Revisiting the lattice pure gauge Landau gauge gluon propagator at zero and finite temperature

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Basic Field and Gauge Transformation:

$$\mathcal{U}_\mu(x) \longrightarrow \Omega(x)\mathcal{U}_\mu(x)\Omega^\dagger(x+\hat{e}_\mu)$$

$$S = \beta \sum_{x,\mu\nu} \left(1 - \frac{1}{N}\mathrm{Tr}\left[\mathcal{U}_\mu(x)\,\mathcal{U}_\nu(x+a\hat{e}_\mu)\,\mathcal{U}^\dagger_\mu(x+a\hat{e}_\nu)\mathcal{U}^\dagger_\nu(x)\right]\right) \qquad \beta = \frac{2N}{g_0^2}$$

No need to gauge fix

Minimal Landau gauge

$$F[g] = \frac{1}{d\,N\,V}\sum_{x,\mu}\mathrm{Re}\left\{\mathrm{Tr}\left[g(x)\,\mathcal{U}_\mu(x)\,g^\dagger(x+a\hat{e}_\mu)\right]\right\}$$

$$\mathcal{U}_\mu(x) = e^{i\,a\,g\,A_\mu(x+\frac{a}{2}\hat{e}_\mu)}$$

$$\partial_\mu A^a_\mu(x) + \mathcal{O}(a^2) = 0$$

$$A_\mu(\hat{q}) = \sum_x e^{-i\hat{q}\cdot(x+a\hat{e}_\mu/2)} A_\mu(x + a\hat{e}_\mu/2)$$

$$\hat{q}_\mu = \frac{2\,\pi}{aL_\mu}\, n_\mu, \qquad n_\mu = 0, \ldots, L_\mu - 1$$

$$\langle A^a_\mu(\hat{q})\, A^b_\nu(\hat{q}')\rangle = D^{ab}_{\mu\nu}(\hat{q})\, V\, \delta(\hat{q} + \hat{q}')$$

$$D^{ab}_{\mu\nu}(\hat{q}) = \delta^{ab}\, P_{\mu\nu}(q)\, D(q^2)$$

$$q_\mu = \frac{2}{a}\,\sin\left(\frac{\pi}{L_\mu} n_\mu\right)$$

$$n_\mu = 0, \ldots, \frac{L_\mu}{2}$$

Quantization → Gribov-Zwanziger Action

$$D(q^2) = \frac{q^2}{q^4 + 2\,g^2\,N_c\gamma^4}$$

no compatible with lattice propagator

Refined Gribov-Zwanziger Action

$$\langle A^a_\mu A^a_\mu \rangle \longrightarrow -m^2$$

$$\langle \overline{\varphi}^a_i \varphi^a_i \rangle \longrightarrow M^2 \qquad \langle \varphi^a_i \varphi^a_i \rangle \longrightarrow \rho \qquad \langle \overline{\varphi}^a_i \,\overline{\varphi}^a_i \rangle \longrightarrow \rho^\dagger$$

$$\frac{q^4 + 2\,M^2 q^2 + M^4 - \rho\rho^\dagger}{q^6 + (m^2 + 2M^2)\,q^4 + (2m^2 M^2 + M^4 + \lambda^4 - \rho\rho^\dagger)\,q^2 + \left[m^2\,(M^4 - \rho\rho^\dagger) + M^2\lambda^4 - \frac{\lambda^2}{2}\,(\rho + \rho^\dagger)\right]}$$

$$= \frac{P_4(q)}{P_6(q)}$$

$$\lambda^4 = 2\,g^2\,N_c\,\gamma^4$$

$$\langle \overline{\varphi}_i^a \, \overline{\varphi}_i^a \rangle = \langle \varphi_i^a \, \varphi_i^a \rangle = \rho = \rho^\dagger$$

$$D(q^2) = \frac{q^2 + M^2 + \rho_1}{q^4 + (M^2 + \rho_1 + m^2)q^2 + m^2(M^2 + \rho_1) + \lambda^4}$$

$$= \frac{P_2(q)}{P_4(q)}$$

Can one use the RGZ prediction to describe the lattice data ?

$$D(q^2) = Z \frac{q^2 + \mathcal{M}_1^2}{q^4 + \mathcal{M}_2^2 \, q^2 + \mathcal{M}_3^4}$$

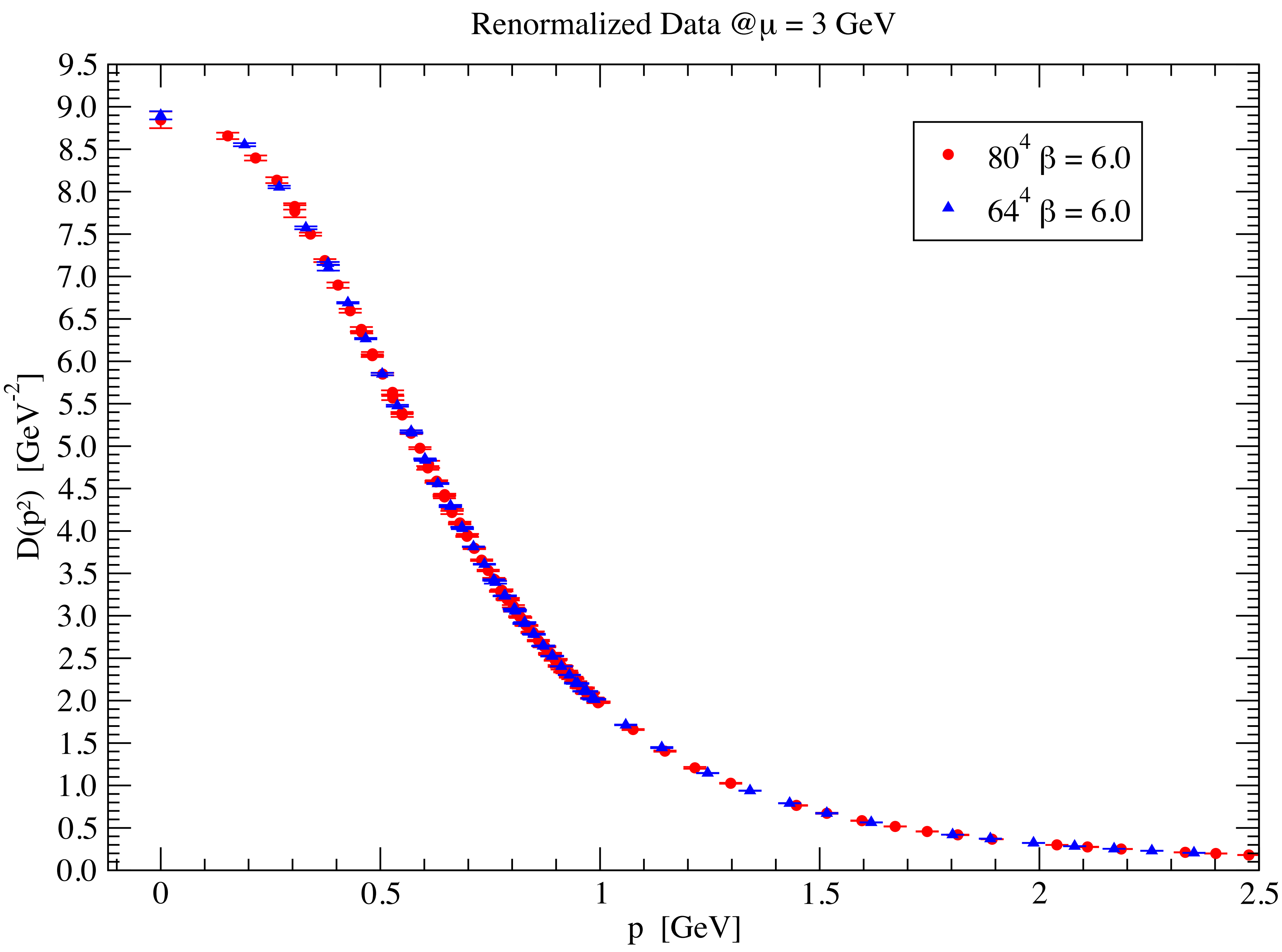
or other polynomial combinations?

D Dudal, O Oliveira, N Vandersickel, PRD 81, 074505 (2010)
D Dudal, O Oliveira, J Rodríguez-Quintero, PRD 86, 105005 (2012)
O Oliveira, P J Silva, PRD 86, 114513 (2012)
D Dudal, A Cucchieri, T Mendes, B Vandersickel, PRD 85, 094513 (2012)
E Rojas, J P B Melo, B El-Bennich, O Oliveira, T Frederico, JHEP 1310, 193 (2013)

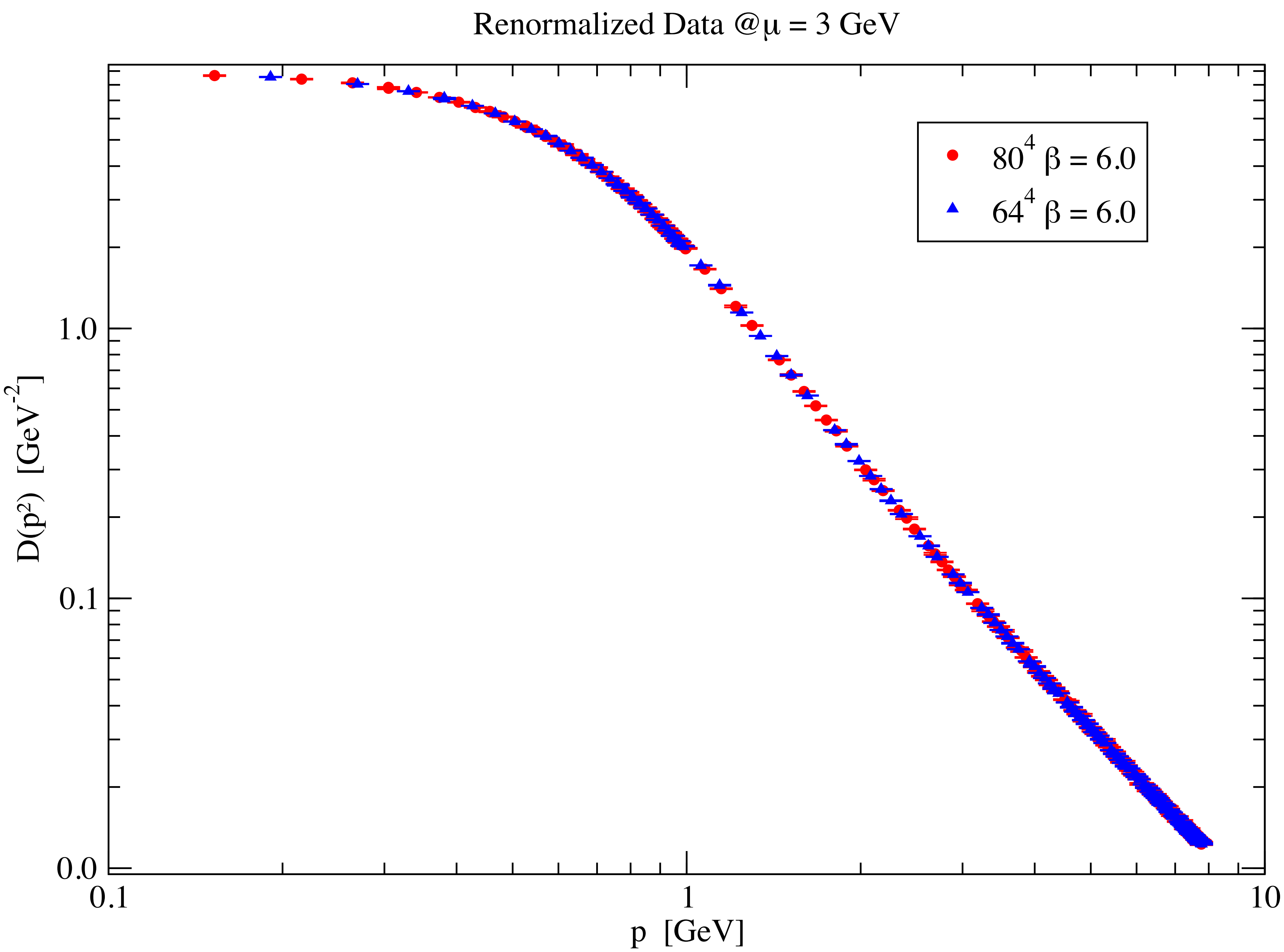
Double Complex Conjugate poles
Spectral Representation

$\beta = 6.0$ Configurations rotated to the Landau gauge

80^4 550 configurations
64^4 2000 configurations



Renormalized Data @μ = 3 GeV
D(p²) [GeV⁻²]
p [GeV]
80⁴ β = 6.0
64⁴ β = 6.0
0
0.5
1
1.5
2
2.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0
9.5



Renormalized Data @μ = 3 GeV
80^4 β = 6.0
64^4 β = 6.0
D(p²) [GeV^-2]
1.0
0.1
0.0
0.1
1
10
p [GeV]

$$D(q^2) = Z\,\frac{q^2 + m_1^2}{q^4 + m_2^2\,q^2 + m_3^4}$$

Refined Gribov-Zwanziger

$$D(q^2) = \frac{q^4 + m_2^2\,q^2 + m_1^2}{q^6 + m_5^2\,q^4 + m_4^2\,q^2 + m_3^6}$$

Very Refined Gribov-Zwanziger

$$D(q^2) = \frac{q^6 + m_3^2\,q^4 + m_2^4\,q^2 + m_1^6}{q^8 + m_7^2\,q^6 + m_6^4\,q^4 + m_5^6\,q^4 + m_4^8}$$

VRGZ-P8

Fit lattice data in [0, p_{cut}]

- **how far can we go?**
- **how stable are the poles?**
- **can we distinguish between the various scenarios?**
- **can we say something about the condensates?**

Refined Gribov-Zwanziger

$$D(q^2) = Z\,\frac{q^2 + m_1^2}{q^4 + m_2^2\,q^2 + m_3^4}$$

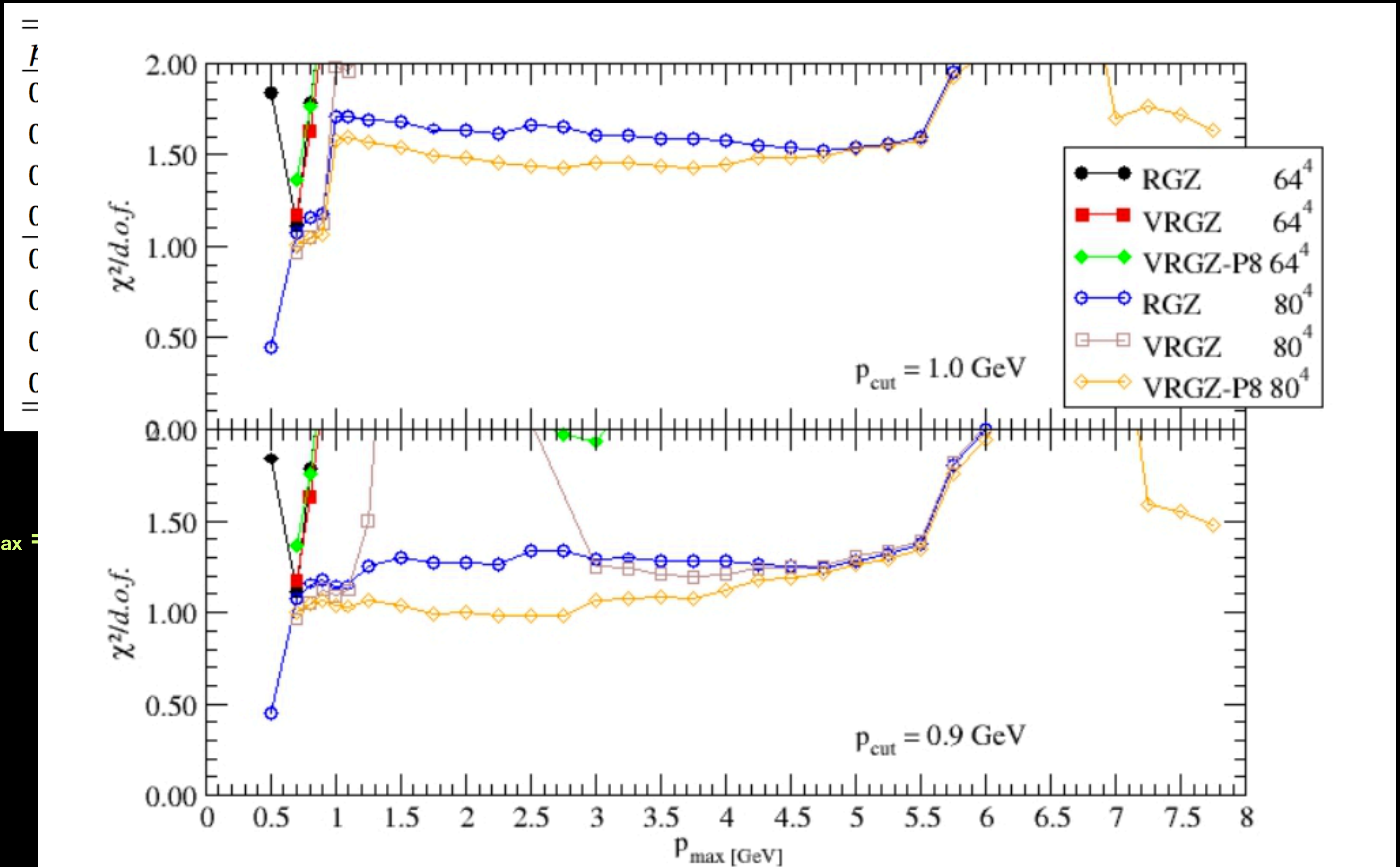
p_{max}	ν	Z	m_1^2	m_2^2	m_3^4	
0.5	1.84	2.3 ± 1.3	0.57 ± 0.78	0.421 ± 0.045	0.14 ± 0.12	64^4
0.7	1.11	1.47 ± 0.17	1.24 ± 0.30	0.422 ± 0.028	0.205 ± 0.026	
0.8	1.78	1.10 ± 0.14	2.12 ± 0.47	0.474 ± 0.033	0.258 ± 0.024	
0.9	2.26	1.072 ± 0.074	2.20 ± 0.29	0.480 ± 0.026	0.263 ± 0.016	
0.5	0.45	2.69 ± 0.35	0.25 ± 0.13	0.362 ± 0.012	0.077 ± 0.028	80^4
0.7	1.08	1.66 ± 0.18	0.97 ± 0.26	0.401 ± 0.031	0.179 ± 0.027	
0.8	1.15	1.22 ± 0.13	1.79 ± 0.37	0.465 ± 0.034	0.243 ± 0.024	
0.9	1.18	1.000 ± 0.079	2.53 ± 0.36	0.512 ± 0.029	0.281 ± 0.017	

PRD 81, 074505 (2010)

p_{max} = 0.948 GeV	1.00	2.579(60)	0.536(23)	0.2828(52)	64^4
	1.00	2.15(13)	0.34(19)		Ext.

Refined Gribov-Zwanziger

$$D(q^2) = Z \frac{q^2 + m_1^2}{q^4 + m_2^2\, q^2 + m_3^4}$$



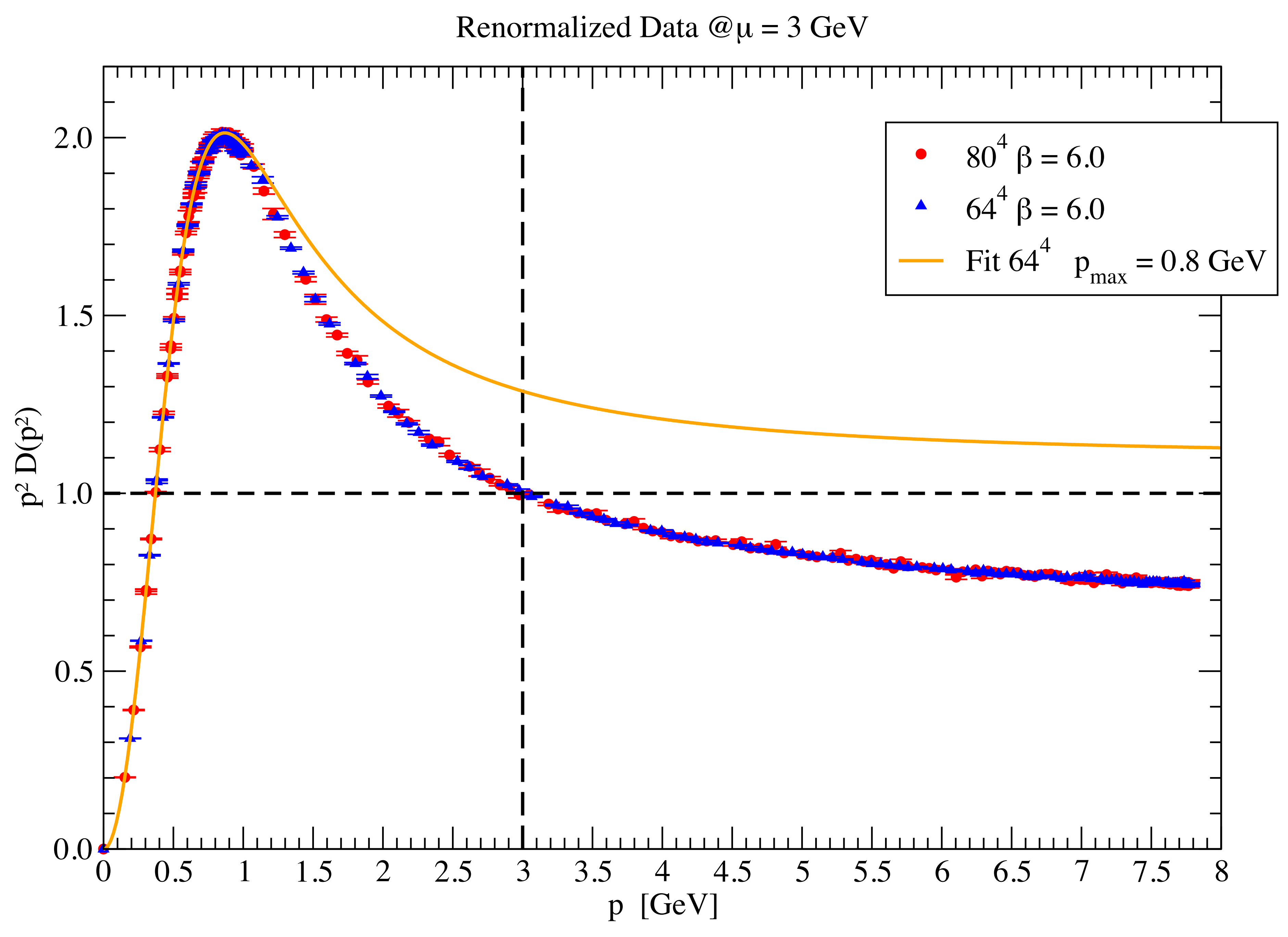
p_{max} =

Very Refined Gribov-Zwanziger

$$D(q^2) = \frac{q^4 + m_2^2\, q^2 + m_1^2}{q^6 + m_5^2\, q^4 + m_4^2\, q^2 + m_3^6}$$

p_{max}	ν	m_1^4	m_2^2	m_3^6	m_4^4	m_5^2
0.80	1.63	0.003 ± 0.052	2.505 ± 0.080	0.0003 ± 0.0058	0.278 ± 0.014	0.506 ± 0.041
0.90	2.19	0.019 ± 0.062	2.537 ± 0.066	0.0021 ± 0.0069	0.283 ± 0.014	0.521 ± 0.040
0.70	0.96	0.11 ± 0.13	2.85 ± 0.25	0.012 ± 0.015	0.326 ± 0.042	0.65 ± 0.12
0.80	1.05	0.067 ± 0.078	2.70 ± 0.11	0.0076 ± 0.0087	0.305 ± 0.021	0.589 ± 0.059
0.90	1.13	0.032 ± 0.047	2.583 ± 0.057	0.0036 ± 0.0053	0.290 ± 0.011	0.540 ± 0.033
1.00	1.98	-0.2921 ± 0.0087	2.328 ± 0.024	-0.0326 ± 0.0010	0.2156 ± 0.0022	0.364 ± 0.011
1.00	1.09	0.032 ± 0.046	2.583 ± 0.055	0.0036 ± 0.0052	0.290 ± 0.011	0.540 ± 0.032
1.10	1.12	0.023 ± 0.039	2.560 ± 0.049	0.0026 ± 0.0045	0.2866 ± 0.0094	0.529 ± 0.028
1.25	1.50	0.004 ± 0.033	2.504 ± 0.045	0.0004 ± 0.0038	0.2789 ± 0.0082	0.504 ± 0.025

		~ 0		~ 0		
$p_{max} = 0.80$			2.12(47)		0.258(29)	0.474(33)
$p_{max} = 0.90$			2.53(36)		0.281(17)	0.512(29)



Renormalized Data @μ = 3 GeV
80^4 β = 6.0
64^4 β = 6.0
Fit 64^4 p_{max} = 0.8 GeV
$p^2 D(p^2)$
p [GeV]
0.0
0.5
1.0
1.5
2.0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8

VRGZ-P8

$$D(q^2) = \frac{q^6 + m_3^2\, q^4 + m_2^4\, q^2 + m_1^6}{q^8 + m_7^2\, q^6 + m_6^4\, q^4 + m_5^6\, q^4 + m_4^8}$$

p_{max}	ν	m_1^6	m_2^4	m_3^2	m_4^8	m_5^6	m_6^4	m_7^2
0.80	1.04	0.019 ± 0.029	-0.21 ± 0.27	2.40 ± 0.20	0.0021 ± 0.0032	-0.021 ± 0.029	0.248 ± 0.057	0.40 ± 0.1
0.90	1.07	0.028 ± 0.034	-0.16 ± 0.46	2.34 ± 0.14	0.0032 ± 0.0038	-0.014 ± 0.051	0.261 ± 0.099	0.39 ± 0.1
2.25	1.46	2.2 ± 1.8	54 ± 32	11.8 ± 7.0	0.25 ± 0.20	6.2 ± 3.6	12.0 ± 6.6	16 ± 10
2.50	1.44	2.2 ± 1.7	55 ± 19	12.0 ± 4.4	0.25 ± 0.19	6.2 ± 2.1	12.2 ± 3.9	16.1 ± 6.2
2.75	1.43	2.4 ± 1.9	67 ± 21	14.6 ± 4.9	0.27 ± 0.21	7.6 ± 2.3	14.6 ± 4.1	19.8 ± 6.8
3.00	1.46	$3. \pm 3.0$	141 ± 51	31 ± 12	0.41 ± 34	15.8 ± 5.6	30 ± 10	43 ± 17
2.75	1.98	11.3 ± 6.1	72 ± 19	13.5 ± 4.5	1.26 ± 0.68	9.4 ± 2.1	19.0 ± 3.9	19.3 ± 6.3
3.00	1.93	11.3 ± 5.9	65 ± 12	12.1 ± 3.0	1.26 ± 0.66	8.7 ± 1.4	17.6 ± 2.5	17.2 ± 4.1
1.00	1.04	0.028 ± 0.032	-0.15 ± 0.45	2.34 ± 0.13	0.0032 ± 0.0036	-0.014 ± 0.050	0.261 ± 0.096	0.39 ± 0.1
2.25	0.98	2.7 ± 1.6	40 ± 17	8.4 ± 3.7	0.30 ± 0.18	4.7 ± 1.9	9.5 ± 3.5	11.2 ± 5.4
2.50	0.98	2.7 ± 1.6	43 ± 12	9.1 ± 2.8	0.30 ± 0.18	5.1 ± 1.3	10.2 ± 2.3	12.2 ± 3.9
3.50	1.08	5.2 ± 3.8	238 ± 67	53 ± 16	0.59 ± 0.42	26.7 ± 7.4	50 ± 13	74 ± 22

~ 0 ~ 0 ~ 0 ~ 0

p_{max} = 0.80	2.12(47)	0.258(29)	0.474(33)
p_{max} = 0.90	2.53(36)	0.281(17)	0.512(29)

✳ how far can we go?

RGZ works fine up to ~ 0.9 GeV

✳ how stable are the poles?

fits seem to be stable

$p^2 = -0.237 \pm i\, 0.449$ GeV^2 $p^2 = -0.256 \pm i\, 0.464$ GeV^2

✳ can we distinguish between the various scenarios?
✳ can we say something about the condensates?

$$\langle \overline{\varphi}_i^a \, \overline{\varphi}_i^a \rangle = \langle \varphi_i^a \, \varphi_i^a \rangle = \rho = \rho^\dagger$$

$$M^2 + \rho_1 = 2.12 \pm 0.47 \text{ GeV}^2$$ 2.15(13) GeV^2

$$m^2 = -1.65 \pm 0.47 \text{ GeV}^2$$ -1.81(14) GeV^2

$$2\, g^2 N_c \gamma^4 = 3.27 \pm 0.78 \text{ GeV}^4$$ 4.16(38) GeV^4

Gluon Spectral Representation

$$D(p^2) = \int d\mu \; \frac{\rho(\mu)}{p^2 + \mu}$$

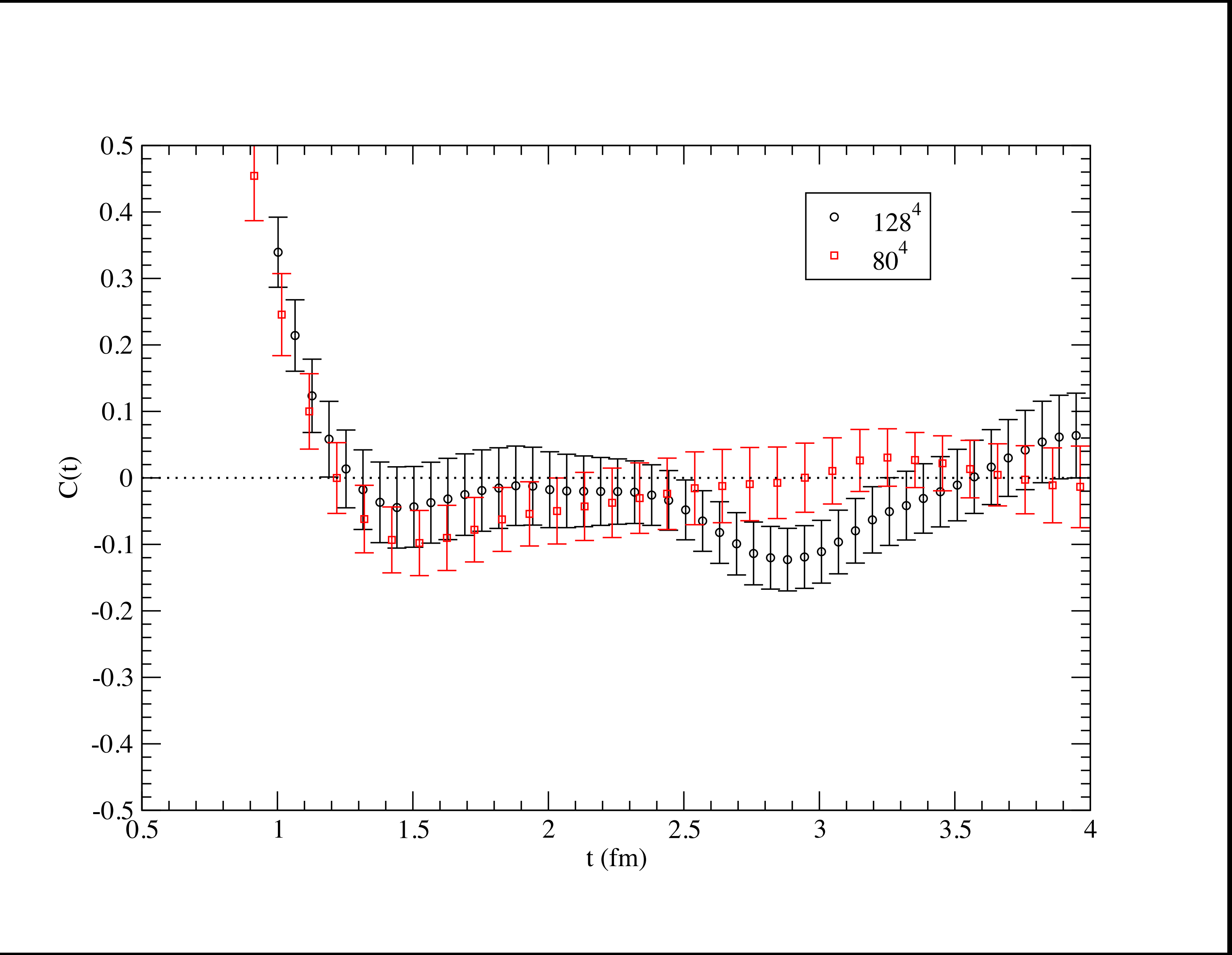
$\rho(\mu)$ **positive defined if contributes to S matrix**

$$\frac{d\,D(p^2)}{d\,p^2} = -\int d\mu \; \frac{\rho(\mu)}{\left(p^2 + \mu\right)^2}$$

Spectral density is not positive defined for the gluon

Can we "measure" the positivity violation?

$$C(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dp \, e^{-ipt} D(p^2) = \int_0^{+\infty} dy \; \rho(y^2) \; e^{-ty}$$



$t \sim 1.3$ fm

~ 150 MeV

$$D(p^2) = \int_{\mu_0}^{+\infty} d\mu \; \frac{\rho(\mu)}{p^2 + \mu}$$

To Invert

✓ needs regularisation

✓ data is not perfect

Tikhonov regularisation

$$\mathcal{J}_\lambda = \sum_{i=1}^{N} \left[\frac{\int_{\mu_0}^{+\infty} d\mu \; \frac{\rho(\mu)}{p_i^2 + \mu} - D(p_i^2)}{\Delta D(p_i^2)} \right]^2 + \lambda \int_{\mu_0}^{+\infty} d\mu \; \rho^2(\mu)$$

linear variations wrt to spectral density

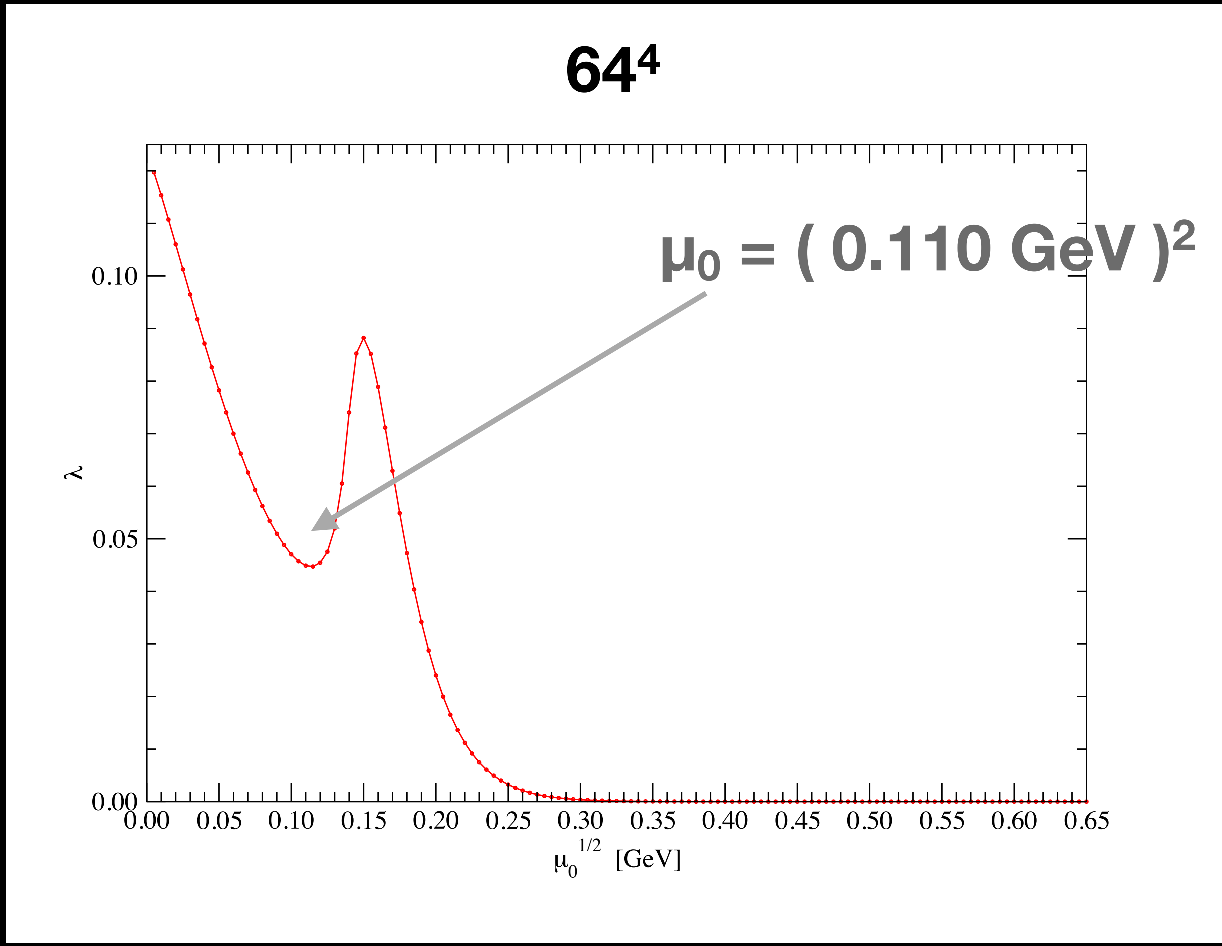
$$\frac{1}{\lambda}\mathcal{M}c + c = -D$$

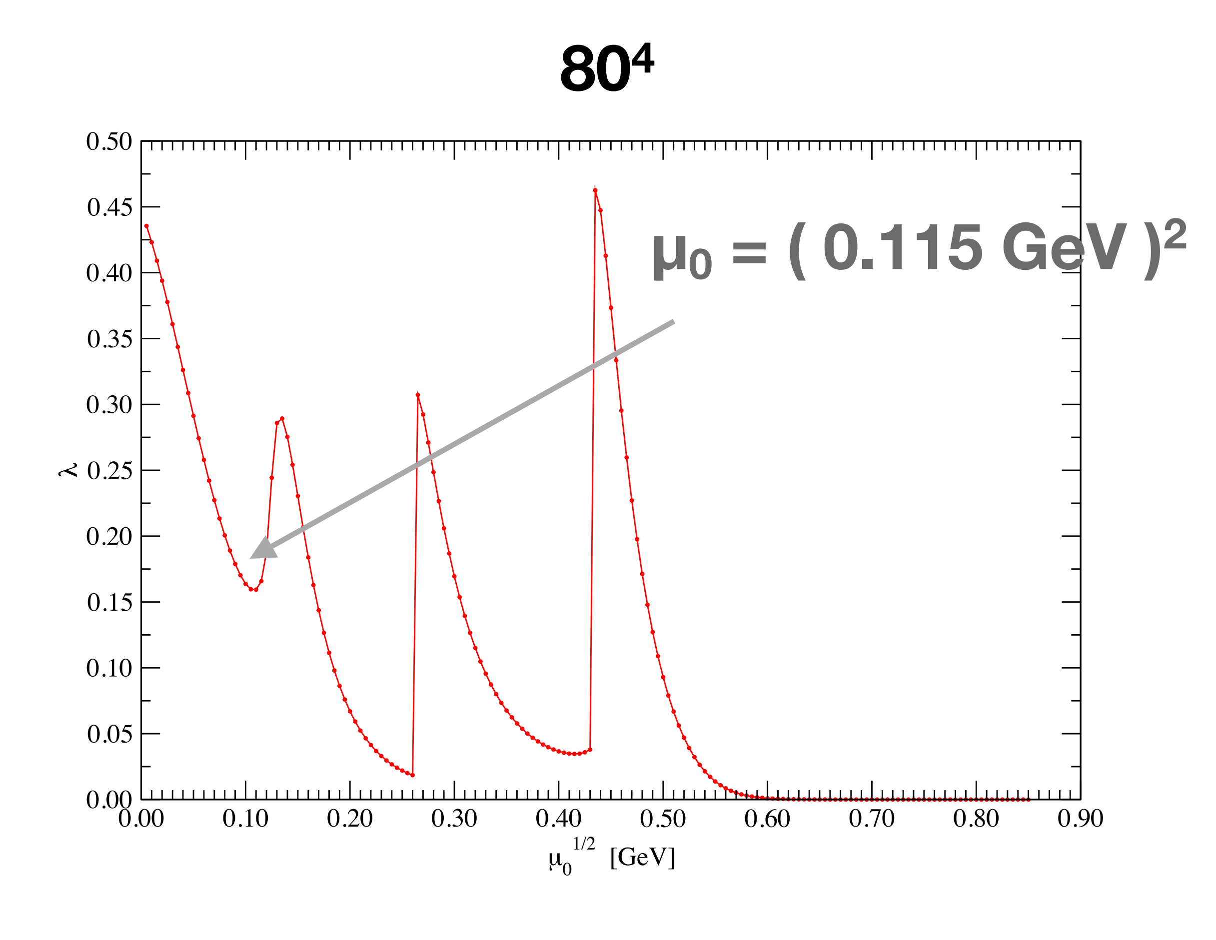
rebuild the spectral density and propagator

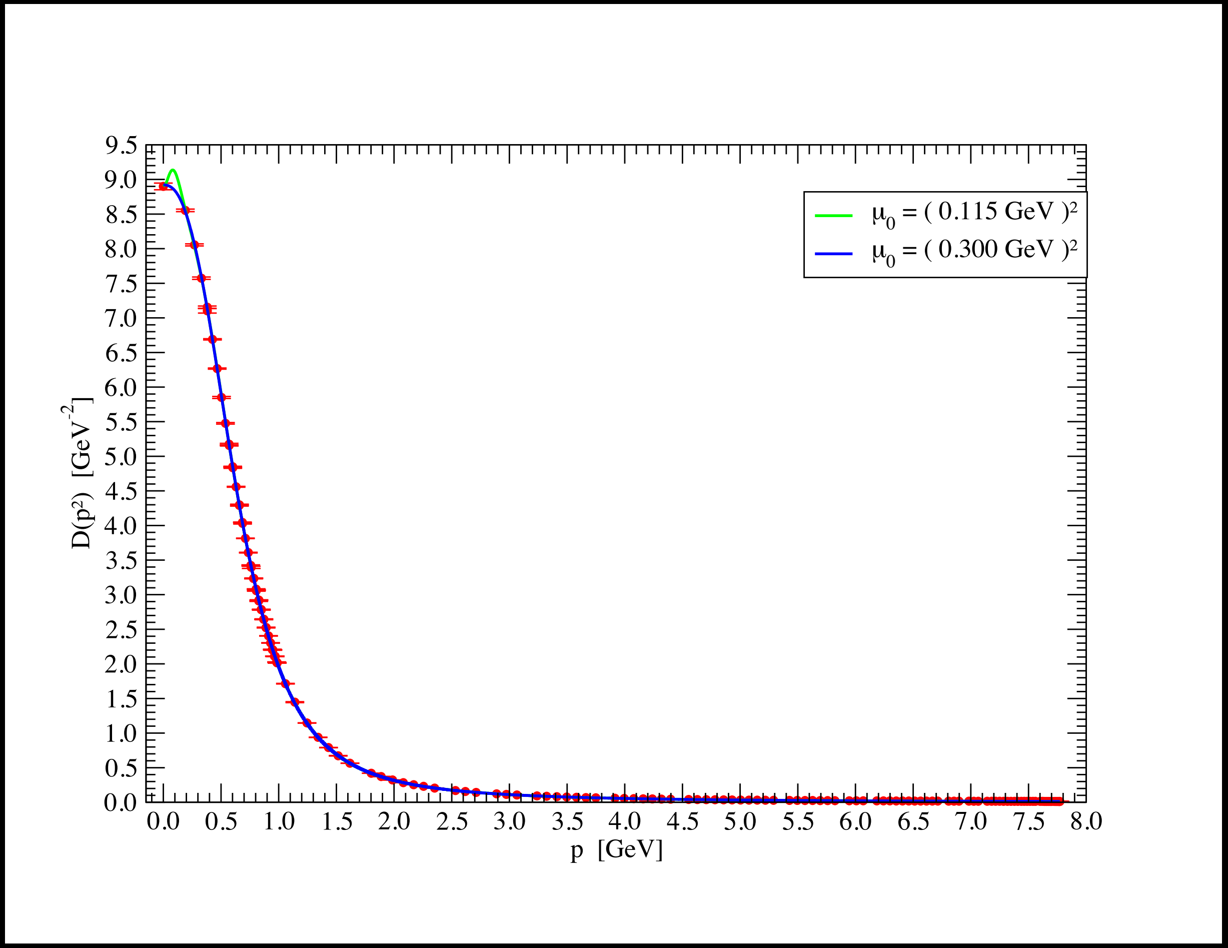
Morozov discrepancy principle

$$\sqrt{\sum_i \left[D_{inv}(p_i^2) - D(p_i^2)\right]^2} \lesssim \sqrt{\sum_i \left[\Delta D(p_i^2)\right]^2}$$ optimal λ

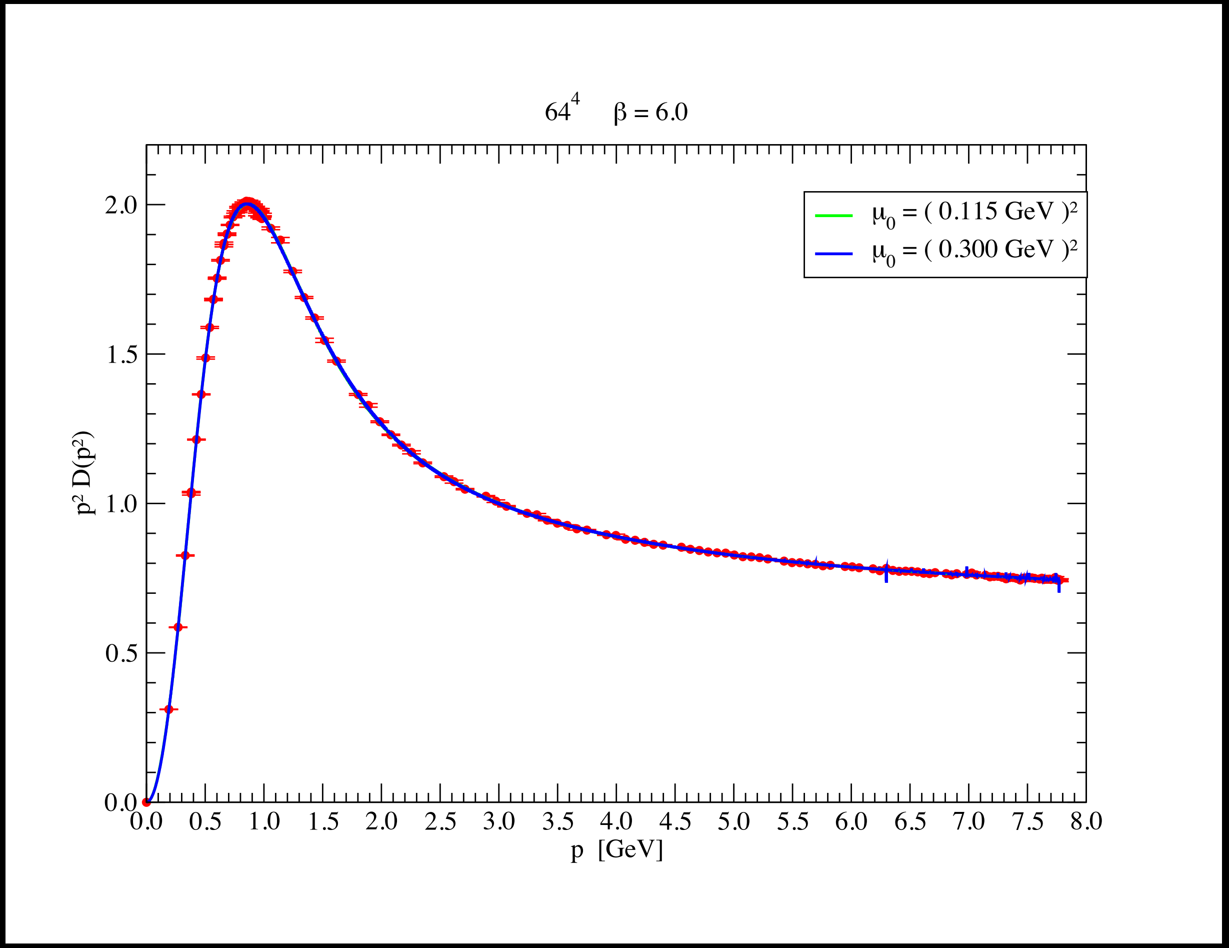
IR cutoff look at minima of λ(μ)



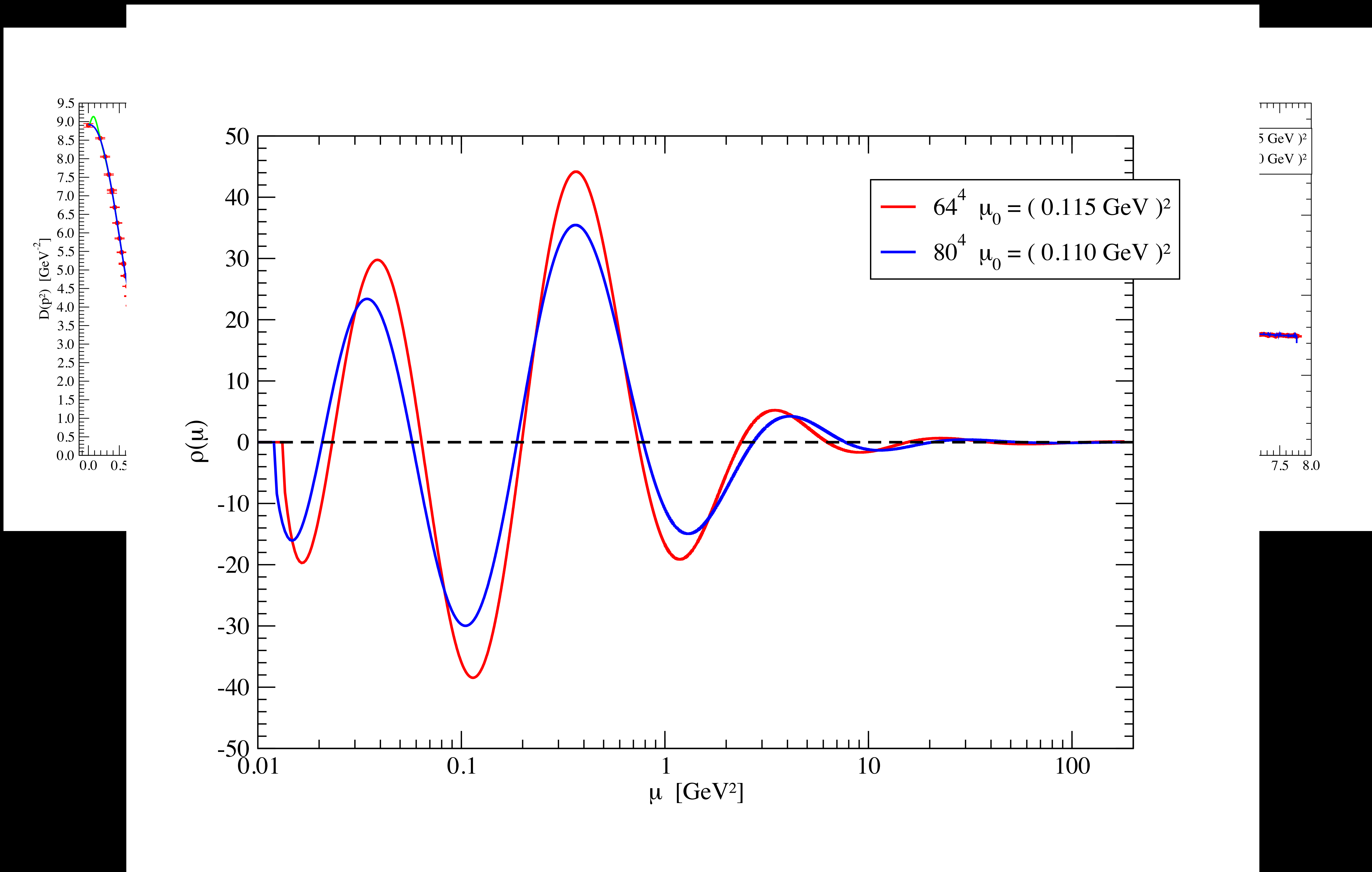




D(p²) [GeV⁻²]
p [GeV]
μ0 = (0.115 GeV)²
μ0 = (0.300 GeV)²

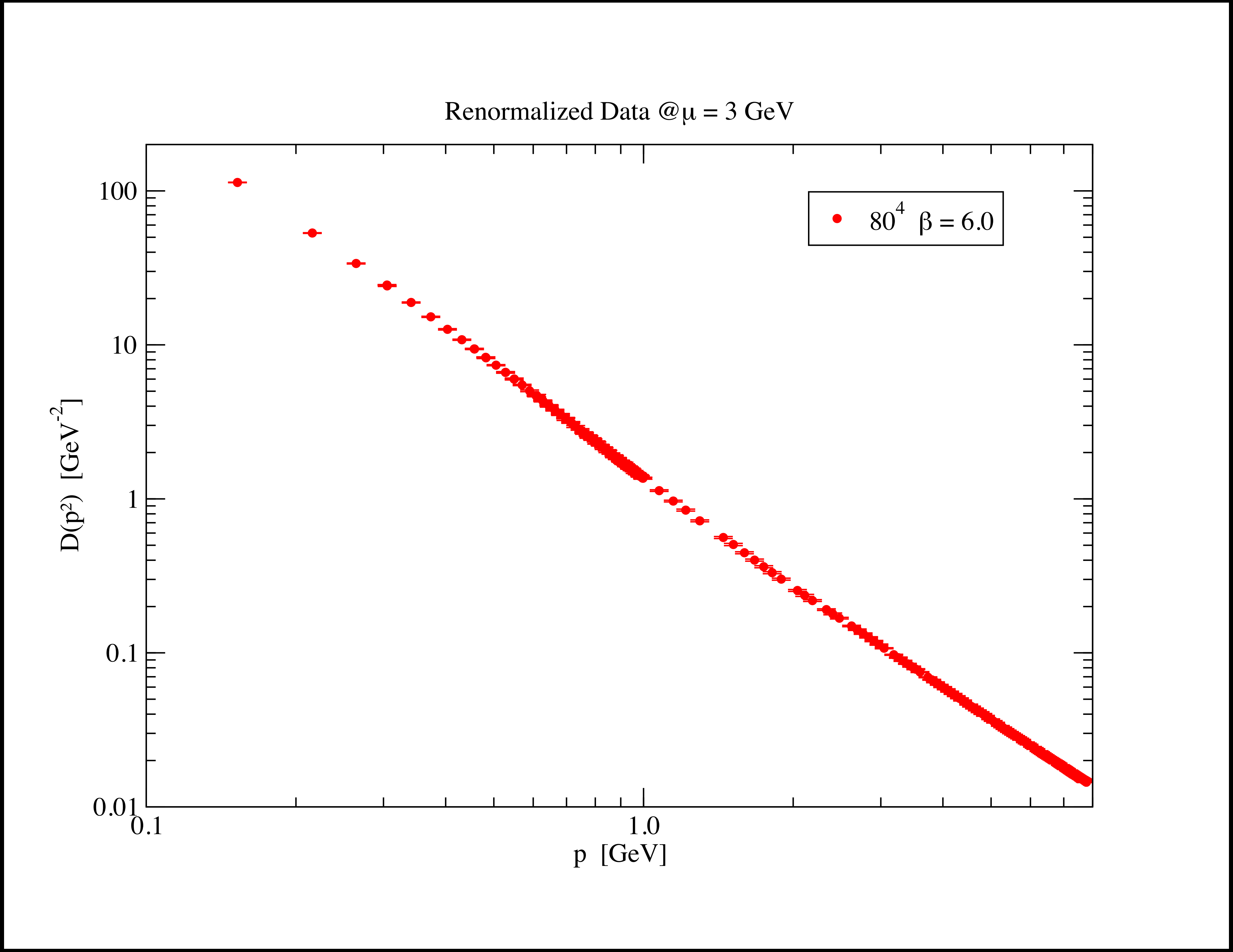


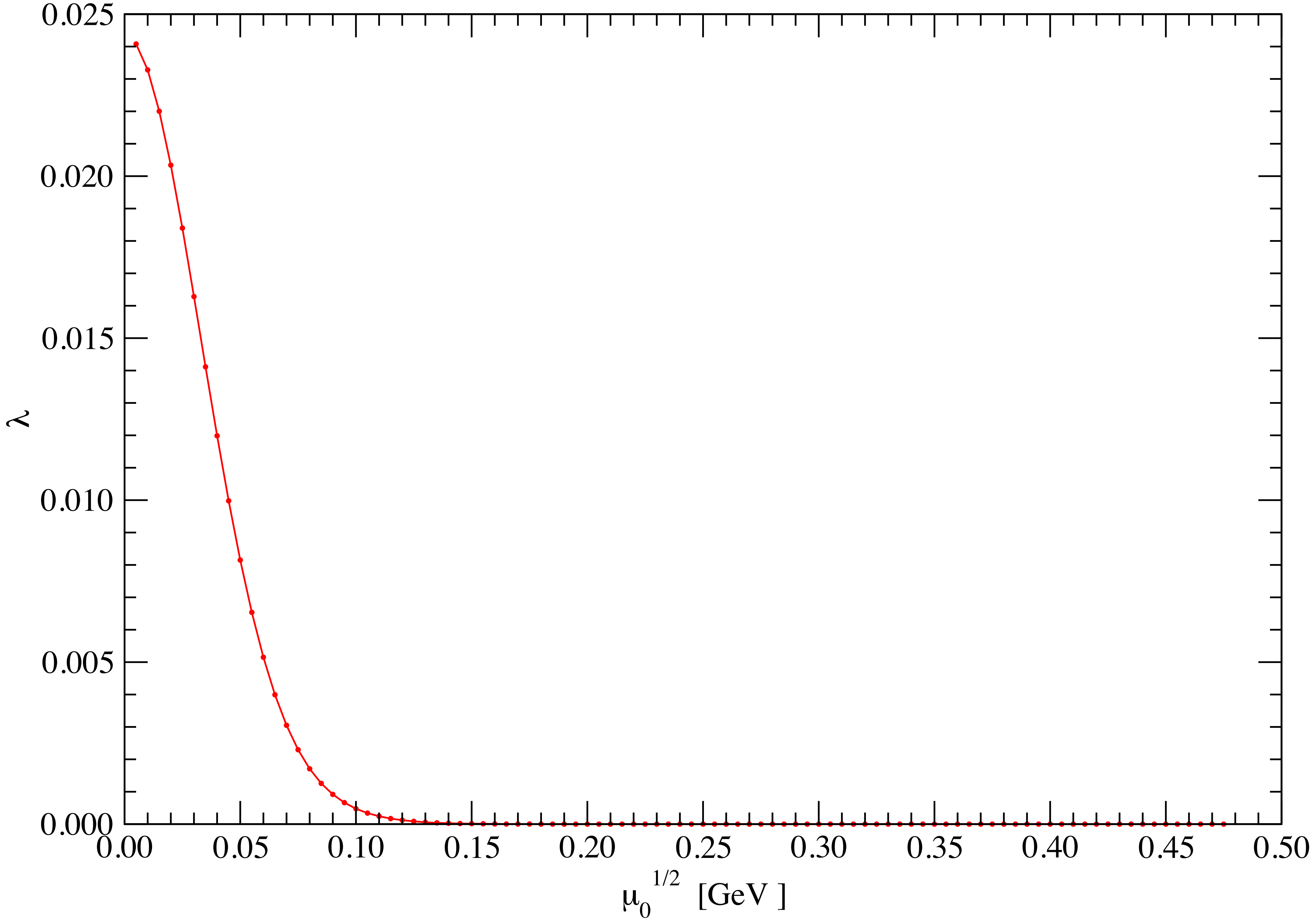
64⁴ β = 6.0
p² D(p²)
p [GeV]
μ0 = (0.115 GeV)²
μ0 = (0.300 GeV)²



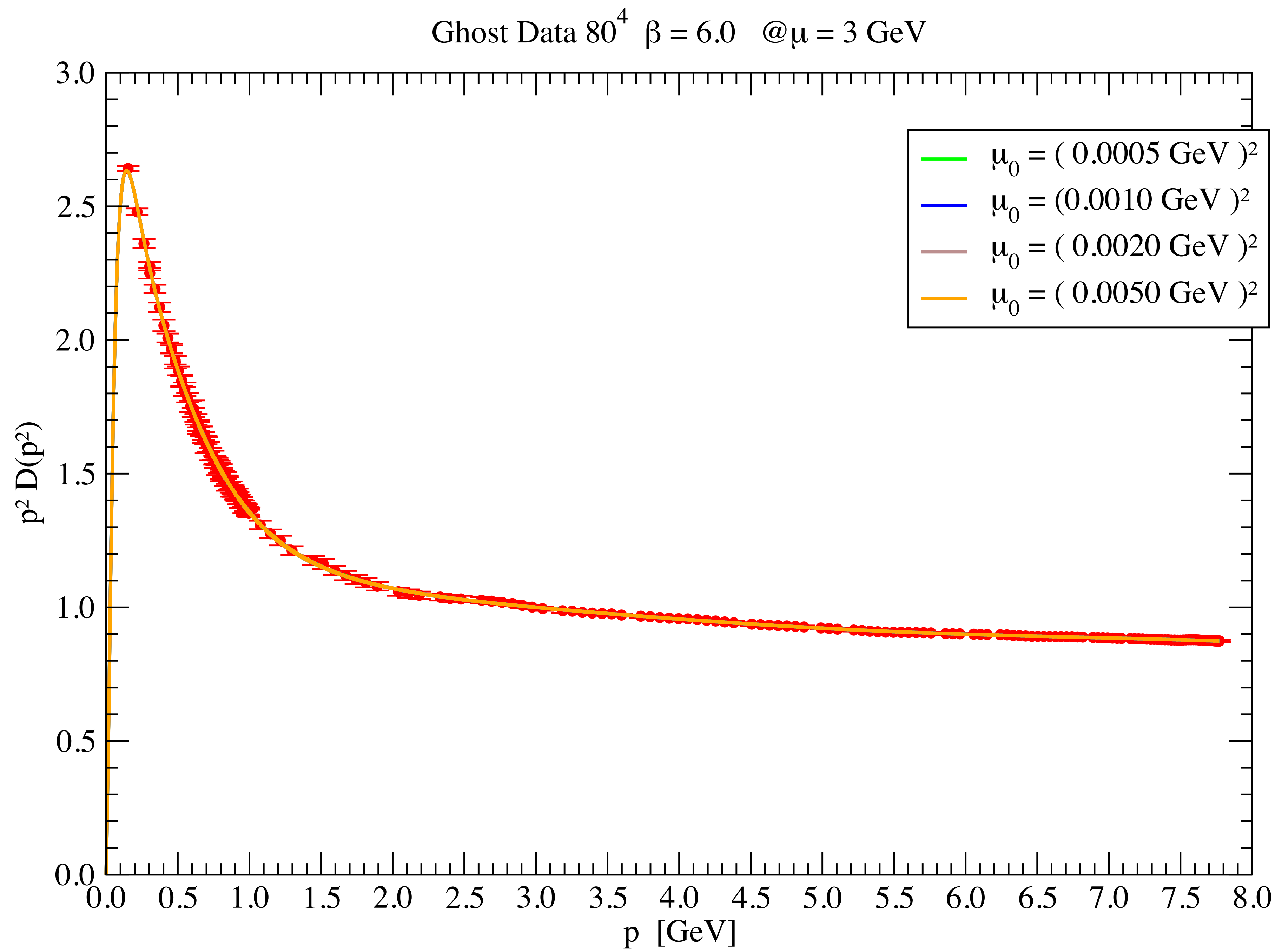
ρ(μ)
μ [GeV²]
64⁴ μ0 = (0.115 GeV)²
80⁴ μ0 = (0.110 GeV)²
50
40
30
20
10
0
-10
-20
-30
-40
-50
0.01
0.1
1
10
100
D(p²) [GeV⁻²]

Ghost Propagator

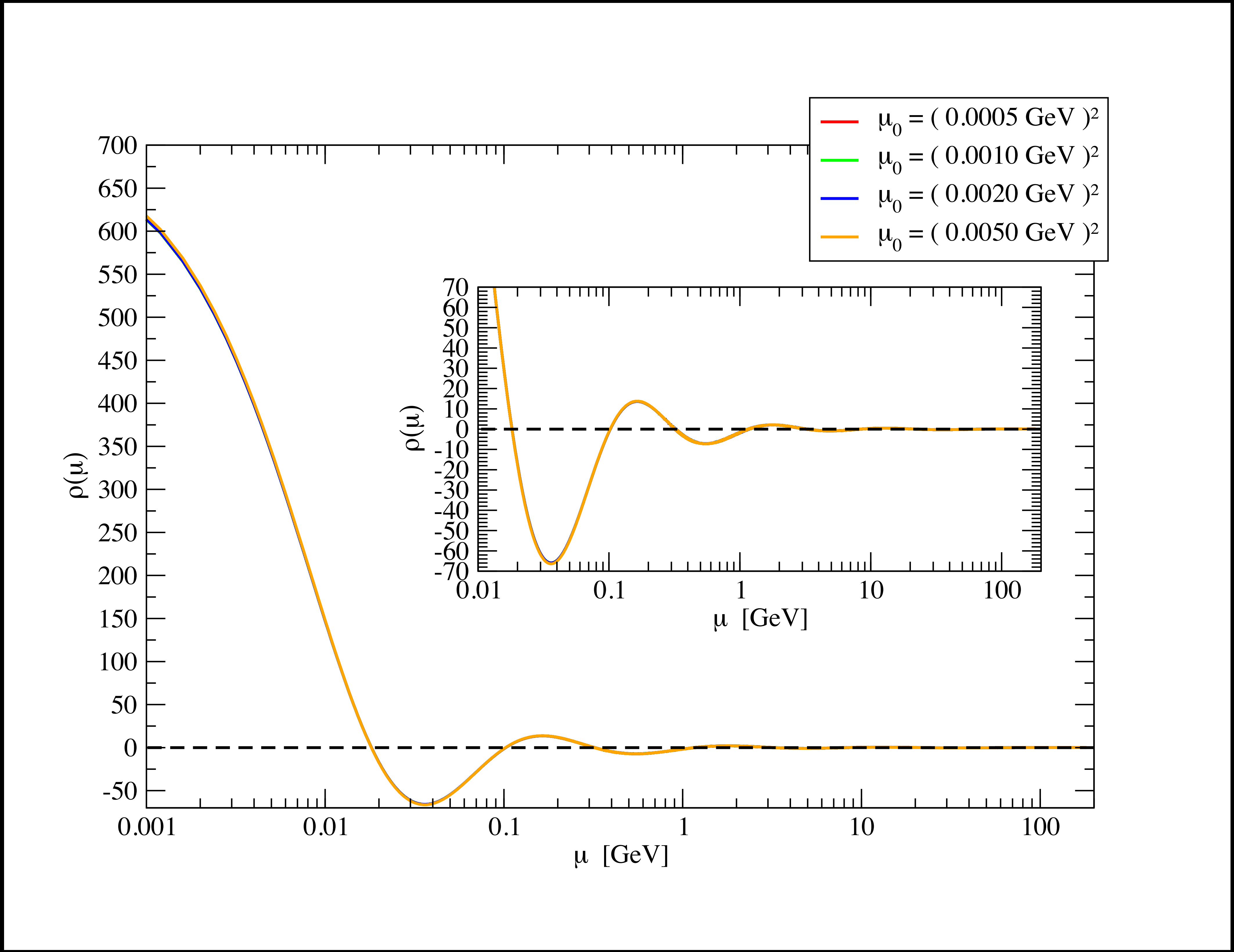




0.025
0.020
0.015
0.010
0.005
0.000
λ
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
μ0^1/2 [GeV]



Ghost Data 80^4 β = 6.0 @μ = 3 GeV
μ0 = (0.0005 GeV)²
μ0 = (0.0010 GeV)²
μ0 = (0.0020 GeV)²
μ0 = (0.0050 GeV)²
p² D(p²)
p [GeV]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0



μ_0 = (0.0005 GeV)²
μ_0 = (0.0010 GeV)²
μ_0 = (0.0020 GeV)²
μ_0 = (0.0050 GeV)²
ρ(μ)
μ [GeV]
ρ(μ)
μ [GeV]

Sum Rule:

$$\int d\mu \; \rho(\mu) = 0$$

-2.7 GeV^2	64^4	Gluon
-3.5 GeV^2	80^4	
5.6 GeV^2	80^4	Ghost

- ✓ **Positivity Violation gluon and ghost**
- ✓ **Inversions seem to be numerically stable**
- ✓ **Ghost points towards a term proportional to $\delta(\mu)$ + oscillations**

Functional Description of Lattice Data

$$D(p^2) = z\frac{p^2 + m_1^2}{p^4 + m_2^2 p^2 + m_3^4}$$

Refined Gribov-Zwanziger

D. Dudal et al, Phys. Rev. D78, 065047 (2008)

D. Dudal, OO, N. Vandersickel, Phys. Rev. D81, 074505 (2010)

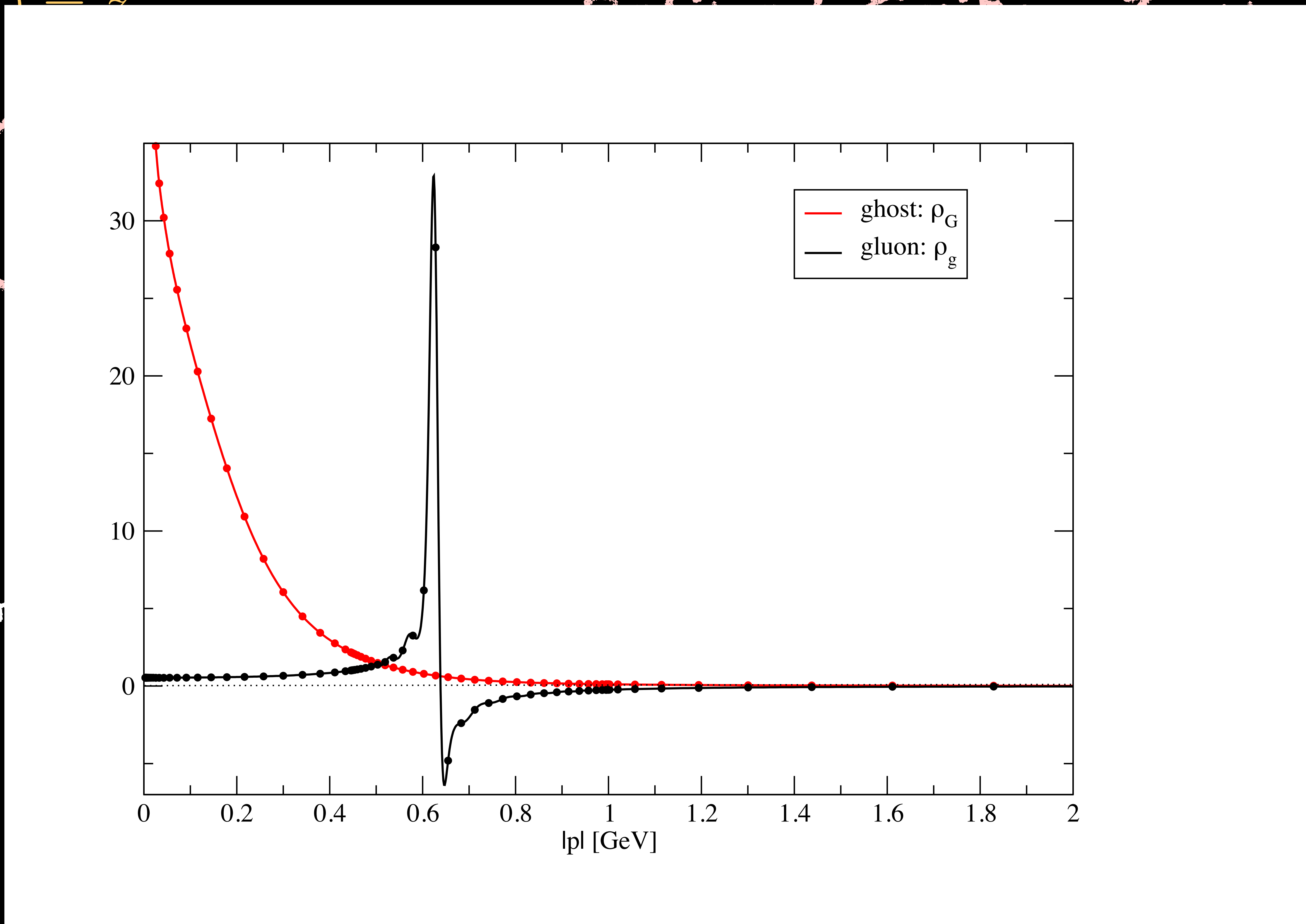
A. Cucchieri et al, Phys. Rev. D85, 094513 (2012)

→ up to two complex conjugate poles

not seen in the Dyson-Schwinger equations

S. Strauss, C. S. Fischer, C. Kellermann, Phys. Rev. Lett. 109, 252001 (2012)

Functional Description of Lattice Data



Functional Description of Lattice Data

$$D(p^2) = z \frac{p^2 + m_1^2}{p^4 + m_2^2 p^2 + m_3^4}$$

Refined Gribov-Zwanziger

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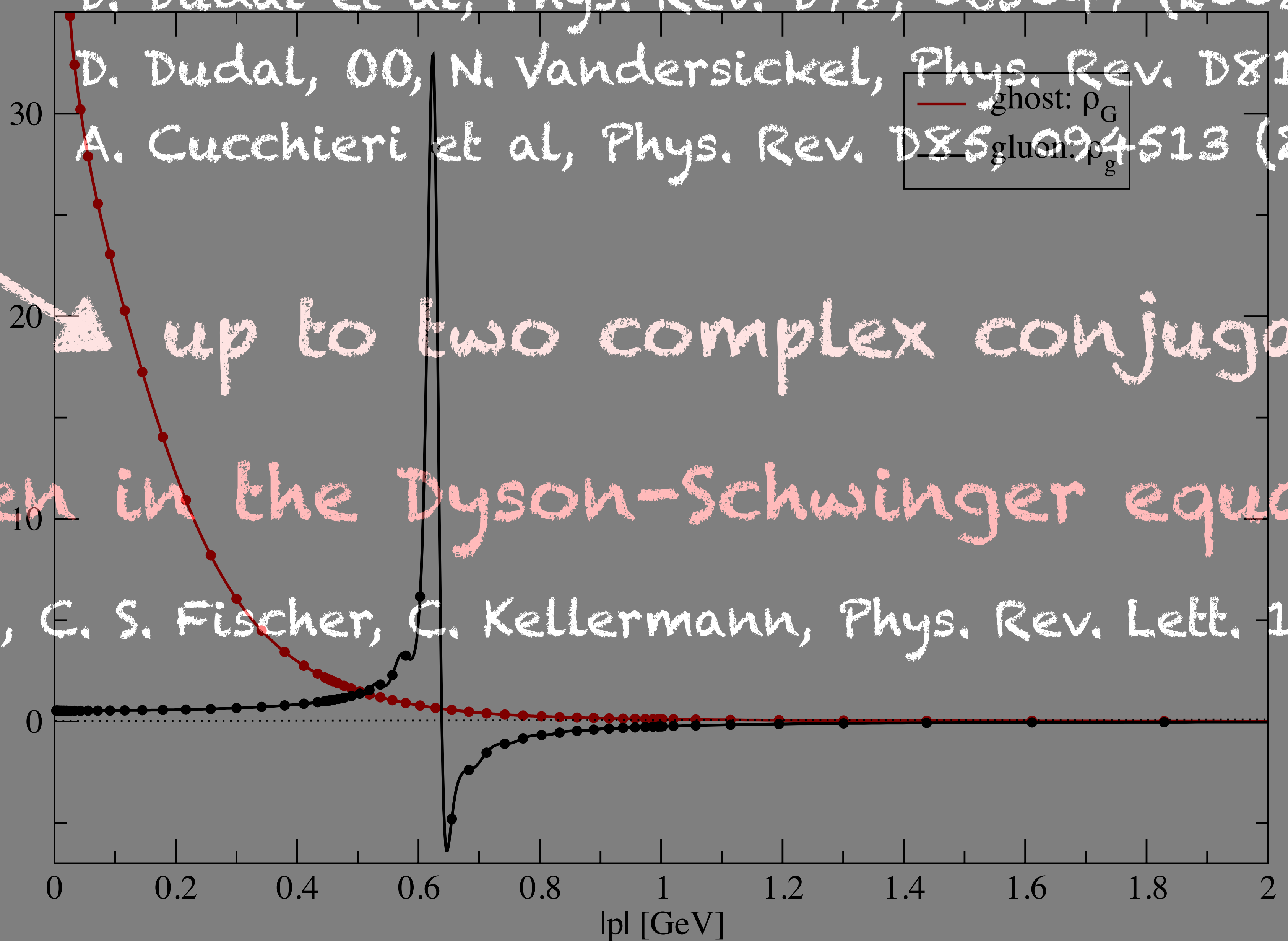
up to two complex conjugate poles



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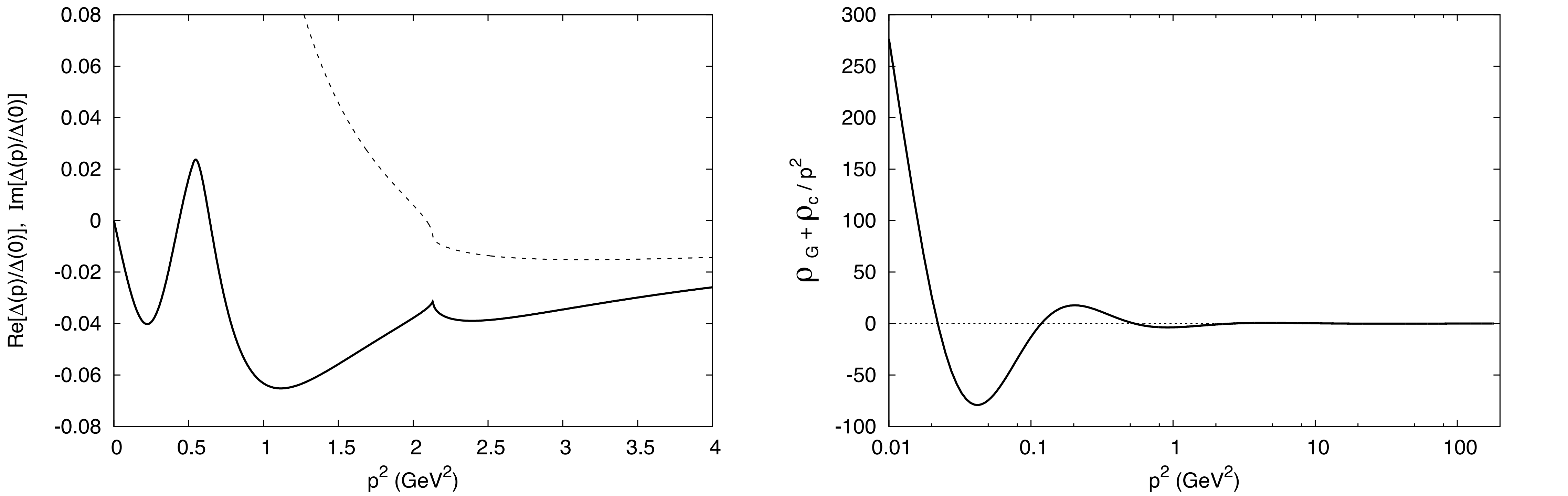


F Siringo, PRD94, 114036 (2016)

Gluon at Finite T

$$N^3 \times N_t \qquad N_t < N \qquad T = 1/aN_t$$

$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\left\{ P^T_{\mu\nu} D_T(p_4, \vec{p}) + P^L_{\mu\nu} D_T(p_4, \vec{p}) \right\}$$

Gluon at Finite T

$N^3 \times N_t \qquad N_t < N \qquad T = 1/aN_t$

$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\left\{ P^T_{\mu\nu} D_T(p_4,\vec{p}) + P^L_{\mu\nu} D_T(p_4,\vec{p}) \right\}$$

$\sim (6.5fm)^3$

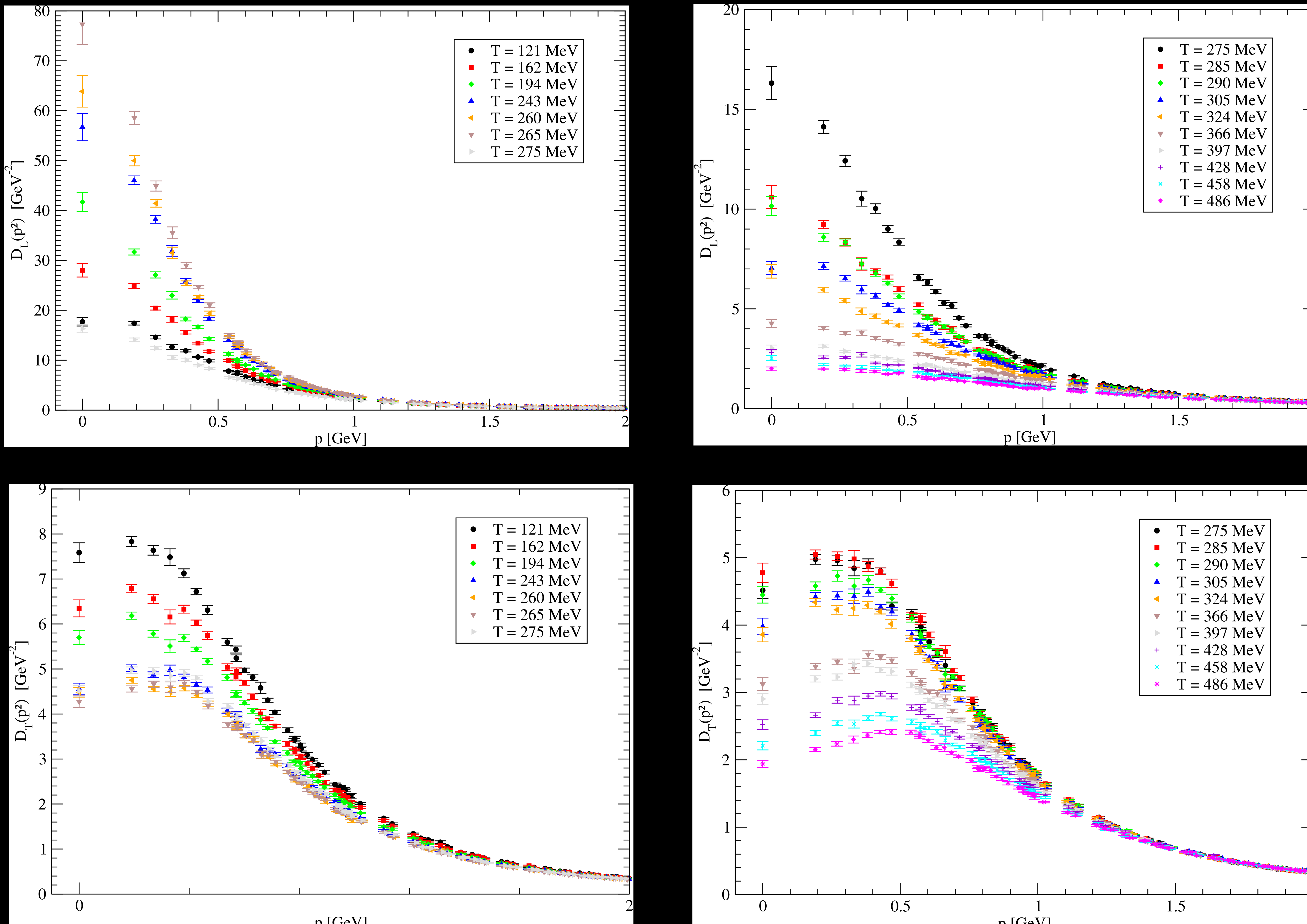
Gluon at Finite T

$N^3 \times N_t \qquad N_t < N \qquad T = 1/aN_t$

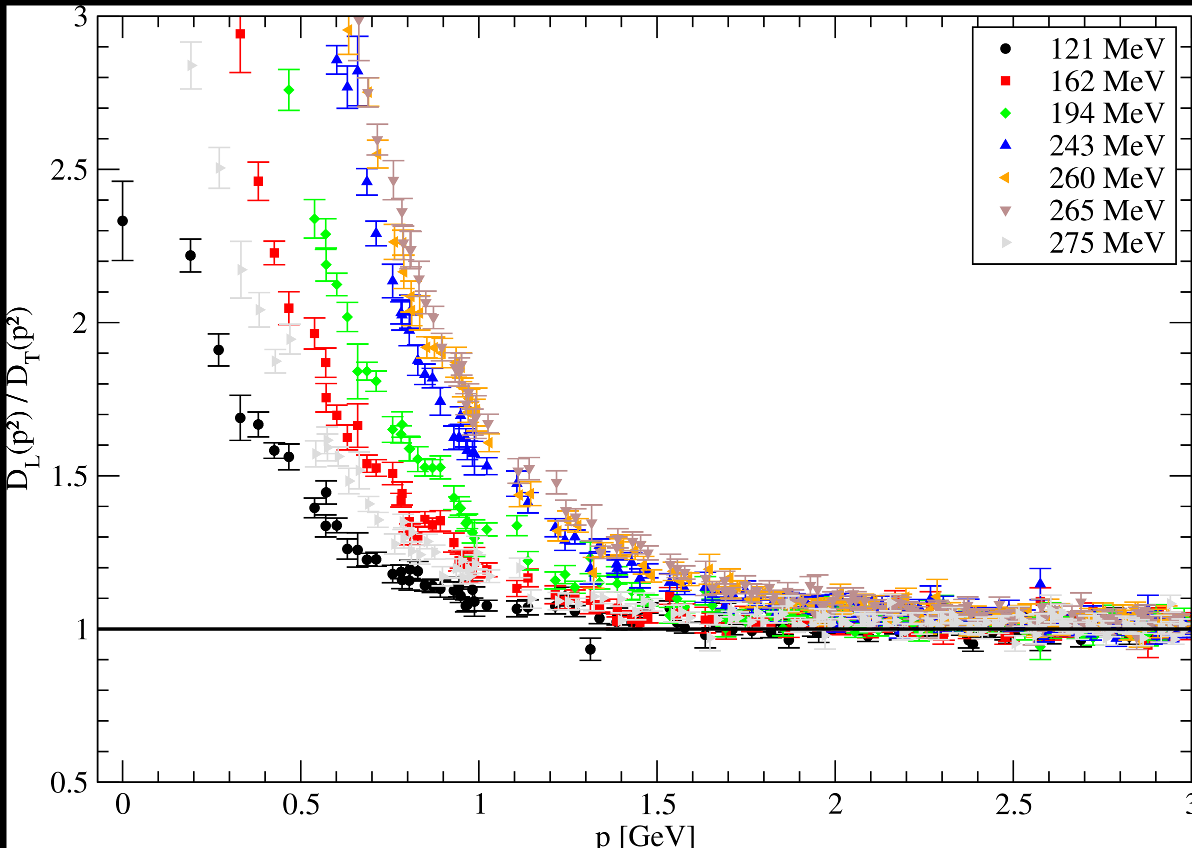
$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\left\{P^T_{\mu\nu}D_T(p_4,\vec{p}) + P^L_{\mu\nu}D_T(p_4,\vec{p})\right\}$$

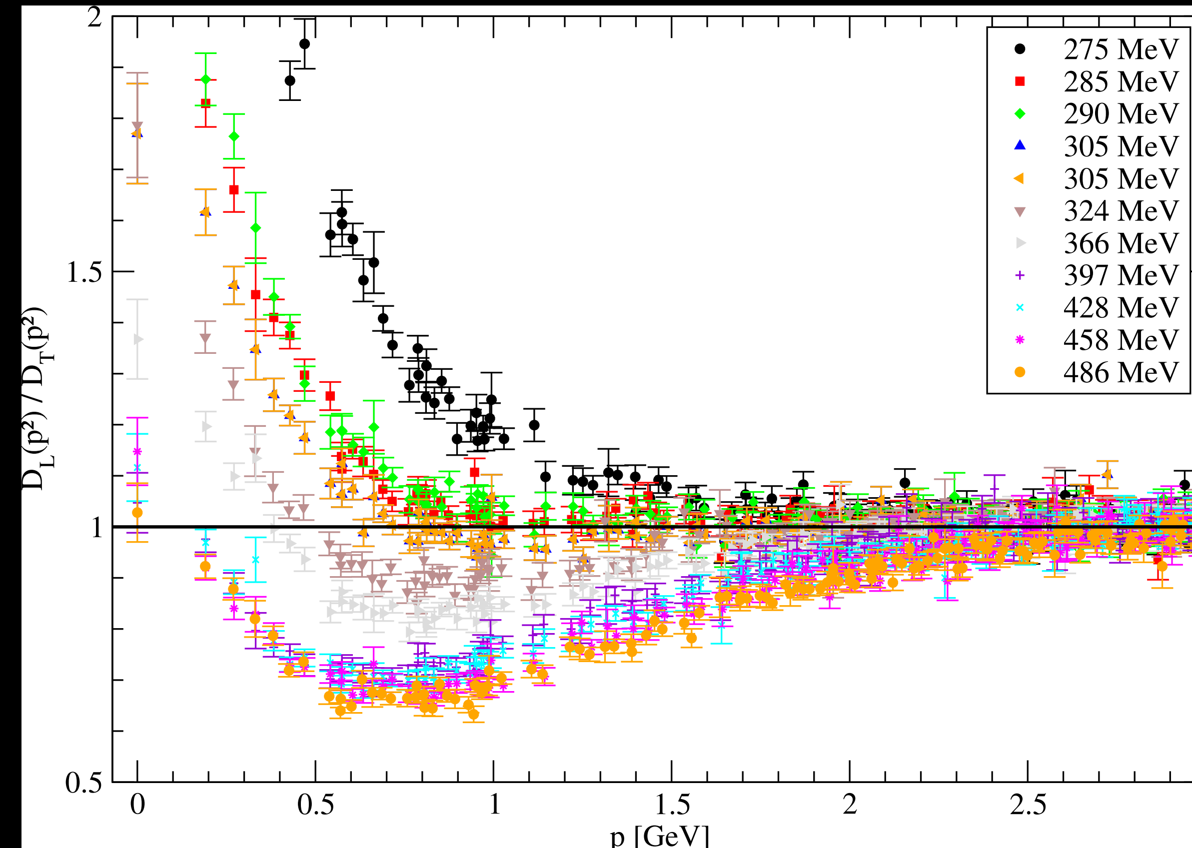
$\sim (6.5fm)^3$

Temp. (MeV)	β	L_s	L_t	a (fm)	$1/a$ (GeV)
121	6.0000	64	16	0.1016	1.9426
162	6.0000	64	12	0.1016	1.9426
194	6.0000	64	10	0.1016	1.9426
243	6.0000	64	8	0.1016	1.9426
260	6.0347	68	8	0.09502	2.0767
265	5.8876	52	6	0.1243	1.5881
275	6.0684	72	8	0.08974	2.1989
285	5.9266	56	6	0.1154	1.7103
290	6.1009	76	8	0.08502	2.3211
305	6.1326	80	8	0.08077	2.4432
324	6.0000	64	6	0.1016	1.9426
366	6.0684	72	6	0.08974	2.1989
397	5.8876	52	4	0.1243	1.5881
428	5.9266	56	4	0.1154	1.7103
458	5.9640	60	4	0.1077	1.8324
486	6.0000	64	4	0.1016	1.9426

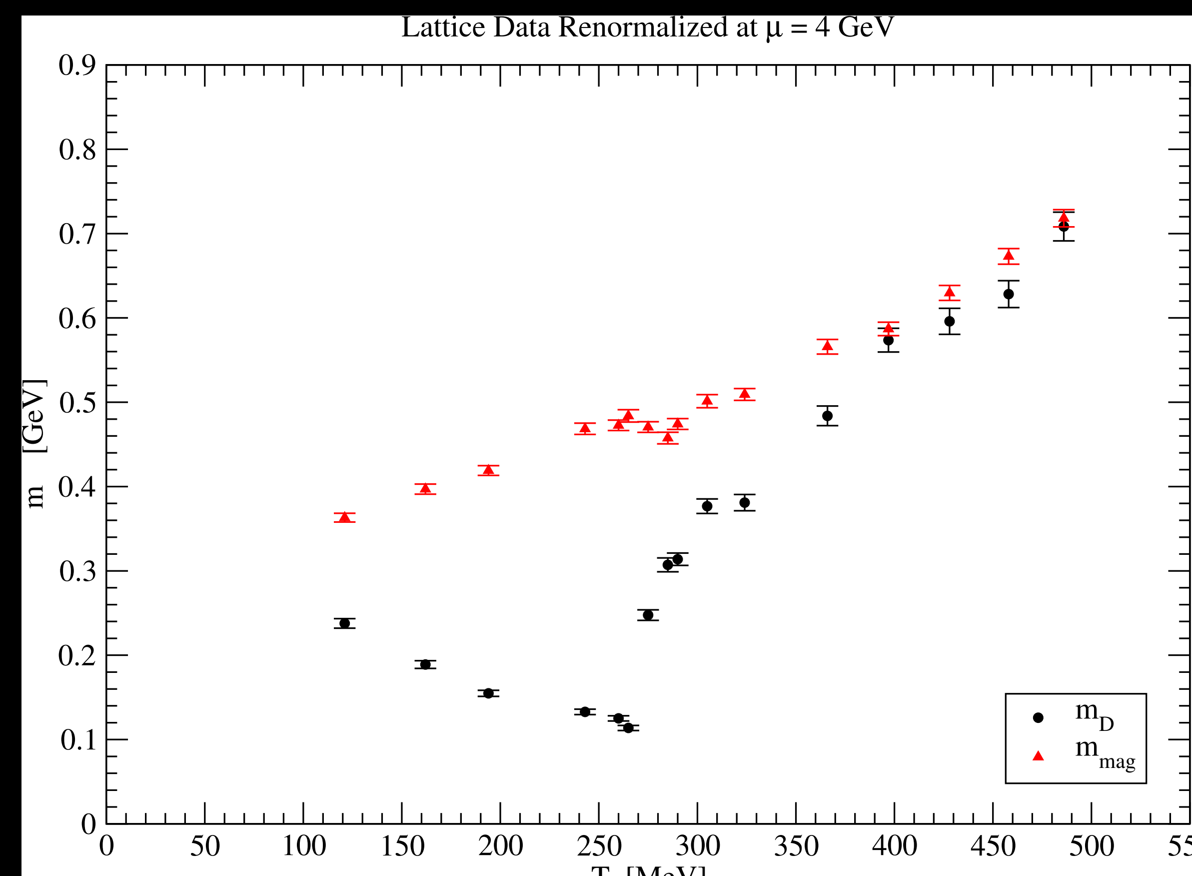


T = 121 MeV
T = 162 MeV
T = 194 MeV
T = 243 MeV
T = 260 MeV
T = 265 MeV
T = 275 MeV
$D_L(p^2)$ [GeV^{-2}]
p [GeV]
T = 275 MeV
T = 285 MeV
T = 290 MeV
T = 305 MeV
T = 324 MeV
T = 366 MeV
T = 397 MeV
T = 428 MeV
T = 458 MeV
T = 486 MeV
$D_L(p^2)$ [GeV^{-2}]
p [GeV]
T = 121 MeV
T = 162 MeV
T = 194 MeV
T = 243 MeV
T = 260 MeV
T = 265 MeV
T = 275 MeV
$D_T(p^2)$ [GeV^{-2}]
p [GeV]
T = 275 MeV
T = 285 MeV
T = 290 MeV
T = 305 MeV
T = 324 MeV
T = 366 MeV
T = 397 MeV
T = 428 MeV
T = 458 MeV
T = 486 MeV
$D_T(p^2)$ [GeV^{-2}]
p [GeV]





$$m = \frac{1}{\sqrt{D(0;T)}}$$

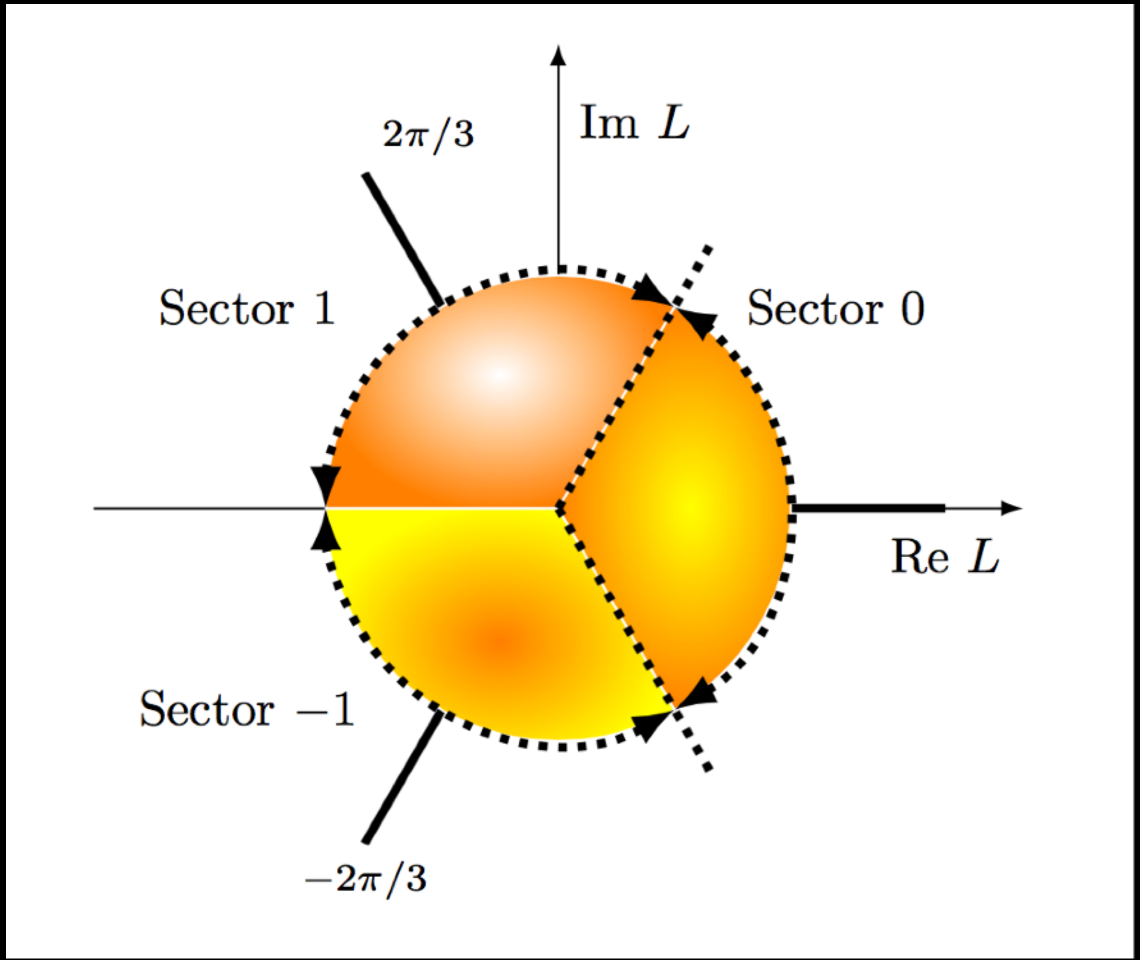


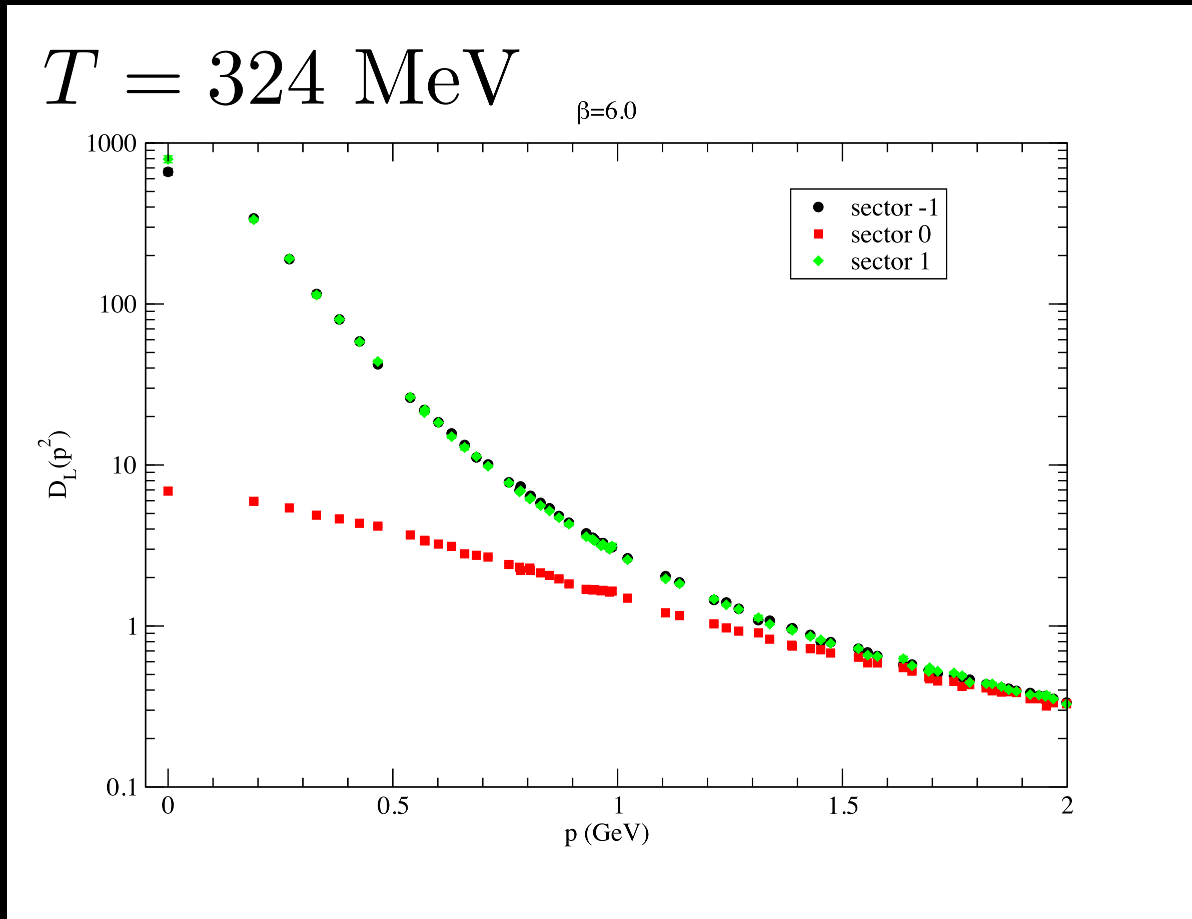
What about center symmetry

$$\mathcal{U}_\mu(x) \longrightarrow z\,\mathcal{U}_\mu(x) \qquad z \in Z_3 = \left\{1,\, e^{\pm i\frac{2\pi}{3}}\right\}$$

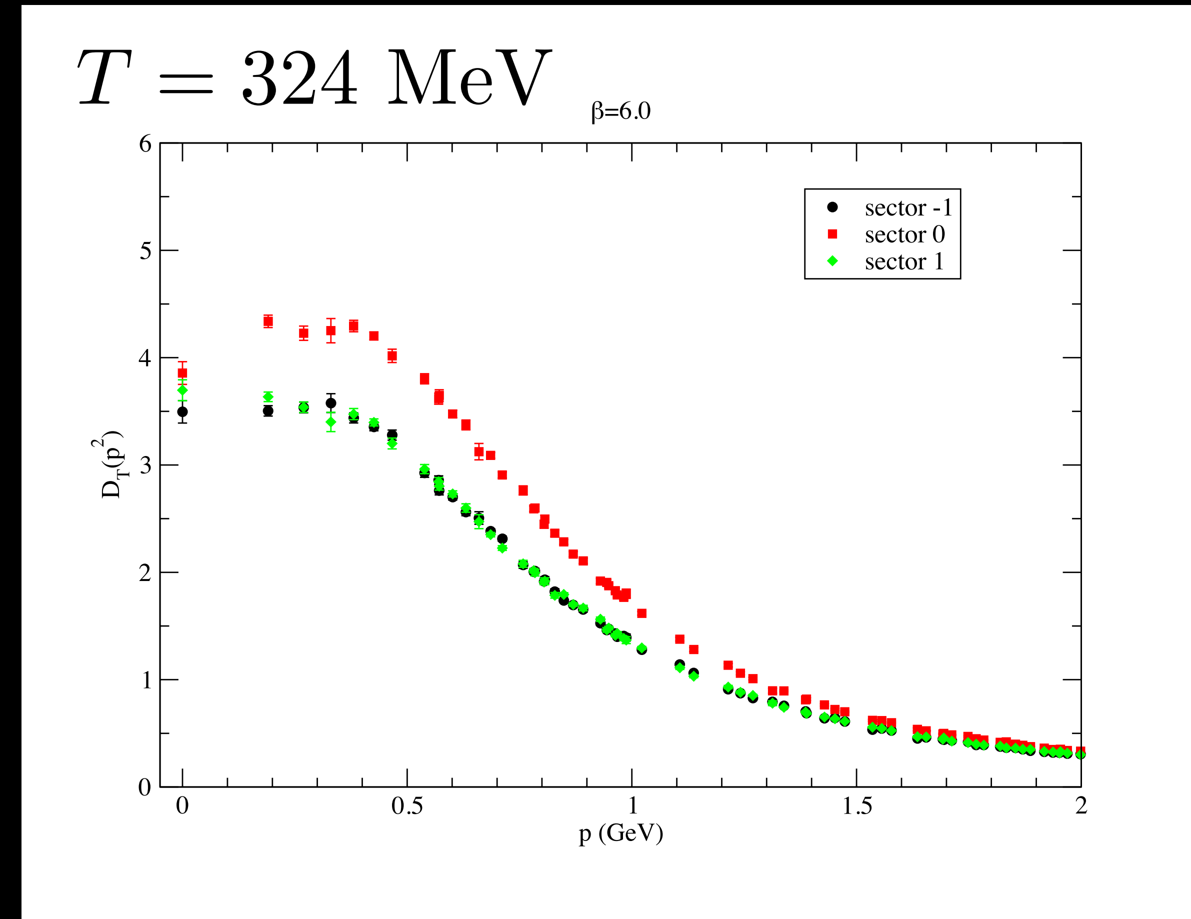
$$L = \langle L(\vec{x})\rangle = \mathrm{Tr}\left\{\mathcal{P}\exp\left[ig\int_0^{1/T} dx_0\, A_0(x_0,\vec{x})\right]\right\} \propto e^{-F_q/T}$$

$$L \longrightarrow L e^{i\theta}$$

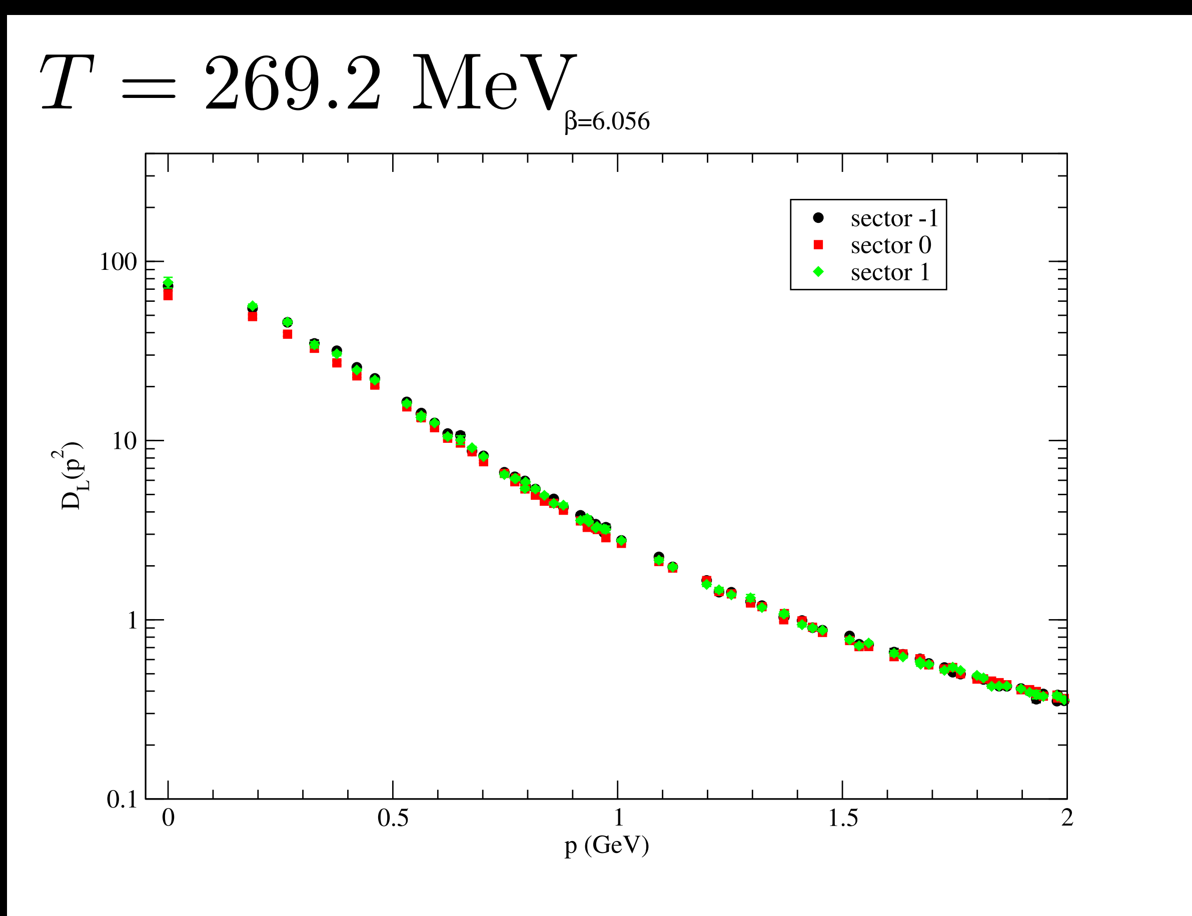




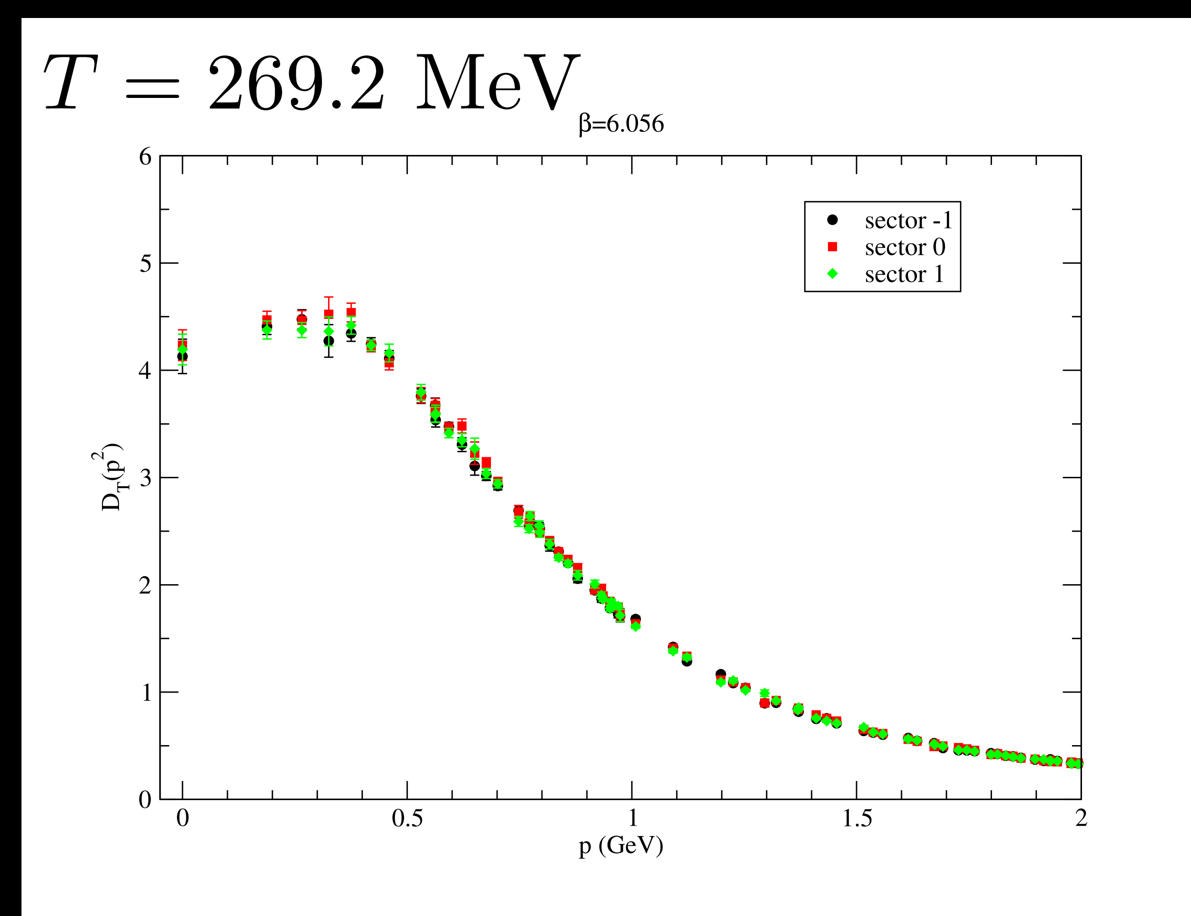
$T = 324$ MeV
β=6.0
sector -1
sector 0
sector 1
$D_L(p^2)$
p (GeV)



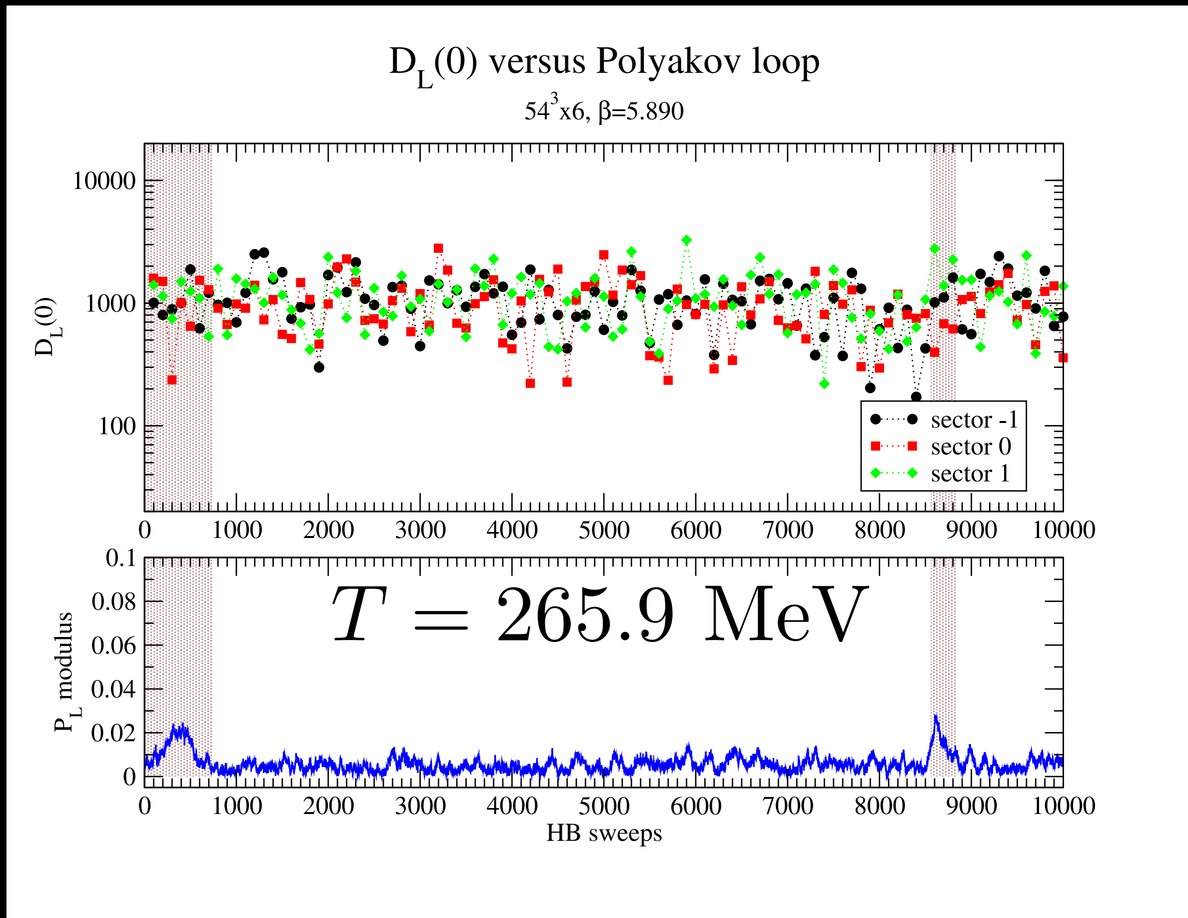
$T = 324$ MeV
β=6.0
sector -1
sector 0
sector 1
$D_T(p^2)$
p (GeV)

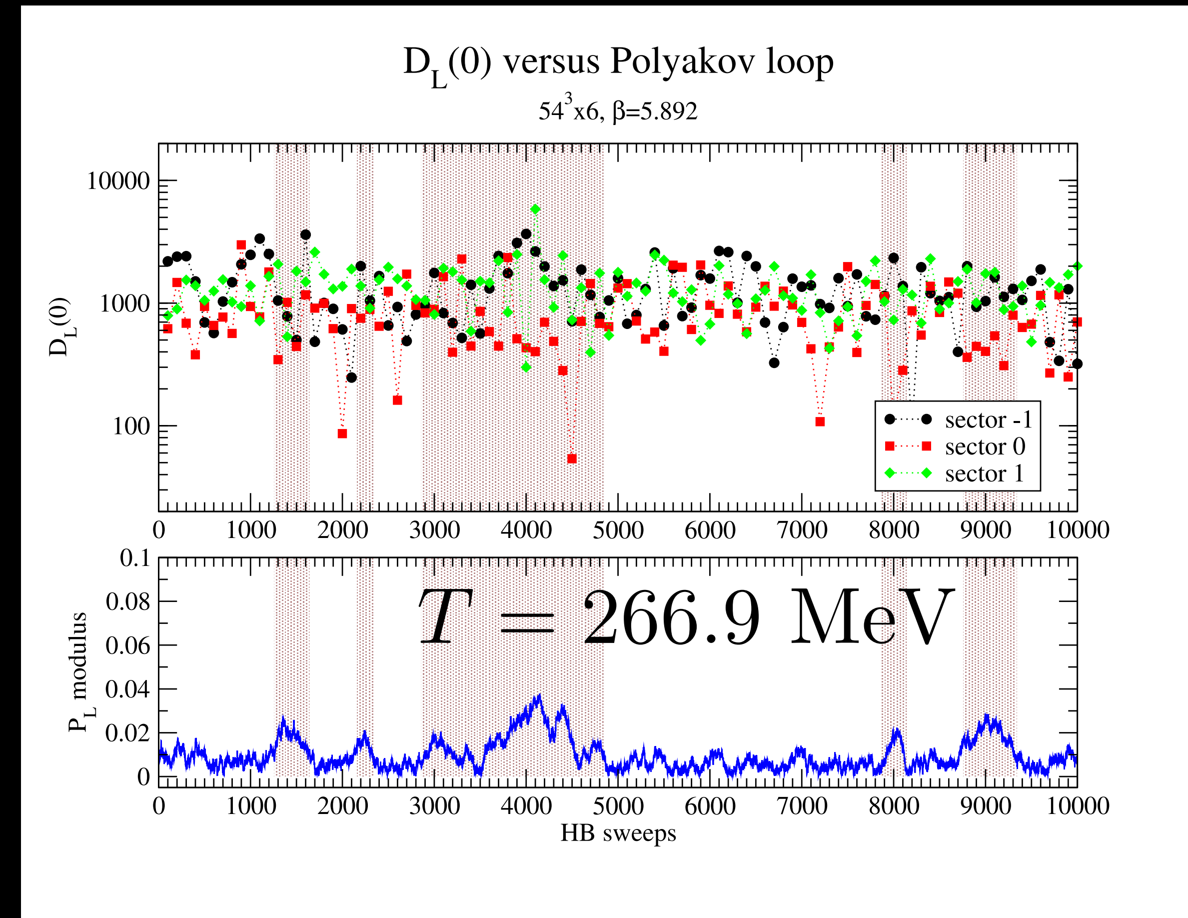


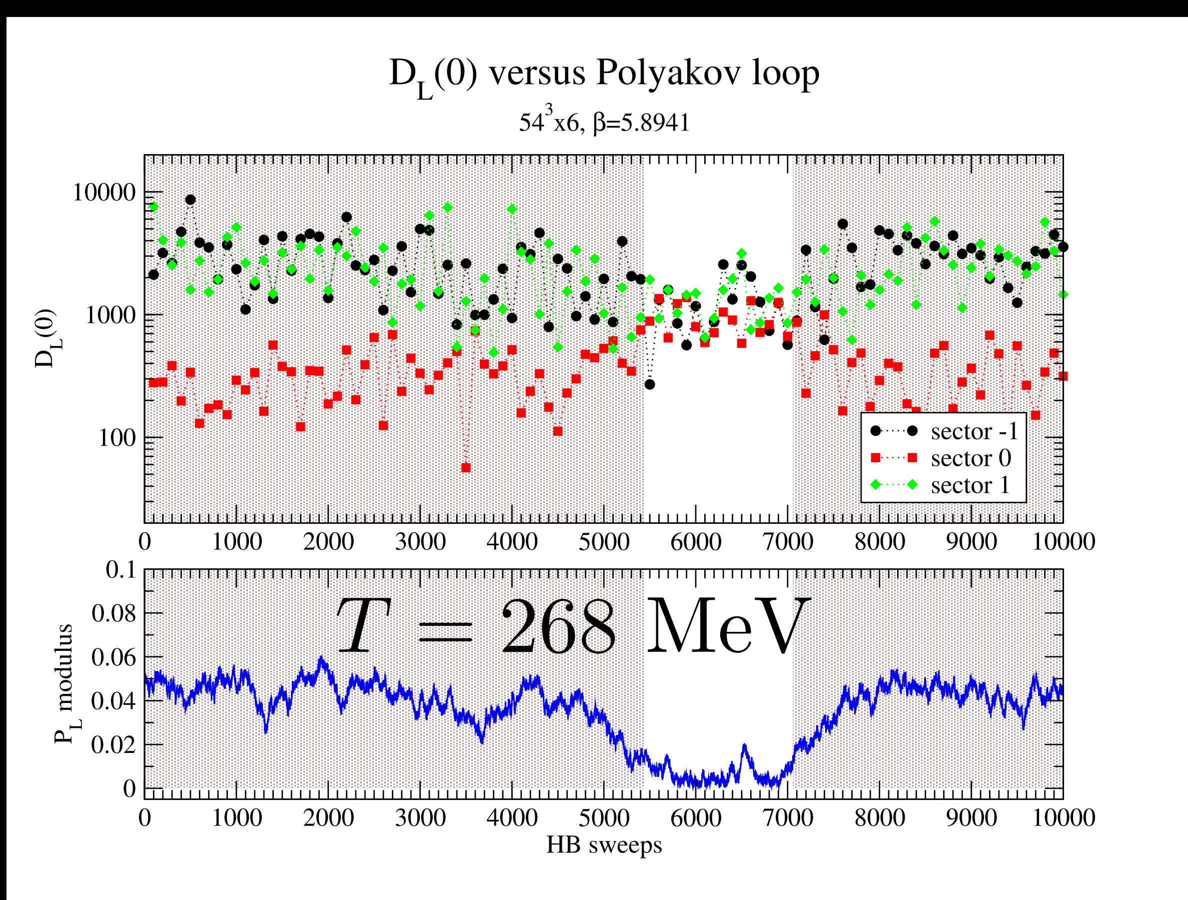
$T = 269.2$ MeV
β=6.056
sector -1
sector 0
sector 1
$D_L(p^2)$
p (GeV)

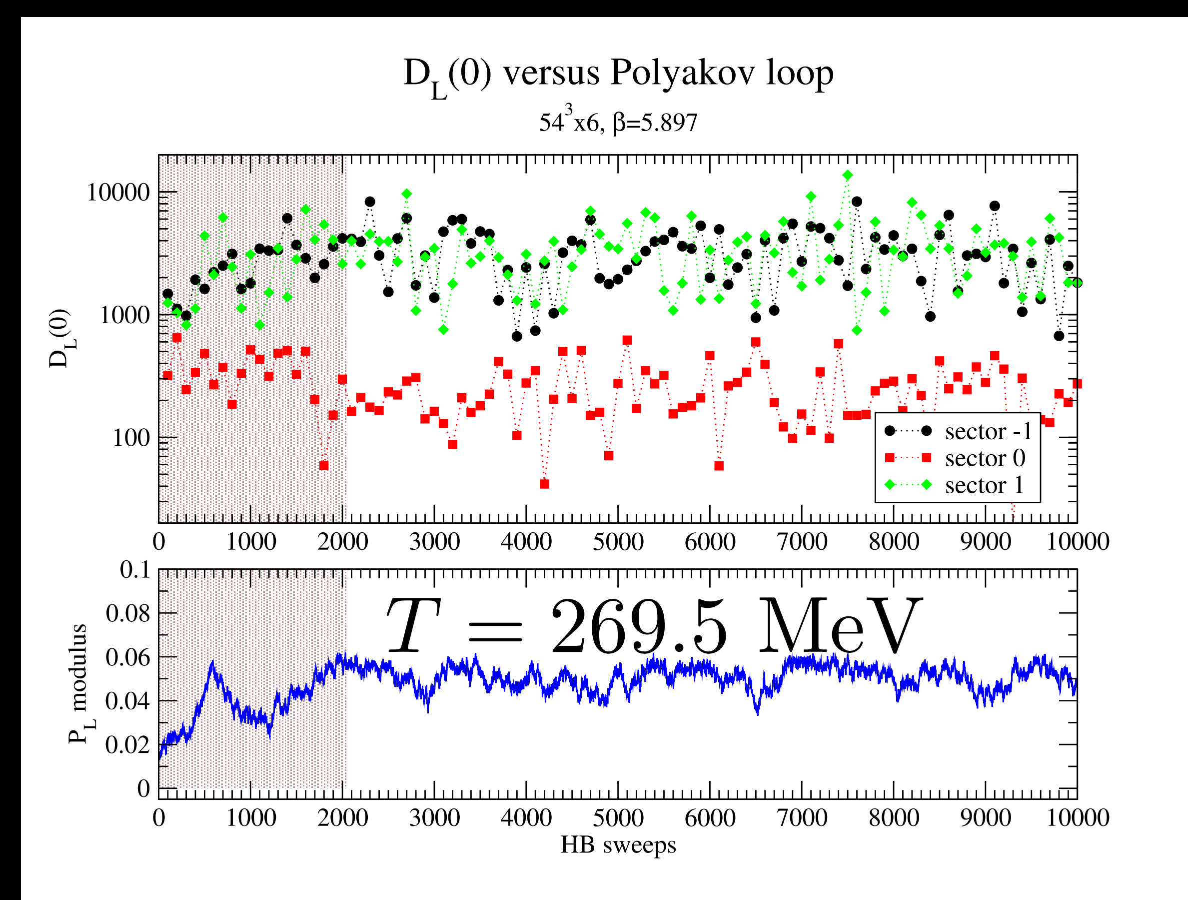


$T = 269.2$ MeV
β=6.056
sector -1
sector 0
sector 1
$D_T(p^2)$
p (GeV)


D_L(0) versus Polyakov loop
54^3x6, β=5.890
D_L(0)
sector -1
sector 0
sector 1
T = 265.9 MeV
P_L modulus
HB sweeps


D_L(0) versus Polyakov loop
54^3x6, β=5.892
D_L(0)
sector -1
sector 0
sector 1
T = 266.9 MeV
P_L modulus
HB sweeps


D_L(0) versus Polyakov loop
54^3x6, β=5.8941
D_L(0)
sector -1
sector 0
sector 1
T = 268 MeV
P_L modulus
HB sweeps


D_L(0) versus Polyakov loop
54^3x6, β=5.897
D_L(0)
sector -1
sector 0
sector 1
T = 269.5 MeV
P_L modulus
HB sweeps

	Coarser Lattice	Finer Lattice
$a(\text{fm})$	0.1237	0.09163
Starts at $T(\text{MeV})$	$267(D_L)$	$269(D_L)$
Seen above $T(\text{MeV})$	$269(D_L);\ 268(D_T)$	271

$$T_c \sim 268 - 271\ \text{MeV}$$

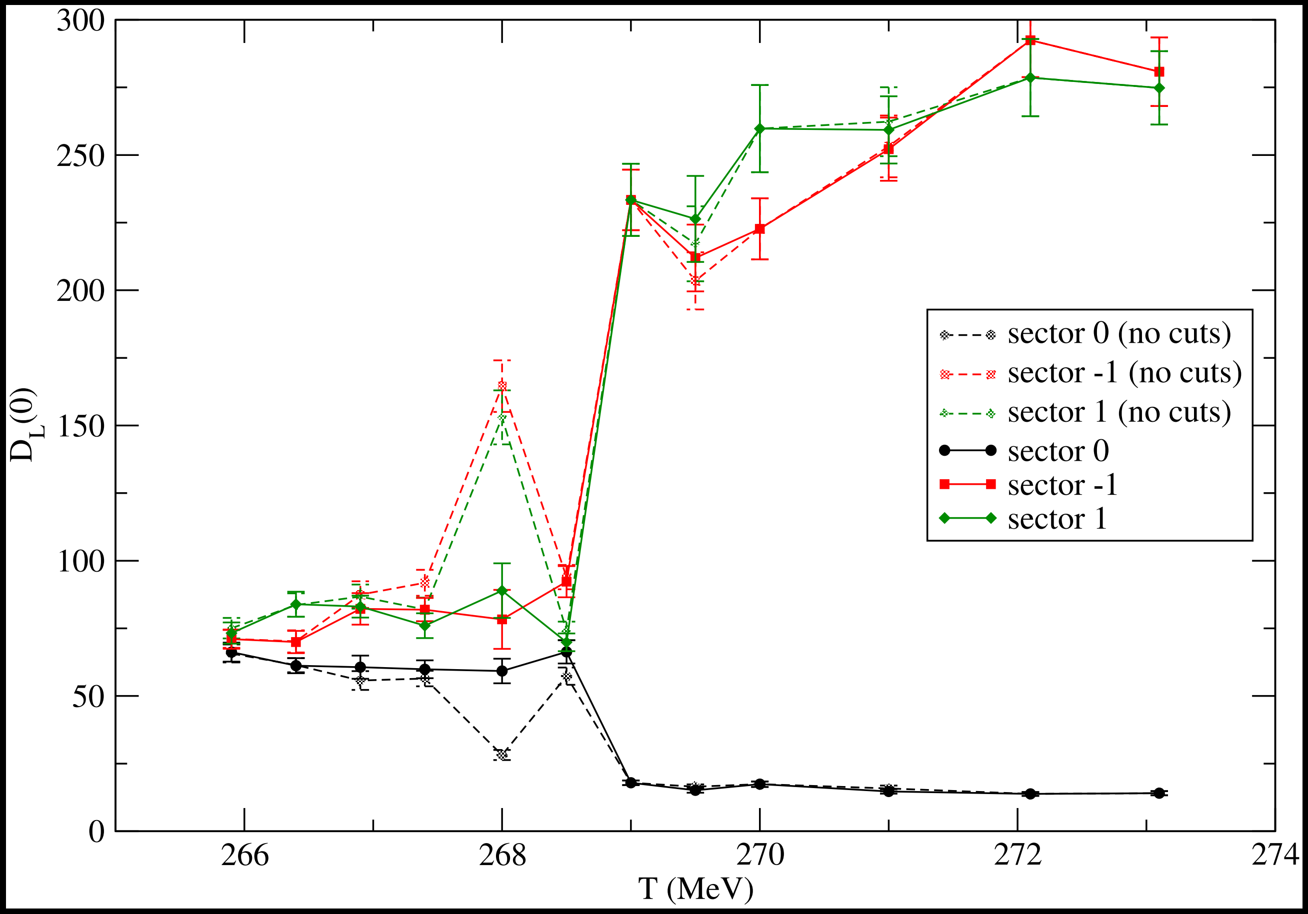
$$\mathcal{U}_\mu(x) = e^{iagA_\mu(x+a\,\hat{e}_\mu/2)}$$

$$\mathcal{U}_\mu(x) = u_0 e^{iag(A_\mu(x+a\,\hat{e}_\mu/2)+a_\mu)}$$

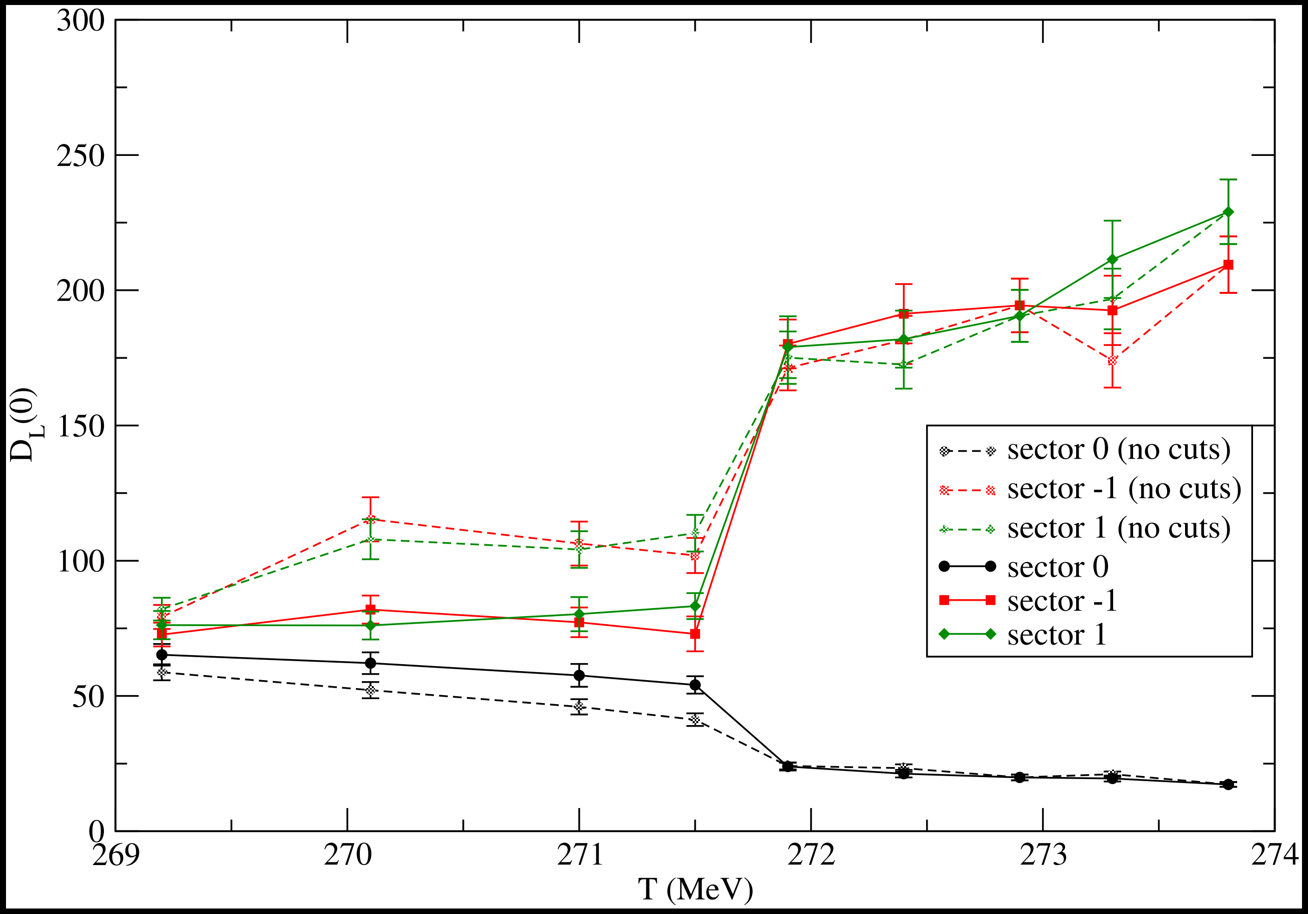
Temp. (MeV)	$L_s^3 \times L_t$	β	a (fm)	$L_s a$ (fm)
265.9	$54^3 \times 6$	5.890	0.1237	6.68
266.4	$54^3 \times 6$	5.891	0.1235	6.67
266.9	$54^3 \times 6$	5.892	0.1232	6.65
267.4	$54^3 \times 6$	5.893	0.1230	6.64
268.0	$54^3 \times 6$	5.8941	0.1227	6.63
268.5	$54^3 \times 6$	5.895	0.1225	6.62
269.0	$54^3 \times 6$	5.896	0.1223	6.60
269.5	$54^3 \times 6$	5.897	0.1220	6.59
270.0	$54^3 \times 6$	5.898	0.1218	6.58
271.0	$54^3 \times 6$	5.900	0.1213	6.55
272.1	$54^3 \times 6$	5.902	0.1209	6.53
273.1	$54^3 \times 6$	5.904	0.1204	6.50
269.2	$72^3 \times 8$	6.056	0.09163	6.60
270.1	$72^3 \times 8$	6.058	0.09132	6.58
271.0	$72^3 \times 8$	6.060	0.09101	6.55
271.5	$72^3 \times 8$	6.061	0.09086	6.54
271.9	$72^3 \times 8$	6.062	0.09071	6.53
272.4	$72^3 \times 8$	6.063	0.09055	6.52
272.9	$72^3 \times 8$	6.064	0.09040	6.51
273.3	$72^3 \times 8$	6.065	0.09025	6.50
273.8	$72^3 \times 8$	6.066	0.09010	6.49

P J Silva, O Oliveira PRD 93, 114509 (2016)

Coarser Lattice

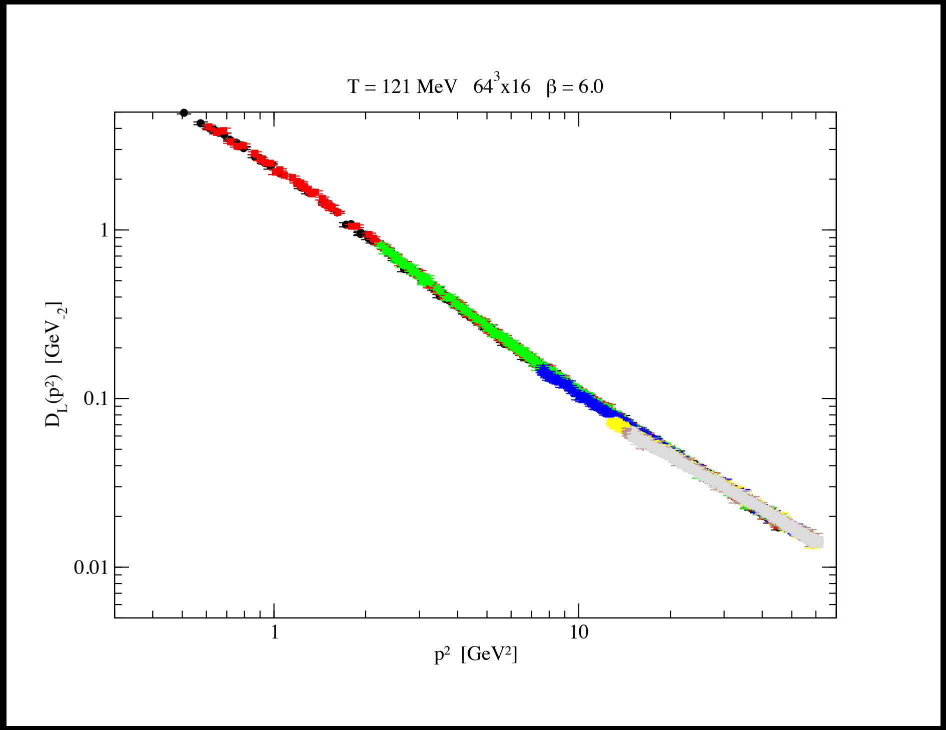


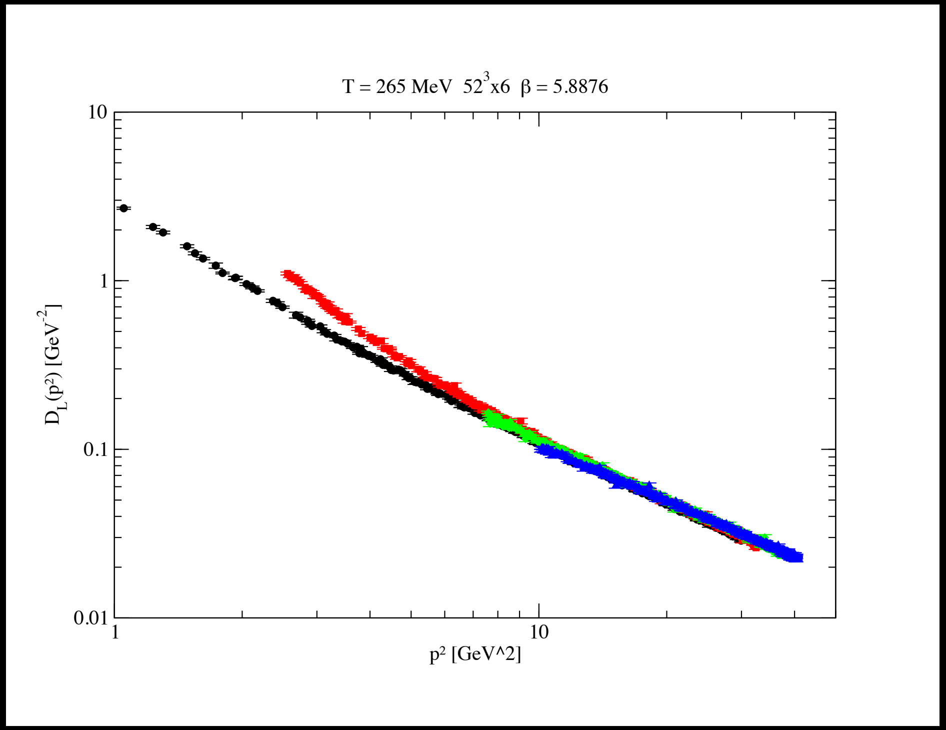
Finer Lattice

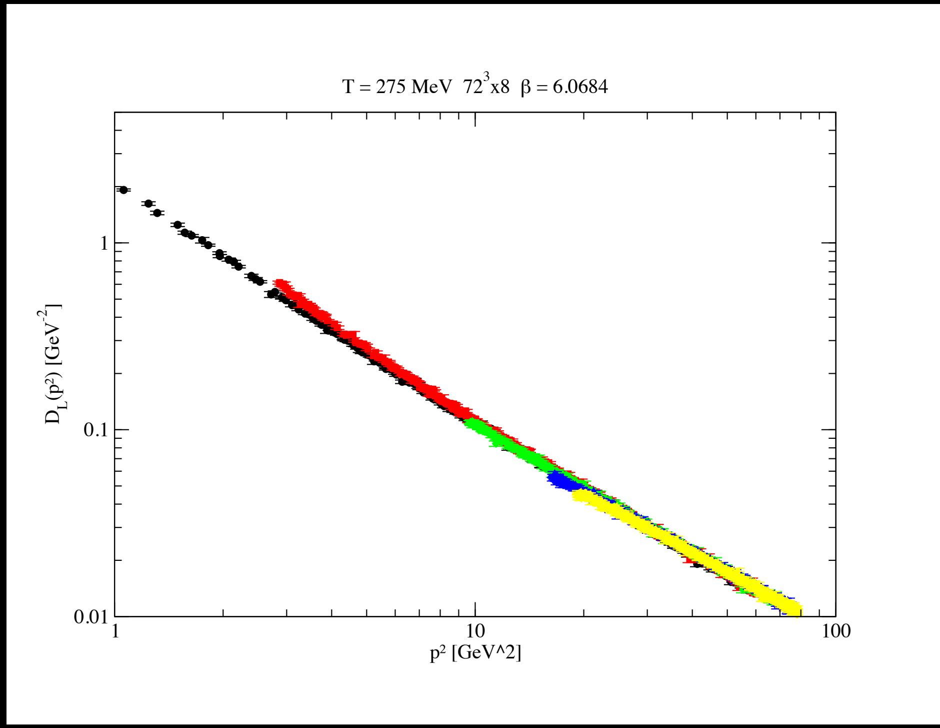


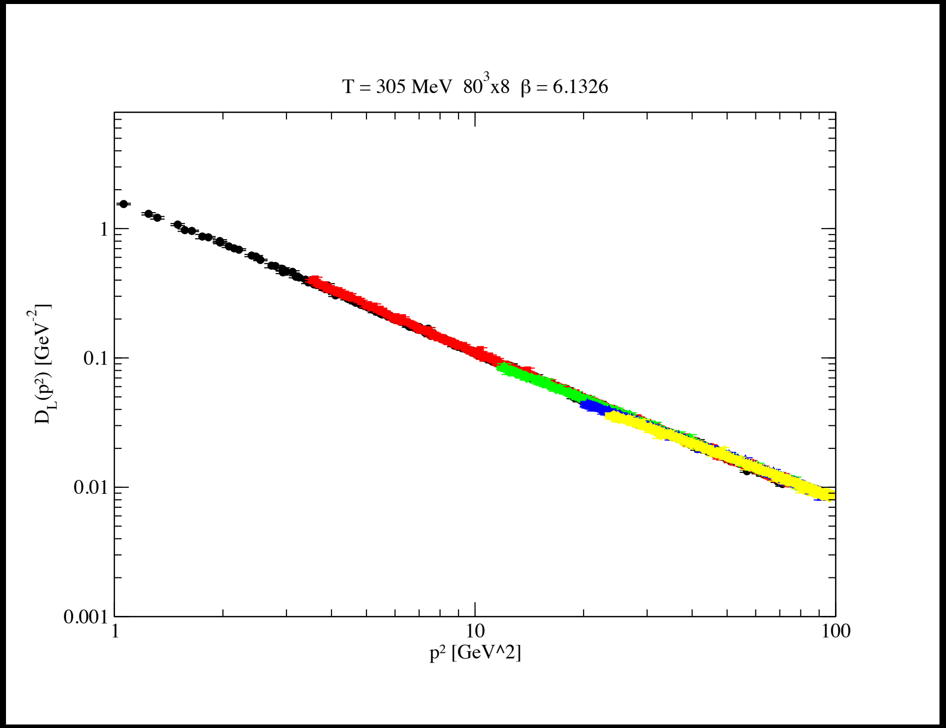
O(4) Scaling: $$D(q_4, \vec{q}) = D(p^2 = q_4^2 + \vec{q} \cdot \vec{q})$$

Electric Form Factor



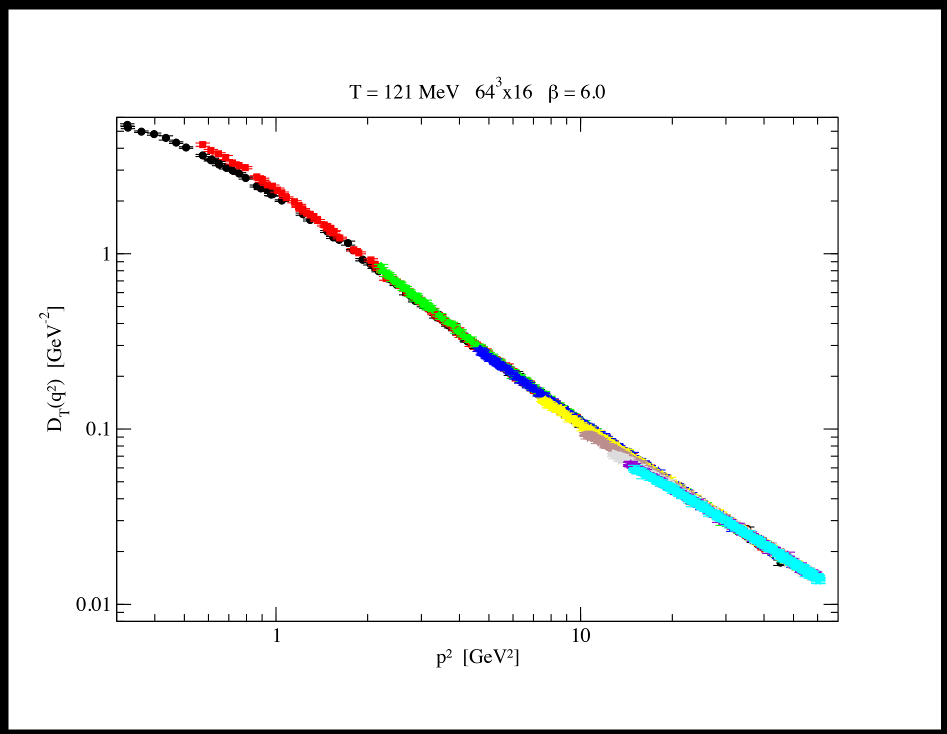


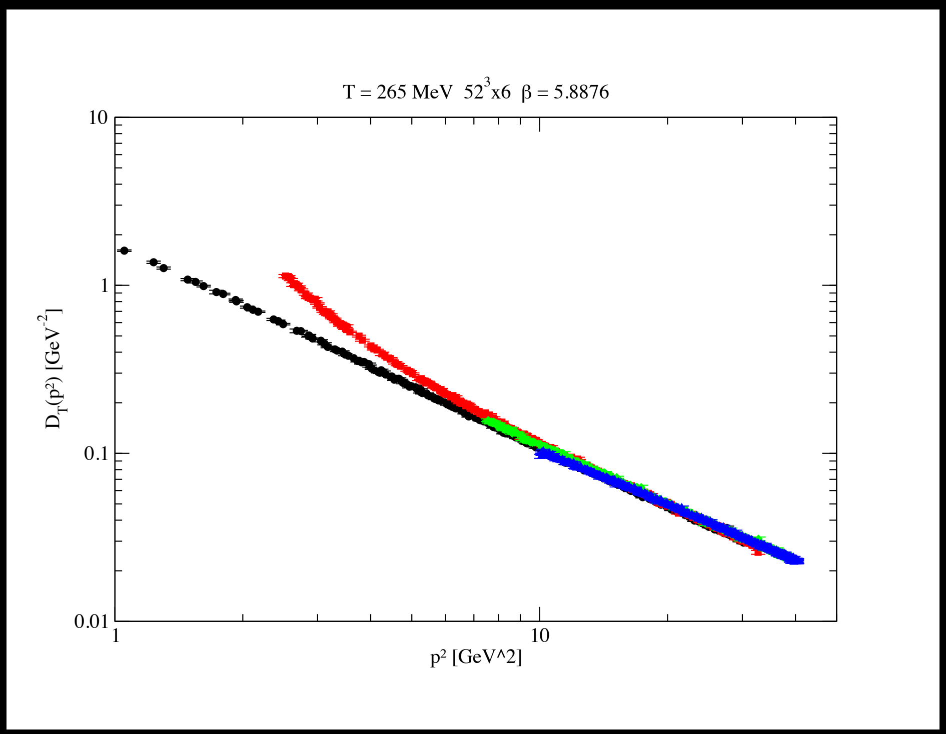


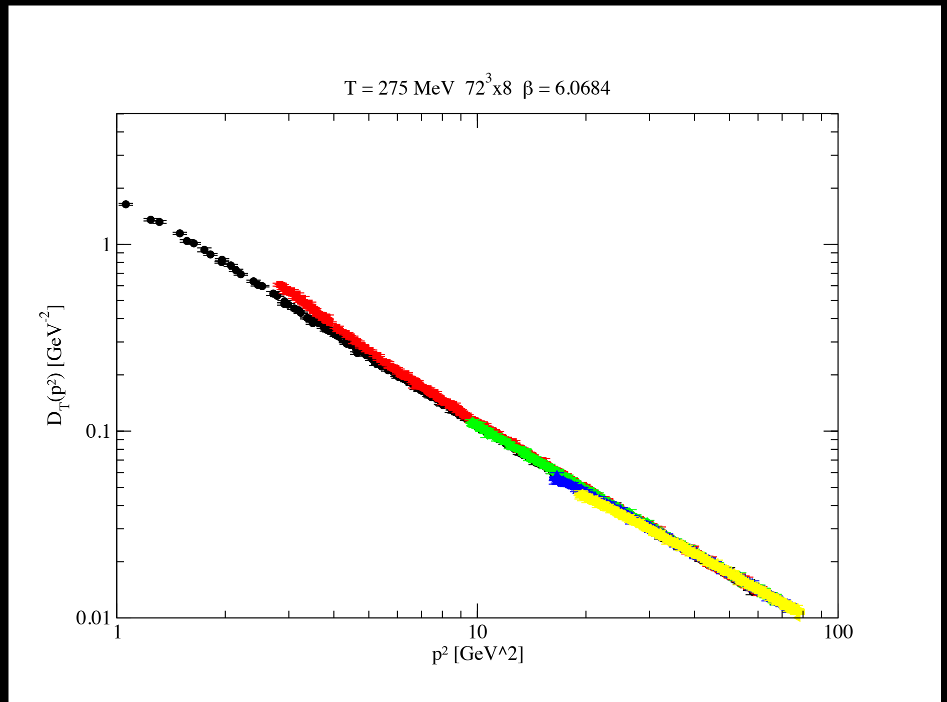


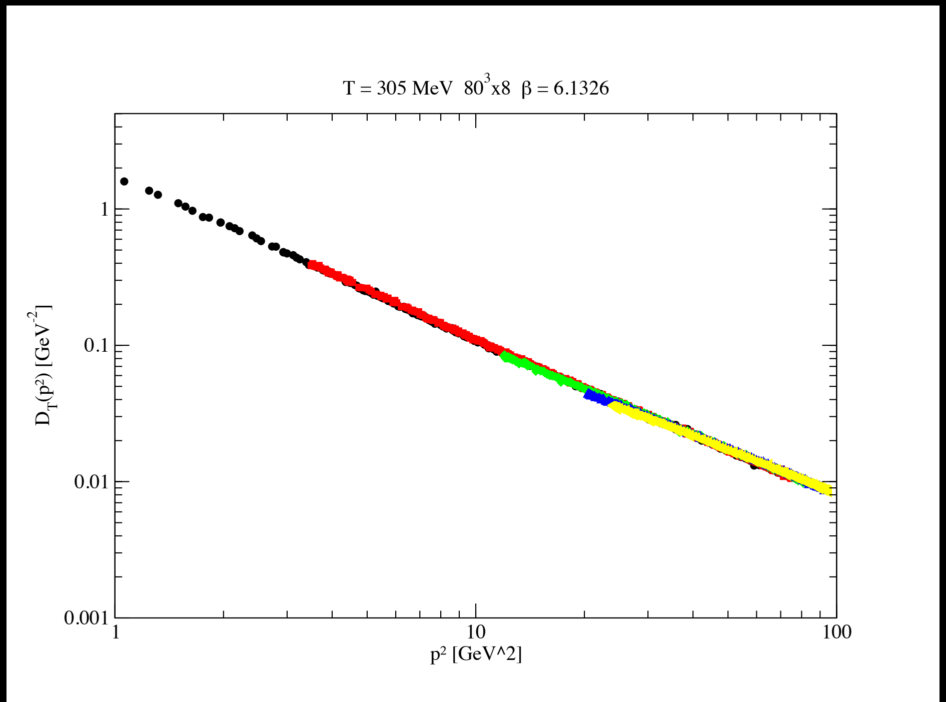
O(4) Scaling: $$D(q_4, \vec{q}) = D(p^2 = q_4^2 + \vec{q} \cdot \vec{q})$$

Magnetic Form Factor









Quasi-Particle approximation :

$$D(q_4 = 0, \vec{q}) = \frac{Z}{q^2 + m_g^2}$$

only the electric part form factor D_L

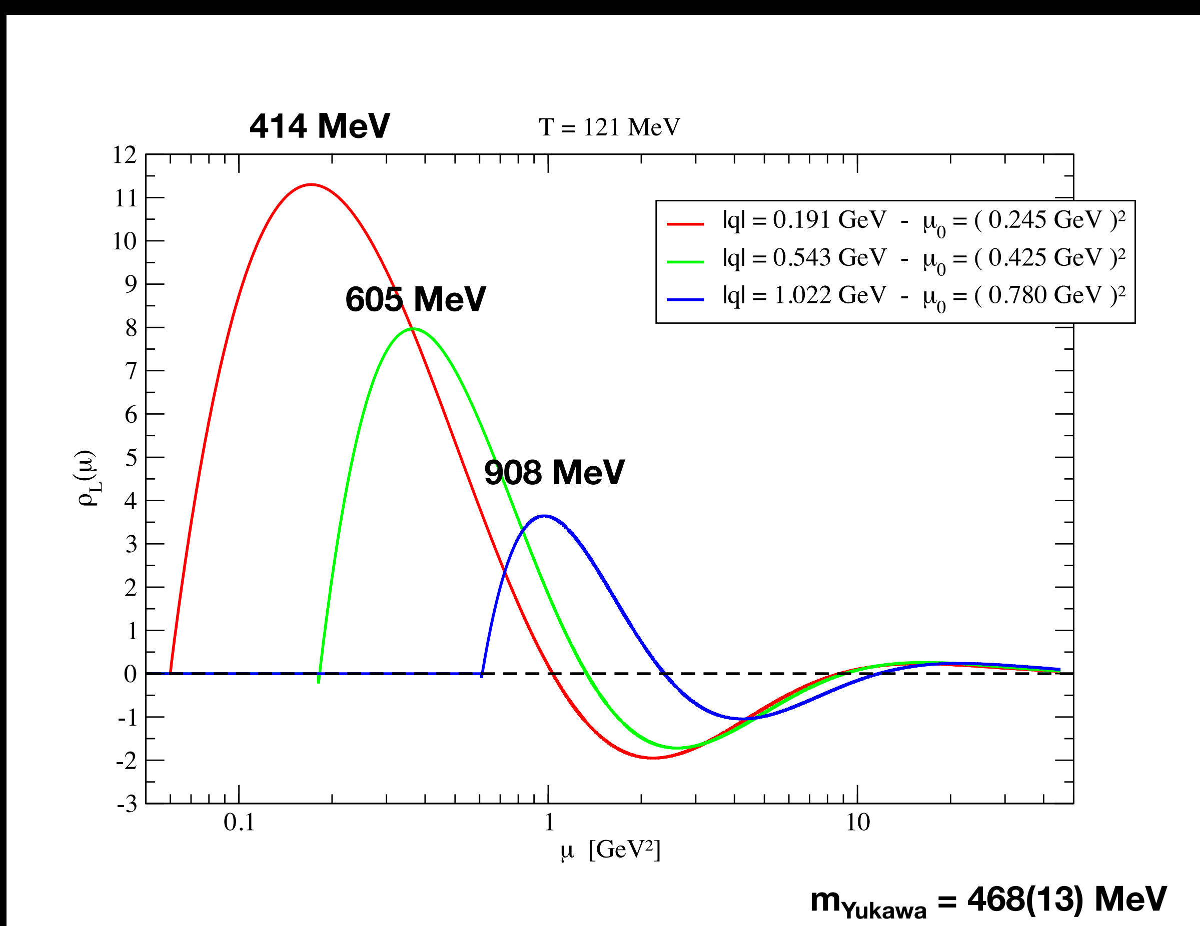
Temp. (MeV)	p_{max} (GeV)	Z	$m_g(T)$ (GeV)	χ^2/d.o.f.
121	0.467	4.28(16)	0.468(13)	1.91
162	0.570	4.252(89)	0.3695(73)	1.66
194	0.330	5.84(50)	0.381(22)	0.72
243	0.330	8.07(67)	0.374(21)	0.27
260	0.271	8.73(86)	0.371(25)	0.03
265	0.332	7.34(45)	0.301(14)	1.03
275	0.635	3.294(65)	0.4386(83)	1.64
285	0.542	3.12(12)	0.548(16)	0.76
290	0.690	2.705(50)	0.5095(85)	1.40
305	0.606	2.737(80)	0.5900(32)	1.30
324	0.870	2.168(24)	0.5656(63)	1.36
366	0.716	2.242(55)	0.708(13)	1.80
397	0.896	2.058(34)	0.795(11)	1.03
428	1.112	1.927(24)	0.8220(89)	1.30
458	0.935	1.967(37)	0.905(13)	1.45
486	1.214	1.847(24)	0.9285(97)	1.55

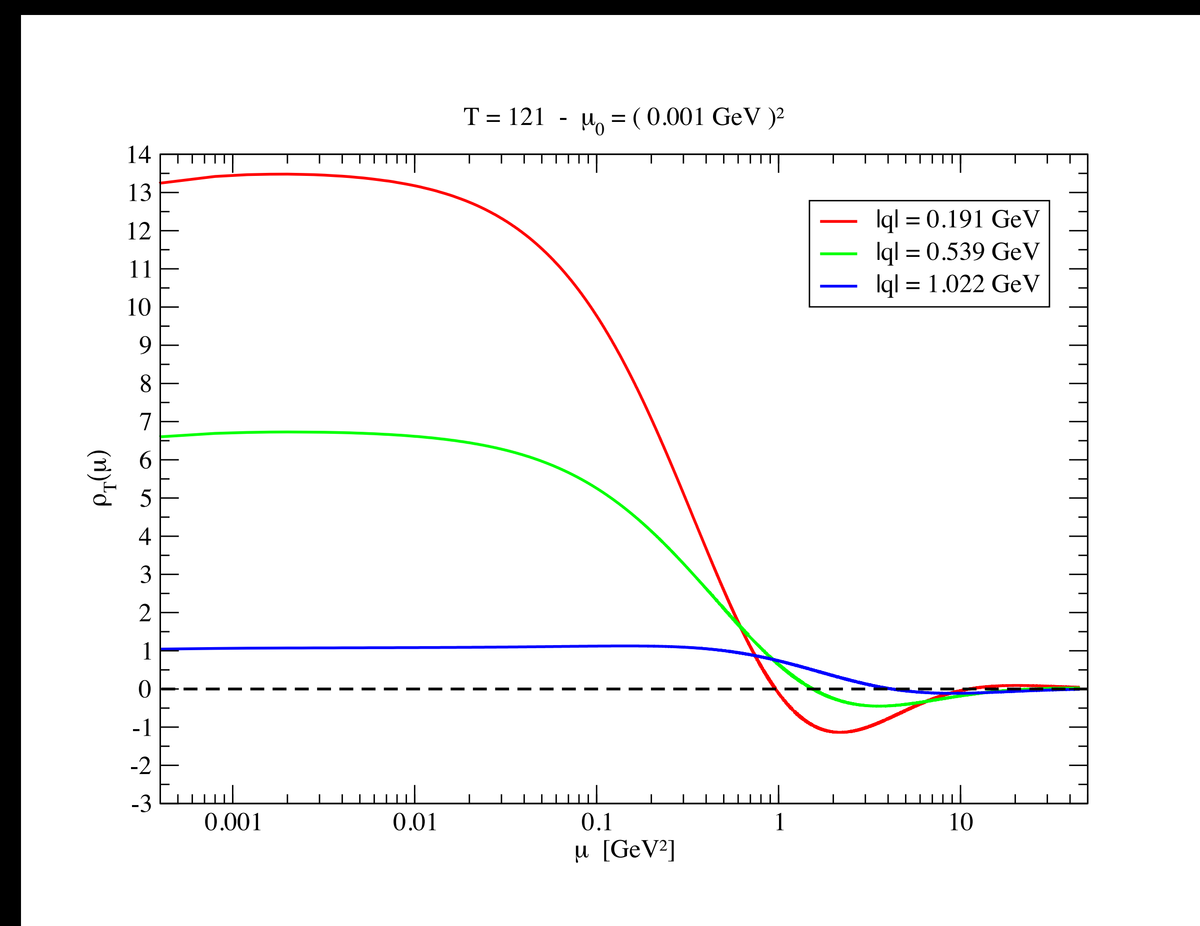
compatible with linear grow with T

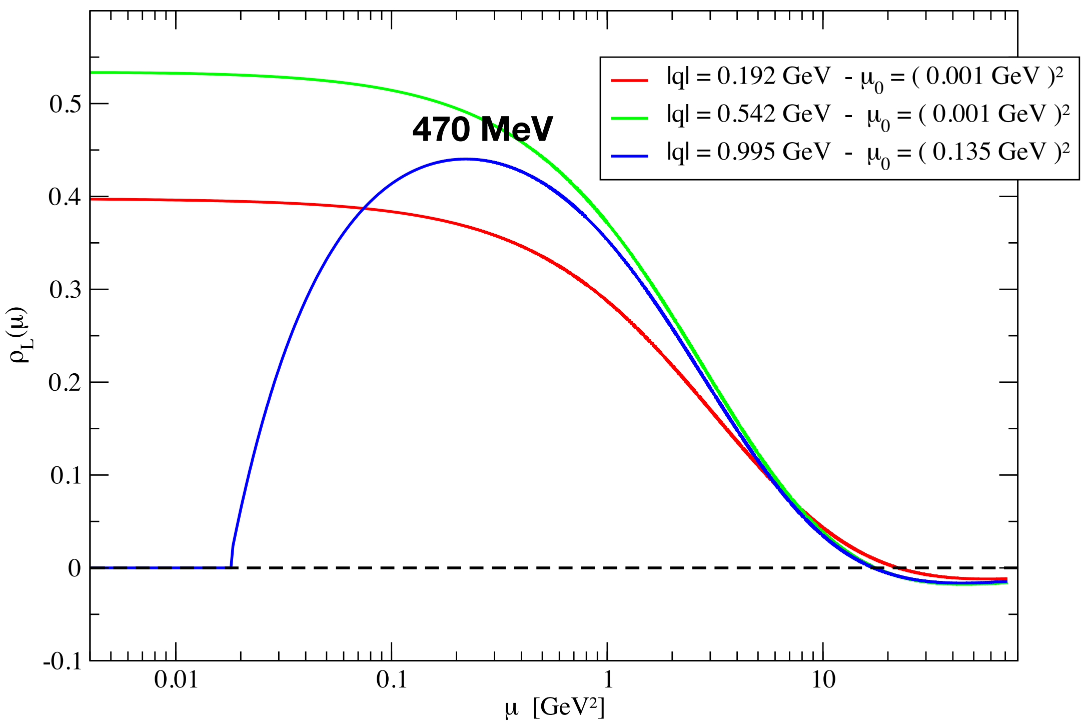
P J Silva, O Oliveira, P Bicudo, N Cardoso, PRD89, 074503 (2014)

Spectral Representation @Finite T

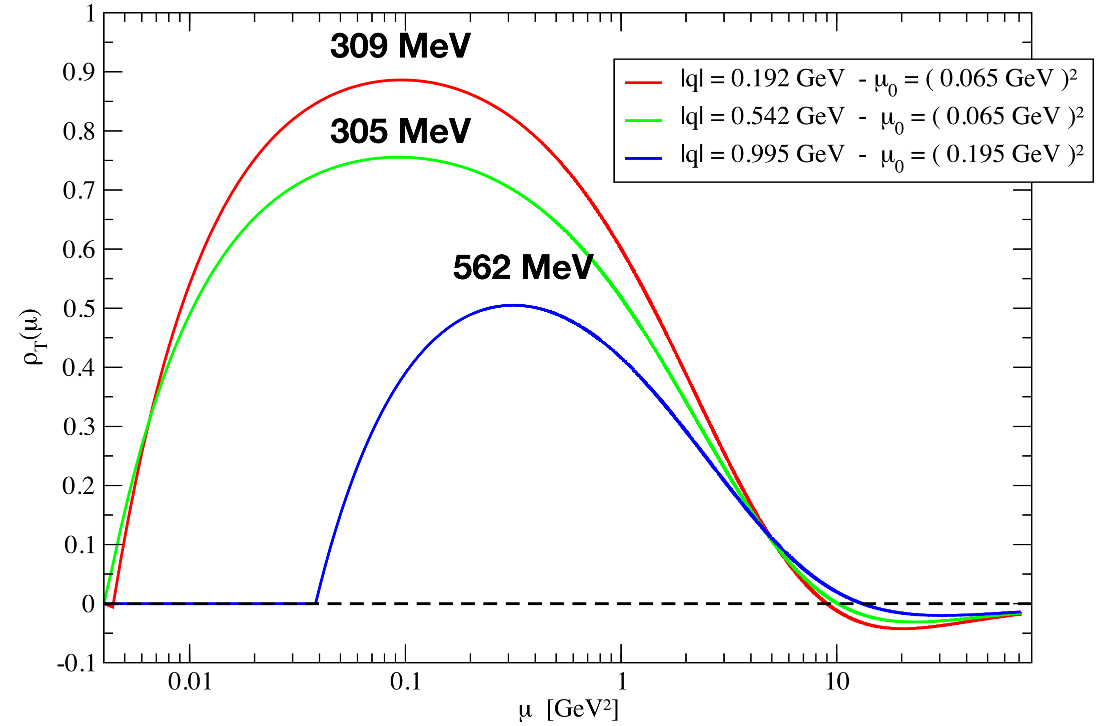
$$D(q_4,\vec{q}) = \int_{\mu_0}^{+\infty} d\mu \; \frac{\rho(\mu,\vec{q})}{q_4^2+\mu}$$

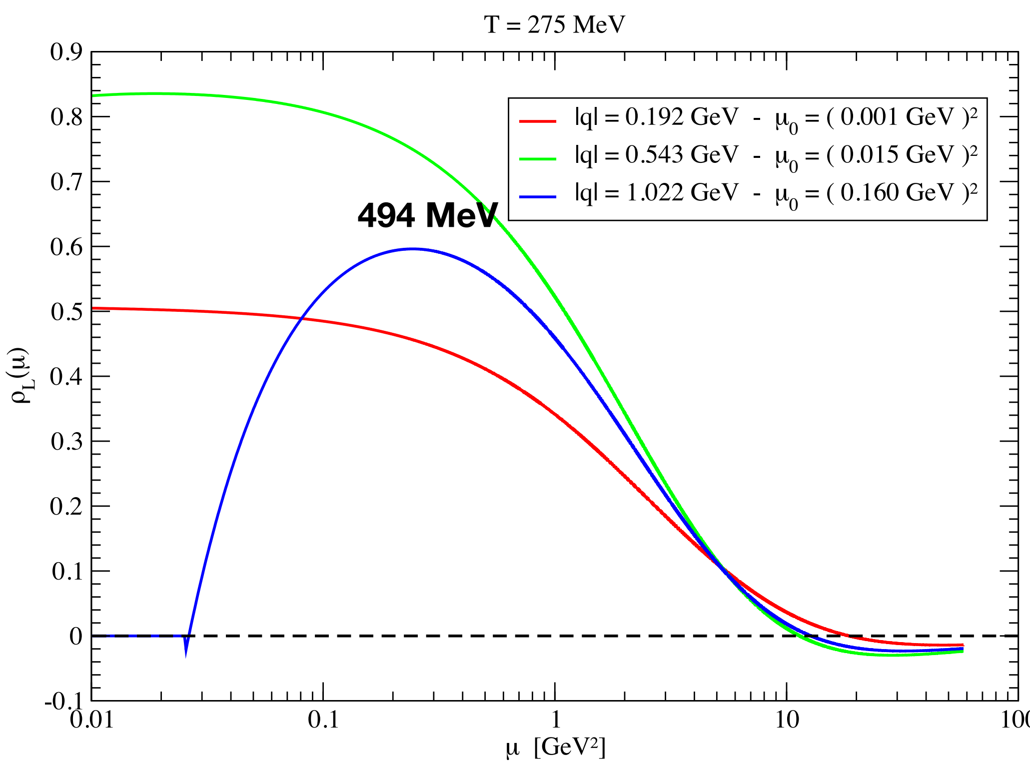




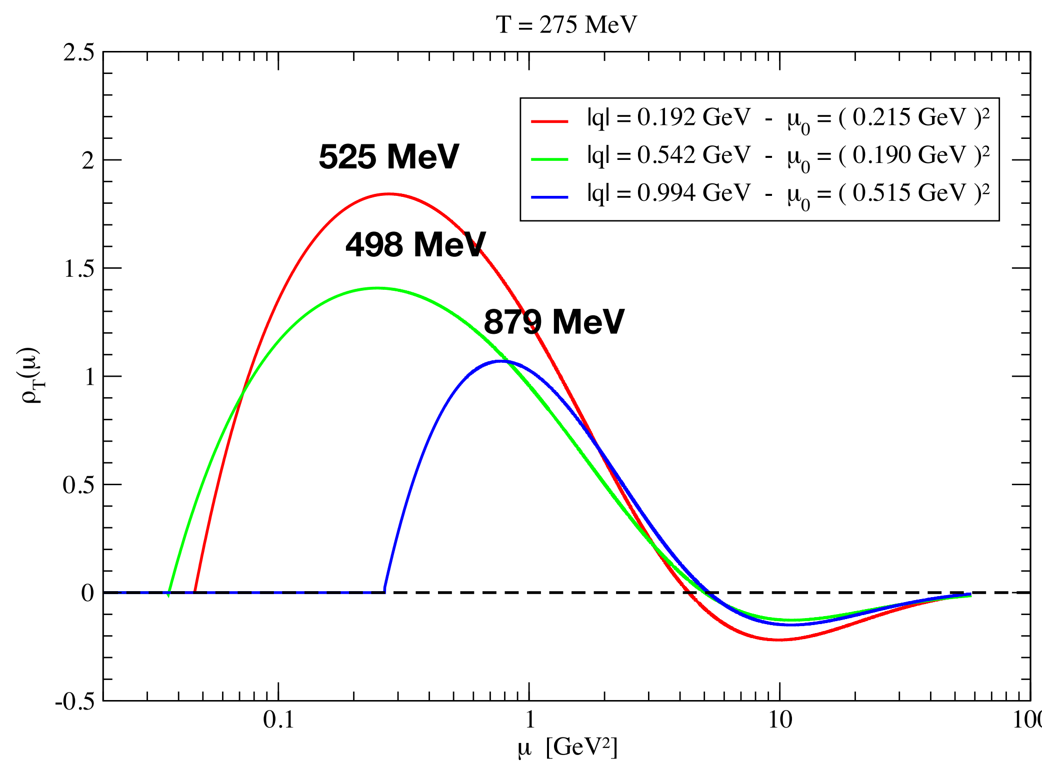


m_{Yukawa} = 590(3) MeV

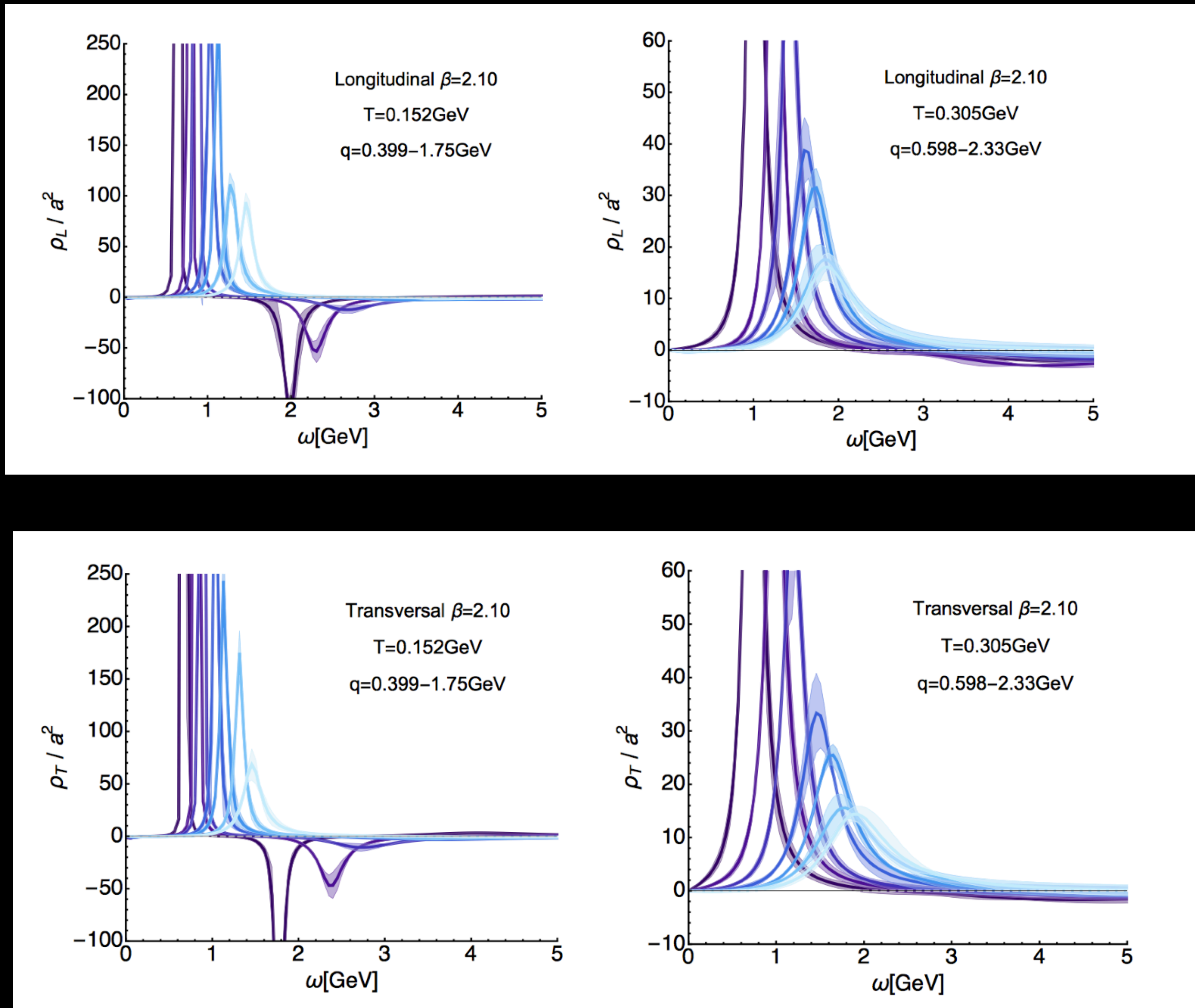


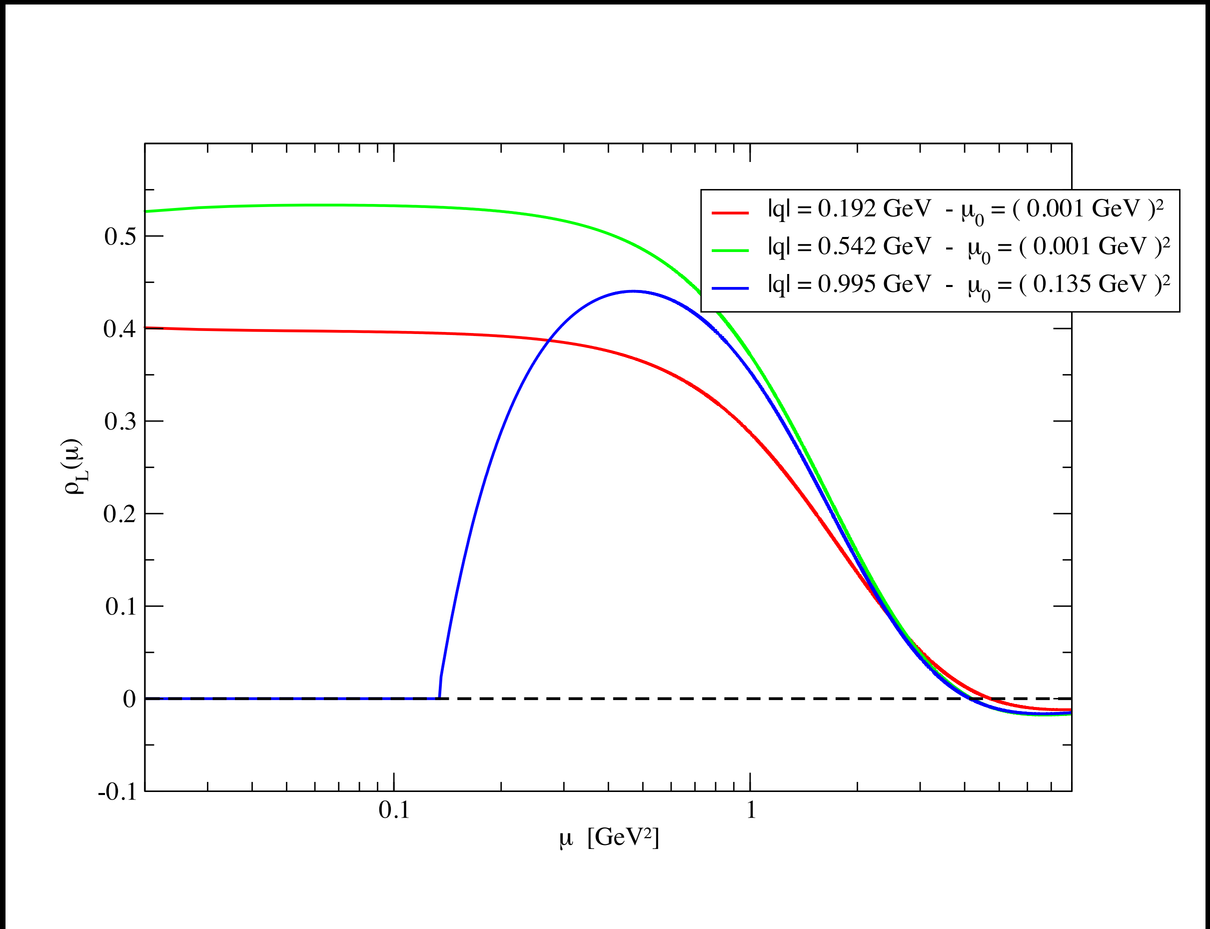


m_{Yukawa} = 439(8) MeV

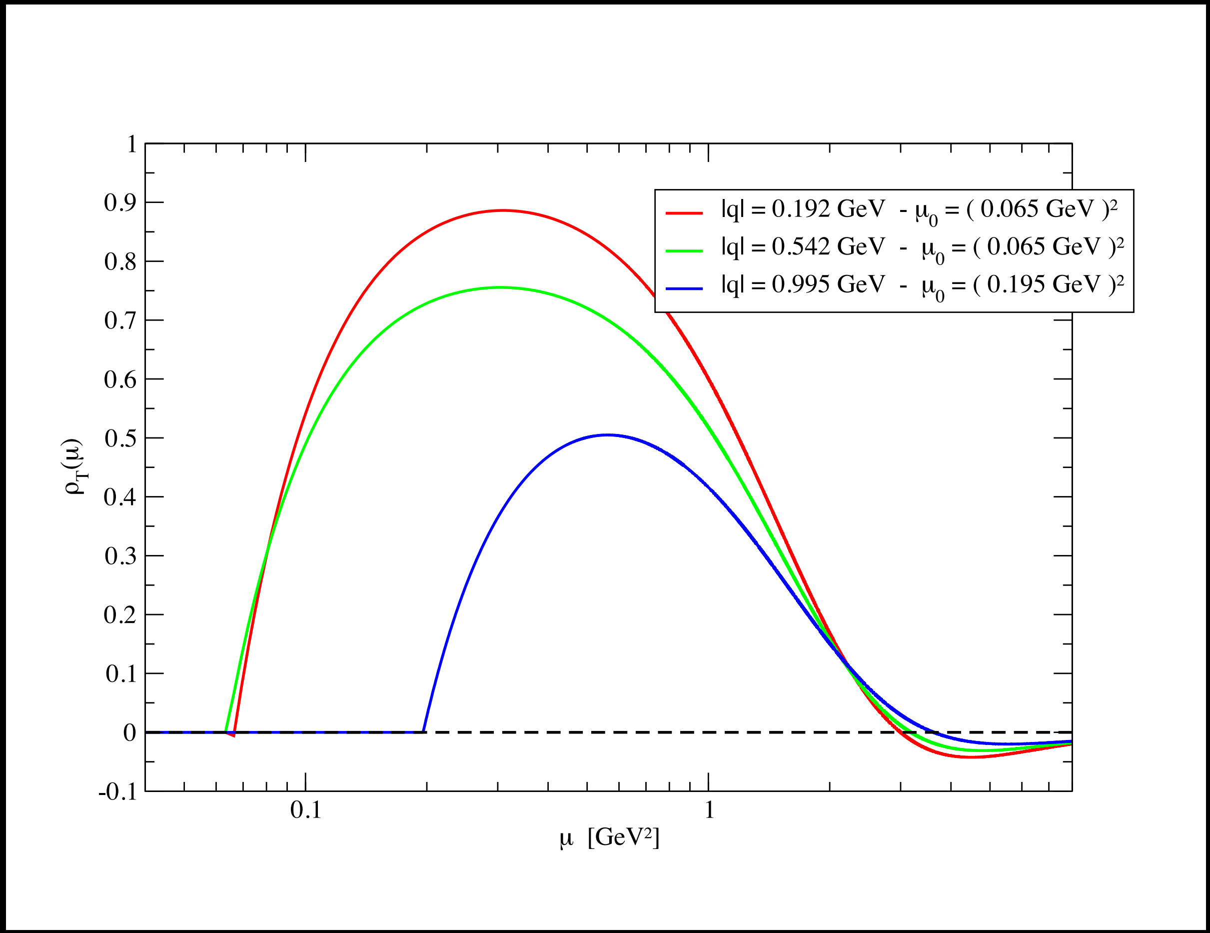


E-M Ilgenfritz, J M Pawlowski, A Rothkopf, A Trunin, arXiv:1701.08610





|q| = 0.192 GeV - μ_0 = (0.001 GeV)²
|q| = 0.542 GeV - μ_0 = (0.001 GeV)²
|q| = 0.995 GeV - μ_0 = (0.135 GeV)²
$\rho_L(\mu)$
μ [GeV²]
0.5
0.4
0.3
0.2
0.1
0
-0.1
0.1
1



|q| = 0.192 GeV - μ_0 = (0.065 GeV)²
|q| = 0.542 GeV - μ_0 = (0.065 GeV)²
|q| = 0.995 GeV - μ_0 = (0.195 GeV)²
$\rho_T(\mu)$
μ [GeV²]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0.1
1

Thank you !!!