Taming Gribov copies via the horizon restriction: done and to do

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Overview

- Gribov copies and how to deal with them in the path integral
- (Softly broken) standard BRST
- BRST symmetric formulation of Gribov-Zwanziger theory in Landau gauge
- Comparison with gauge fixed lattice theory
- Generalization to the linear covariant gauge
- Application of BRST symmetry: Nielsen identities in the Gribov-Zwanziger theory
- (Non-Abelian) Landau-Khalatnikov-Fradkin transformations
- Gribov-Zwanziger with background fields

Faddeev-Popov quantization of QCD

Classical level

- Classical $SU(N)$ Yang-Mills action in $d=4$ Euclidean space time

$$S_{YM}=\frac{1}{4}\int \mathrm{d}^4x F^a_{\mu\nu}F^a_{\mu\nu}$$

- S_{YM} possesses enormous local invariance,

$$A_\mu \quad\to\quad A^S_\mu \;=\; S^+\partial_\mu S+S^+A_\mu S \qquad S\in SU(N)$$

or in infinitesimal form

$$\begin{aligned} A^a_\mu &\to A^a_\mu+D^{ab}_\mu\omega^b \\ D^{ab}_\mu &\equiv \partial_\mu\delta^{ab}-gf^{abc}A^c_\mu \end{aligned}$$

- Freedom to choose gauge

Faddeev-Popov quantization of QCD

- Faddeev and Popov implemented a linear covariant gauge choice as follows

$$Z_{FP} = \int [dA]\delta(\partial A - B) \det \mathcal{M}^{ab} e^{-S_{YM}}$$

$$\mathcal{M}^{ab} = -\partial_\mu(\partial_\mu \delta^{ab} - g f^{abc} A^c_\mu) \quad = \text{Faddeev-Popov operator}$$

+ Gaussian sampling over B via $\int [dB] e^{-\frac{B^2}{2\alpha}}$.

- The FP trick is based on the functional version of

$$\int dx \delta[f(x) - y] g(x) = \left\{ g(x) \left| \frac{\partial f}{\partial x} \right|^{-1} \right\}_{f(x)=y}$$

- Notice that this *assumes* that $f(x) = y$ only has a single solution! Otherwise one needs

$$\int dx \delta[f(x) - y] g(x) = \sum_i \left\{ g(x) \left| \frac{\partial f}{\partial x} \right|^{-1} \right\}_{f(x_i)=y}$$

Faddeev-Popov quantization of QCD

The Faddeev-Popov action

- Faddeev and Popov implemented a gauge choice as follows

$$Z_{FP} = \int [dA] \underbrace{\delta(\partial A - B) \det \mathcal{M}^{ab}}_{\rightarrow \text{ unity}} e^{-S_{YM}}$$

- $\delta(\partial A) \Rightarrow \partial A = 0 \equiv$ Landau gauge if $\alpha = 0$
- Faddeev-Popov determinant $\det \mathcal{M}^{ab}$ is corresponding Jacobian
- This form is not suitable to work/compute with (we want Feynman rules from local action)

Faddeev-Popov quantization of QCD

The Faddeev-Popov action in the Landau gauge

- We shall work with linear covariant gauge $\partial_\mu A^a_\mu = \alpha b^a$.
- Very popular gauge, as it has many nice (quantum) properties.
- The eventual gauge fixed action reads

$$S_{YM} + S_{gf} = \int \mathrm{d}^4 x \left(\frac{1}{4} F^2_{\mu\nu} + b^a \partial_\mu A^a_\mu - \frac{\alpha}{2} b^a b^a + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

The BRST symmetry

an important (crucial) symmetry

- Quantum action enjoys nilpotent BRST symmetry, $s(S_{YM}+S_{gf}) = 0$

$$sA_\mu^a = -D_\mu^{ab}c^b\,, \qquad sc^a = \frac{g}{2}f^{abc}c^bc^c$$
$$s\bar{c}^a = b^a\,, \qquad sb^a = 0\,, \qquad s^2 = 0$$

- Quantum replacement for classical gauge invariance
- Used for proofs of
  - (perturbative) renormalizability via *Slavnov-Taylor identity*
  - (perturbative) unitarity: e.g. ghost, antighost, longitudinal and time-like gluon polarizations cancel, only 2 transverse gluon degrees of freedom survive!
  - BRST symmetry is an important concept

Perturbative unitarity (Kugo-Ojima)

- Define $|\psi_p\rangle$ physical state $\Leftrightarrow Q_{BRST}|\psi_p\rangle = 0$, $|\psi_p\rangle \neq Q_{BRST}|\ldots\rangle$ using *asymptotic* Fock states/free BRST charge
- $|\psi_p\rangle \in \mathrm{cohom}(Q_{BRST})$
- unphysical states $|\psi_u\rangle$ always come in quartets, and decouple from the physical spectrum.
- Indeed: If $\mathcal{N}$ = counting operator of longitudinal and temporal gauge polarization, ghost, antighost, then $\mathcal{N} = \{Q_{BRST}, \mathcal{R}\}$
- If $\mathcal{N}|\psi_p\rangle = n|\psi_p\rangle$, then automatically $|\psi_p\rangle = 1/nQ_{BRST}[\mathcal{R}|\psi_p\rangle]$
- So, we find a trivial (zero norm) state, so no unphysical particles in subspace! The 2 transverse gluons remain, with positive norm (as is checked explicitly, this is not following from the BRST).
- BRST analysis thus guarantees perturbative unitarity (Q_{BRST} commutes with H as symmetry)
- Situation beyond perturbation theory:more complicated. Well-defined BRST charge? Sign of norm of states (never related to BRST itself)?

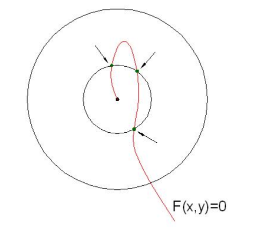
Potential flaw in FP quantization

The Gribov problem

- Take A_μ in linear covariant gauge $\Leftrightarrow \partial_\mu A_\mu = \alpha b$
- Consider (infinitesimal) gauge transform: $A'_\mu = A_\mu + D_\mu \omega$
- $\partial_\mu A'_\mu = \alpha b$ if $\partial_\mu D_\mu \omega = 0$
  GAUGE COPY if FP operator has (normalizable) zero modes!
- Was investigated (and drawn attention to) by Gribov in late seventies for Landau ($\alpha = 0$) and Coulomb gauge ($\partial_i A_i = 0$).

Potential flaw in FP quantization

The Gribov problem



- There is still some overcounting when using FP action (which is mathematically seen “wrong”)
- Solution: use the more correct functional version of δ-function where the argument has multiple zeros.
- Unfortunately: cannot be put in useful partition function

Treating the copy problem

From Faddeev-Popov to Gribov-Zwanziger: first Landau gauge $\partial A = 0$

- A class of copies was related to zero modes of Faddeev-Popov operator $M = -\partial D$
- Let us restrict path integral to region Ω where $\partial A = 0$ and $M > 0$. Only sensible if $\partial A = 0$ as only then $M = -\partial D$ is Hermitian!
- Ω corresponds to local minima of the functional $\int d^4 x A_\mu^2$!
- $\Rightarrow$ This is already an improvement of Faddeev-Popov!
- Compare with lattice where one seeks for (in theory) global minima of $\int d^4 x A_\mu^2$
- How to implement restriction to Ω in continuum?

The Gribov-Zwanziger action

Gribov-Zwanziger

- Gribov and later on Zwanziger worked out this problem and proved many properties of region Ω, together with Dell'Antonio.
- Example: every gauge orbit passes through Ω, it is convex and bounded in every direction ($\rightarrow$ nice integration region).
- Warning: Ω, region of *relative* minima, is still plagued by Gribov copies. One should work in region of absolute minima, but to my knowledge, not so easy to deal with this.

Gribov-Zwanziger

The Gribov restriction: semiclassical level

- Partition function

$$Z=\int[D\Phi]e^{-S_{YM+FP}}\to Z=\int[D\Phi]\theta(M)e^{-S_{YM+FP}}$$

- how to characterize $\theta(M)$? We look at its inverse, schematically written as

$$M^{-1}=(-\partial D)^{-1}=k^2(1+\sigma(k^2,A))$$

- Gribov no pole condition: $\boxed{\sigma(0,A)<1}$
- Partition function

$$Z=\int[D\Phi]e^{-S_{YM+FP}}\to Z=\int[D\Phi]\theta(1-\sigma(0,A))e^{-S_{YM+FP}}$$

Gribov-Zwanziger

The Gribov restriction: semiclassical level

- Semiclassical analysis in thermodynamic limit: $\theta \to \delta$
- Partition function (at the quadratic level) [saddle point evaluation]

$$Z = \int [D\Phi]\delta(1-\sigma(0,A))e^{-S_{YM+FP}} \to \int [D\Phi]e^{-S_{YM+FP}+\gamma^4\int d^4xA\frac{1}{\partial^2}A}$$

- $\gamma =$ thermodynamic parameter, fixed by gap equation,

$$\frac{3}{4}g^2N\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^4+\gamma^4} = 1 \qquad (1)$$

- **New partition function** (or action) $\to$ **perfect tool to study theory with**

Gribov-Zwanziger

- Restricts the integration to the Gribov region to all orders (Zwanziger, NPB323 (1989), NPB399 (1993))
- The Gribov-Zwanziger action is given by

$$S_{GZ} = S_{YM} + S_{gf} + \gamma^4 \int d^4x \, h(x)$$

with the horizon function

$$h(x) = g^2 f^{abc} A_\mu^b \left(\mathcal{M}^{-1}\right)^{ad} f^{dec} A_\mu^e$$

horizon condition (= gap equation) $\langle h(x) \rangle = d(N^2-1)$

- For $\gamma = 0$, everything reduces to Faddeev-Popov.

Gribov-Zwanziger

- Somewhat easier derivation than Zwanziger: generalization of Gribov's analysis to all orders Capri et al, PLB719 (2013)
- We considered a classical A_μ^a, and formally computed the inverse via

$$\langle \bar{c}^a(x) c^b(y) \rangle = \frac{\int \mathcal{D}c\mathcal{D}\bar{c}\; \bar{c}^a(x)c^b(y)e^{\int \bar{c}^c \mathcal{M}^{cd} c^d}}{\int \mathcal{D}c\mathcal{D}\bar{c}\; e^{\int \bar{c}^c \mathcal{M}^{cd} c^d}}$$

  Then

$$\mathcal{G}(k,A) = \frac{1}{V(N^2-1)} \langle \bar{c}^a(p) c^a(-q) \rangle|_{p=q=k} = \frac{1}{k^2}(1+\sigma(k,A))$$

  Notice we are *tracing* here, which is a simplification, also present in Zwanziger's derivation.
- At zero momentum, the series can be resummed and leads to

$$\sigma(0,A) = -\frac{g^2}{VD(N^2-1)} \int \frac{d^D p}{(2\pi)^D} \int \frac{d^D q}{(2\pi)^D} A_\mu^{ab}(-p) \left(\mathcal{M}^{-1}\right)^{bc}_{pq} A_\mu^{ca}(q)$$
$$= \frac{H(A)}{DV(N^2-1)} .$$

Gribov-Zwanziger

- No-pole condition: $\sigma(0, A) \leq 1$

$\Rightarrow$ $\theta(1-\sigma) \Rightarrow \delta(1-\sigma) \Rightarrow \langle\sigma\rangle = 1 \Rightarrow \langle H\rangle = DV(N^2-1)$

- So no-pole gives exactly what Zwanziger also obtained, at smaller cost.
- Notice that actually, only a posteriori, we can check that

$$\langle\sigma(k, A)\rangle \leq \langle\sigma(0, A)\rangle = 1$$

  i.e. after averaging.
- Likewise,

$$1 + \langle\sigma(k, A)\rangle_{\text{conn}} = \frac{1}{1 - \langle\sigma(k, A)\rangle_{1\text{PI}}}$$

Gribov-Zwanziger

- We replace the action with a local (equivalent) action

$$S_{GZ} = S_{YM} + S_{gf} + S_h$$

with now

$$S_h = \int d^4x \left(\overline{\varphi}_\mu^{ac} \partial_\nu \left(\partial_\nu \varphi_\mu^{ac} + g f^{abm} A_\nu^b \varphi_\mu^{mc} \right) - \overline{\omega}_\mu^{ac} \partial_\nu \left(\partial_\nu \omega_\mu^{ac} + g f^{abm} A_\nu^b \omega_\mu^{mc} \right) - g \left(\partial_\nu \overline{\omega}_\mu^{ac} \right) f^{abm} \left(D_\nu c \right)^b \varphi_\mu^{mc} \right.$$
$$\left. -\gamma^2 g \left(f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \overline{\varphi}_\mu^{bc} + \frac{4}{g} \left(N^2 - 1 \right) \gamma^2 \right) \right)$$

- horizon condition (= gap equation)

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0 \Leftrightarrow \underbrace{\left\langle g f^{abc} A_\mu^a (\varphi + \overline{\varphi})_\mu^{bc} \right\rangle = 2d(N^2-1)\gamma^2}_{d=2\,\text{condensate}}$$

- $\gamma \propto \Lambda_{QCD}$: source of dimensional transmutation.

Gribov-Zwanziger

Gribov-Zwanziger quantization

The GZ formalism is a geometrically inspired path-integral construction (cf. boundary condition to stay within the Gribov region in e.g. DSE approach) with good quantum properties that improves upon the standard FP quantization.

Gribov-Zwanziger quantization

Nice property: closely related to lattice formulation, as in both cases minimization of $\int A^2$ along the gauge orbit is used to define the (a) nonperturbative Landau gauge.

There is the technical issue of having multiple local minima, unknown how to be dealt with at the quantitative level.

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The Gribov-Zwanziger action

What about the BRST symmetry?

- The (naturally extended) BRST symmetry

$$s\overline{\omega}_{\mu}^{ab} = \overline{\varphi}_{\mu}^{ab}\,, \qquad s\overline{\varphi}_{\mu}^{ab} = 0\,, \qquad s\varphi_{\mu}^{ab} = \omega_{\mu}^{ab}\,, \qquad s\omega_{\mu}^{ab} = 0\,,$$

  is softly broken

$$sS_{GZ} = g\gamma^2 \int d^4x \left(f^{abc} A_{\mu}^{a} \omega_{\mu}^{bc} - (D_{\mu}^{am} c^m)(\overline{\varphi}_{\mu}^{bc} + \varphi_{\mu}^{bc}) \right)$$

- Apparently: treating Gribov copy leads to soft breaking of BRST.
- What about gauge parameter independence of correlation functions of BRST invariant operators if we were to generalize GZ to other gauges?

  Let us first find a BRST!

The Gribov-Zwanziger action vs. BRST

- If nilpotent BRST is found, we can define/study cohomology. Also good news for renormalization proof (but in se not necessary) Invariant subspace under time evolution. This is good news, but notice that gluon propagator behaves as

$$D(p^2) = \frac{p^2}{p^4 + \lambda^4}$$

$\rightarrow$ poles at $p^2 = \pm i\lambda^2$. The Kugo-Ojima analysis makes use of *asymptotic* states. To us, it is unclear how to define these in presence of complex conjugate (*cc*) masses. For example, canonical quantization leads to exploding/dampened Fourier expansion in terms of time t.
- In general (known to us), there is no rigorous way to deal with constructing a physical sector with positivity/unitarity using such constituent propagators. This is beyond simply having a BRST invariance.

The Gribov-Zwanziger action vs. BRST

- As studied in Baulieu et al, PRD82 (2010); Dudal et al, PRL106 (2011); Windisch et al, PRD87 (2013); Felix et al, PRD96 (2017) at leading order, it is possible to combine *cc* masses pairwise to get a physical spectral representation

$$D(p^2) = \int_0^\infty \frac{\rho(t)dt}{t+p^2}, \qquad \rho(t) \geq 0$$

  of gauge invariant composite operator correlation functions (correct branch cut location + positivity), but also other cuts emerge, in e.g. $\langle F^2_{\mu\nu}(p)F^2_{\mu\nu}(-p)\rangle$. Open problem how to "massage away" the unphysical cuts when working with basic propagators with *cc* poles.
- See also talk of Oliveira on lattice gluon/ghost spectral function.
- Also non-perturbative context, e.g. by modelling (lattice) quark propagator via series of *cc* masses Alkofer et al, PRD70 (2004), study Bethe-Salpeter bound state eq. in such settings, etc, with good results on the (investigated) physical Minkowski half-axis, but the "unphysical sectors" are not discussed Bhagwat, Pichowsky, Tandy, PRD 67(2003), work of Roberts et al..

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Preliminaries

- Consider A^2-functional

$$f_A[u] \equiv \mathrm{Tr}\int d^4x\, A^u_\mu A^u_\mu = \mathrm{Tr}\int d^4x \left(u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u\right)^2$$

and set $v = he^{ig\omega}$.

- Working up to 2nd order to identify minima:

$$f_A[v] = f_A[h] + 2\mathrm{Tr}\int d^4x\, \left(\omega \partial_\mu A^h_\mu\right) - \mathrm{Tr}\int d^4x\, \omega \partial_\mu D_\mu(A^h)\omega + O(\omega^3)$$

$\Rightarrow$ $\partial_\mu A^h_\mu = 0 \quad \& \quad -\partial_\mu D_\mu[A^h] > 0$

We recognize the Landau gauge and defining condition of the Gribov region (positive FP operator).

Preliminaries

- The "minimum configuration" can be solved for

$$A^h_\mu = A_\mu - \frac{1}{\partial^2}\partial_\mu\partial A - ig\frac{\partial_\mu}{\partial^2}\left[A_\nu, \partial_\nu\frac{\partial A}{\partial^2}\right] - i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\partial A, \frac{1}{\partial^2}\partial A\right] + ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] + i\frac{g}{2}\left[\frac{1}{\partial^2}\partial A, \frac{\partial_\mu}{\partial^2}\partial A\right] + O(A^3)$$

  It is transverse and gauge invariant order by order. See also Lavelle, McMullan, Phys. Rep. 279 (1997).

- Observation: if $\partial A = 0$, $A = A^h$. More precisely

$$A = A^h + \text{non-local power series in } (A, \partial A)$$

Rewriting GZ action

- $$A = A^h + \text{non-local power series in } (A, \partial A)$$
- Consider GZ action with non-local horizon action

$$H(A) = g^2 \int d^4x d^4y \ f^{abc} A^b_\mu(x) \left[\mathcal{M}^{-1}(x,y)\right]^{ad} f^{dec} A^e_\mu(y)$$

$$\begin{aligned} S_{GZ} &= S_{YM} + \int d^4x \, (b\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c) + \gamma^4 H(A) \\ &= S_{YM} + \int d^4x \, (b\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c) + \gamma^4 H(A^h) - \gamma^4 R(A)(\partial A) \\ &= S_{YM} + \int d^4x \left(b^h \partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) + \gamma^4 H(A^h) \end{aligned}$$

with a new field b^h

$$b^h = b - \gamma^4 R(A)$$

Rewriting GZ action

- Introduce auxiliary fields to obtain

$$
\begin{aligned}
S_{GZ} &= S_{YM} + \int d^4x \left(b^h \partial_\mu A_\mu + \bar{c} \partial_\mu D_\mu c \right) \\
&\quad + \int d^4x \left(\bar{\phi} \mathcal{M}(A^h) \phi - \bar{\omega} \mathcal{M}(A^h) \omega + \gamma^2 A^h (\bar{\phi} + \phi) \right)
\end{aligned}
$$

  and rename $b^h \to b$ again.

- This new (equivalent) GZ action in the Landau gauge enjoys a nilpotent BRST symmetry

$$
\begin{aligned}
sA_\mu^a &= -D_\mu^{ab} c^b \,, \; sc^a = \frac{g}{2} f^{abc} c^b c^c \,, \; s\bar{c}^a = b^a \,, \; sb^a = 0 \,, \\
s\phi_\mu^{ab} &= s\omega_\mu^{ab} = s\bar{\omega}_\mu^{ab} = s\bar{\phi}_\mu^{ab} = 0
\end{aligned}
$$

  thanks to $sA^h = 0$.

Rewriting GZ action

- Very nonlocal because of $A^h \to$ hard to discuss renormalizability etc.
- Locality of A^h_μ via introduction of Stückelberg field

$$A^h_\mu = (A^h)^a_\mu T^a = h^\dagger A^a_\mu T^a h + \frac{i}{g} h^\dagger \partial_\mu h, \quad h = e^{ig\xi^a t^a}$$

- Addition of $\int d^4x \left[\tau \partial A^h - \overline{\eta}\mathcal{M}(A^h)\eta\right]$ [1] to action gives rise to equivalent action as before upon solving the ξ-EOM
- $sA^h = 0$ under

$$h \to u^\dagger h\,, \quad h^\dagger \to h^\dagger u\,, \quad A_\mu \to u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u$$

or

$$\begin{aligned} sh^{ij} &= -igc^a (T^a)^{ik} h^{kj} \Rightarrow \\ s\xi^a &= -c^a + \frac{g}{2} f^{abc} c^b \xi^c - \frac{g^2}{12} f^{amr} f^{mpq} c^p \xi^q \xi^r + O(g^3) \end{aligned}$$

[1] Thanks to M. Tissier for alerting the rôle of $(\eta, \overline{\eta})$.

Rewriting GZ action

- Can be combined with algebraic renormalization formalism, following Dragon et al, NPB Proc.Suppl.56B (1997). A fully renormalizable framework, see Capri et al, PRD94 (2016) & PRD96 (2017)
- Important comment about renormalization: Addition of $\tau\partial A^h$ saves the day.
  - Standard Stückelberg propagator (via addition of gauge invariant mass term $m^2 A^h A^h$):

    $$\langle\xi\xi\rangle \sim \frac{1}{m^2 p^2}$$

    $\rightarrow$ leads to power counting non-renormalizibility!
  - Here (even for $m \neq 0$):

    $$\langle\xi\xi\rangle \sim \frac{1}{p^4}$$

- Final (important) point of interest

$$\langle A \ldots \overline{c} \ldots c\rangle_{\text{new GZ}} \equiv \langle A \ldots \overline{c} \ldots c\rangle_{\text{old GZ}} \qquad \text{(Landau gauge)}$$

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The Refined Gribov-Zwanziger action in linear covariant gauge: extra dynamical effects

- We include extra dynamical effects due to non-perturbative gauge invariant $d = 2$ condensates $\to$ lower vacuum energy Dudal et al, PRD78 (2008), PRD84 (2011) & work in progress
- Ghost propagator $G(p^2) \sim \frac{1}{p^2}$ for $p^2 \sim 0$, $G(p^2) \neq \frac{1}{p^4}$.
- Gluon propagator

$$D(p^2) \propto \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + \lambda^4}, \qquad D(0) \neq 0 .$$

  m^2 and M^2 are mass scales corresponding to condensates, in particular

$$m^2 \sim \left\langle A^h A^h \right\rangle, \qquad M^2 \sim \langle \overline{\varphi}\varphi - \overline{\omega}\omega \rangle$$

- Works pretty well to describe Landau lattice data Oliveira et al, PRD81 (2010) 074505; Cucchieri al, PRD85 (2012) 094513; Bornyakov et al, PRD85 (2012) Rodriguez-Quintero et al, PRD88 (2013) . Massive-type gluon propagator with *cc* poles.

  This is of course nice, but also not fully satisfactory: there is still a (direct) dependence on lattice input.

The Refined Gribov-Zwanziger action

Effective action approach (sketch)

- We add $d = 2$ Local Composite Operators (LCO) $\mathcal{O}$ to the action with sources: $S \to S + \int J\mathcal{O} + \int \zeta J^2$
- Renormalizability can be proven; ζJ^2 needed to cure vacuum divergences, as well as consistency with renormalization group.
- Via Hubbard-Stratonovich transformation with field σ, one gets an effective action $\Gamma(\sigma)$ with $\langle\sigma\rangle \propto \langle\mathcal{O}\rangle$. Γ obeys homogenous renormalization group equation by construction (role of ζ), and contains a term $\sim \sigma\mathcal{O}$ and $J\sigma$. No more quadratic J^2-source term.
- We apply generalization of this to GZ $+$ relevant $d = 2$ operators (Dudal et al, PD84 (2011) for unfinished attempt, now being further developed) to construct $\Gamma(m^2, M^2, \gamma^2)$. Notice presence of Gribov parameter $\gamma^2 \neq 0$. Somewhat more complicated due to mixing of operators.

The Refined Gribov-Zwanziger action

Effective action approach (sketch)

- We must solve gap equations $\frac{\partial\Gamma}{\partial\text{scale}} = 0$, leading to scale $\propto \Lambda_{QCD}$.
- As $\gamma \neq 0$, *very difficult* to have e.g. $\left.\frac{\partial\Gamma}{\partial M^2}\right|_{M^2=0} = 0$
- GZ transforms almost automatically into RGZ on dynamical grounds. Though, it remains to be seen how well the computed values will compare with lattice fits. Our goal is to make the current RGZ independent from lattice input for the non-perturbative scales.
- Notice that $m^2 \propto \left\langle A^h A^h \right\rangle = \left\langle AA \right\rangle_{\text{Landau}}$, leading to $m^2 A^2$ term in action. Connection with work of Serreau, Tissier, Reinosa, Peláez, Wschebor? To be explored!

Lattice gluon propagator in the linear covariant gauge

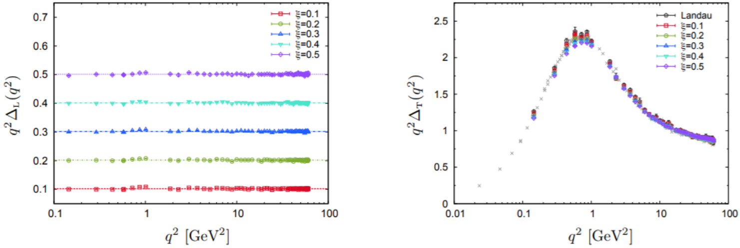

Figure: taken from Bicudo et al, PRD92 (2015)

We notice almost no dependence on gauge parameter. Nonperturbatively, the longitudinal propagator still fixed by $\frac{\alpha}{p^2}\frac{p_\mu p_\nu}{p^2}$.

Lattice gluon propagator in the linear covariant gauge

We notice almost no dependence on gauge parameter. Compare with Dyson-Schwinger equations (DSE) output, with a more prominent dependence. Less good modeling of vertices for $\alpha \neq 0$ than in Landau?

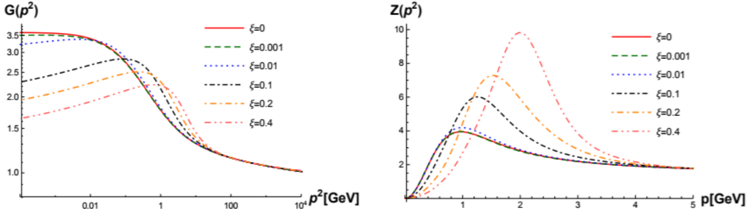

Figure: taken from Huber, PRD91 (2015). Similar results as in Aguilar et al, PRD 91 (2015). Left: ghost form factor $p^2 G(p^2)$; right: transversal gluon form factor $p^2 D(p^2)$. Outside Landau gauge, the ghost is suppressed in IR with gauge parameter? What does GZ teach us?

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How to tame Gribov copies in generic linear covariant gauge

- We will restrict to

$$\Omega^h = \{ A^a_\mu | \, \partial_\mu A^a_\mu = -i\alpha b^a, \, \mathcal{M}^{ab}(A^h) > 0 \}$$

Due to $\partial A^h = 0$, $\mathcal{M}^{ab}(A^h) > 0$ makes sense.

- In Dudal et al, PRD92 (2015), it was shown that this removes all infinitesimal gauge copies that are connected via Taylor expansion in α to Landau gauge zero modes.
- The (BRST symmetric) RGZ action reads

$$\begin{aligned} S &= S_{YM} + \int d^4x \left(\alpha \frac{b^a b^a}{2} + i b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu(A) c^b \right) \\ &+ \int d^4x \, \tau^a \partial_\mu (A^h)^a_\mu + \int d^4x \bar{\eta}^a \partial_\mu D^{ab}_\mu (A^h) \eta^b + \int d^4x \frac{m^2}{2} A^{h,a}_\mu A^{h,a}_\mu \\ &+ \int d^4x \left(-\bar{\varphi}^{ac}_\mu \mathcal{M}(A^h)^{ab} \varphi^{bc}_\mu + \bar{\omega}^{ac}_\mu \mathcal{M}(A^h)^{ab} \omega^{bc}_\mu + g\gamma^2 f^{abc} (A^h)^a_\mu (\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu) + M^2 (\bar{\varphi}^{bc}_\mu \varphi^{bc}_\mu - \bar{\omega}^{bc}_\mu \omega^{bc}_\mu) \right) \end{aligned}$$

How to tame Gribov copies in generic linear covariant gauge

Sketch of the argument

- From

$$\partial A = \alpha b \Rightarrow A = A^h + \tau \quad \text{with} \quad \partial\tau = \partial A = \alpha b = \mathcal{O}(\alpha)$$

- Assume that ζ is such that

$$\partial D\zeta = 0 \qquad \text{(zero mode of standard FP operator)}$$

- Then also

$$\partial D^h\zeta = \partial(\tau\zeta) \Rightarrow \zeta = \left(\partial D^h\right)^{-1}\left(\partial(\tau\zeta)\right)$$

- Linear covariant gauge = continuous deformation around Landau gauge $\alpha = 0$:

$$\zeta = \sum_{n=0}^{\infty} \zeta_n \alpha^n$$

- Recursively,

$$\zeta_n = 0 \;\text{ since }\; \partial D^h = \mathcal{O}(1)\,, \tau = \mathcal{O}(\alpha)$$

Ferrari, Picariello, Quadri, AnnPhys 294 (2001); Grassi, Hurth, Quadri, PRD70 (2004)

Intermediate summary

Gauge invariant GZ action

- Constructed a local (non-polynomial) GZ action
- Kills off (large class of) infinitesimal copies in generic linear covariant gauge
- Fully gauge invariant and renormalizable to all orders, including gauge invariant Gribov mass

$$\left\langle g f^{abc} A_\mu^{h,a} (\varphi + \overline{\varphi})_\mu^{bc} \right\rangle = 2d(N^2-1)\gamma^2$$

and other gauge invariant $d = 2$ condensates.

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Back to RGZ propagator in Landau gauge

- $D(p^2) = \frac{p^2+M^2}{p^4+(M^2+m^2)p^2+\lambda^4} \to$ 2 complex conjugate mass poles
  Cannot correspond to physical gluon $\to$ sort of "effective confinement".
- What about situation in other linear gauges?
- *cc* poles are RG and gauge independent $\to$ follows from Nielsen identities
- Much more can be learnt from GZ Nielsen identities!

Nielsen identities (NPB101 (1975))

- BRST invariance $\to$ functional Slavnov-Taylor identity (STI) on $1PI$ generator Γ:

$$\mathcal{S}(\Gamma) = \int d^4x \left(\frac{\delta\Gamma}{\delta\Omega^a_\mu}\frac{\delta\Gamma}{\delta A^a_\mu} + \frac{\delta\Gamma}{\delta L^a}\frac{\delta\Gamma}{\delta c^a} + \frac{\delta\Gamma}{\delta K^a}\frac{\delta\Gamma}{\delta\xi^a} + ib^a\frac{\delta\Gamma}{\delta\bar{c}^a}\right) + \chi\frac{\partial\Gamma}{\partial\alpha}$$

  Ω is source coupled to BRST variation of A (Dc), L to that of c (Lcc) and $s\alpha = \chi$, χ couples to $\bar{c}b$.
- Acting with test operators on STI allows to derive functional relations between n-point functions.

  In particular, deriving w.r.t. χ allows to control α-dependence! Sibold, Piguet, NPB253 (1985)
- After a long technical analysis in GZ theory (due to mixed propagators, using properties of SU(N) tensors):

$$\frac{\partial}{\partial\alpha}[\text{gluon poles}] = 0$$

Nielsen identities

- $$\frac{\partial}{\partial\alpha}[\text{gluon poles}] = 0$$
  gives an *a posteriori* argument why it works to fit lattice Landau gauge propagator with
  $$D(p^2) = Z\frac{p^2+M^2}{p^4+(M^2+m^2)p^2+\lambda^4}$$
  and the observed α-independence of lattice gluon propagator. Indeed, same (tree level propagator) fit will work for any α.

RGZ strategy

To avoid confusion: RGZ has a nontrivial vacuum, encoded in a few gauge invariant $d=2$ condensates. On top of that non-perturbative, we assume perturbation theory to work, with a small effective coupling g^2_{eff}, with the IR screened due to the mass scales, so that g^2_{eff} does not get too large.
The RGZ tree level propagator is thus never supposed to be exact, it will receive (presumably) controllable corrections, leading to logs etc.
Though, to capture first essential physics, one (we) can fit the tree level propagator to the “full” lattice data, and “feed” such propagator into other quantities.

Longitudinal gluon propagator

- Slavnov-Taylor identity, next to Ward identity $\frac{\delta\Gamma}{\delta b} = \alpha\partial A$ (gauge condition) allows to prove, also in GZ, that

$$\frac{p_\mu p_\nu}{p^2}\left\langle A_\mu A_\nu \right\rangle \equiv \frac{\alpha}{p^2}$$

(consistent with lattice)

- Interesting comment: gluon self energy, $\langle AA \rangle_{1PI}$, is *not* transverse, despite exact BRST invariance. This is only true in absence of propagator mixing!

GZ ghost propagator in the linear covariant gauge

$$G(k^2) = \frac{1}{k^2}\frac{1}{1-\omega(k^2)}$$

where

$$\omega(k^2) = \frac{Ng^2}{k^2(N^2-1)}\int\frac{d^4q}{(2\pi)^4}\frac{k_\mu(k-q)_\nu}{(k-q)^2}\langle A^a_\mu(q)A^a_\nu(-q)\rangle = \omega^T(k^2)+\omega^L(k^2)$$

$\omega^T(k^2)$ comes from transverse component of gluon propagator, with

$$\omega^T(k^2) = c + \mathcal{O}(k^2)$$

$\omega^L(k^2)$ stems from the (standard perturbative) longitudinal component, at 1-loop

$$\omega^L(k^2) = \alpha\frac{Ng^2}{64\pi^2}\log\frac{k^2}{\bar{\mu}^2}$$

as we have exactly $D_{L,\mu\nu}(k^2) = \frac{\alpha}{k^2}\frac{k_\mu k_\nu}{k^2}$. So, logarithmic IR suppression is there, and will remain to be there at higher orders.

Lattice ghost propagator in the linear covariant gauge

- No predictions yet, because the FP operator $-\partial D$ is non-Hermitian for $\alpha \neq 0 \rightarrow$ numerically not so easy nut to crack how to invert this FP operator.
- Work in progress by Roelfs, Cucchieri, Oliveira et al, based on lattice linear covariant gauge minimizing functional Cucchieri & Mendes, PRL103 (2009), Binosi et al, PRD92(2015).

$$\min_U \operatorname{Tr} \int d^4x \left(A_\mu^U A_\mu^U + \frac{2}{g} \operatorname{Re}(iU\Lambda) \right)$$

  Λ^a Gaussian sampled with width related to gauge parameter.
- In conjunction with Gribov copy question based on this functional.

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Nielsen identities

- In a compact notation, for a propagator the Nielsen identity looks like
$$\frac{\partial}{\partial\alpha}G_{\phi\phi} = G_{\phi\phi}M$$
where M corresponds to a composite operator correlation function like $\Gamma_{\chi\Omega\phi}$ (1PI insertion of Dc and $\int \overline{c}b$). This leads to
$$G_{\phi\phi}^{(\alpha)} = G_{\phi\phi}^{(\alpha=0)} e^{\int_0^\alpha d\alpha' M(\alpha')}$$
- This relates propagator in one gauge to that of Landau gauge via an exponential prefactor containing *all* gauge dependent stuff.
- Work in progress to use the Nielsen identities to “resum” the gauge dependence.

  Schematically: $\alpha \ln k^2 \to e^{\alpha \ln k^2} = (k^2)^\alpha$.

  Such approach does exist already, but not via Nielsen identities.

(Abelian) Landau-Khalatnikov-Fradkin transformations

- Landau & Khalatnikov, Sov. Phys. JETP **2** (1956) & Fradkin, idem, see also Johnson & Zumino, PRL3 (1959)
- (Abelian!) transformation that relates photon and electron propagator in different gauges,

$$\langle\bar{\psi}(x)\psi(y)\rangle_\alpha = \langle\bar{\psi}(x)\psi(y)\rangle_{\alpha=0}\left\langle e^{-ig\xi(x)}e^{ig\xi(y)}\right\rangle_{\alpha=0}$$

  with

$$\langle\xi(p)\xi(-p)\rangle_\alpha = -\alpha\frac{1}{p^4}$$

- Useful to check validity of e.g. vertex Ansätze: consistent with gauge (LKF) transformation law, see e.g. Bashir & Raya, PRD66 (2002) and other works. Related to gauge invariance of physical estimates, far from trivial, even in Abelian case, see Kızılersü et al, PRD91 (2015).
- Would be most interesting to apply to QCD, to see how "gauge covariance" manifests itself based on employed vertex modelling in Landau gauge.
- Problem: only Abelian LKFs are known! (at least to our knowledge)

(non-Abelian) Landau-Khalatnikov-Fradkin

- The Abelian LKF can be derived via path integral manipulations.
- An alternative proof was given in Sonoda, PLB 499 (2001) using Stückelberg-like trick (to construct gauge invariant photon).
- Since we have by now access to non-Abelian gauge invariant A^h, without destroying renormalizability $\rightarrow$ possibility to derive non-Abelian LKF? $\rightarrow$ YES
- Gauge invariant A^h, but also gauge invariant

$$\psi^h = h^\dagger \psi$$

with $h = e^{ig\xi^a t^a}$, t^a in adjoint or fundamental rep. For the moment, we solve for ξ:

$$\xi = \frac{1}{\partial^2}\partial_\mu A_\mu + i\frac{g}{\partial^2}\left[\partial A, \frac{\partial A}{\partial^2}\right] + i\frac{g}{\partial^2}\left[A_\mu, \partial_\mu \frac{\partial A}{\partial^2}\right] + \frac{i}{2}\frac{g}{\partial^2}\left[\frac{\partial A}{\partial^2}, \partial A\right] + \mathcal{O}(A^3)$$

(non-Abelian) Landau-Khalatnikov-Fradkin

work in progress by De Meerleer, Bashir, et al

- Crux of the matter

$$
\begin{aligned}
&\left\langle A^h \dots \overline{\psi}^h \dots \psi^h \right\rangle = \text{gauge invariant} \\
&= \langle A \dots \overline{\psi} \dots \psi \rangle_{\alpha=0} \\
&= \langle A \dots \overline{\psi} \dots \psi \rangle_{\alpha} + \text{ series in } \xi \text{ (or } \partial A)
\end{aligned}
$$

Application: LKF for quark propagator

Using the Landau gauge properties, we found for example

$$
\langle \bar{\psi}(x)\psi(y)\rangle_{\alpha} \quad = \quad \langle \bar{\psi}(x)\psi(y)\rangle_{\alpha=0} \left\langle h^{\dagger}(x)h(y)\right\rangle_{\alpha=0}
$$

(non-Abelian) Landau-Khalatnikov-Fradkin

Alternative derivation via path integral transformation (leading to same LKFTs)

$$
\begin{aligned}
A_\mu \to A'_\mu &= U^\dagger A_\mu U + \frac{i}{g} U^\dagger \partial_\mu U\,, \\
\psi &\to U^\dagger \psi\,, \\
U &= e^{ig\xi} = 1 + ig\xi - \frac{g^2}{2}\xi^2 + \mathcal{O}(\xi^3) \\
c &\to U^\dagger c U\,, \\
\bar{c} &\to U^\dagger \bar{c} U\,,
\end{aligned}
$$

the Jacobian is trivial, and for example

$$Z[\alpha, J, \bar{J}] = Z[\alpha', UJ, \bar{J}U^\dagger]$$

with $J, \bar{J}$ sources coupled to $\bar{\psi}, \psi$. Deriving w.r.t. sources gives desired LKF transformations in the matter sector.

(non-Abelian) Landau-Khalatnikov-Fradkin

Main conceptual requirement: we want to keep the action invariant, up to gauge parameter $\alpha \to \alpha'$. We have that

$$A'_\mu = -\partial_\mu \xi + ig[A_\mu, \xi] + \frac{ig}{2}[\partial_\mu \xi, \xi] + \mathcal{O}(\text{fields}^3)\,,$$

thence we must impose that

$$\begin{aligned}\partial_\mu A'_\mu &= \partial_\mu A_\mu - \partial^2 \xi + ig[A_\mu, \partial_\mu \xi] + ig[\partial_\mu A_\mu, \xi] + \frac{ig}{2}\left[\xi, \partial^2 \xi\right] + \mathcal{O}(\text{fields}^3) \\ &\equiv (1+X)\partial_\mu A_\mu\end{aligned}$$

which sends

$$\int \mathrm{d}^4 x \left(ib\partial_\mu A_\mu + \frac{\alpha}{2} b^2\right) \to \int \mathrm{d}^4 x \left(ib'\partial_\mu A'_\mu + \frac{\alpha'}{2}(b')^2\right)$$

with

$$b \to b' = \frac{1}{1+X} b\,, \qquad \alpha \to \alpha' = (1+X)^2 \alpha\,,$$

(non-Abelian) Landau-Khalatnikov-Fradkin

- We get

$$\xi = -X\frac{1}{\partial^2}\partial_\mu A_\mu - igX\frac{1}{\partial^2}\left[A_\mu, \frac{1}{\partial^2}\partial_\mu\partial_\nu A_\nu\right] - igX\frac{1}{\partial^2}\left[\partial_\nu A_\nu, \frac{1}{\partial^2}\partial_\mu A_\mu\right] + \frac{igX^2}{2}\frac{1}{\partial^2}\left[\frac{1}{\partial^2}\partial_\mu A_\mu, \partial_\nu A_\nu\right] + \mathcal{O}(A^3)\,.$$

  with the LKF-propagator given by

$$\langle\xi(p)\xi(-p)\rangle_{\alpha'} = -X^2\alpha\frac{1}{p^4}\,.$$

- $X \to -1$ corresponds to LKF ending up in Landau gauge.
- The LKF field ξ is *not* free, but interacts quite complicatedly in the non-Abelian case.

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Gribov-Zwanziger with background

Why?

- As discussed in Marhauser, Pawlowski, arXiv:0812.1144; talk and work of Reinhardt et al; talk and work of our hosts, access to Polyakov loop *vev* via $\langle A_0 \rangle$, more precisely [SU(2)]

$$\langle \mathcal{P} \rangle = \cos\frac{r}{2}\,, \qquad r = g\beta\bar{A}_0\,, \qquad \bar{A}^a_\mu = \delta_{\mu 0}\delta^{a3}\bar{A}_0 = \text{background}$$

- Access to (de)confinement transition and thermodynamics in both phases using functional techniques for $\Gamma(r)$.
- Evidently, we must then face the introduction of $\bar{A}_0$ into the GZ theory.

Gribov-Zwanziger with background

- Work in Landau-DeWitt gauge (Landau background gauge)

$$\bar{D}a = 0 \Leftrightarrow \bar{D}A = 0 \ \text{ if } \ \partial\bar{A} = 0$$

with

$$a = \bar{A} + A$$

- How to generalize (standard) GZ action in presence of $\bar{A}$? Natural guess, based on standard "rule" $\partial \to \bar{D} \equiv \partial + \bar{A}$:

$$S_{\text{GZ+PLoop}} = \int d^dx \left\{ \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - \frac{\left(\bar{D}A\right)^2}{2\xi} + \bar{c}^a\bar{D}^{ab}_{\mu}D^{bd}_{\mu}(a)c^d + \bar{\varphi}^{ac}_{\mu}\bar{D}^{ab}_{\nu}D^{bd}_{\nu}(a)\varphi^{dc}_{\mu} \right.$$
$$\left. -\bar{\omega}^{ac}_{\mu}\bar{D}^{ab}_{\nu}D^{bd}_{\nu}(a)\omega^{dc}_{\mu} - g\gamma^2 f^{abc}A^a_{\mu}\left(\varphi^{bc}_{\mu} + \bar{\varphi}^{bc}_{\mu}\right) - \gamma^4 d(N^2-1) \right\} .$$

- Corresponds to $-\bar{D}(\bar{D}+A) > 0$, i.e. positive background FP related to excluding infinitesimal copies Zwanziger, NPB209 (1982).
- So far so good (it seems).

Gribov-Zwanziger with background: hopeful result

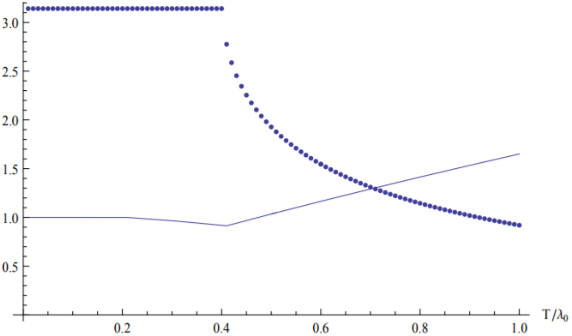
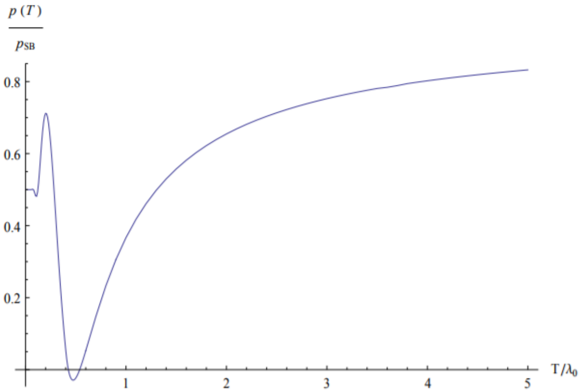


Figure: taken from Canfora, Pais et al, EPJC75(2015). Gribov parameter (line) and r (dots) in terms of temperature T. There is a phase transition, with cusp behaviour for Gribov mass λ^2. Value for T_c is lower than compared to "lattice experiment", notice no RGZ yet since we are still working on $T = 0$ effective action approach!

Gribov-Zwanziger with background: less hopeful result



- Pressure P relative to $P(T=0)$ is non-monotonous at low T, and becomes even negative over small region.

Gribov-Zwanziger with background: less hopeful result

- The "oscillatory behaviour" at low T is expected and not clear how to avoid, see also Benic, Blaschke, Buballa, PRD86 (2012).

  Roughly speaking:

  $$\text{massive DOF} \Rightarrow e^{-m/T} \text{ at low } T \Rightarrow \text{if } m \text{ complex, then } \sin \& \cos$$

- Also $P < 0$ at low T has a "deep cause".

  First without background: massless ghosts versus massive gluons (supported by *all* functional approaches): the negative norm ghosts will always dominate at low T. (Having) BRST does not help here, unlike in the large T regime. See also Chernodub, Zakharov, PRL100 (2008).

  On general grounds, one expects that for $T < T_c$, as the confined spectrum is massive, that P is exponentially suppressed in $1/T$.

- Coupling to a background makes this problem milder, see e.g. Reinosa et al, PRD91 (2015), but it persists.

Gribov-Zwanziger with background: another conceptual issue

- Current background GZ action does not enjoy background gauge invariance.

$$\delta\bar{A} = \bar{D}\omega, \qquad \delta A = gfA\omega, \ldots$$

background gauge invariance is the big merit of using a background gauge fixing

- Not merely a technical problem, also conceptual problem.

  At $T = 0$, $\bar{A}_0$ is gauge equivalent to zero. Canfora, Pais et al, PLB763 (2016); PLB772 (2017); Dudal, Vercauteren, to appear. So we should expect $\Gamma(\bar{A}_0) \equiv 0$, but we even find nontrivial minimum at $\bar{A}_0 \neq 0$. Of course related to lack of background gauge invariance. Evidently, spontaneous Lorentz symmetry breaking is also not desirable.

Gribov-Zwanziger with background: background invariant formulation

- BRST is crucially related to background gauge invariance Grassi, Hurth, Quadri, PRD70 (2004). STI + ghost Ward identity $\frac{\delta\Sigma}{\delta c} = \ldots$ actually gives background gauge invariance encoded in Ward identity.
- Use GZ action based on a^h

$$\Sigma = \ldots + \int d^4x \left(\bar{\phi}_\mu^{ac}\partial_\nu (D^h)_\nu^{ab}\phi_\mu^{bc} - \bar{\omega}_\mu^{ac}\partial_\nu (D^h)_\nu^{ab}\omega_\mu^{bc} + \gamma^2 f^{abc}(a^h)_\mu^a(\bar{\phi}_\mu^{bc} + \phi_\mu^{bc})\right)$$

  implementing $-\partial D(a^h) > 0$. This is sufficient to kill infinitesimal gauge copies related to Landau-DeWitt gauge $\bar{D}A = 0$.
- By construction, it is background gauge invariant!

Gribov-Zwanziger with background: background invariant formulation

- Quite technical analysis, for SU(2) (lowest order in quantum field expansion):

$$\begin{aligned}(a^h)^a_\mu =& \mathrm{R}_3^{ab}(g\mathcal{A}x_0)A^b_\mu - \partial_\mu \frac{1}{\partial^2}\partial_\nu(\mathrm{R}_3^{ab}(g\mathcal{A}x_0)A^b_\nu) + \cdots \\ =& \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)(\mathrm{R}_3^{ab}(g\mathcal{A}x_0)A^b_\nu) + \cdots \,.\end{aligned}$$

  Interpretation: construct gauge transform that minimizes $\int \bar{A}^2$ (i.e. that gives $\bar{A}^h$), then use this “gauge angle” to adjointly transform A, and then project on the transverse subspace.

- By construction now at $T = 0$, $\Gamma(A_0) \equiv 0$. We also checked it explicitly at one loop (non-trivial computation to find a trivial zero).
- At $T > 0$, $\bar{A}_0$ cannot necessarily be transformed to 0 due to periodicity constraint,

$$g(\bar{A}^h)^a_\mu = rT\delta_{\mu 0}\delta^{a3}\,, \qquad r \in [-\pi, \pi[.$$

- Effective action computation using now correct background GZ action in progress.

The End!

Thanks!