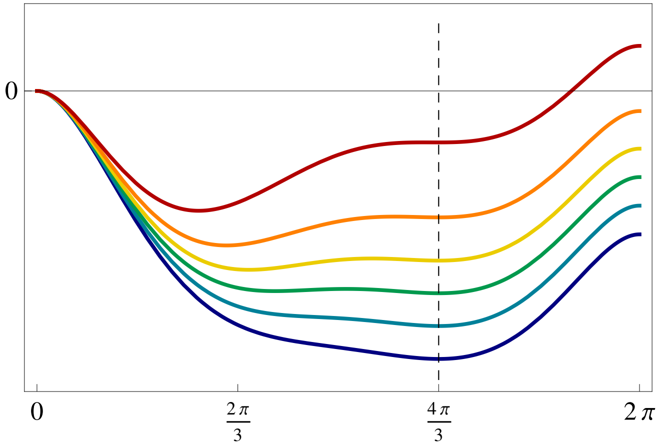


Pure YM: massive LDW gauge at one-loop



Pure YM: massive LDW gauge at one-loop

We obtain a first order phase transition:



[UR, J. Serreau, M. Tissier and N. Wschebor, Phys.Lett. B742 (2015) 61-68]

Pure YM: massive LDW gauge at one-loop

Why is this working?

$$V(r) = \frac{3}{2}\sum_\kappa T\sum_{n\in\mathbb{N}}\int\frac{d^3q}{(2\pi)^3}\ln\left[(\omega_n + T\kappa\cdot r)^2 + q^2 + m^2\right] \leftarrow \text{gluons}$$

$$- \frac{1}{2}\sum_\kappa T\sum_{n\in\mathbb{N}}\int\frac{d^3q}{(2\pi)^3}\ln\left[(\omega_n + T\kappa\cdot r)^2 + q^2\right] \leftarrow \text{ghosts}$$

$T \gg m$: Weiss potential, the confining point is the absolute maximum

$$\left(\frac{3}{2}-\frac{1}{2}\right)\sum_\kappa T\sum_{n\in\mathbb{N}}\int\frac{d^3q}{(2\pi)^3}\ln\left[(\omega_n + T\kappa\cdot r)^2 + q^2\right]$$

$T \ll m$: Inverted Weiss potential, the confining point becomes the minimum

$$-\frac{1}{2}\sum_\kappa T\sum_{n\in\mathbb{N}}\int\frac{d^3q}{(2\pi)^3}\ln\left[(\omega_n + T\kappa\cdot r)^2 + q^2\right]$$

In line with the confinement scenario of [J. Braun, H. Gies, J.M. Pawlowski]

Pure YM: Summary of one-loop results

order	lattice	fRG	variational	CF at 1-loop
SU(2)	2nd	2nd	2nd	2nd
SU(3)	1st	1st	1st	1st
SU(4)	1st	1st	??	1st
Sp(2)	1st	1st	??	1st

T_c (MeV)	lattice	fRG(*)	variational(**)	CF at 1-loop(***)
SU(2)	295	230	239	238
SU(3)	270	275	245	185

(*) L. Fister and J. M. Pawlowski, Phys.Rev. D88 (2013) 045010.

(**) M. Quandt and H. Reinhardt, Phys.Rev. D94 (2016) no.6, 065015.

(***) UR, J. Serreau, M. Tissier and N. Wschebor, Phys.Lett. B742 (2015) 61-68.

Pure YM: Summary of two-loop results

order	lattice	fRG	variational	CF at 1-loop	CF at 2-loop
SU(2)	2nd	2nd	2nd	2nd	2nd
SU(3)	1st	1st	1st	1st	1st
SU(4)	1st	1st	??	1st	1st
Sp(2)	1st	1st	??	1st	1st

T_c (MeV)	lattice	fRG(*)	variational(**)	CF at 1-loop	CF at 2-loop(***)
SU(2)	295	230	239	238	284
SU(3)	270	275	245	185	254

(*) L. Fister and J. M. Pawlowski, Phys.Rev. D88 (2013) 045010.

(**) M. Quandt and H. Reinhardt, Phys.Rev. D94 (2016) no.6, 065015.

(***) UR, J. Serreau, M. Tissier and N. Wschebor, Phys.Rev. D93 (2016) 105002.

Adding quarks: Symmetries

$$S = S_{\rm mLDW} + \sum_{f=1}^{N_f} \int_x \left\{ \bar{\psi}_f (\partial\!\!\!/ - ig A\!\!\!/^a t^a + M_f - \mu\gamma_0)\psi_f \right\}$$

Quarks transform in the fundamental representation:

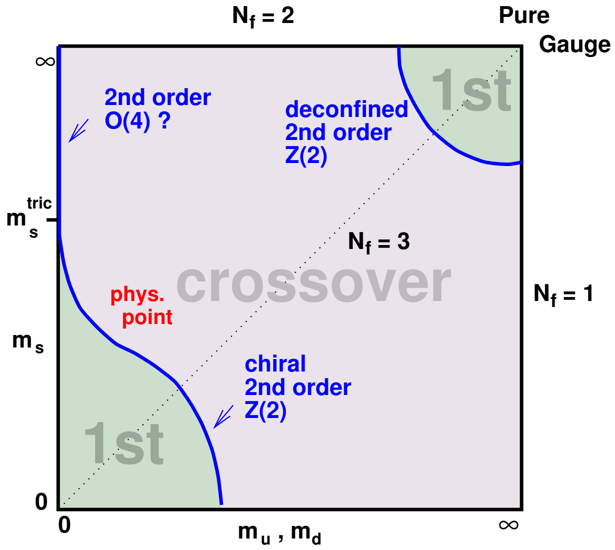
$$\psi^U(\tau, \vec{x}) = U(\tau, \vec{x})\psi(\tau, \vec{x})$$

Center symmetry is broken due to the boundary conditions:

$$U(\beta, \vec{x}) = e^{i2\pi/3} U(0, \vec{x}) \Rightarrow \psi^U(\beta, \vec{x}) = -e^{i2\pi/3}\psi^U(0, \vec{x})$$

Charge conjugation is also broken as soon as $\mu \neq 0$. Then $r_8 \neq 0$.

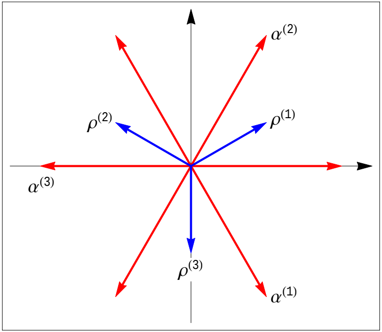
Adding quarks: Columbia Plot at $\mu = 0$



$\mu = 0$: massive LDW gauge at one-loop

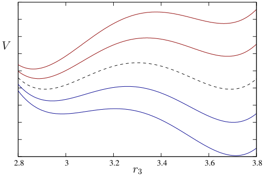
$$\delta V(r) = -\sum_f \sum_\rho T \sum_{n \in \mathbb{N}+1/2} \int \frac{d^3p}{(2\pi)^3} \ln \left[(\omega_n + i\mu + T\rho \cdot r)^2 + p^2 + M_f^2 \right]$$

with $r \equiv (r_3, r_8)$ and $\rho \in \{\rho^{(1)}, \rho^{(2)}, \rho^{(3)}\}$:



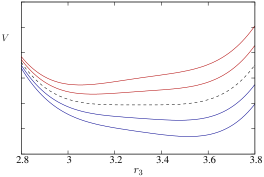
$\mu = 0$: massive LDW gauge at one-loop

$\underline{M > M_c}$



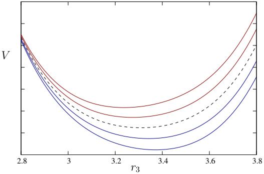
first order

$\underline{M = M_c}$



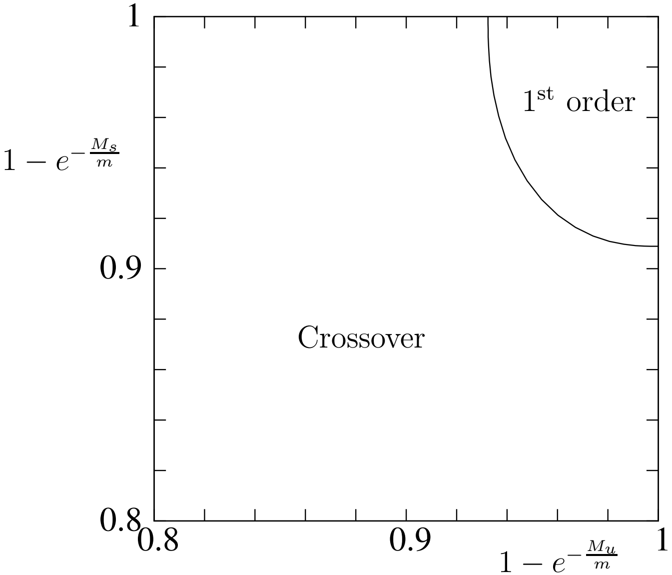
second order

$\underline{M < M_c}$



crossover

$\mu = 0$: massive LDW gauge at one-loop



$\mu = 0$: massive LDW gauge at one-loop

N_f	$(M_c/T_c)^{\mathrm{CF}}$ (i)	$(M_c/T_c)^{\mathrm{lattice}}$ (ii)	$(M_c/T_c)^{\mathrm{matrix}}$ (iii)	$(M_c/T_c)^{\mathrm{SD}}$ (iv)
1	6.74	7.22	8.04	1.42
2	7.59	7.91	8.85	1.83
3	8.07	8.32	9.33	2.04

(i) UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015).

(ii) M. Fromm, J. Langelage, S. Lottini and O. Philipsen, JHEP 1201 (2012) 042.

(iii) K. Kashiwa, R. D. Pisarski and V. V. Skokov, Phys.Rev. D85 (2012) 114029.

(iv) C. S. Fischer, J. Luecker and J. M. Pawlowski, Phys.Rev. D91 (2015) 1, 014024.

$\mu = 0$: massive LDW gauge at two-loop

At two-loop order, the comparison to lattice data is tricky since the quark mass M is scheme dependent. To reduce scheme dependences we can compare ratios of $R_{N_f} = M_c(N_f)/T_c(N_f)$ at various values of N_f.

R_{N_f}	$N_f = 1$	$N_f = 2$	$N_f = 3$	R_2/R_1	R_3/R_1
CF 1-loop$^{(i)}$	6.74	7.59	8.07	1.12	1.20
CF 2-loop$^{(ii)}$	7.53	8.40	8.90	1.11	1.18
Lattice$^{(iii)}$	7.23	7.92	8.33	1.10	1.15
Matrix$^{(iv)}$	8.04	8.85	9.33	1.10	1.16
DS$^{(v)}$	1.42	1.83	2.04	1.29	1.43

$^{(i)}$ UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015).

$^{(ii)}$ J. Maelger, UR and J. Serreau, arXiv:1710.01930.

$^{(iii)}$ M. Fromm, J. Langelage, S. Lottini and O. Philipsen, JHEP 1201 (2012) 042.

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CF 2-loop[(ii)]	7.53	8.40	8.90	1.11	1.18
Lattice[(iii)]	7.23	7.92	8.33	1.10	1.15
Matrix[(iv)]	8.04	8.85	9.33	1.10	1.16
DS[(v)]	1.42	1.83	2.04	1.29	1.43

(i) UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015).

(ii) J. Maelger, UR and J. Serreau, arXiv:1710.01930.

(iii) M. Fromm, J. Langelage, S. Lottini and O. Philipsen, JHEP 1201 (2012) 042.

(iv) K. Kashiwa, R. D. Pisarski and V. V. Skokov, Phys.Rev. D85 (2012) 114029.

(v) C. S. Fischer, J. Luecker and J. M. Pawlowski, Phys.Rev. D91 (2015) 1, 014024.

Imaginary chemical potential

$$S = S_{\mathrm{mLDW}} + \sum_{f=1}^{N_f} \int_x \left\{ \bar{\psi}_f (\not{\partial} - ig \not{A}^a t^a + M_f - (\quad \mu \qquad\qquad) \gamma_0) \psi_f \right\}$$

Center symmetry is explicitely broken: $\psi^U(\beta, \vec{x}) = -e^{i2\pi/3} \psi^U(0, \vec{x})$.

However this can be compensated by an abelian transformation

$$(\psi^U)'(\tau, \vec{x}) = e^{-i(2\pi/3)(\tau/\beta)} \psi^U(\tau, \vec{x})$$

The generated abelian gauge field corresponds to a shift of μ.

Combining this with a C transformation that changes μ to $-\mu$, one finds a symmetry $\mu \to i2\pi/3 - \mu$ with a fixed-point at $\mu = i\pi/3$.
$\Rightarrow$ Roberge Weiss symmetry.

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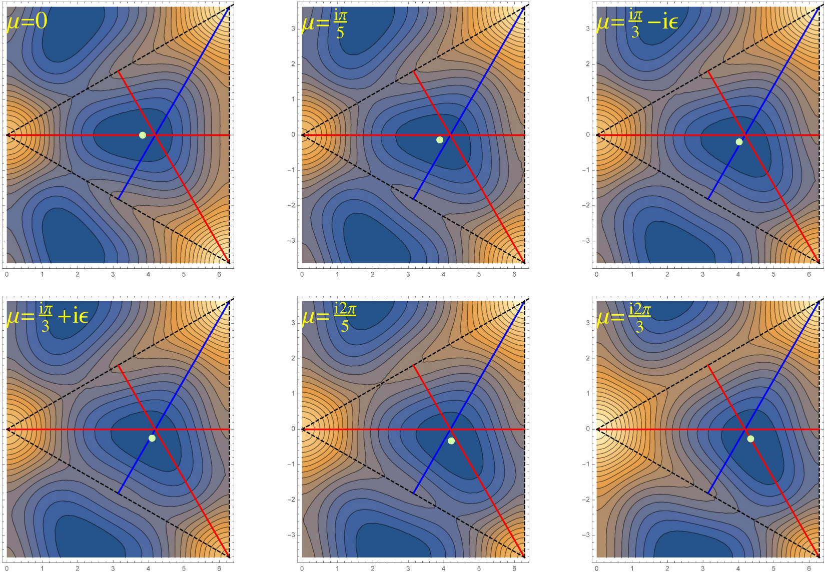
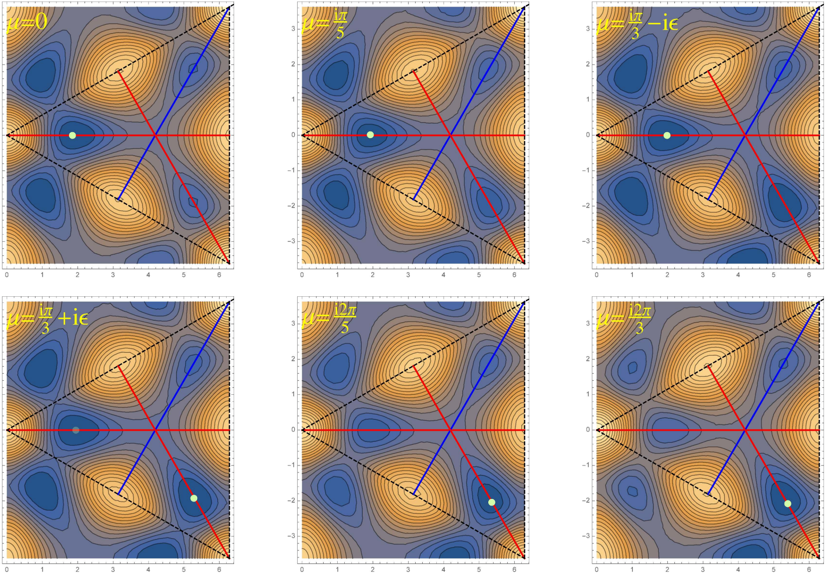
Imaginary μ: massive LDW gauge at one-loop

Low T:

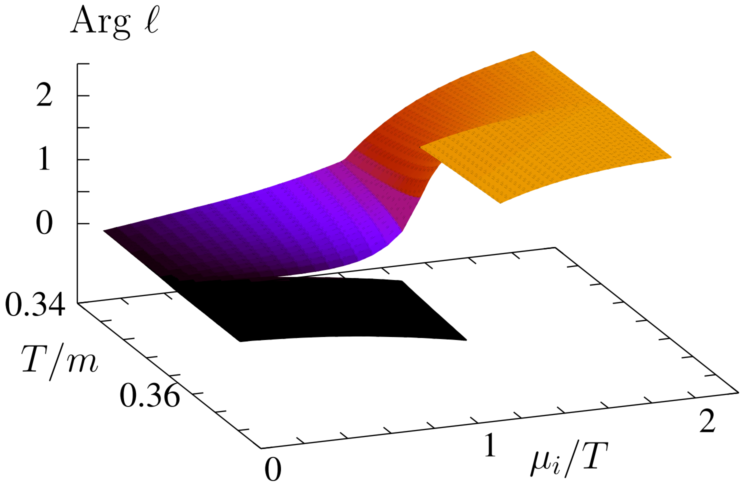


Imaginary μ: massive LDW gauge at one-loop

High T:

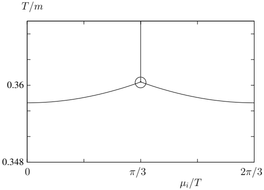


Imaginary μ: massive LDW gauge at one-loop

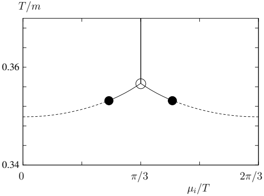


Imaginary μ: massive LDW gauge at one-loop

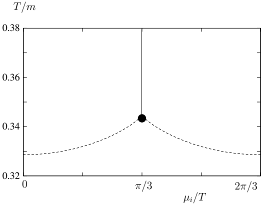
$M > M_c(0)$:



$M \in [M_c(i\pi/3), M_c(0)]$:

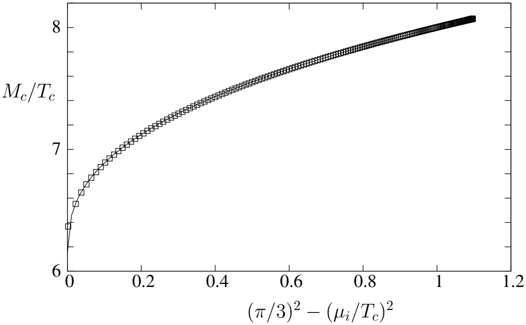


$M = M_c(i\pi/3)$:



Agrees with lattice [P. de Forcrand, O. Philipsen, Phys.Rev.Lett. 105 (2010)]

Imaginary μ: massive LDW gauge at one-loop



$$\frac{M_c}{T_c} = \frac{M_{\text{tric.}}}{T_{\text{tric.}}} + K\left[\left(\frac{\pi}{3}\right)^2 + \left(\frac{\mu}{T}\right)^2\right]^{2/5}$$

	our model(*)	lattice(**)	SD(***)
K	1.85	1.55	0.98
$\frac{M_{\text{tric.}}}{T_{\text{tric.}}}$	6.15	6.66	0.41

(*) UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015).
(**) Fromm et.al., JHEP 1201 (2012) 042.
(***) Fischer et.al., Phys.Rev. D91 (2015) 1, 014024.

Real chemical potential

For $\mu \in \mathbb{R}$, the potential $V(r_3, r_8)$ becomes complex over a given Weyl chamber, which seems incompatible with the fact that one should be able to extract (real) thermodynamical quantities from $V(r_3, r_8)$.

The Polyakov loops $\ell_q(r_3, r_8)$ and $\ell_{\bar{q}}(r_3, r_8)$ for quarks and anti-quarks become complex, in contradiction with their interpretation in terms of free energies $-T \ln \ell_{q,\bar{q}} = \Delta F_{q,\bar{q}} \equiv F_{q,\bar{q}} - F_{\rm bath}$.

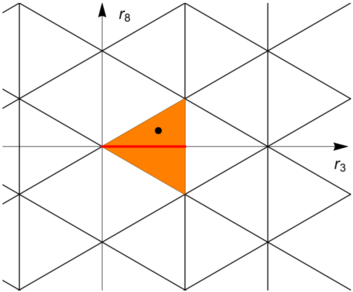
A widespread fix is to restrict to $r_8 = 0$ because then $V(r_3, r_8 = 0)$, $\ell_q(r_3, r_8)$ and $\ell_{\bar{q}}(r_3, r_8)$ become again real.

This cannot be the solution because $r_8 = 0$ selects C-invariant states, in contradiction with the explicit breaking of C that a non-zero μ implies. In particular, one finds $F_q = F_{\bar{q}}$, whereas one expects instead $F_q \neq F_{\bar{q}}$.

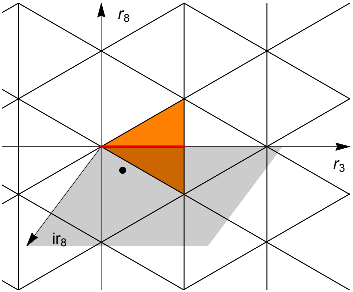
Real chemical potential

Solution: take $r_8 \neq 0$ but r_8 imaginary, not real!

[UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015)]



$\mu \in i\mathbb{R} : (r_3, r_8) \in \mathbb{R} \times \mathbb{R}$



$\mu \in \mathbb{R} : (r_3, ir_8) \in \mathbb{R} \times i\mathbb{R}$

Real chemical potential

Justification: Recall that the background is self-consistent:

$$(\bar{A}_3, \bar{A}_8) = \langle (A_3, A_8) \rangle_{(\bar{A}_3, \bar{A}_8)}$$

In other words, it is a fixed-point of the mapping

$$F : (\bar{A}_3, \bar{A}_8) \mapsto (\langle A_3 \rangle_{(\bar{A}_3, \bar{A}_8)}, \langle A_8 \rangle_{(\bar{A}_3, \bar{A}_8)})$$

For imaginary μ, the fermion determinant is real (even > 0) and F maps $\mathbb{R} \times \mathbb{R}$ into itself. This is a favorable situation for the existence of real-valued fixed points.

For real μ, $\mathbb{R} \times \mathbb{R}$ is not anymore mapped into itself because the fermion determinant becomes complex. However, there is a new stable subspace, the "Wick-rotated" plane $\mathbb{R} \times i\,\mathbb{R}$.

Sign problem

The stability of $\mathbb{R} \times i\mathbb{R}$ is shown using the following property of the Dirac operator (combined with a Weyl transformation)

$$\gamma_1\gamma_3(\not{\partial} - ig\not{A} + M + \mu\gamma_0)\gamma_3\gamma_1 = (\not{\partial} - ig\not{A}^C + M + \mu^*\gamma_0)^*$$

which deals with the non-real valuedness of the fermion determinant.

It implies also that $Z_{\mathrm{QCD}}, \ell_q, \ell_{\bar{q}} \in \mathbb{R}$. But we need to show more, namely that $Z_{\mathrm{QCD}}, \ell_q, \ell_{\bar{q}} > 0$, so that $V(r_3, r_8) \leftrightarrow \ln Z_{\mathrm{QCD}} \in \mathbb{R}$ and $\Delta F_{q,\bar{q}} = -T \ln \ell_{q,\bar{q}} \in \mathbb{R}$.

This is way more difficult due to the non-positivity of the fermion determinant $\Rightarrow$ sign problem in a functional approach.

Sign problem

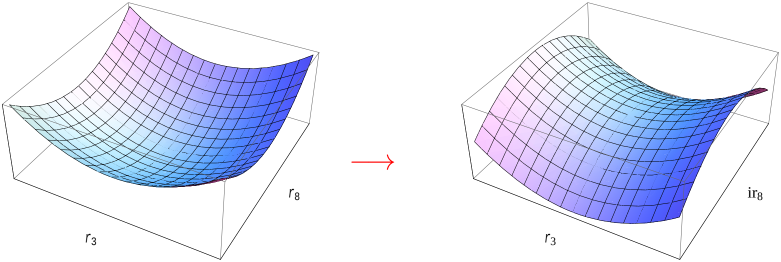
We find nevertheless that the one and two-loop expressions for $V(r_3, r_8)$ are indeed real after Wick rotating r_8. At one-loop

$$
\begin{aligned}
\delta V(r_3, ir_8) \;=\; & -\sum_n \int_p \ln\left[\left(\omega_n - i\mu + T\left(\frac{r_3}{2} + \frac{ir_8}{2\sqrt{3}}\right)\right)^2 + p^2 + M^2\right] \\
& -\sum_n \int_p \ln\left[\left(\omega_n + i\mu + T\left(\frac{r_3}{2} - \frac{ir_8}{2\sqrt{3}}\right)\right)^2 + p^2 + M^2\right] \\
& -\frac{1}{2}\sum_n \int_p \ln\left[\left(\omega_n - i\mu + T\frac{ir_8}{\sqrt{3}}\right)^2 + p^2 + M^2\right] \\
& -\frac{1}{2}\sum_n \int_p \ln\left[\left(\omega_n + i\mu - T\frac{ir_8}{\sqrt{3}}\right)^2 + p^2 + M^2\right] \in \mathbb{R}
\end{aligned}
$$

Similarly, we find that the Polyakov loops are positive.

Sign problem

More serious impact of the sign problem: the non-positivity of the fermion determinant jeopardizes the principle of minimization of the potential.

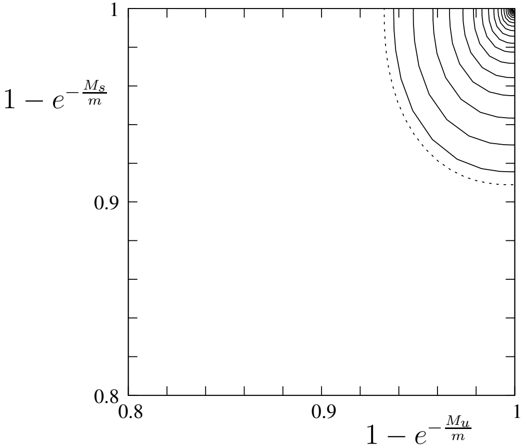


When various saddles are present, which one should we choose?

In what follows we take the rule to choose always the deepest one.

Real μ: massive LDW gauge at one-loop

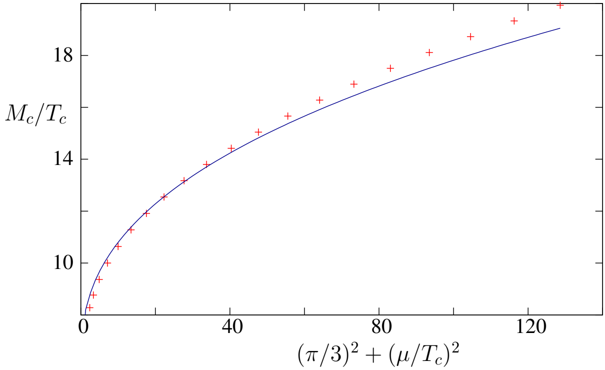
The boundary line in the Columbia plot moves towards larger quark masses:
[UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015)]



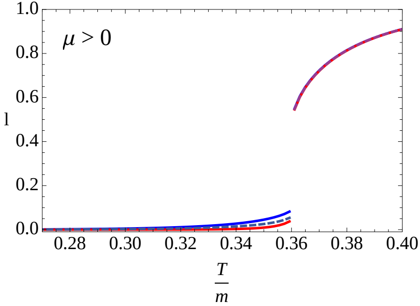
Real μ: massive LDW gauge at one-loop

The tricritical scaling survives deep in the $\mu^2 > 0$ region:

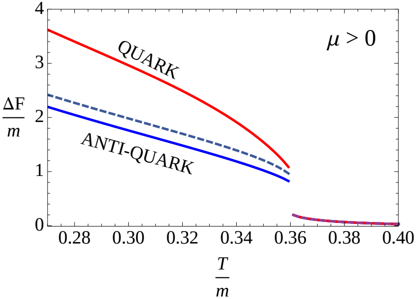
[UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015)]



Real μ: massive LDW gauge at one-loop

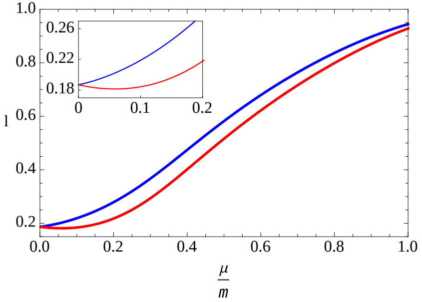


$$\Delta F_{q,\bar{q}} = -T \ln \ell_{q,\bar{q}}$$

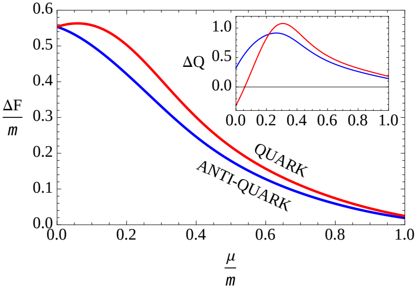


[UR, J. Serreau and M. Tissier, Phys.Rev. D92 (2015)]

Real μ: massive LDW gauge at one-loop



$$\left.\frac{\partial \Delta F}{\partial \mu}\right|_{\mu=0} = -\langle Q \rangle \begin{cases} > 0 \text{ (quark)} \\ < 0 \text{ (anti-quark)} \end{cases}$$



[J. Maelger, UR and J. Serreau, arXiv:1710.01930]

III. Some open issues

Question 1:

=====================

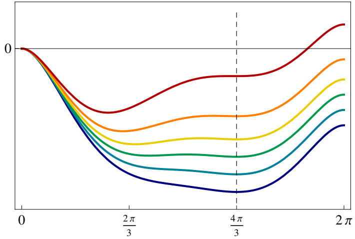
Can we trust our methods
in the Landau gauge at finite T?

=====================

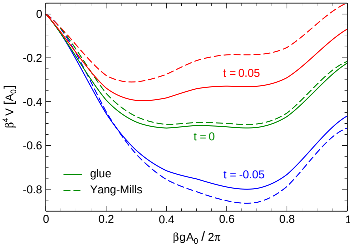
The LDW potential from various approaches

The physics of the deconfinement transition is obtained from the absolute minimum of the LDW potential (order parameter).

Moreover, the potential evaluated at that minimum gives, in principle, the free-energy of the system.



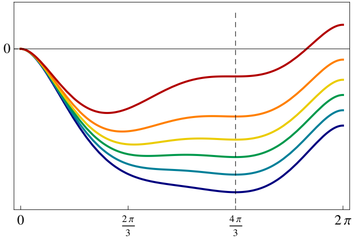
(massive extension)



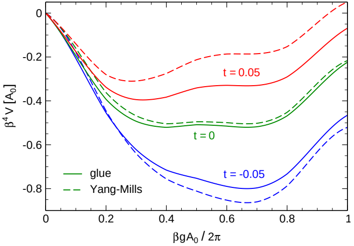
(functional RG)

No one would trust information obtained at a maximum of that potential.

Landau gauge from the LDW gauge



(massive extension)



(functional RG)

Paradox: for a wide range of temperatures, including the confining phase, $\bar{A} = 0$ is a maximum. But the potential evaluated at $\bar{A} = 0$ is also the free-energy of the system, computed in another gauge (Landau gauge).

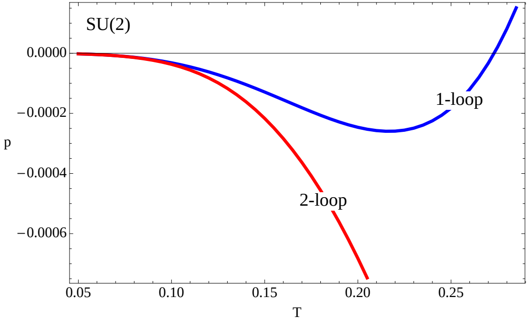
After all, $\bar{A} = 0$ is a self-consistent background because $\langle A \rangle_{\bar{A}=0} = 0 = \bar{A}$. Therefore the potential should also be minimal at $\bar{A} = 0$, implying that $\tilde{\Gamma}[\bar{A} = 0] = \tilde{\Gamma}[\bar{A}_{\rm LDW}]$.

"Instability" of the Landau gauge

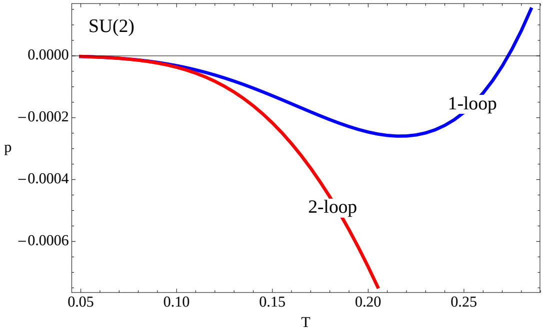
Maybe the difference between the free-energies computed in the LDW gauge and in the Landau gauge is an higher order effect that gets reduced as one includes more loops.

The situation seems to be more dramatic however.

In the massive extension for instance, the potential at $\bar{A} = 0$ gives a decreasing pressure at small T (negative entropy!). This unphysical behavior is not cured, but rather worsened, by two-loop corrections.



Generic problem?



This unphysical feature seems generic since it has to do with the fact that ghost dominate at small temperatures and enter the free-energy with negative bosonic statistics (in the Landau gauge).

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What have other approaches in the Landau gauge to say on this question?

===

[screened PT by Siringo et al. also leads to a negative entropy]

Question 2:

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Do we control basic thermodynamical principles?

==================

From negative to positive entropy

We have just seen that the entropy computed in the massive Landau gauge at one and two loop orders is negative at small T.

The massive LDW gauge cures this in a magical way: despite the unphysical ghosts dominating at small temperatures, the confining background turns their negative bosonic distributions into positive distributions.

For $SU(2)$ for instance, the confining background is $r = \pi$:

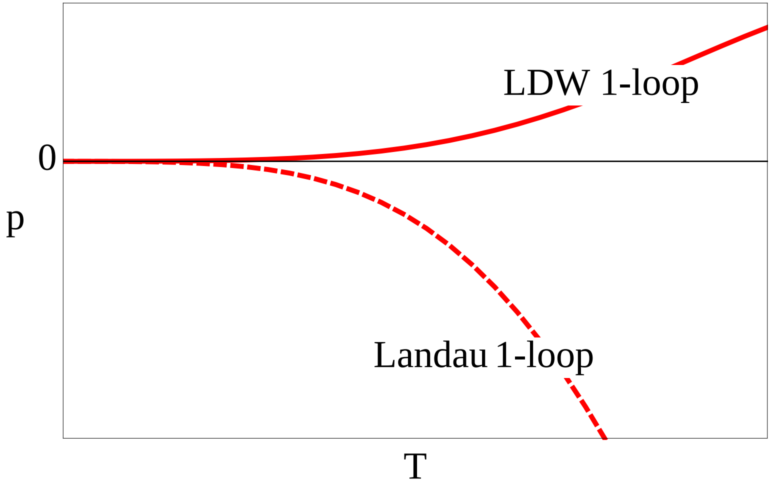
$$\frac{s(T \ll m)}{4T^3} = (N_c^2 - 1) \int \frac{d^3q}{(2\pi)^3} \underbrace{\ln(1 - e^{-q})}_{<0}$$

$\downarrow$ (from Landau to LDW)

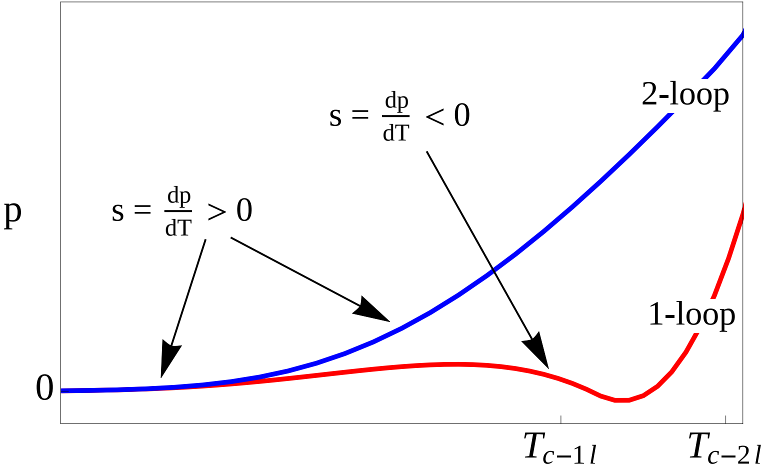
$$\frac{s(T \ll m)}{4T^3} = \int \frac{d^3q}{(2\pi)^3} \left[\ln(1 - e^{-q}) + \ln(1 - e^{-q+ir}) + \ln(1 - e^{-q-ir})\right]$$

$$= \int_q \frac{d^3q}{(2\pi)^3} \left[\underbrace{\ln(1 - e^{-q})}_{<0} + 2\underbrace{\ln(1 + e^{-q})}_{>0}\right] > 0$$

From negative to positive entropy



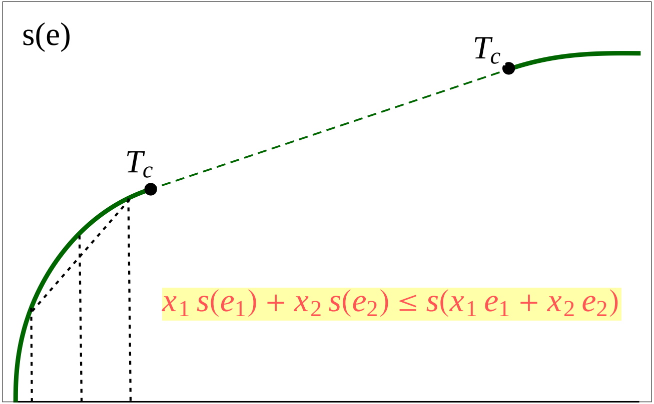
From negative to positive entropy



Two-loop corrections help making the entropy > 0 over the whole range of T.

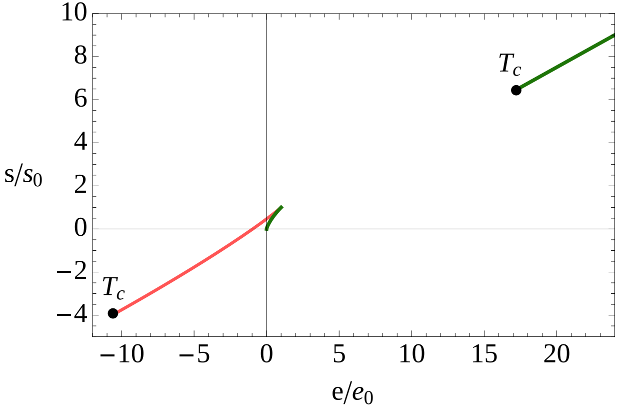
Thermodynamical stability

What about stability? Stability can be discussed by means of $s(e)$:



Thermodynamical stability

At one-loop, we find an unstable region when approaching T_c from below:

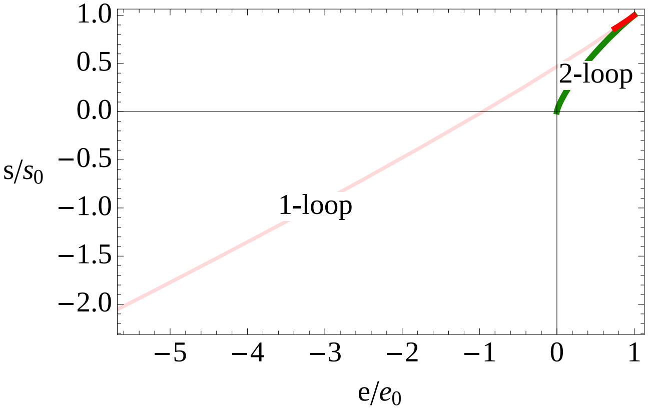


Characterized by $de/dT < 0$. Contradicts fluctuation-dissipation:

$$\frac{de}{dT} = \frac{d\langle u \rangle}{dT} = \left(\frac{\Delta u}{T}\right)^2 > 0$$

Thermodynamical stability

The situation seems to improve at two-loop order:



The interval of instability in temperature shrinks from $\Delta T_{1\mathrm{loop}}/m = 0.133$ to $\Delta T_{2\mathrm{loop}}/m = 0.095$.

Thermodynamical stability

Do these features signal an artefact of the perturbative expansion?

Or do they signal an improper exclusion of negative norm states from physical quantities?

$$\frac{dE}{dT} = \frac{1}{T^2}\frac{\sum_n (E_n - E)^2 e^{-\beta E_n}\langle n|n\rangle}{\sum_n e^{-\beta E_n}\langle n|n\rangle}$$

==

What have other approaches (in the LDW gauge) to say on this question?

==

[some discussion exists within the GZ approach but in the Landau gauge]

Question 3:

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What do we do with ghosts at low T?

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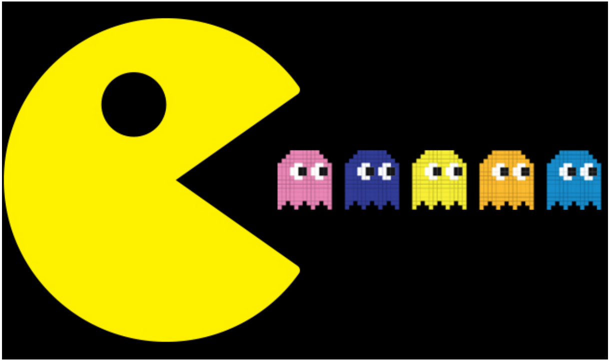
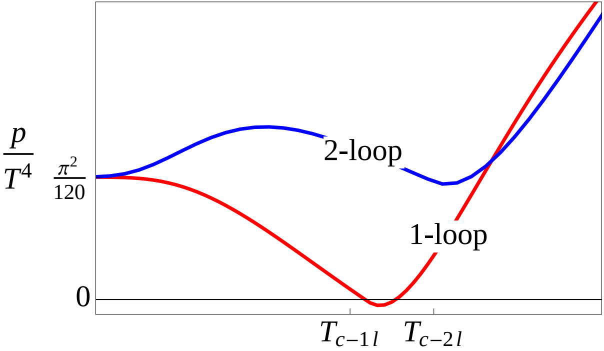


Low temperature behavior

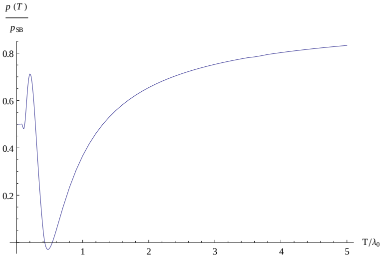
The lattice predicts a strongly suppressed pressure at low temperatures. In fact, we expect $p/T^4 \sim (M_{glueball}/T)^{3/2} e^{-M_{glueball}/T}$.

The massive Landau-DeWitt gauge predicts $p/T^4 \to \pi^2/120$ for $SU(2)$ and $p/T^4 \to 4\pi^2/405$ for $SU(3)$. Two-loop corrections do not help:

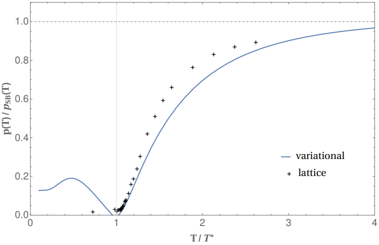


Low temperature behavior

The problem is not restricted to the massive Landau-DeWitt approach since it is present in other approaches [fRG/DSE?]:



(GZ)



(Variational)

It originates from the dominance of (massless) ghosts at low temperatures.

Ghost dominance at low T

We have seen that the ghosts play an important role in obtaining an inverted Weiss potential at low temperature and thus in triggering the transition.

Despite their unphysical behavior, and thanks to the confining background, the thermodynamics seems to be consistent at low T (positive entropy, . . .)

On the other hand, they lead to a wrong p/T^4 behavior at low T.

=====================================

How could one eliminate the ghosts at low temperatures without affecting the restoration of center symmetry?

=====================================

Could the answer lie in the background?

So far we have restricted to configurations $\bar{A}_\mu(\tau,\vec{x}) = \delta_{\mu 0}\bar{A}$ which are explicitly homogeneous and isotropic.

But more generally, we could have considered configurations that are homogeneous and isotropic modulo periodic gauge transformations.

We have classified these configurations in the $SU(2)$ case and found a new class of configurations with $\bar{A}_0^a(\tau,\vec{x}) = 0$ and $\bar{A}_i^a(\tau,\vec{x}) = \bar{A}\,\delta_i^a$:

- ☺ inequivalent to the previous class since $F_{ij}^a = \bar{A}^2\varepsilon_{ija}$;
- ☺ the ghosts acquire massive dispersion relations $\begin{cases}\sqrt{k^2+2\bar{A}^2}\\ \sqrt{(|k|+|\bar{A}|)^2+\bar{A}^2}\\ \sqrt{(|k|-|\bar{A}|)^2+\bar{A}^2}\end{cases}$;
- ☹ not center-symmetric, so should not survive at low T.

Summary

The massive Landau-DeWitt gauge approach seems to capture many aspects of the deconfinement transition in pure Yang-Mills theory and in QCD with heavy quark flavours.

Some crucial problems persist but they seem to be generic to most continuum approaches:

	"Unstable Landau"	Unstable thermo	Wrong low T
fRG/DSE	??	??	??
Variational	??	??	YES
GZ (unref)	??	YES (Landau)	YES
Screened PT	YES	YES (Landau)	??
mL/LDW	YES	YES	YES

YES ≡ yes, there seems to be a problem.
?? ≡ not investigated, but probably there is a problem.

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Can we trust our methods in the Landau gauge at finite T?

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Do we control basic thermodynamical principles?

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What do we do with ghosts at low T?

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