Extracting infrared properties of QCD from the Curci-Ferrari model

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Introduction

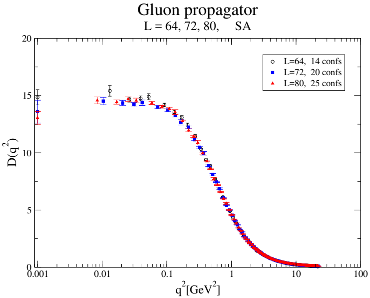
Yang-Mills theory, gauge-fixed in the Landau gauge with the Faddeev-Popov procedure, is described by a set of massless fields: Gluons (A_μ^a), ghosts (c^a and $\bar{c}^a$) and a Lagrange multiplyer (h^a)

$$\mathcal{L} = \frac{1}{4}(F_{\mu\nu}^a)^2 + \partial_\mu \bar{c}^a (D_\mu c)^a + h^a \partial_\mu A_\mu^a$$

However, lattice simulations see unambiguously a gluon propagator that saturates at low momentum.

However...

Gluon propagator is massive!



The origin of the mass could:

- result from solving Dyson-Schwinger or Functional Renormalization-group equations;
- be a consequence of a gluon condensate;
- be related to Gribov ambiguity;
- none of those/ a combination of those...

Phenomenological model

The mass generation is a difficult issue. Once we are convinced it exists, how much physics can we understand?
Introduce a mass for the gluon by hand in the (gauge-fixed) Lagrangian:

$$\mathcal{L} = \frac{1}{4}(F_{\mu\nu}^a)^2 + \partial_\mu \bar{c}^a (D_\mu c)^a + h^a \partial_\mu A_\mu^a + \frac{1}{2} m^2 \left(A_\mu^a\right)^2$$

This is one particular representative of the Curci-Ferrari lagrangian.

Overview

- General properties of the model;
- Systematic comparison with lattice correlation functions.
- Comparison with other approaches.

Bad news

- The mass term breaks standard BRST invariance.

$$sS = -m^2 \int c^a \partial_\mu A_\mu^a \neq 0$$

  However there exists a (non-nilpotent) BRST symmetry.
- Unitarity is not under control.

good news

- The theory is renormalizable to all orders (De Boer et al).
- UV physics ($p \gg m$) unaffected by the gluon mass.
- the (running) gluon mass tends to zero in the ultraviolet [$m^2(\mu) \propto g(\mu)^{35/22}$ when $g \ll 1$].
- Feynman rules are identical to usual ones, except for the massive gluon propagator:

$$\langle A_\mu A_\nu \rangle_0(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{1}{p^2 + m^2}$$

  perturbation calculations are easy to perform.
- Low momentum physics regularized by the gluon mass.

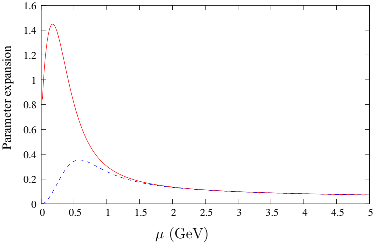
Infrared behavior

At very low momenta, gluons are frozen. Ghost loops dominate.

- $\Gamma_{A_\mu^a A_\nu^b} \sim \delta^{ab}(\delta_{\mu\nu})(\mathrm{cte} + p^{d-2})$
- $\Gamma_{A_\mu^a A_\nu^b A_\rho^c} \sim -f^{abc}(ip_\mu \delta_{\nu\rho} + \cdots)p^{d-4}$
- in $d = 4$, leads to log divergences, hard to see...
- in $d = 3$, gluon propag $cte + |p|$, 3-gluon vertex changes sign (zero crossing), consistent with lattice data.

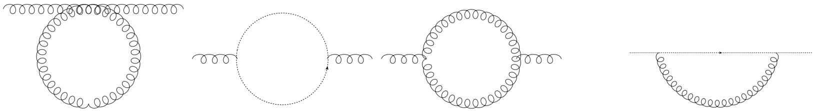
At low p, interaction between ghosts is mediated by heavy gluons (see also Weber). Effective interaction suppressed by some positive power of p at low momentum. The right expansion parameter is

$$\frac{Ng^2(\mu)}{16\pi^2} \frac{\mu^2}{m(\mu)^2 + \mu^2}$$



Ghost and gluon propagators I

At 1-loop, need to compute 4 Feynman diagrams:



Define

$$\langle A_\mu A_\nu \rangle(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) G(p) \qquad \langle c\bar{c}\rangle(p) = \frac{1}{p^2} F(p)$$

To obtain finite correlation functions, we need 4 renormalization factors Z_A, Z_c, Z_g and Z_{m^2}, defined, eg, such that (IRS scheme)

$$G(p=\mu) = \mu^2 + m^2 \qquad F(p=\mu) = \mu^2$$

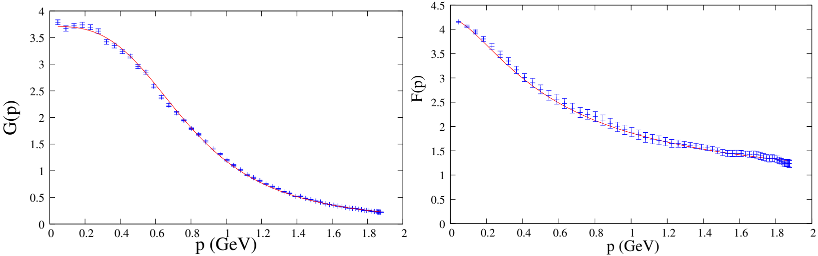
$$Z_A \sqrt{Z_c} Z_g = Z_{m^2} Z_A Z_c = 1$$

Ghost and gluon propagators II

$$G^{-1}(p)/m^2 = s + 1 + \frac{g^2 N}{384\pi^2} s \Big\{ 111 s^{-1} - 2s^{-2} + (2 - s^2)\log s$$
$$+ (4s^{-1} + 1)^{3/2}\left(s^2 - 20s + 12\right)\log\left(\frac{\sqrt{4+s} - \sqrt{s}}{\sqrt{4+s} + \sqrt{s}}\right)$$
$$+ 2(s^{-1} + 1)^3\left(s^2 - 10s + 1\right)\log(1+s) - (s \to \mu^2/m^2)\Big\},$$
$$F^{-1}(p) = 1 + \frac{g^2 N}{64\pi^2}\Big\{ - s\log s + (s+1)^3 s^{-2}\log(s+1)$$
$$- s^{-1} - (s \to \mu^2/m^2)\Big\},$$

with $s = p^2/m^2$.

For SU(2) (Cucchieri, Mendes '08), we fit the gluon mass to over the lattice data.



Agreement too good to be true, for a 1-loop calculation...

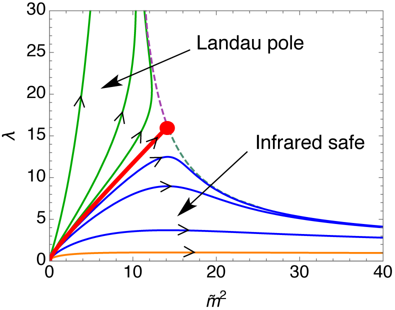
Renormalization-group flow

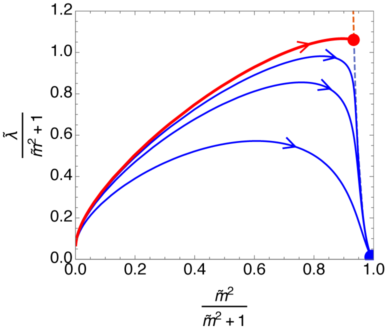
From renormalization factors, deduce a set of coupled β functions for g and m:

In the UV ($\mu \gg m$) $\beta_g \simeq -\frac{g^3 N}{16\pi^2}\frac{11}{3}$

In the IR ($\mu \ll m$) $\beta_g \simeq +\frac{g^3 N}{16\pi^2}\frac{1}{6}$

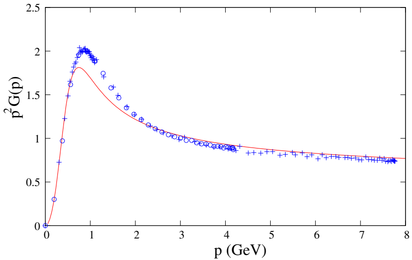
This opens the way to infrared-safe trajectories [$\tilde{\lambda} = g^2 N/(16\pi^2)$, $\tilde{m} = m/\mu$].

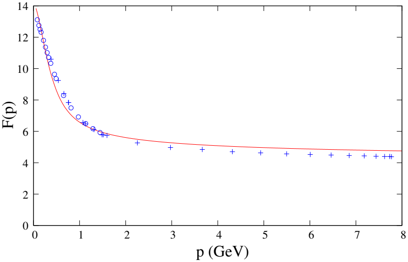




Comparison with lattice data

For SU(3) (Bogolubsky '09, Dudal '10), momentum range is larger and the RG effects cannot be neglected. Fix the gluon mass to specify a trajectory (for both curves):

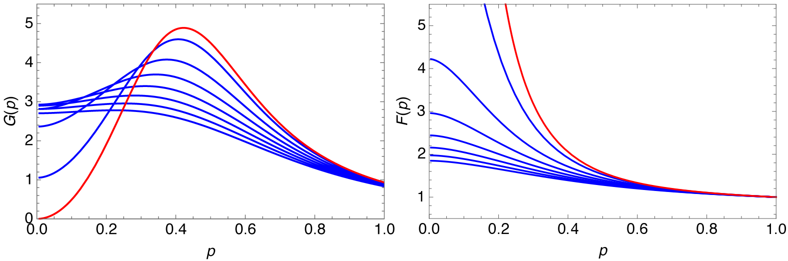




The error is about 10-15% at most, in the whole range.
We can do slightly better by changing the renormalization scheme.

Other RG trajectories

We can move around the best fit trajectory.



Red curve: scaling solution, with Gribov scaling.

$$G(p) \sim p^2 \qquad F(p) \sim p^{-2}$$

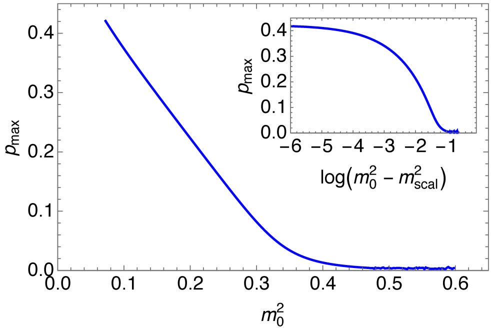
The exponent only depend on the existence of a fixed point, not on its precise location.

For general $d > 2$

$$G \sim p^{d-2} \qquad F \sim p^{2-d}$$

Positivity violation

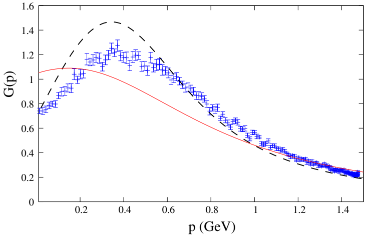
Källen-Lhemann representation + positive spectral function implies that the propagator is a monotonously decreasing function of p.
We can study the position of the maximum of $G(p)$ when changing the renormalization-group trajectory.

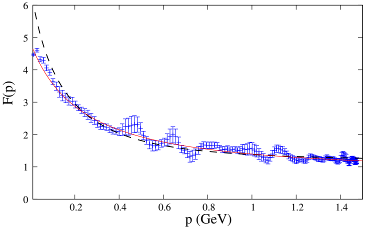


We always find a nonvanishing p_{max}.

Other dimensions

- In $d = 3$, the picture is qualitatively similar. The fits are less convincing:





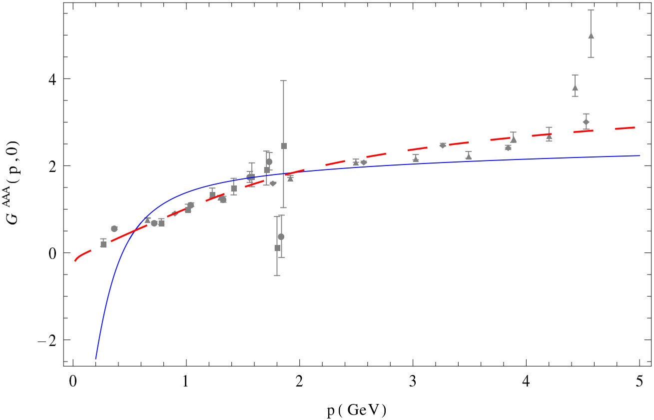
Red curve perturbative results. Black: RG-improved.

- In $d = 2$, we do not recover the lattice results.

Other correlation functions

- By the same technique, we have computed (all tensorial components) and compared with lattice data of Maas, Cuccieri, Mendes:
  - 3 gluon vertex and ghost-gluon vertex;
  - quark propagator;
  - quark-gluon vertex;
- In general, satisfactory agreement (maximal error of 15-20%) in the quenched approximation.
- 1-loop compares badly to lattice for the quark renormalization factor and for one of the structure tensors of the quark-gluon vertex (λ_2).
- For unquenched calculations, see Marcela's talk on friday.

1 example



To be done, at a methodological level

- 2-loop calculation of the propagators (in collaboration with Gracey, Reinosa, Peláez).
- Comparison of 1-loop 3-point correlation functions with more recent lattice simulations.
- Make contact with other approaches. Status of the "gluon mass" in Dyson-Schwinger, FRG?
- Understand the origin of the gluon mass. Gribov copies? Gluon condensate?