Variational approach to QCD

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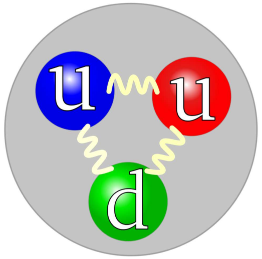
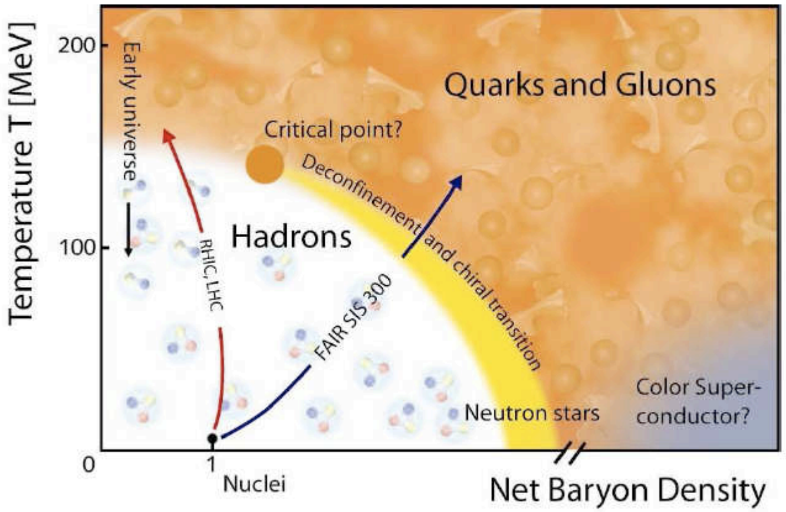
QCD

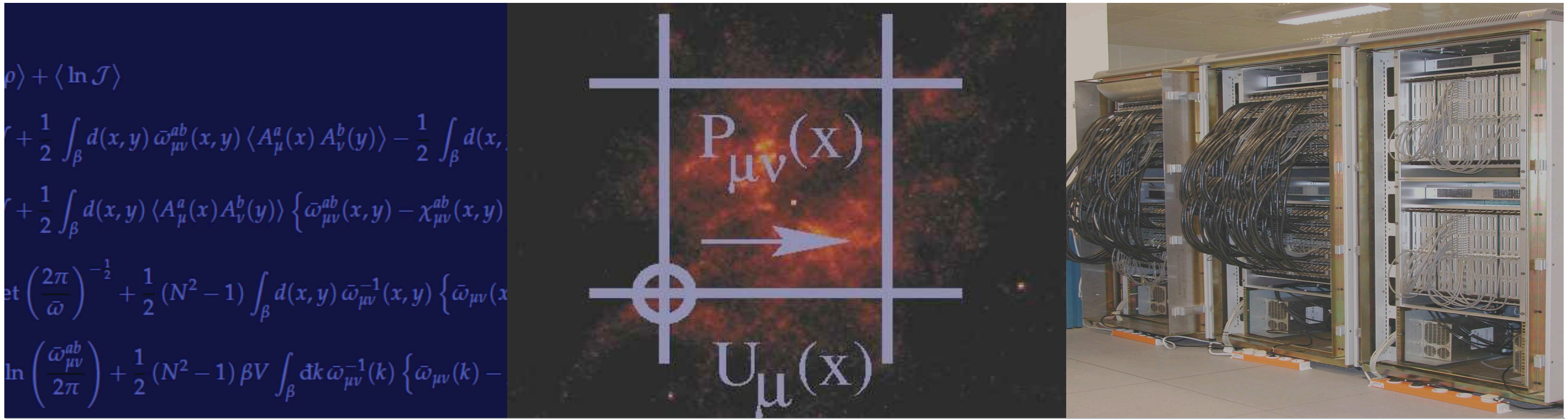
- *vacuum*
  - confinement
  - SB chiral symmetry



- *phase diagram*
  - deconfinement
  - rest. chiral symm



- latticeMC-fail at large chemical potential
  continuum approaches required



Non-perturbative Continuum Approaches

Non-perturbative Continuum Approaches

- Dyson-Schwinger equations
  - Landau(+Coulomb)gauge

  Alkofer, Fischer, von Smekal, Huber,...Aguilar, Papavasiliuo, Rodrigues-Quintero,... Zwanziger, P. Watson, H.R.

- FRG flow equations
  - Landau gauge

  Pawlowski, Gies, Braun, Mitter,...

- Covariant variational approach
  - Landau gauge

  Quandt, H. R...

- Hamiltonian variational approach
  - Coulomb gauge

  Feuchter, Campagnari, H. R...

Alternative Continuum Approaches

- massive gluon propagator (Curci-Ferrari model)
  - Landau gauge — Reinosa, Serreau, Tissier, Wschebor, Siringo...
  - pertubation theory
- eff. Gribov-Zwanziger action
  - Landau gauge — Dudal, Sorella, Oliveira,...

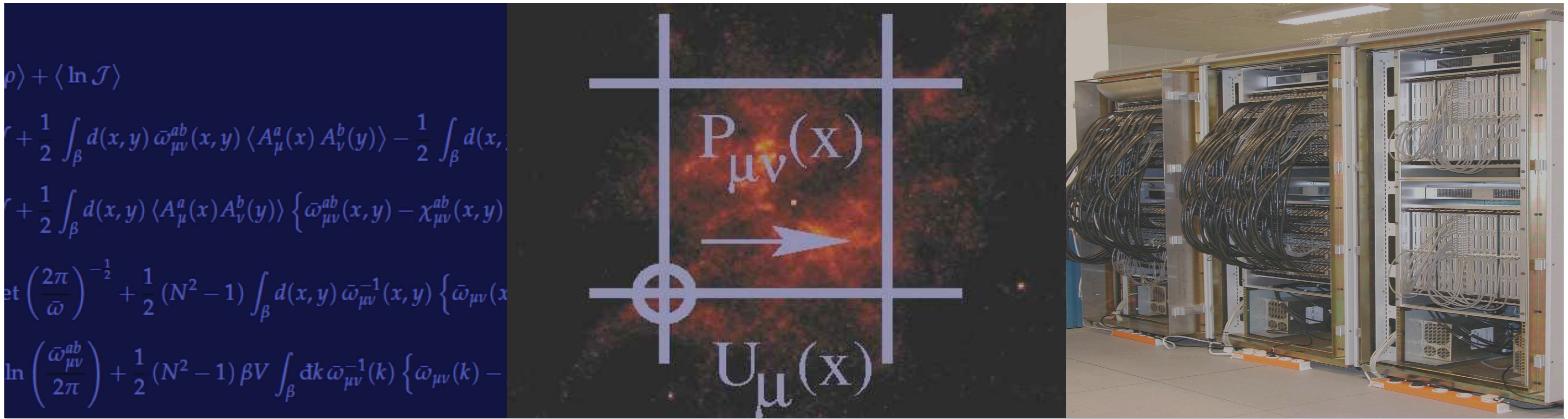
Non-perturbative Continuum Approaches

- Dyson-Schwinger equations
  - Landau(+Coulomb)gauge
- FRG flow equations
  - Landau gauge
- Covariant variational approach
  - Landau gauge
- Hamiltonian variational approach
  - Coulomb gauge

Outline

- introduction
- basics of the covariant variational approach
    - analogy to the Hamiltonian approach
- YMT at T =0
    - gluon & ghost propagator
- YMT at finite T
    - effective potential of the Polyakov loop
    - pressure & energy density
- ---
- Hamiltonian approach QCD in Coulomb gauge (quark sector)
  - finite temperature by compactification of a spatial dimension
    - Polyakov loop
    - chiral & dual quark condensate
- conclusions

in collaboration with M. Quandt, D. Campagnari and J. Heffner



Basics of the Covariant Variational Approach

Statistical Systems

- *density operator (matrix)* D

  -knowing D is equivalent to knowing
  the expectation values of all observables

  -exact D - not known
  - to much information

>>> reduced statistical description

- **entropy**

$$\Sigma = -Tr(D \ln D)$$

- *measure for the missing information*

Principle of Maximum Entropy

- *relevant observables* Ω^i

H. R. , Balian, Alhassid
Nucl.Phys.A422(1984)349

Balian, Alhassid, H. R.
Phys. Rept. 131(1986)1

$$\langle \Omega^i \rangle = \omega^i \qquad \langle \ldots \rangle = Tr(D \ldots)$$

- *maximize the entropy under the constraints*

$$\Sigma = -Tr(D \ln D)$$

$$\tilde{\Sigma} = \Sigma - \lambda_i \langle \Omega^i \rangle = -Tr[D(\ln D + \lambda_i \Omega^i)] \rightarrow \max$$

- *solution*

$$D = \exp\left(-\lambda_i \Omega^i\right)$$

- *least bias density*

normalization $\Omega^0 = 1$ $\quad \exp\left(\lambda_0 \Omega^0\right) = Tr \exp\left(-\sum_{i \neq 0} \lambda_i \Omega^i\right)$

Covariant variational approach to QFT

- ***relevant observables*** $\Omega = S[\phi]$ ***action*** $Tr\ldots = \int D\phi\ldots$
- ***maximize the entropy under the constraint*** $\langle S \rangle = \sigma$

$$\tilde{\Sigma} = \Sigma - \lambda \langle S \rangle = -\lambda F \rightarrow \max \qquad \Sigma = -Tr(D \ln D)$$

- ***free action*** $F = \langle S \rangle - \lambda^{-1} \Sigma \rightarrow \min$
- ***solution***

  Gibbs measure

$$D = \frac{\exp(-\lambda S[\phi])}{\int D\phi \exp(-\lambda S[\phi])}$$

$$\lambda = \hbar^{-1}$$

- ***variational principle***

$$F = \langle S \rangle - \hbar \, \Sigma \rightarrow \min$$

Quandt, Reinhardt, Heffner, PRD(2014)065037

Hamiltonian variational approach to QFT

- *relevant observables* $\Omega = H$ **Hamiltonian** $Tr\ldots = \sum_n \langle n|\ldots|n\rangle$
- *maximize the entropy under the constraint* $\langle H \rangle = E$

$$\tilde{\Sigma} = \Sigma - \lambda \langle H \rangle = -\lambda F \to \max$$

- *free energy* $F = \langle H \rangle - \lambda^{-1}\Sigma \to \min$
- *solution*

$$D = \frac{\exp(-\lambda H)}{\sum_n e^{(-\lambda E_n)}}$$

- *ground state*

$$\lambda \to \infty$$

- *variational principle*

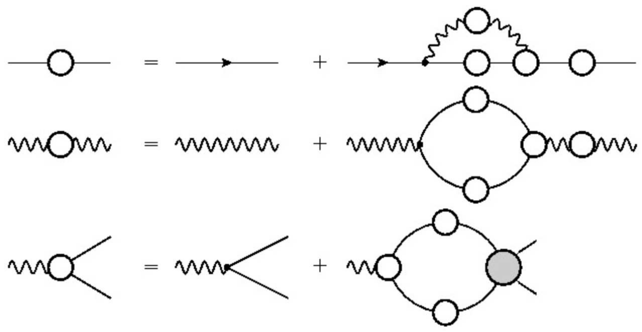
$$F = \langle H \rangle \to \min$$

Covariant variational approach to QFT

- ansatz for the trial action $$S[\phi] = \sum_n \gamma^{(n)}(1,2,\ldots,n)\phi(1)\phi(2)\ldots\phi(n)$$
- density $$D = \frac{1}{Z}\exp\left(-S[\phi]/\hbar\right) \qquad Z = \int D\phi \exp\left(-S[\phi]/\hbar\right)$$
- free action $$F = \left\langle S_{QFT} \right\rangle - \hbar\Sigma = \left\langle S_{QFT} \right\rangle + \hbar \left\langle \ln D \right\rangle$$ $$= \left\langle S_{QFT} \right\rangle - \left\langle S \right\rangle - \hbar \ln Z$$
- use DSEs to express $\langle \phi(1)\phi(2)\ldots\phi(n) \rangle$ in terms of the „bare“ vertices $\gamma^{(n)}(1,2,\ldots,n)$ = are variational kernels
- variational (gap) equations for the bare vertices
- $$\delta F / \delta\gamma^{(n)} = 0$$

Covariant variational approach to QFT
= optimized DSE approach

- DSEs(standard except:)
  - bare vertices are variational kernels



- variational (gap) equations for the bare vertices
  - depend on the truncation of the DSEs
  - try to „make up“ for truncation
- ***optimal bare vertices for each truncation of the DSE***
- ***„auto-tuned“ DSE***

Covariant variational approach to QCD
= optimized DSE approach

- *2 extreme cases:*
  - ***S[A] = polynomial form of the true action***

$$S[A] = \tfrac{1}{2}\omega(1,2)A(1)A(2) + \tfrac{1}{3!}\gamma^{(3)}(1,2,3)A(1)A(2)A(3) + \tfrac{1}{3!}\gamma^{(4)}(1,2,3,4)A(1)A(2)A(3)A(4)$$

-untruncated DSE

>variational equations =trivial

- bare vertices =vertices of the true action

$$\omega \sim -\Box\delta(\ldots), \quad \gamma^{(3)} \sim g f^{a_1 a_2 a_3}\delta(\ldots), \quad \gamma^{(4)} \sim g^2 f^{a_1 a_2 b} f^{a_3 a_4 b}\delta(\ldots)$$

  - ***S[A] = quadratic*** $S[A] = \tfrac{1}{2}\omega(1,2)A(1)A(2)$

>DSE not needed (Wick´s theorem)

-variational (gap) equation for $\omega(1,2)$

Hamiltonian variational approach to QFT

- vacuum wave functional $\Psi[\phi] = \exp\left(-\tfrac{1}{2} S[\phi]\right)$

$\langle \ldots \rangle = \int D\phi \ldots \exp\left(-S[\phi]\right)$

- *3-dim Euclidean QFT*

- ansatz for the „action" $S[\phi] = \sum_n \gamma^{(n)}(1,2,\ldots,n)\phi(1)\phi(2)\ldots\phi(n)$

- use DSEs to express $\langle \phi(1)\phi(2)\ldots\phi(n) \rangle$ in terms of the „bare" vertices $\gamma^{(n)}(1,2,\ldots,n)$ = are variational kernels

- energy $\langle H \rangle \to \min$

- variational (gap) equations for the bare vertices $\gamma^{(n)}(1,2,\ldots,n)$

Variational Hamiltonian Approach	Covariant Variational Approach
non-covariant	fully covariant
Coulomb gauge	Landau gauge
gauge fixed Hamiltonian	gauge fixed action
non-standard renormalization	standard renormalization
Gauss‘ law resolved exactly	BRST symmetry broken
extension to T>0: -canonical approach(complicated)	extension to T>0: -compactification of Euclidean time
-compactification a spatial dimension	

H.Reinhardt, PRD94 $S^1 \times \mathbb{R}^2$

Covariant variational approach to QCD in Landau gauge

- conceptually and technically very similar to the variational Hamiltonian approach in Coulomb gauge

Covariant variational approach to YMT

- density $D = \frac{1}{Z} J[A] \exp\left(-S_{gf}[A]/\hbar\right)$ $Z = \int DAJ[A] \exp\left(-S_{gf}[A]/\hbar\right)$
- Faddeev-Popov determinant $J[A] = Det(-\hat{D}\partial)$
- modified action $\tilde{S}_{gf} = S_{gf} - \hbar \ln J$
- free action
$$F = \left\langle \tilde{S}_{gf} \right\rangle - \hbar \Sigma \rightarrow \min$$
$$= \left\langle S_{gf} \right\rangle - \hbar \left\langle \ln J \right\rangle - \hbar \Sigma$$
$$= \left\langle S_{gf} \right\rangle - \hbar \tilde{\Sigma}$$
- modified entropy $\tilde{\Sigma} = \Sigma + \left\langle \ln J \right\rangle = -\left\langle \ln(D/J) \right\rangle$

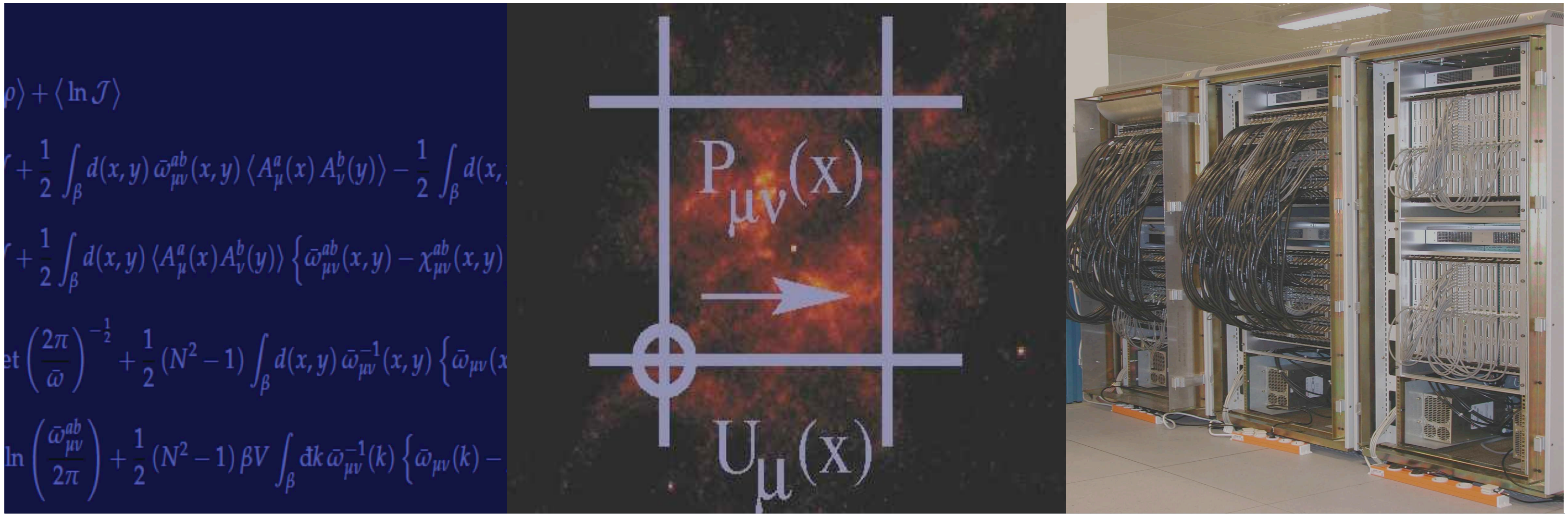
Effective action

-density $D = \mu$

$$F(\mu,\omega) = \min_{\mu} \left\{ F(\mu) \,\middle|\, \langle \Omega \rangle_\mu = \omega \right\} \quad \overset{\min}{\Longrightarrow} \quad \mu = \mu_\omega$$

$$\Gamma(\omega) = F(\mu_\omega,\omega) \overset{!}{=} \min$$ *-effective action*

Note: Usually $\Omega = A \quad (\omega = \mathcal{A})$ and proper functions $\dfrac{\delta\Gamma(\mathcal{A})}{\delta\mathcal{A}(x_1)\cdots\delta\mathcal{A}(x_n)}$



Yang-Mills Theory at T=0

Gaussian ansatz

- UV : gluons weakly interacting
- IR : ghost dominance near Gribov horizon, self-interaction subdominant

$$J(A)\exp\left[-S(A)\right] = N(\omega)\exp\left[-\tfrac{1}{2}\int d^4x d^4y A_\mu^a(x)\omega_{\mu\nu}^{ab}(x,y)A_\nu^b(y)\right]$$

-DSE not needed (Wick´s theorem)

Curvature Approximation

$$\ln \mathcal{J}(A) = -\frac{1}{2}\int d(x,y)\, A_\mu^a(x)\, \chi_{\mu\nu}^{ab}(x,y)\, A_\nu^b(y) + \cdots$$

$$\chi_{\mu\nu}^{ab}(x,y) = -\left\langle \frac{\delta^2 \ln \mathcal{J}}{\delta A_\mu^a(x)\, \delta A_\nu^b(y)} \right\rangle \longrightarrow \delta^{ab}\, t_{\mu\nu}(k)\, \chi(k)$$

curvature

H. Reinhardt & C. Feuchter,PRD71 (2005)

Free action

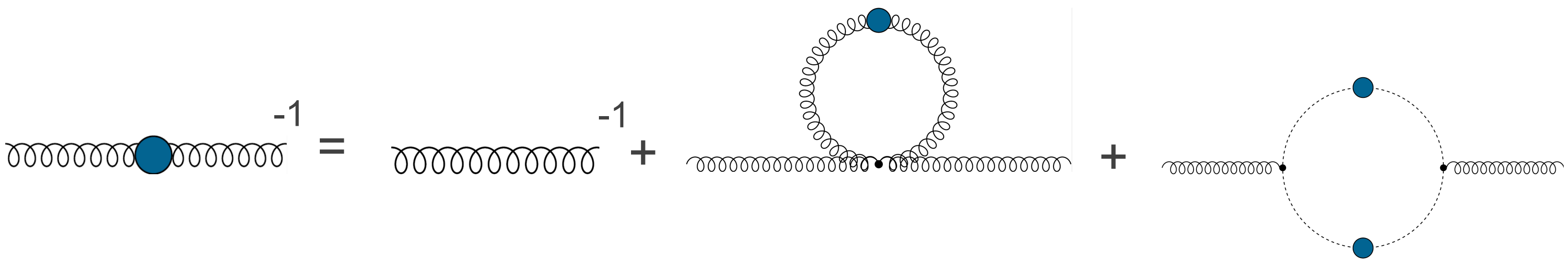
$$F(\omega) = \langle S_{QFT} \rangle_\omega - \Sigma(\omega)$$

Gap Equation

$$\frac{\delta}{\delta\omega(k)} F(\omega) = 0$$

$$\omega(k) = k^2 + M^2 + \chi(k)$$

$$M^2 = I_M$$



=leading order gluon DSE

Ghost sector

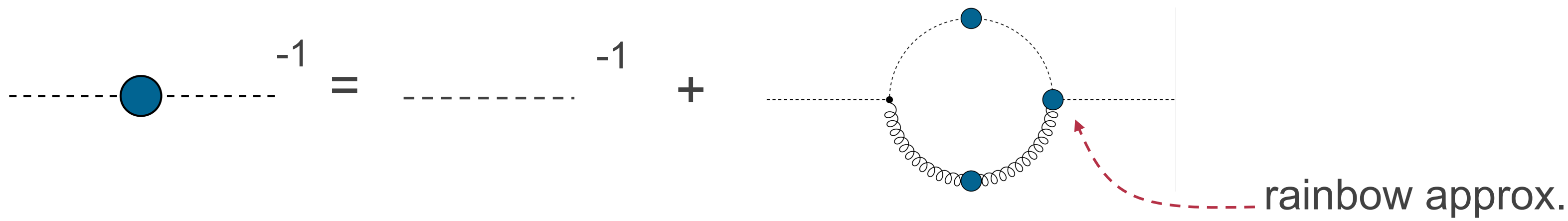
$$G^{-1} = \partial_\mu \hat{D}^\mu = G_0^{-1} + h$$

$$G_0 \langle G^{-1} \rangle = 1 + G_0 \langle hG \rangle \langle G^{-1} \rangle$$

in terms of ghost form factor $G(k) = \frac{\eta(k)}{k^2}$

$$\eta^{-1}(k) = 1 - I_\eta(k)$$

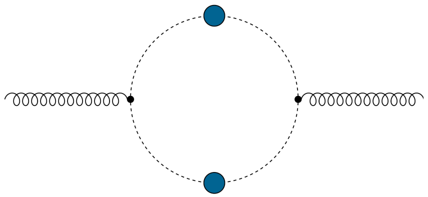
$$I_\chi(k) = Ng^2 \int \frac{d^4q}{(2\pi)^4} \frac{\eta(k-q)}{(k-q)^2} \frac{1-(\hat{k}\cdot\hat{q})}{\bar{\omega}(q)}$$



Curvature Equation

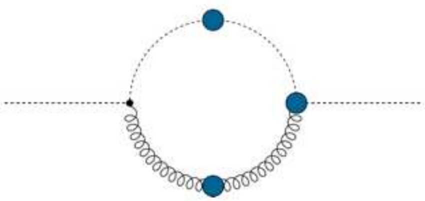
$$\chi(k) = \mathrm{Tr}\left\langle G \frac{\delta(-\partial D)}{\delta A(2)} G \frac{\delta(-\partial D)}{\delta A(1)} \right\rangle \approx \mathrm{Tr}\langle G \rangle \Gamma_0 \langle G \rangle \Gamma_0$$

=



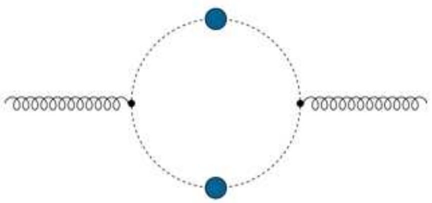
$$\chi(k) = \frac{1}{3} N g^2 \int \frac{d^4 q}{(2\pi)^4} \frac{\eta(k-q)\,\eta(q)}{(k-q)^2} \left[1 - (\hat{k} \cdot \hat{q})\right]$$

UV- behaviour of the loop integrals



$$I_\eta(k) = a_0 \ln(\Lambda^2 / M^2) + \mathit{finite}$$

$$I_M = a_1 \Lambda^2 + b_1 M^2 \ln(\Lambda^2 / M^2) + \mathit{finite}$$



$$\chi(k) = a_2 \Lambda^2 + (b_2 M^2 + c k^2) \ln(\Lambda^2 / M^2) + \mathit{finite}$$

Equations of motion with counter terms

- ghost DSE

$$\eta^{-1}(k) = 1 - I_\eta(k) - \delta Z_c)]$$

- gluon gap eq.

$$\omega(k) = k^2 + [I_M + (\chi(k) + k^2 \delta Z_A)] + \delta M^2$$

Renormalization conditions (3 scales $0 \le \mu_c \le \mu_0 \ll \mu$)

- fix $\eta(\mu_c) \to \delta Z_c$ scaling/decoupling
- fix $\omega(\mu) = Z\,\mu^2$
- fix $\omega(\mu_0) = Z\,M_A^2$ constitutent *mass parameter*

renormalized equations of motion

- **ghost DSE**

$$\eta^{-1}(k) = \eta^{-1}(\mu_c) - [I_\eta(k) - I_\eta(\mu_c)]$$

- **gluon gap equation**

$$\omega(k) = Z\frac{\mu^2 - M_A^2}{\mu^2 - \mu_0^2}k^2 + Z\frac{M_A^2 - \mu_0^2}{\mu^2 - \mu_0^2}\mu^2 + \frac{1}{\mu^2 - \mu_0^2}[\mu^2(\chi(k) - \chi(\mu_0)) - k^2(\chi(\mu) - \chi(\mu_0)) - \mu_0^2(\chi(k) - \chi(\mu))]$$

- **3-dimensionless (finite)renormalization constants**

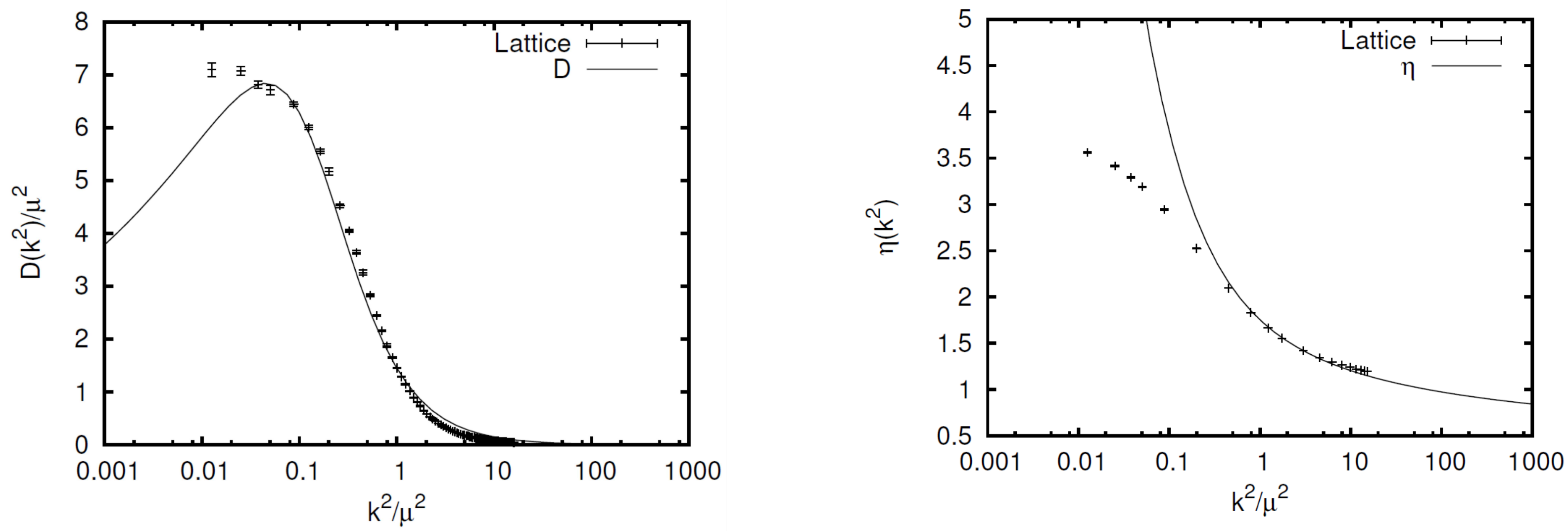
$\eta^{-1}(\mu_c = 0)$ -discriminates between scaling and decoupling sol.

M_A -determines the IR limit of the gluon propagator for the decoupling sol., no effect on scaling sol.

Z -determine overall size of gluon propagator

Scaling Solution (G=SU(2))

MQ, H. Reinhardt, J. Heffner, Phys. Rev. **D89** 035037 (2014)



IR exponents: $\omega(k) \sim (k^2)^{\alpha}$ $\qquad \alpha = \frac{1}{49}\left(44 - \sqrt{1201}\right) \approx 0.1907$

$\eta(k) \sim (k^2)^{-\beta}$ $\qquad \beta = \frac{1}{98}\left(93 - \sqrt{1201}\right) \approx 0.5953$

Lerche, v. Smekal PRD **65**

numerical: $\alpha = 0.191(1)$ $\qquad \beta = 0.595(3)$

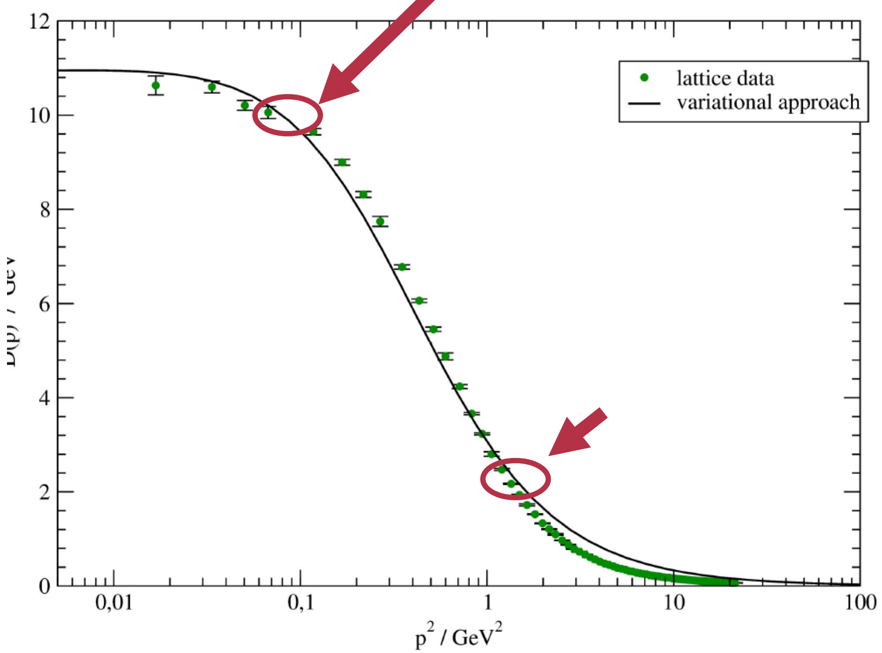
$$\alpha - 2\beta + \left(\frac{d}{2} - 1\right) < 10^{-3}$$

Decoupling Solution (G=SU(2))

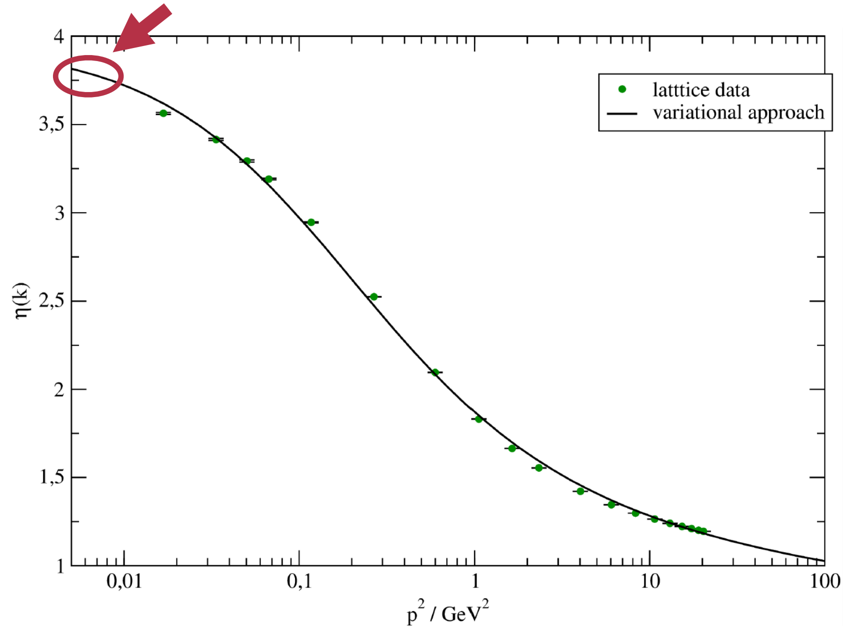
M. Quandt, H. Reinhardt, J. Heffner, Phys. Rev. **D89** 035037 (2014)

Lattice data from Bogolubsky et al., Phys. Lett. **B676** 69 (2009)

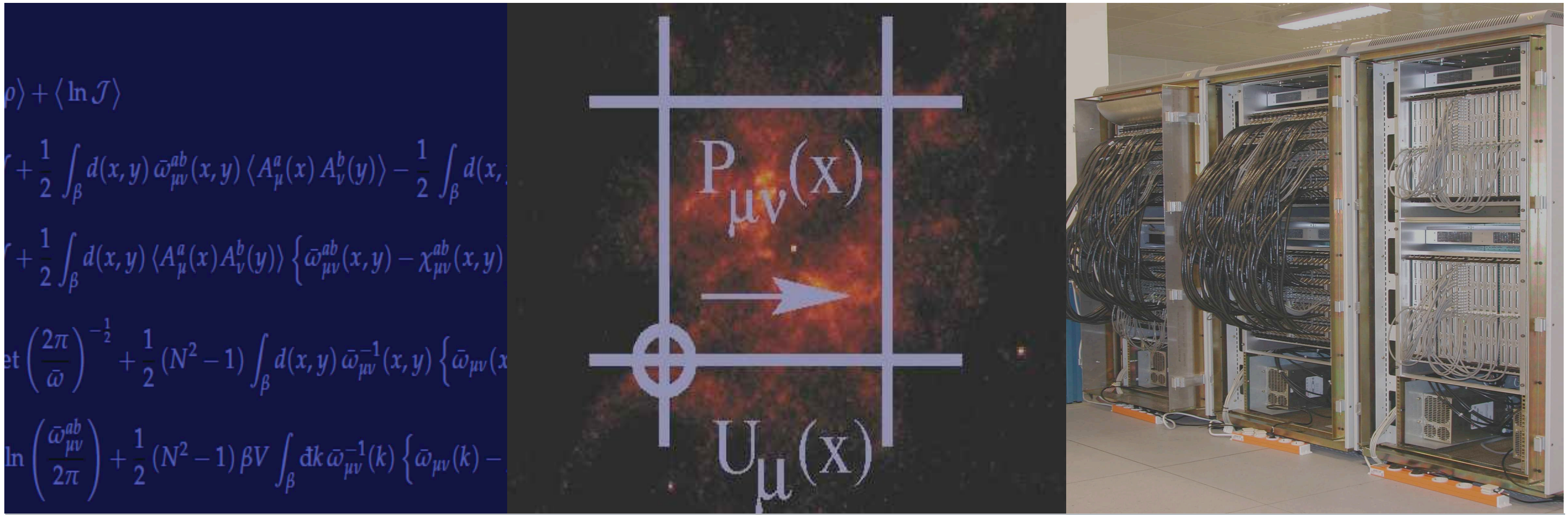
Determines overall scale (not very precise !)



gluon propagator

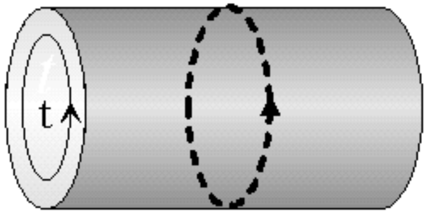


ghost form factor



Yang-Mills Theory at Finite Temperature

Extension to Finite Temperature



compactify euclidean time $t \in [0, \beta]$

periodic b.c. for gluons (up to center twists)

periodic b.c. for ghosts (even though fermions) !

$$A(t, \mathbf{x}) = \beta^{-1} \sum_{n \in \mathbb{Z}} \int \frac{d^3k}{(2\pi)^3} e^{i(\nu_n t + \mathbf{kx})} A_n(\mathbf{k})$$

$$\nu_n = \frac{2\pi}{\beta} n \qquad (n \in \mathbb{Z})$$

⇒ Extension to T>0 straightforward

$$\int \frac{d^4k}{(2\pi)^4} \longrightarrow \int_\beta đq \equiv \beta^{-1} \sum_{n \in \mathbb{Z}} \int \frac{d^3k}{(2\pi)^3}$$

heat bath singles out restframe (1,0,0,0) ⇨ breaks Lorentz invariance

two different 4-transversal projectors

!

$$\mathcal{P}^T_{\mu\nu}(k) = (1-\delta_{\mu 0})\,(1-\delta_{\nu 0})\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{\mathbf{k}^2}\right)$$ ← 3-transversal

$$\mathcal{P}^L_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) - \mathcal{P}^T_{\mu\nu}(k)$$ ← 3-longitudinal

⇨ Two Lorentz structures for kernel and curvature

$$\omega_{\mu\nu}(k) = \omega_\perp(k)\cdot\mathcal{P}^T_{\mu\nu}(k) + \omega_\parallel(k)\cdot\mathcal{P}^L_{\mu\nu}(k) + \text{long.}$$

$$\chi_{\mu\nu}(k) = \chi_\perp(k)\cdot\mathcal{P}^T_{\mu\nu}(k) + \chi_\parallel(k)\cdot\mathcal{P}^L_{\mu\nu}(k) + \text{long.}$$

Gap Equations

$$\omega_\perp(k) = k_0^2 + \mathbf{k}^2 + \chi_\perp(k) + M^2(\beta)$$

$$\omega_\parallel(k) = k_0^2 + \mathbf{k}^2 + \chi_\parallel(k) + M^2(\beta) + \frac{\mathbf{k}^2}{k_0^2 + \mathbf{k}^2}\widetilde{M}^2(\beta)$$

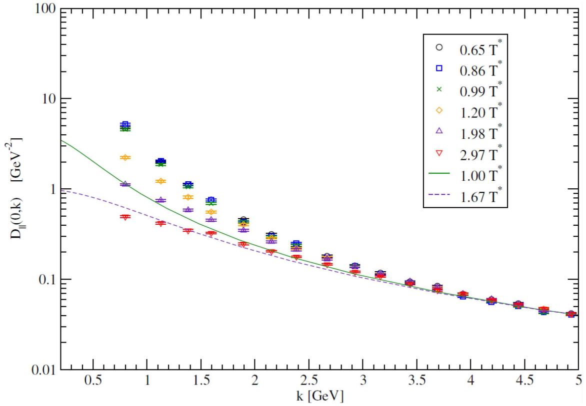
induced gluon masses now temperature-dependent

$$M^2(\beta) = \frac{1}{2} N g^2 \int_\beta đq \left[\frac{A}{\omega_\perp(q)} + \frac{B(q)}{\omega_\parallel(q)}\right]$$

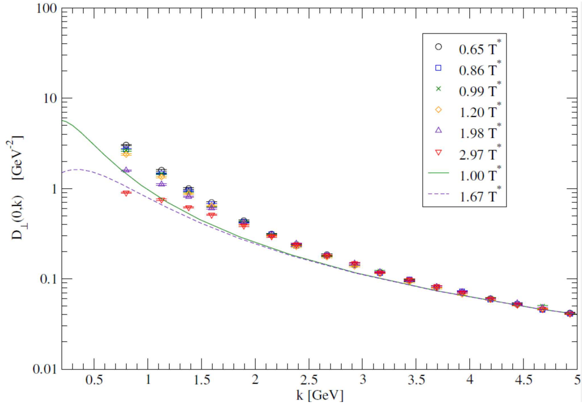
$$\widetilde{M}^2(\beta) = \frac{1}{3} N g^2 \int_\beta đq \left[\frac{2}{\omega_\perp(q)} + \left(\frac{q_0^2 - 3\mathbf{q}^2}{q_0^2 + \mathbf{q}^2}\right)\frac{1}{\omega_\parallel(q)}\right]$$

renormalization by T=0 counter terms (in principle …)

M. Quandt, H. Reinhardt, Phys. Rev. **D92** 025051 (20



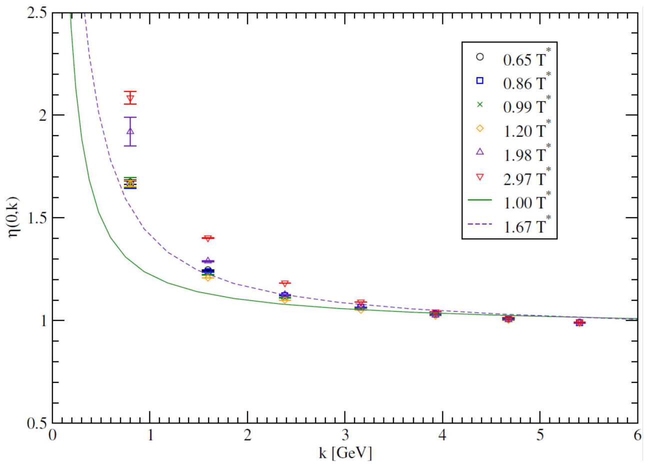
longitudinal gluon $D_{||}(0,p)$



transversal gluon $D_{\perp}(0,p)$

ghost formfactor

$\eta(0,p)$

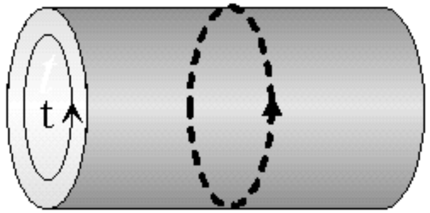


With increasing T

- Slight ghost enhancement
- Mild gluon suppression
- Temperature sensitivity larger in direction longitudinal to the heat bath

Polyakov loop

$$\mathsf{L}(\mathbf{x}) = P\exp\left[-\int_0^\beta dt\, A_0(t,\mathbf{x})\right]$$



Interpretation: free static quark energy

$$\langle \mathrm{tr}\,\mathsf{L}(\mathbf{x})\rangle = \exp\left[-\beta F_q(\mathbf{x})\right]$$

$$\langle \mathrm{tr}\,\mathsf{L}(\mathbf{x})\,\mathsf{L}(\mathbf{y})^\dagger\rangle = \exp\left[-\beta\, F_{q\bar{q}}(\mathbf{x}-\mathbf{y})\right]$$

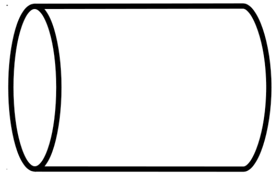
Center symmetry

- maps $\mathsf{L} \to z \cdot \mathsf{L}$
- If unbroken $\langle \mathsf{L} \rangle = 0$ [**confinement**]
- If broken $\langle \mathsf{L} \rangle \neq 0$ [**deconfinement**]

The Polyakov loop -
order parameter of confinement

$$P[A_0](\vec{x}) = \tfrac{1}{d_r} tr P \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

$T^{-1} = L$



Polyakov gauge $\partial_0 A_0 = 0, \quad A_0 = diagonal$

$SU(2):$ $$P[A_0](\vec{x}) = \cos(\tfrac{1}{2} A_0(\vec{x}) L)$$

$P[A_0] - \textit{unique function of } A_0$

alternative order parameters of confinement

$$\langle P[A_0](\vec{x}) \rangle \qquad P\left[\langle A_0 \rangle\right](\vec{x}) \qquad \langle A_0(\vec{x}) \rangle$$

- *F.Marhauser and J. M. Pawlowski, arXiv:0812.11144*
- *J. Braun, H. Gies, J. M. Pawlowski, Phys. Lett. B684(2010)262*

Alternative order parameter

$$x \equiv \frac{\beta \langle A_0^3 \rangle}{2\pi} = \frac{\beta \mathsf{a}}{2\pi} \in [0, 1]$$

G=SU(2)
Polyakov gauge $[\, \partial_0 A_0 = A_0^{\mathrm{ch}} = 0 \,]$
Background gauge $[\, \partial_0 \mathsf{a} = 0 \,]$

Background gauge

$$A_\mu = \mathsf{A}_\mu + Q_\mu = \mathsf{a}\, \delta_{\mu 0} + Q_\mu$$

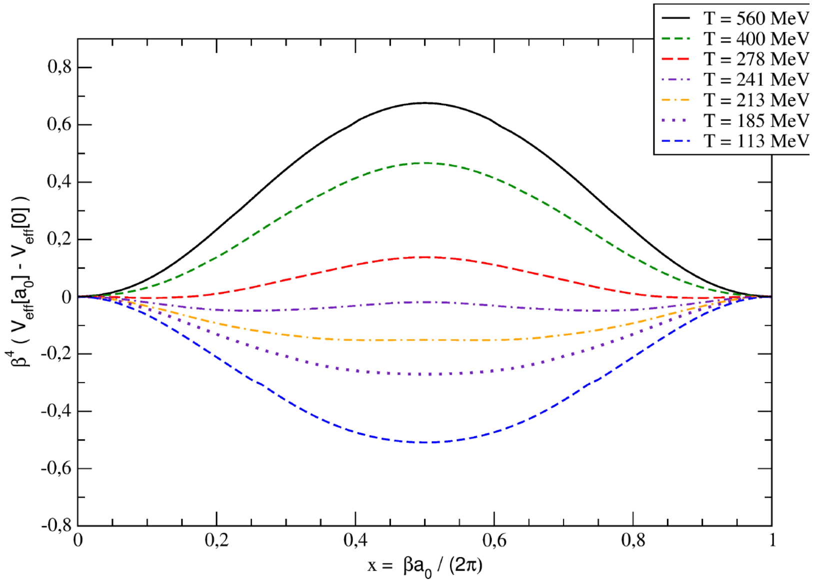
$$[D_\mu(\mathsf{a}), Q_\mu] = [\mathsf{d}_\mu, Q_\mu] = 0$$

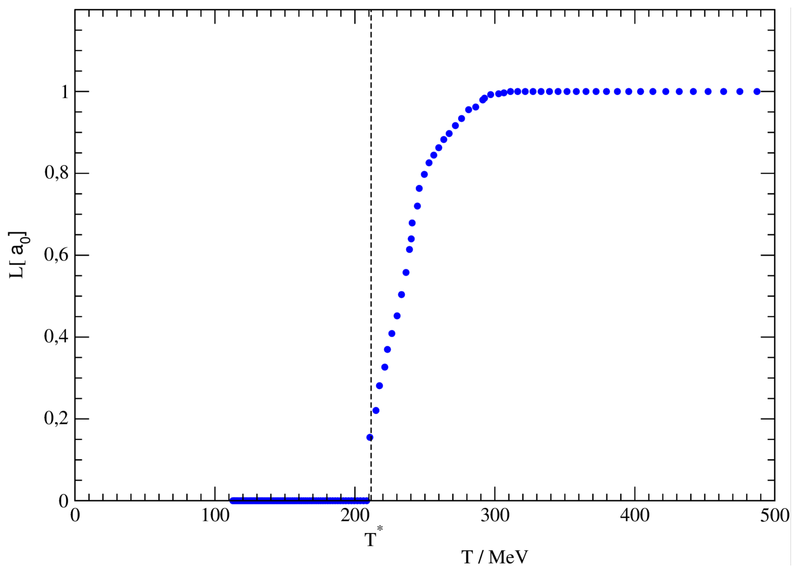
Transfer Landau -- Background

- replace $\partial_\mu \, \delta^{ab} \mapsto \hat{\mathsf{d}}^{ab}_\mu$ in basis where rhs is diagonal
- replace $p_\mu \mapsto p_\mu - \sigma\, \mathsf{a}\, \delta_{\mu 0}$ where σ are the simple roots
- replace $N^2 - 1 = 3 \mapsto \sum_{\sigma = 0, \pm 1}$ sum over simple roots

Phase transition for G=SU(2)

M.Quandt, H. Reinhardt, Phys. Rev. **D94** (2016)





Eff. Potential for Polyakov loop

ghost dominance

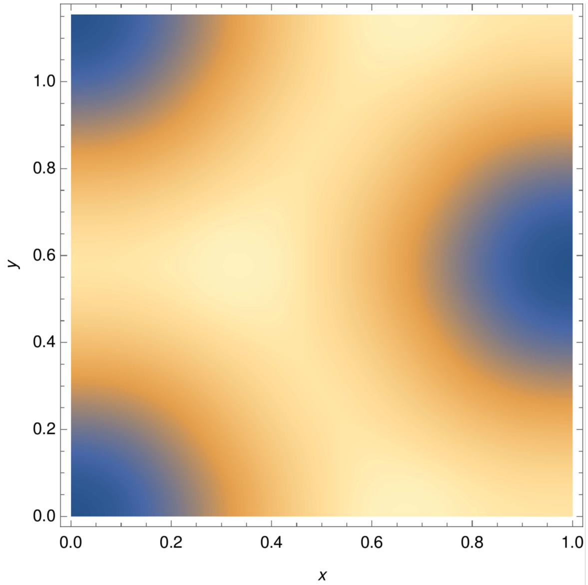
- 2nd order transition
- critical temperature $T^* = 216\,\mathrm{MeV}$

Lattice $T^* = 306\,\mathrm{MeV}$

Lucini, Teper, Wenger, JHEP **01** (2004) 061

Effective potential for Polakov loop in G=SU(3)

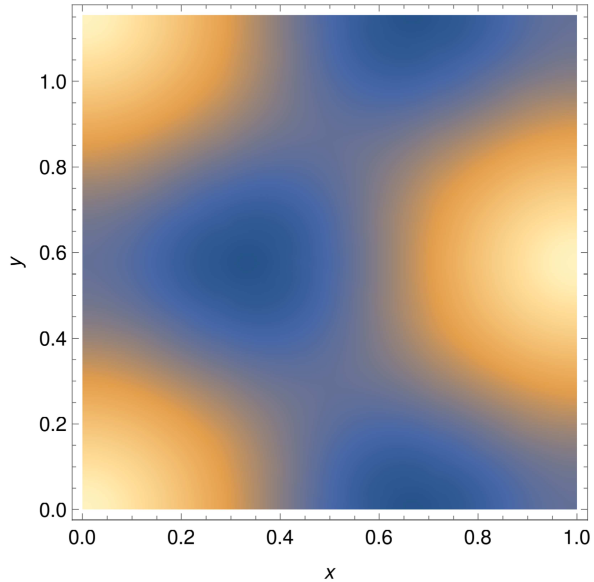
M. Quandt & H. Reinhardt, Phys. Rev. **D9**4(2016)



Deconfined phase

V(x,y) maximal at center symmetrc points

$$T = 400\,\mathrm{MeV}$$



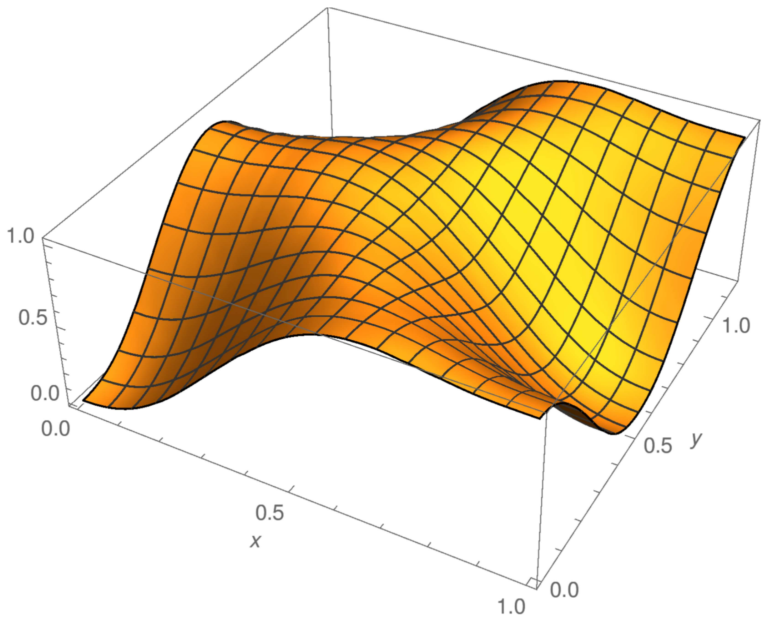
Confined phase

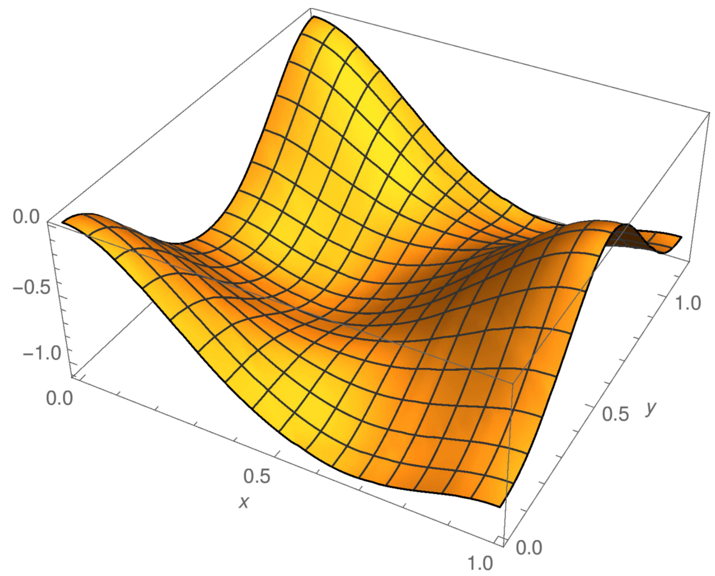
V(x,y) minimal at center symmetrc points

$$T = 141\,\mathrm{MeV}$$

Effective potential for Polakov loop in G=SU(3)

M. Quandt & H. Reinhardt, Phys. Rev. **D9**4(2016)





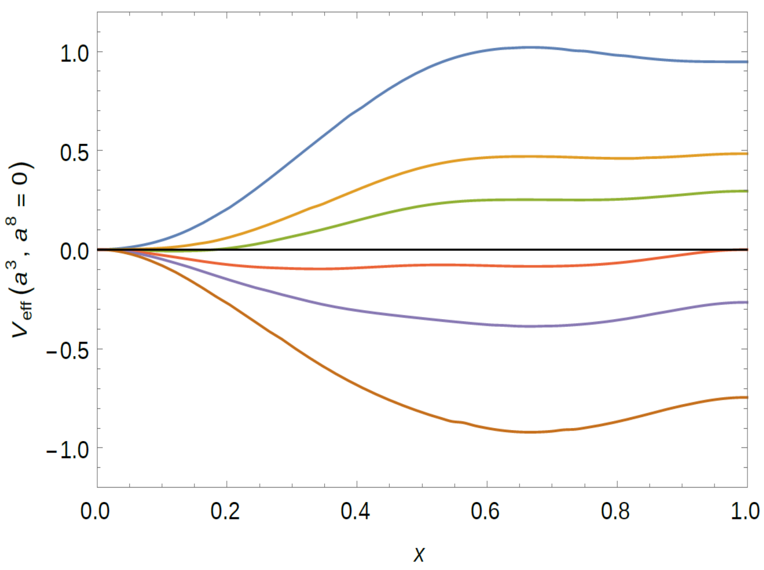
Deconfined phase | Confined phase

V(x,y) maximal at center symmetrc points | V(x,y) minimal at center symmetrc points

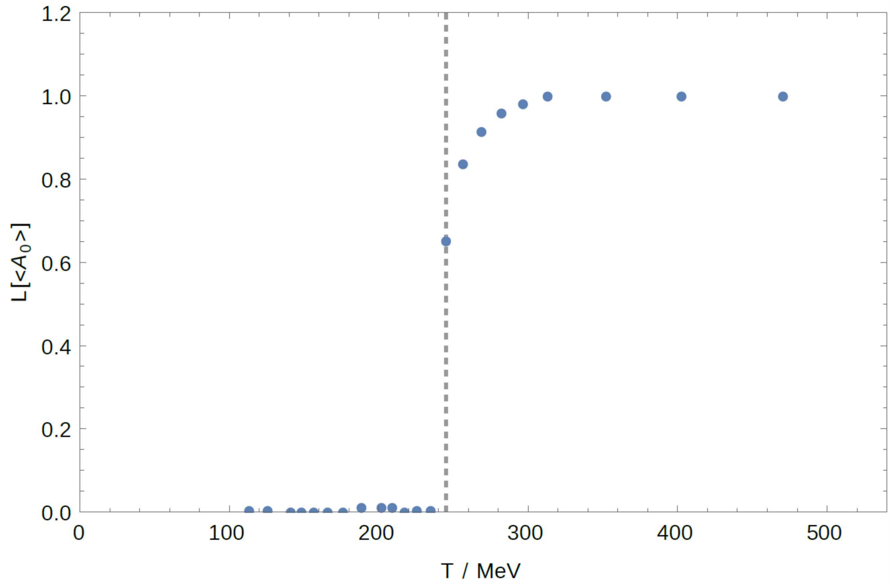
$T = 400\,\mathrm{MeV}$ | $T = 141\,\mathrm{MeV}$

Phase transition for G=SU(3)

M.Quandt, H. Reinhardt, Phys. Rev. **D94** (2016)



slice of eff. Potential for Polyakov loop



- 1st order transition
- critical temperature $T^* = 245\,\mathrm{MeV}$

Lattice $T^* = 284\,\mathrm{MeV}$

Lucini, Teper, Wenger, JHEP **01** (2004) 061

Thermodynamics of the YM plasma

M. .Quandt, H. Reinhardt,
Phys. RevD 96(2017)

- Free energy density:

$$F(\beta) = \min_{\mu} F_{\beta}(\mu) = \min_{a} \Gamma_{\beta}[a] = -\ln Z(\beta) = V_3 \cdot \beta f(\beta)$$

- pressure: $p(\beta) = -f(\beta)$

- energy density: $\epsilon(\beta) = f(\beta) + \beta \partial f / \partial \beta$

- Interaction strength: $\Delta(\beta) = \beta^{-3} \partial(p\beta^4)/\partial\beta$

Free relativstic gas: $p \sim T^4 \quad \Longrightarrow \quad \Delta = 0$

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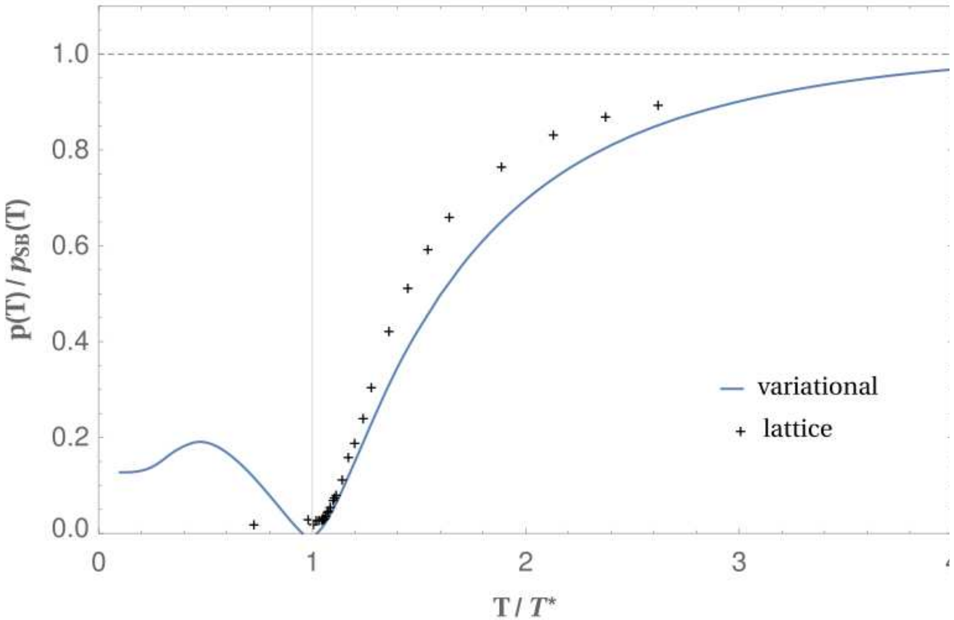
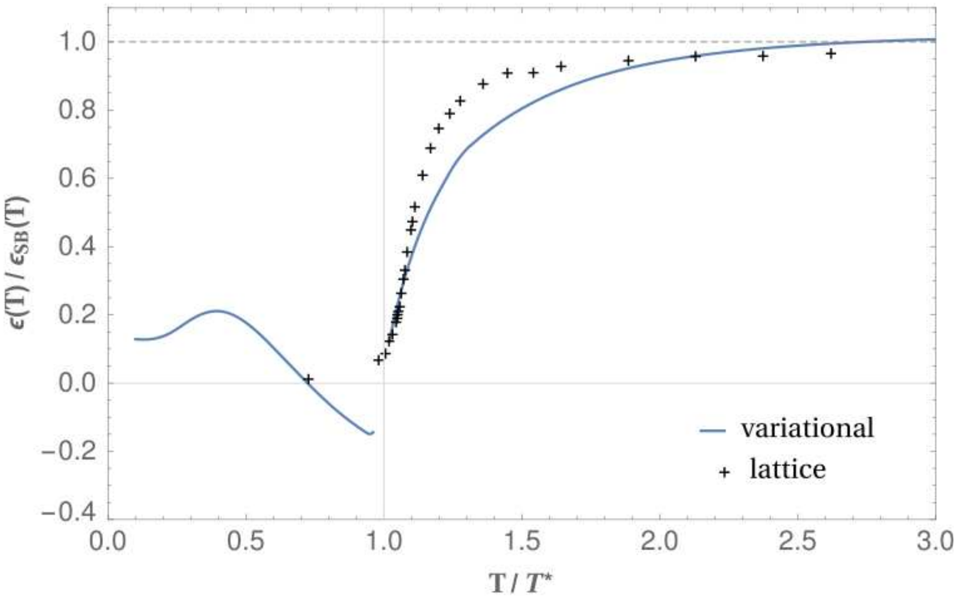
pressure

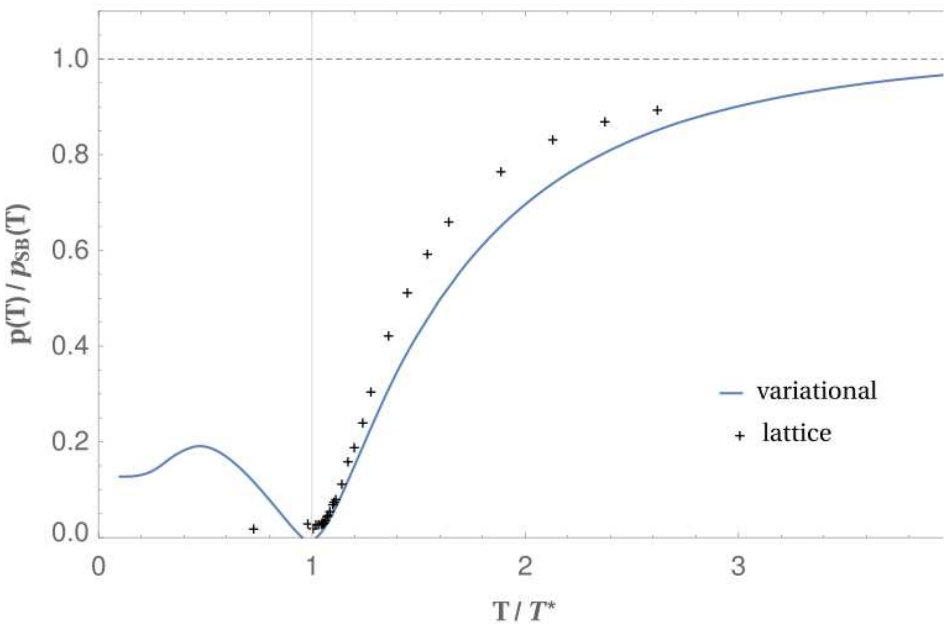


energy density

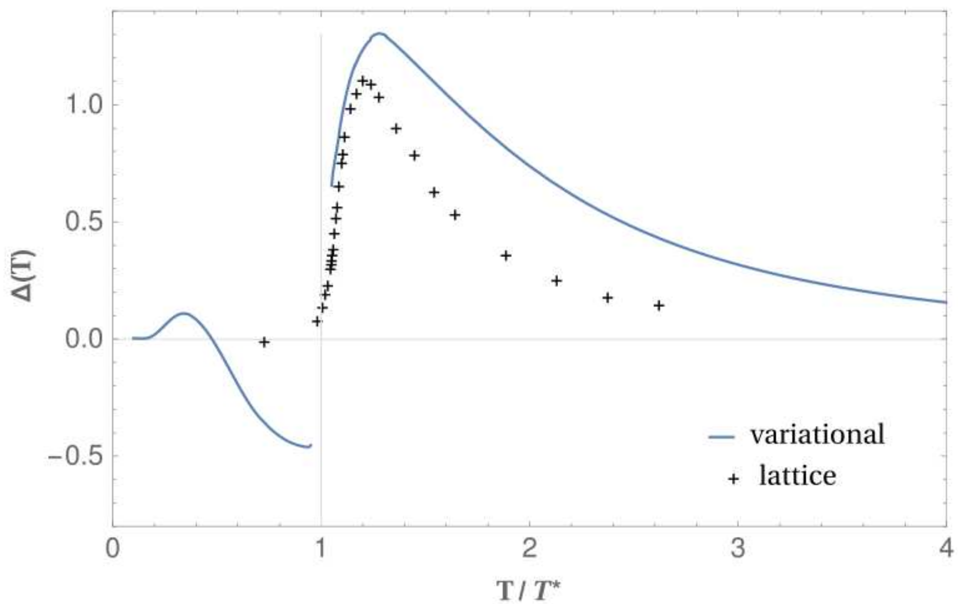


- Correct SB limit
- Energy density $\beta^4 \epsilon$ „steeper“ than pressure $\beta^4 p$ $\Longrightarrow$ $\Delta > 0$
- Lattice data slightly steeper than variational results

pressure



interaction measure



- Correct SB limit
- Energy density $\beta^4\epsilon$ „steeper“ than pressure $\beta^4 p \quad \Longrightarrow \quad \Delta > 0$
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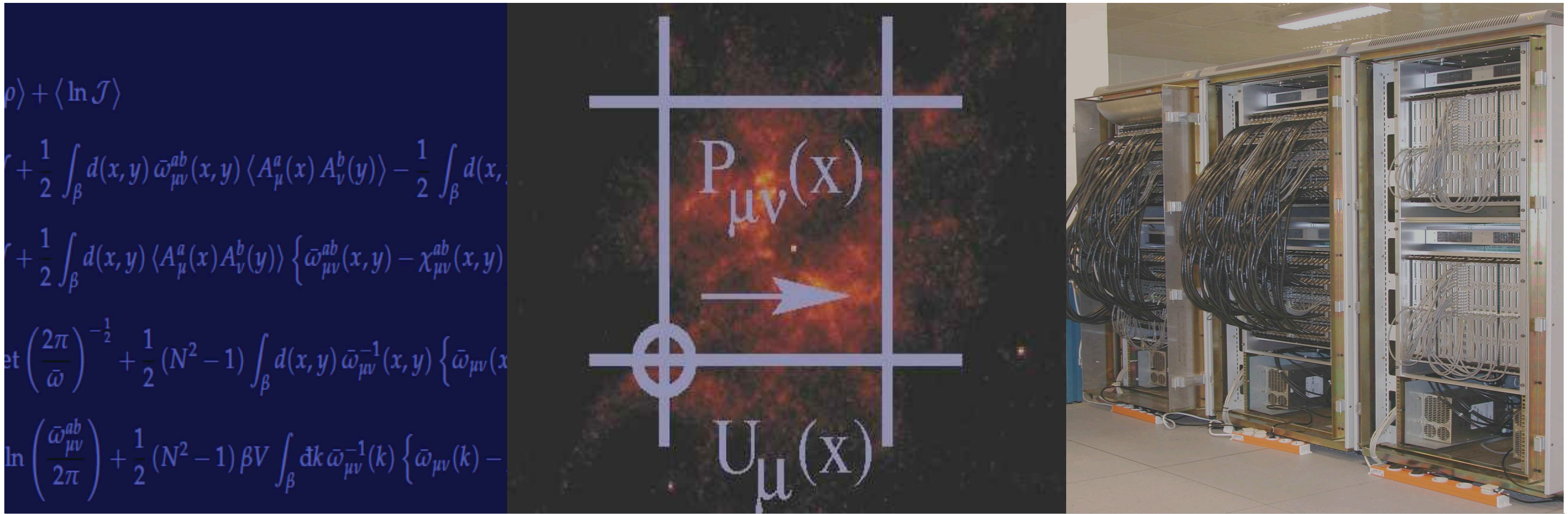


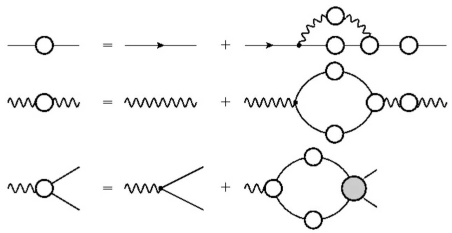
Beyond the Gaussian ansatz

Covariant variational approach to QCD
beyond the Gaussian ansatz

- ansatz for action

$$S[A] = \tfrac{1}{2}\omega(1,2)A(1)A(2) + \tfrac{1}{3!}\gamma^{(3)}(1,2,3)A(1)A(2)A(3) + \tfrac{1}{3!}\gamma^{(4)}(1,2,3,4)A(1)A(2)A(3)A(4)$$

- DSEs needed to express the n-point functions in terms of the variational kernels (bare vertices) $\gamma^{(n)}(1,2,\ldots,n)$



- not yet done in the covariant but done in the Hamiltonian variational approach

D. Campagnari & H. R, Phys.Rev.D82(2010)
Phys.Rev.D92(2015)

Variational approach to YMT with non-Gaussian wave functional

D. Campagnari & H.R,
Phys.Rev.D82(2010)
Phys.Rev.D92(2015)

wave functional

$$|\psi[A]|^2 = \exp(-S[A])$$

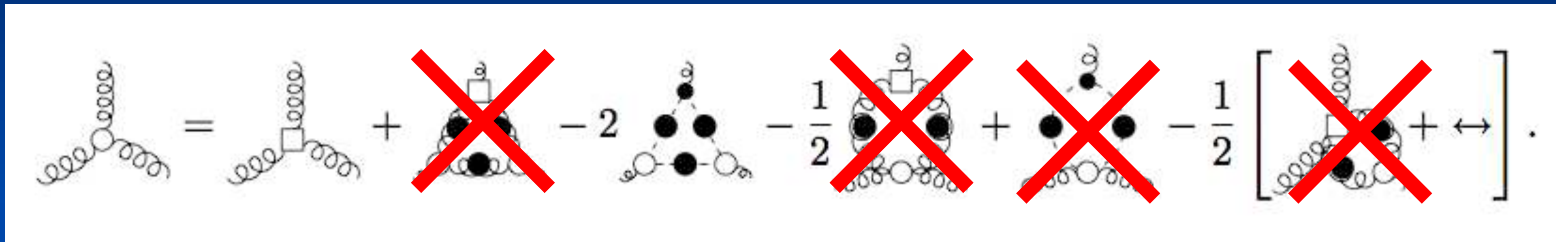
ansatz

$$S[A] = \int \omega A^2 + \frac{1}{3!}\int \gamma^{(3)} A^3 + \frac{1}{4!}\int \gamma^{(4)} A^4$$

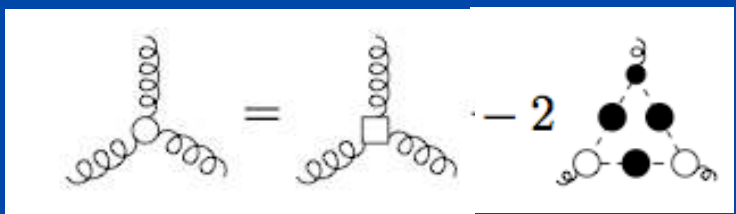
exploit DSE

3-gluon vertex

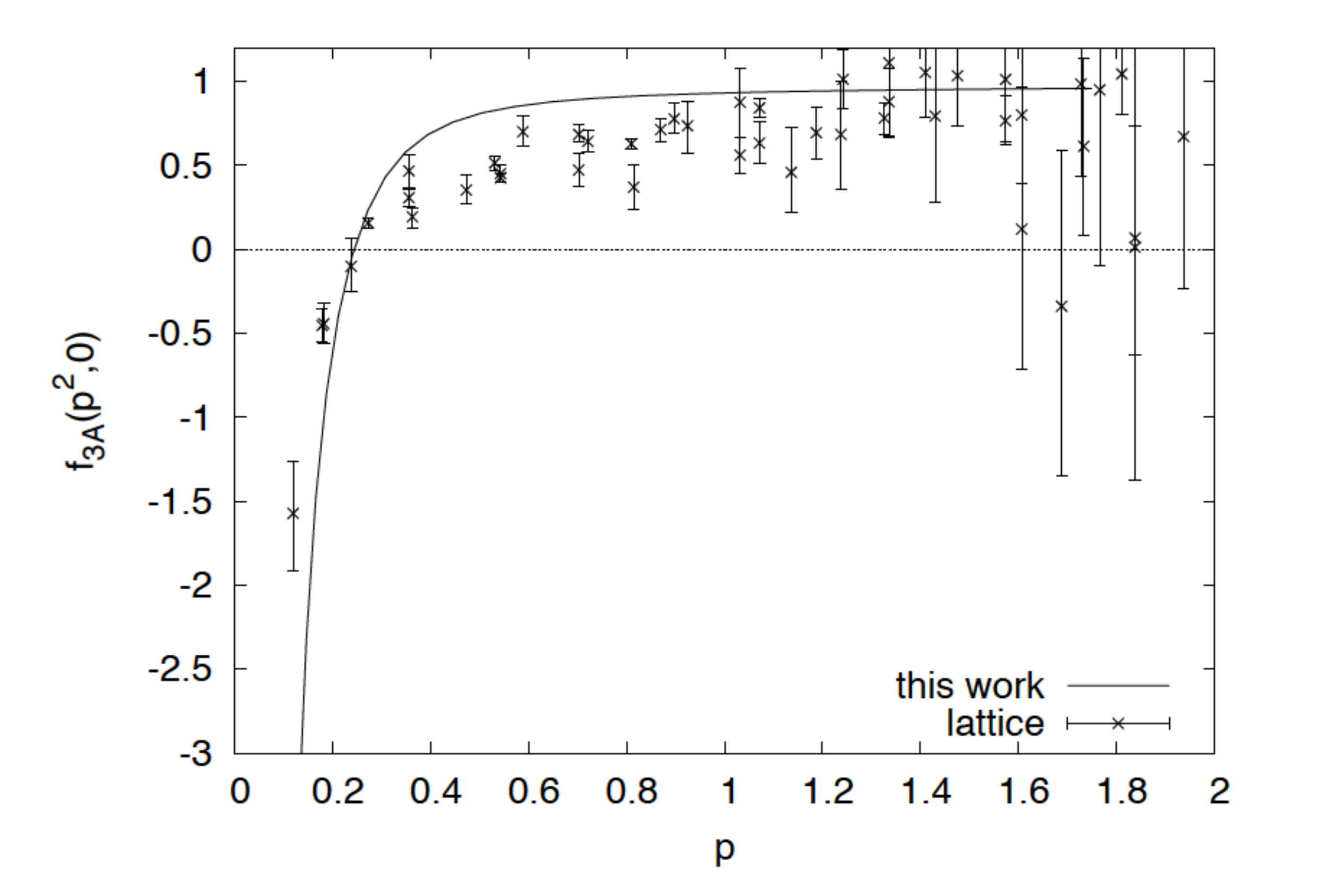
D. Campagnari & H.R, Phys.Rev.D82(2010)



ghost dominance in the IR



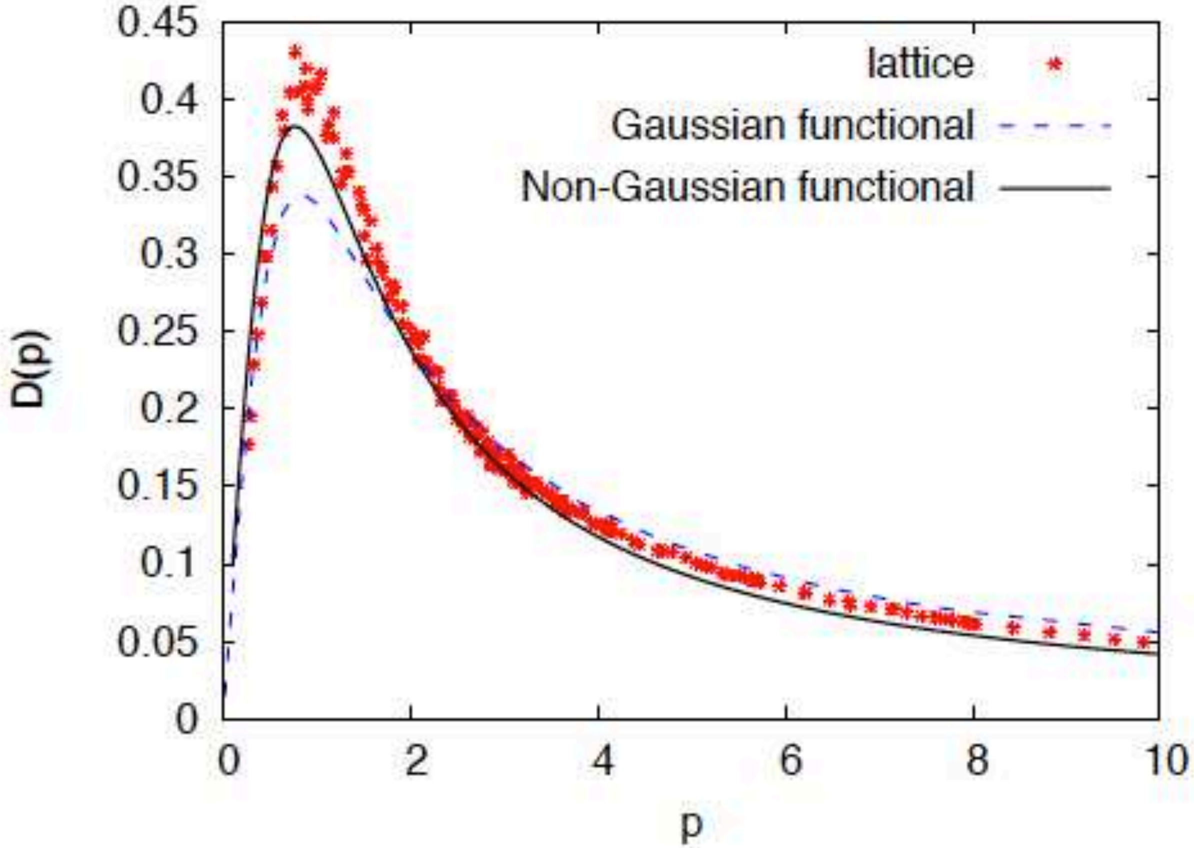
3-gluon vertex



D. Campagnari & H.R, Phys.Rev.D82(2010)

lattice data: A. Cucchieri, A. Maas and T. Mendes, PRD77, 094510 (2008)

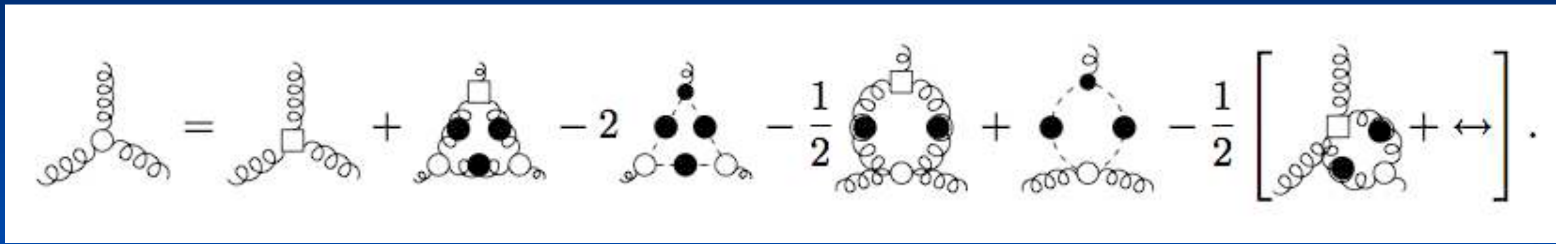
Corrections to the gluon propagator

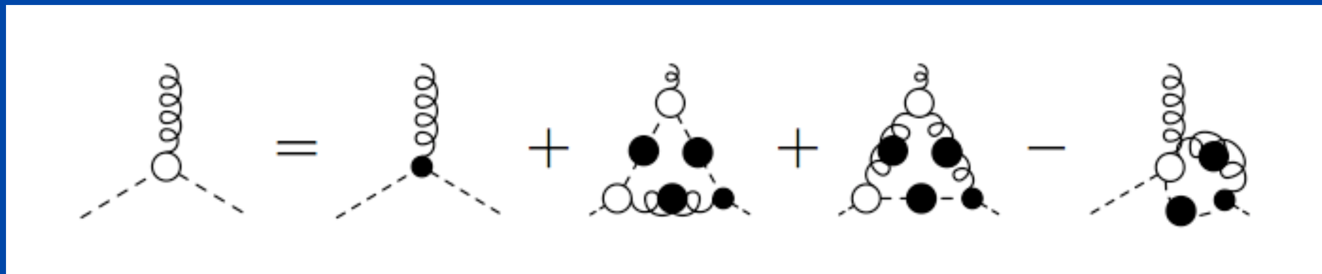


D. Campagnari & H.R, Phys.Rev.D82(2010)

3-gluon & ghost-gluon-vertex

M. Huber, D. Campagnari & H.R, PRD91(2015)

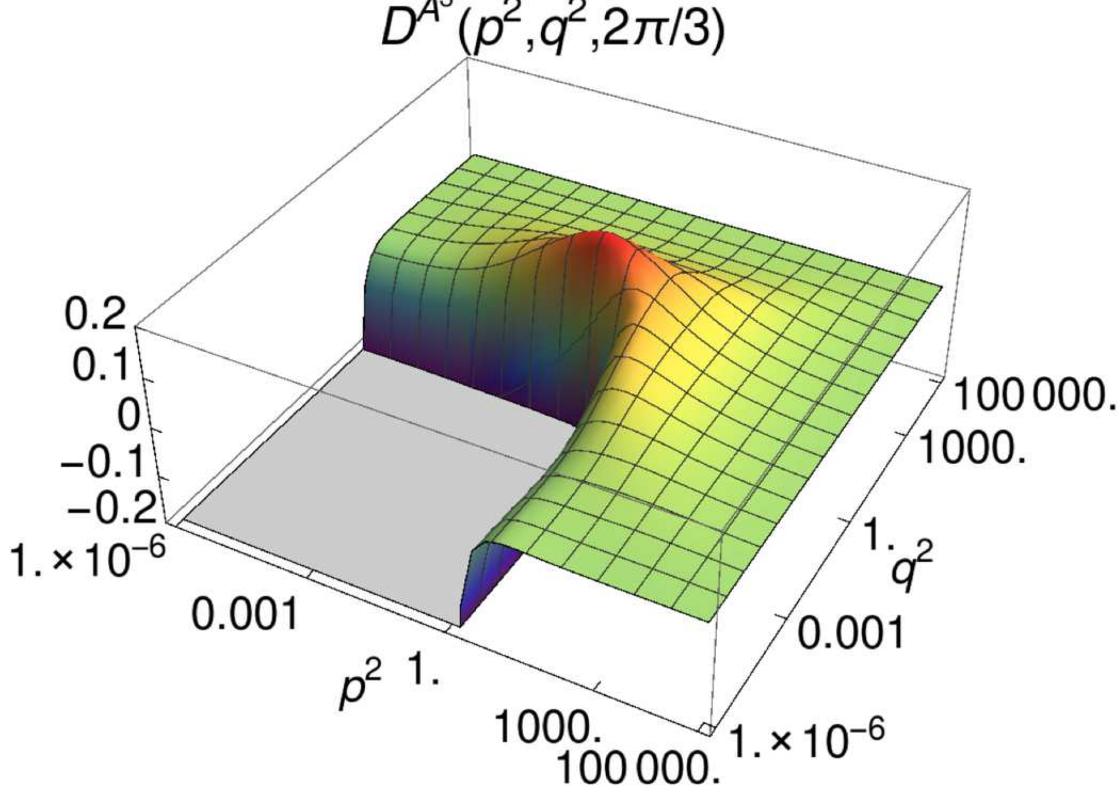




input : Gribov's formula $\omega(k)=\sqrt{k^2+\frac{M^4}{k^2}}$ $M=0.88GeV$

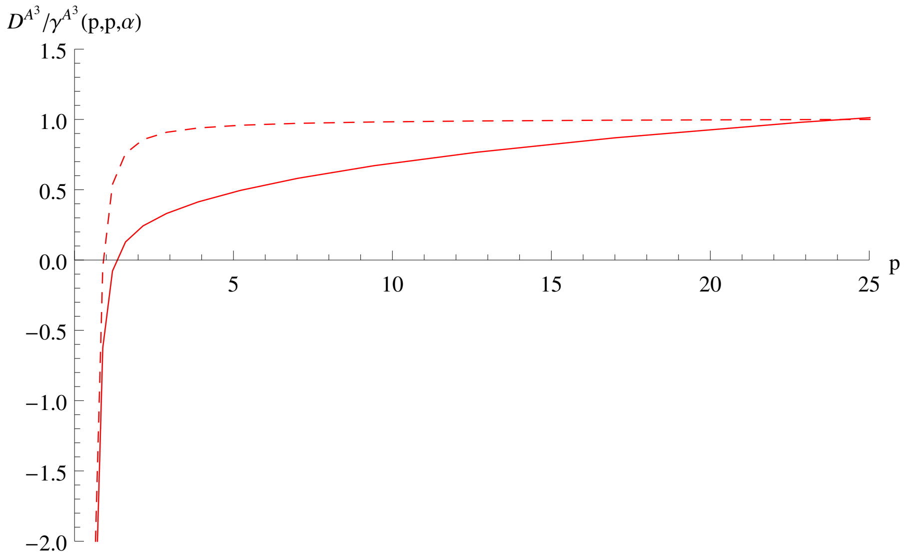
ghost form factor $d(k)-$ *from Gaussian VWF*

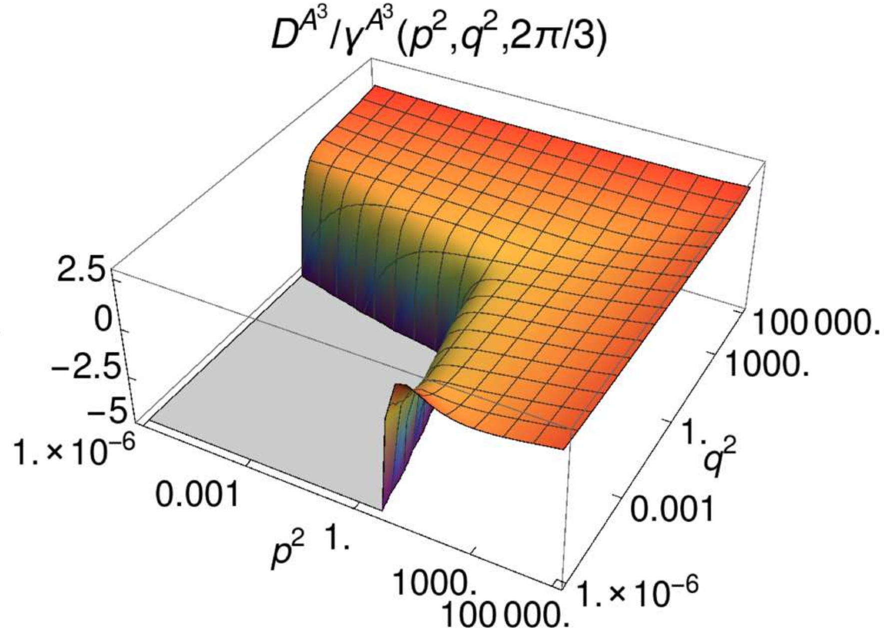
3-gluon-vertex



IR divergent

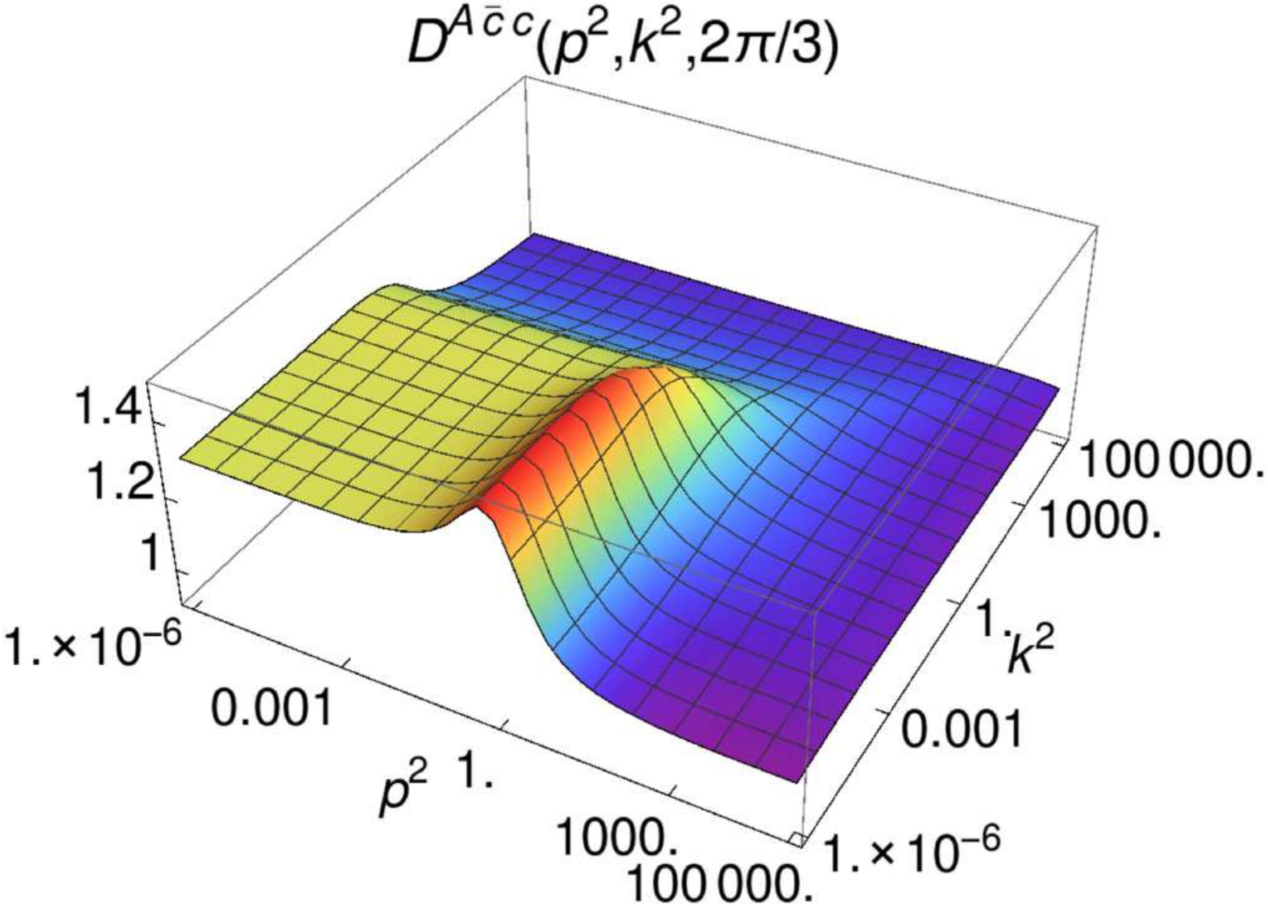
3-gluon-vertex

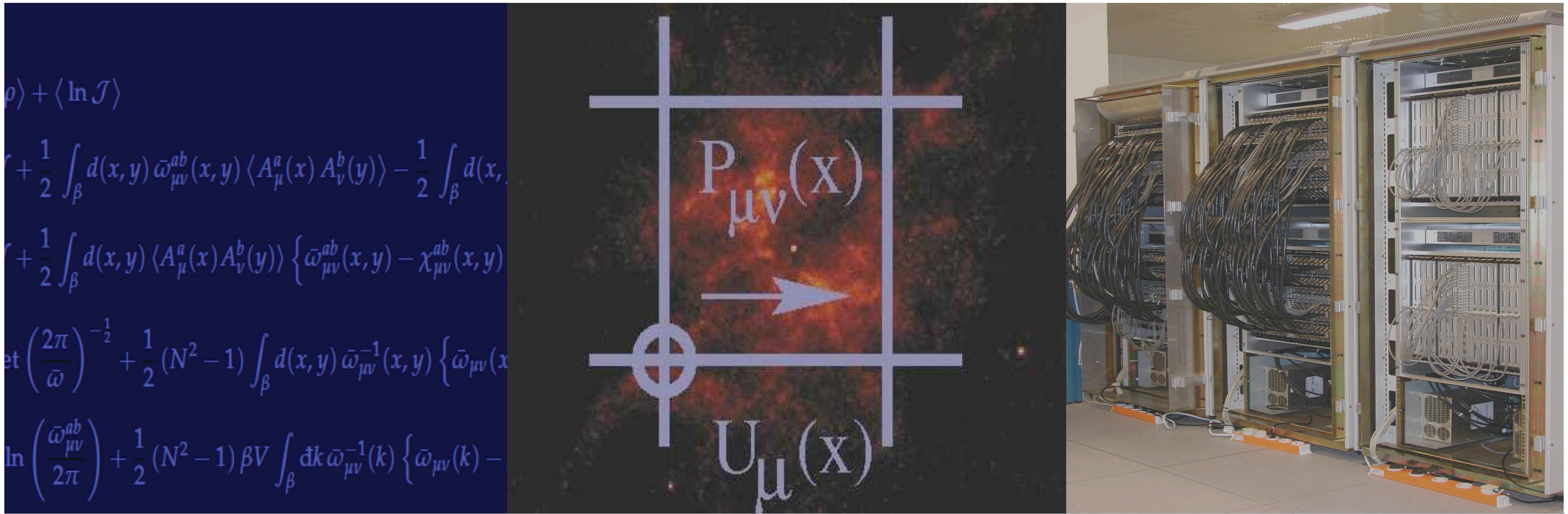




substantial deviations from the variational kernel

ghost-gluon-vertex





Results for QCD (Hamiltonian approach)

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A | \Phi \rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A}) \Psi_- \right] | 0 \rangle$$

s, v, w – variational kernels $\quad$ $\vec{\alpha}, \beta$ – *Dirac matrices*

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A | \Phi \rangle_q = \exp\left[\int \Psi_+^\dagger (\mathrm{s}\beta + \mathrm{v}\vec{\alpha}\cdot\vec{A} + \mathrm{w}\beta\vec{\alpha}\cdot\vec{A})\Psi_- \right] | 0 \rangle$$

$s, v, w -$ variational kernels $\quad \vec{\alpha}, \beta -$ *Dirac matrices*

$\mathrm{v}=\mathrm{w}=0:$ *BCS − wave function*

Finger & Mandula
Adler & Davis,
Alkofer & Amundsen

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A | \Phi \rangle_q = \exp\left[\int \Psi_+^\dagger (\mathrm{s}\beta + \mathrm{v}\vec{\alpha}\cdot\vec{A} + \mathrm{w}\beta\vec{\alpha}\cdot\vec{A})\Psi_- \right] |0\rangle$$

$s, v, w -$ variational kernels $\quad \vec{\alpha}, \beta -$ *Dirac matrices*

$\mathrm{v}=\mathrm{w}=0:$ *BCS – wave function*

Finger & Mandula
Adler & Davis,
Alkofer & Amundsen

$\mathrm{v} \neq 0, \mathrm{w}=0:$ *quark - gluon - coupling*

Pak & Reinhardt,

quark wave functional

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

s, v, w – variational kernels $\vec{\alpha}, \beta$ – *Dirac matrices*

> calculate $\langle H_{QCD}\rangle$ up to 2 loops

> variation w.r.t. **s, v, w**

$$v(p,q) = f_v[s,\omega] \qquad w(p,q) = f_w[s,\omega]$$

$$s(p) = f_s[s,v,w;p]$$

gap equation

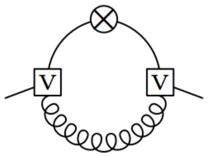
cancelation of all UV-divergencies

cancellation of UV-divergencies

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger(s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

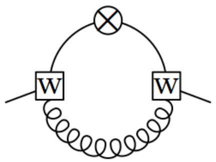
divergent loop contributions to the gap equation

> kernel **V**



$$\frac{C_F}{16\pi^2} g^2 S(k)\left[-2\Lambda + k\ln\frac{\Lambda}{\mu}\left(-\frac{2}{3} + 4P(k)\right)\right]$$

> kernel **W**



$$\frac{C_F}{16\pi^2} g^2 S(k)\left[2\Lambda + k\ln\frac{\Lambda}{\mu}\left(\frac{10}{3} - 4P(k)\right)\right]$$

> Coulomb term V_C

$$-\frac{C_F}{6\pi^2} g^2 k S(k)\ln\frac{\Lambda}{\mu}.$$

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

s, v, w – variational kernels $\quad$ $\vec{\alpha}, \beta$ – Dirac matrices

numerical calculation

D. Campagnari , E. Ebadati, H.R. and P: Vastag,
arXiv:1608.06820,PRD, in press

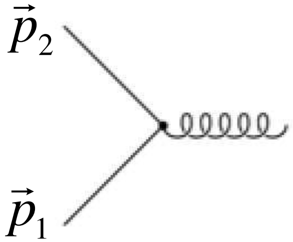
input: $\quad \omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}} \quad M = 0.88 GeV$

lattice: $\sigma_C = 2.5\sigma$

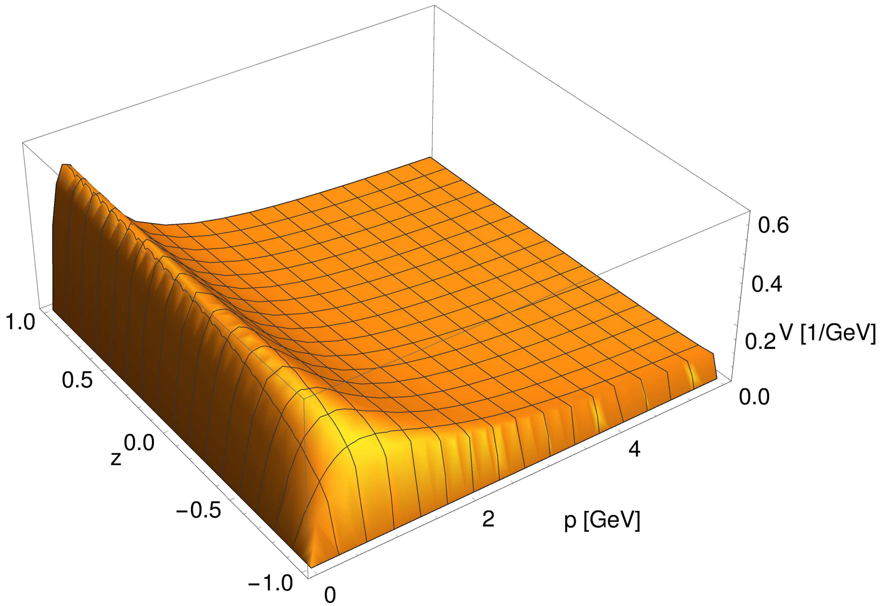
G. Burgio, M.Quandt , H.R.,
PRL102(2009)

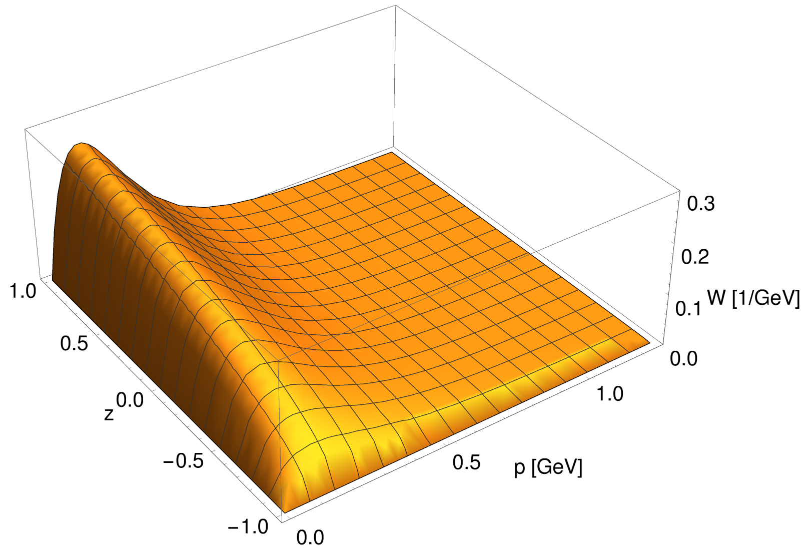
choose g to reproduce $\langle \bar{q}q \rangle = (-235\, MeV)^3 \quad \Rightarrow g \simeq 2.1$

vector form factors v , w

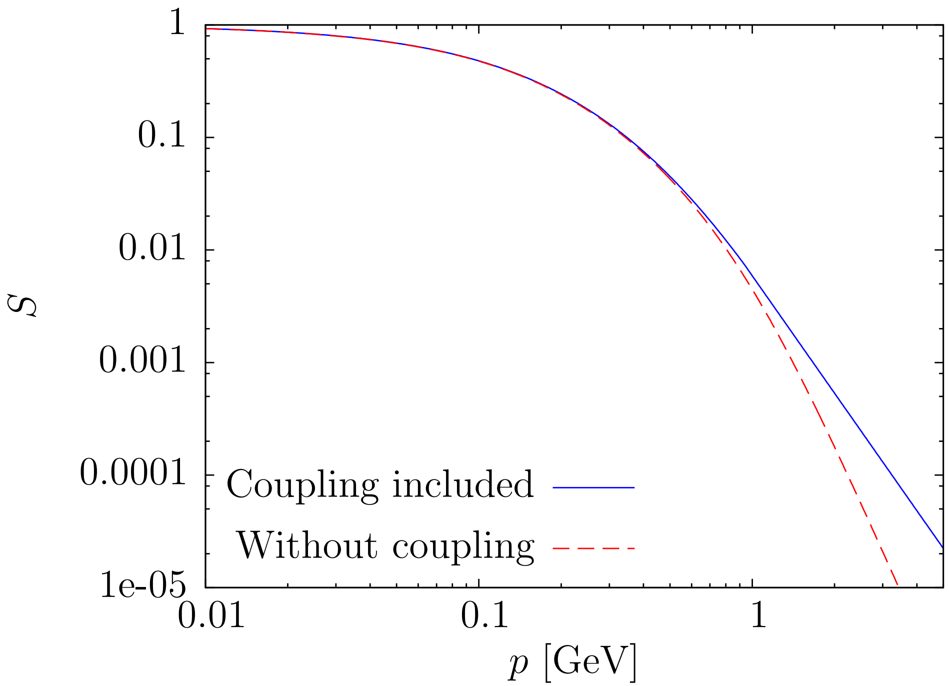


$$v, w(\vec{p}_1, \vec{p}_2): \quad p := |\vec{p}_1| = |\vec{p}_2|, \; z = \cos\sphericalangle(\vec{p}_1, \vec{p}_2)$$





scalar form factor



-quark-gluon coupling modifies only the mid- and high-momentum regime

-low-momentum regime is dominated by Coulomb term

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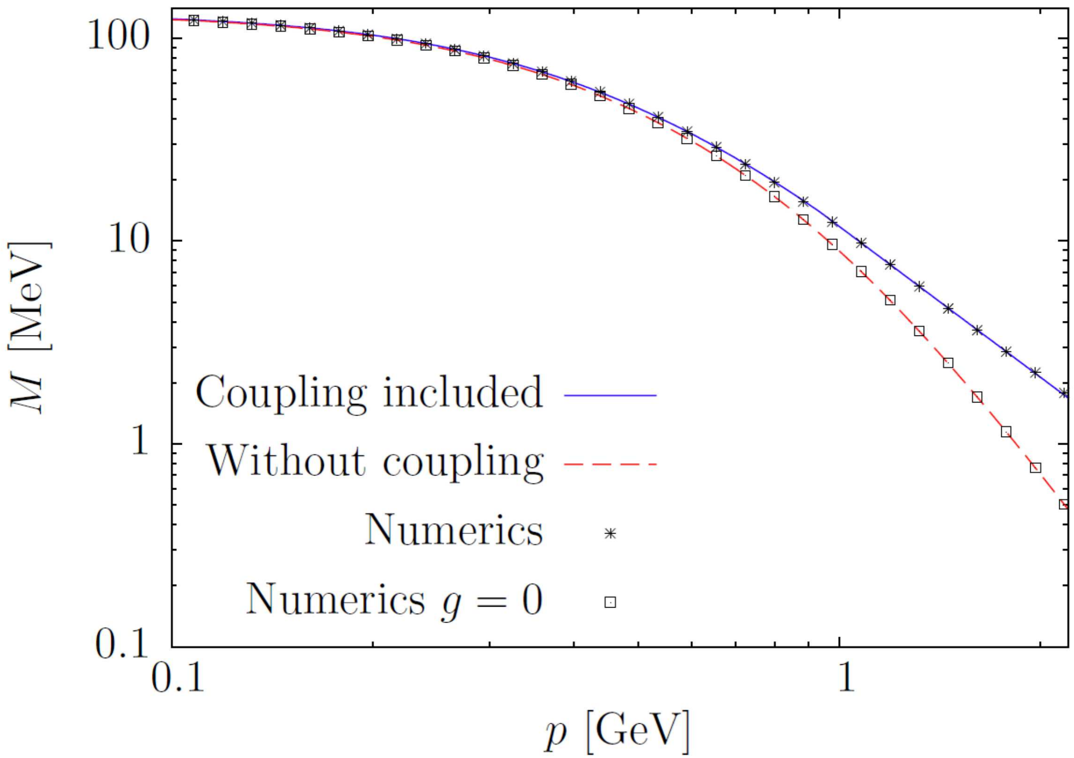
effective quark mass

D.Campagniari, E.Ebadati, H. Reinhardt, P.Vastag, PRD **94** 074027 (2016)



Quark condensate

$$\langle \bar{q}q \rangle = (-236\,\text{Mev})^3$$

$$g = 2.1$$

Adler-Davis:

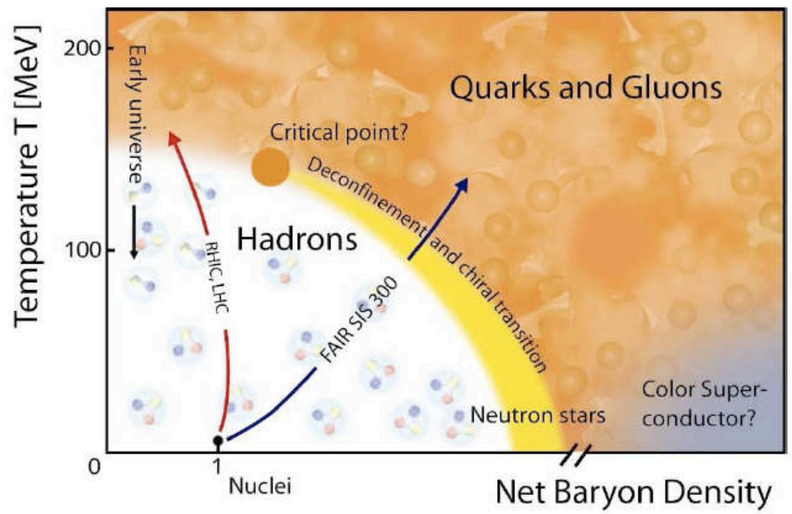
$$\langle \bar{q}q \rangle = (-185\,\text{Mev})^3$$

> coupling to transversal gluons substantially increases chiral symmetry breaking

QCD at finite temperature:
grand canonical ensemble

- quasi-particle ansatz for the density operator

- minimization of the thermodynamic potential



YM sector:

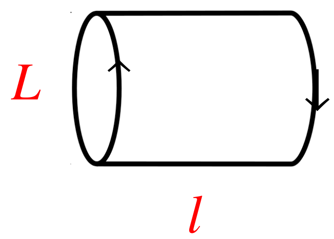
H.Reinhardt, D.Campagnari & A. Szczepaniak, Phys.Rev.D84(2011)

J.Heffner, H.Reinhardt & D.Campagnari, Phys.Rev.D85(2012)

Alternative Hamiltonian approach to finite temperature QFT

H. R. arXiv:1604.06273
Phys.Rev.D94(2016)045016

- no ansatz for the density matrix required
- temperature is introduced by compactification of a spatial dimension
- works for relativistic systems
- free energy - Casimir pressure
- Hamiltonian approach on spatial manifold

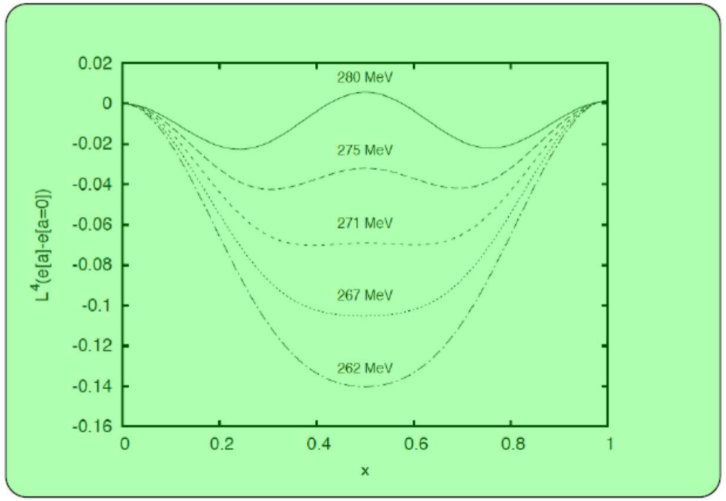


$$S^1(L) \times \mathbb{R}^3$$

- Polyakov loop available although Weil gauge $A_0 = 0$

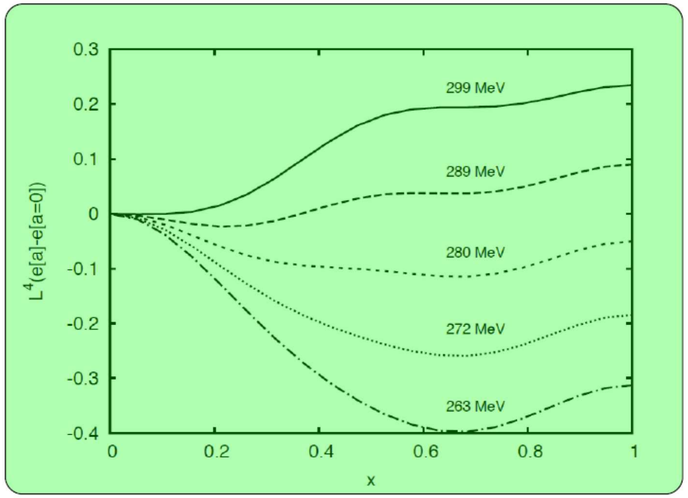
Hamiltonian Approach: Polyakov Loop

G= SU(2)



- Input: $M = 880\,\mathrm{MeV}$
- Second order
- criticality $T^* = 269\,\mathrm{MeV}$

G= SU(3)

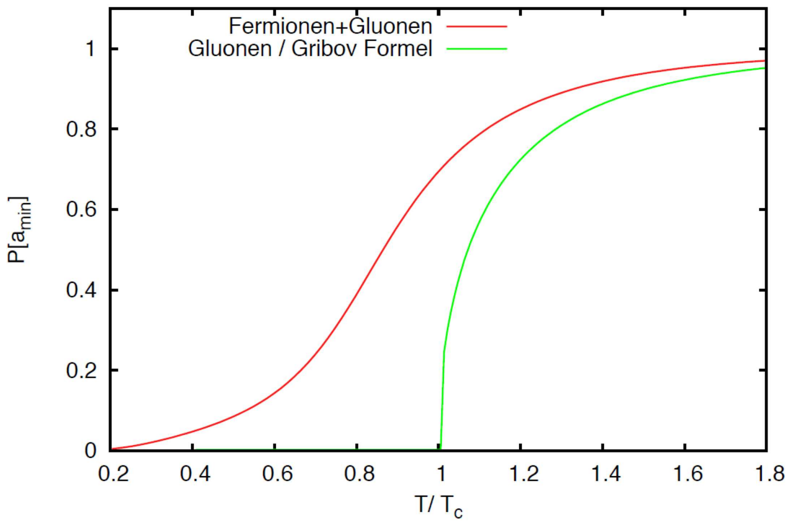


- Input: $M = 880\,\mathrm{MeV}$
- first order
- criticality $T^* = 283\,\mathrm{MeV}$

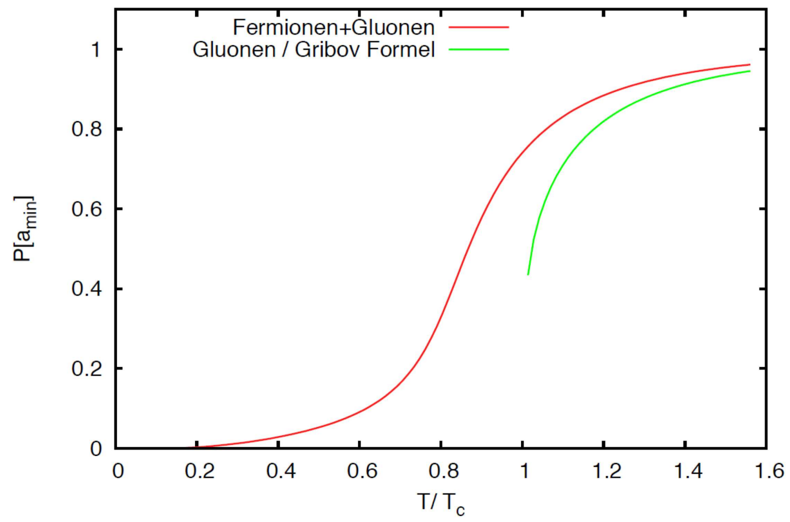
H. Reinhardt, J. Heffner, PRD **88** (2013)

Polyakov loop with dynamical fermions

J. Heffner, H.Reinhardt, P.Vastag, to be published



G=SU(2)



G=SU(3)

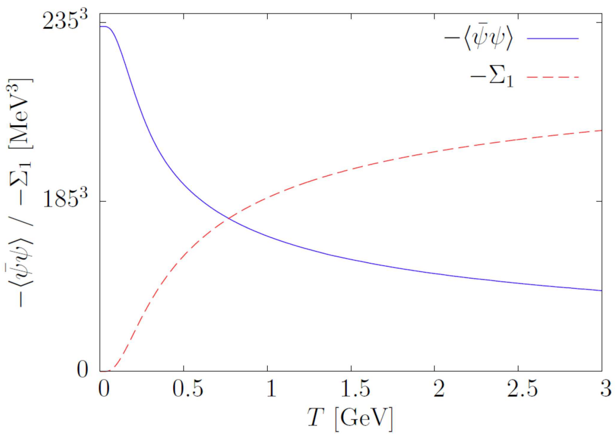
→ Deconfinement phase transition becomes cross-over at smaller T

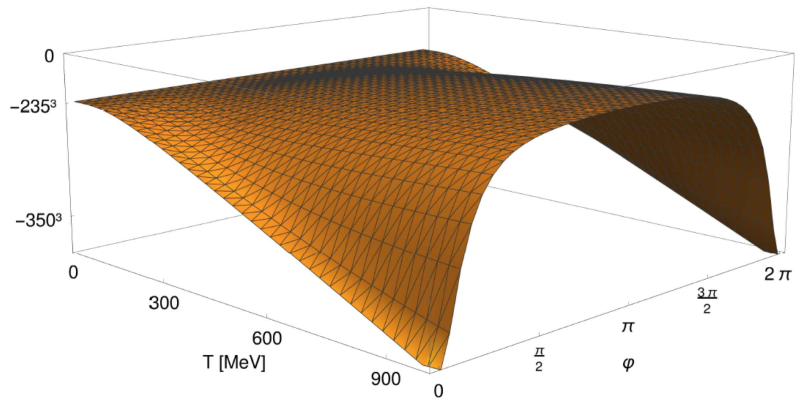
Chiral and dual condensate

Gattringer, PRL **97** (2006)

$$\Sigma_n \equiv \int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{in\varphi} \left\langle (\bar{q}q)_\varphi \right\rangle \qquad q(\beta) = e^{i\varphi}\, q(0)$$

- Loops winding n-times around the compactified time
- Σ_1 dressed Polyakov loop
- Imaginary chemical potential $\mu = i\,\frac{\pi - \varphi}{\beta}$





D.Campagniari, E.Ebadati, H. Reinhardt, P.Vastag, PRD **94** 074027 (2016)

chiral & dual condensate

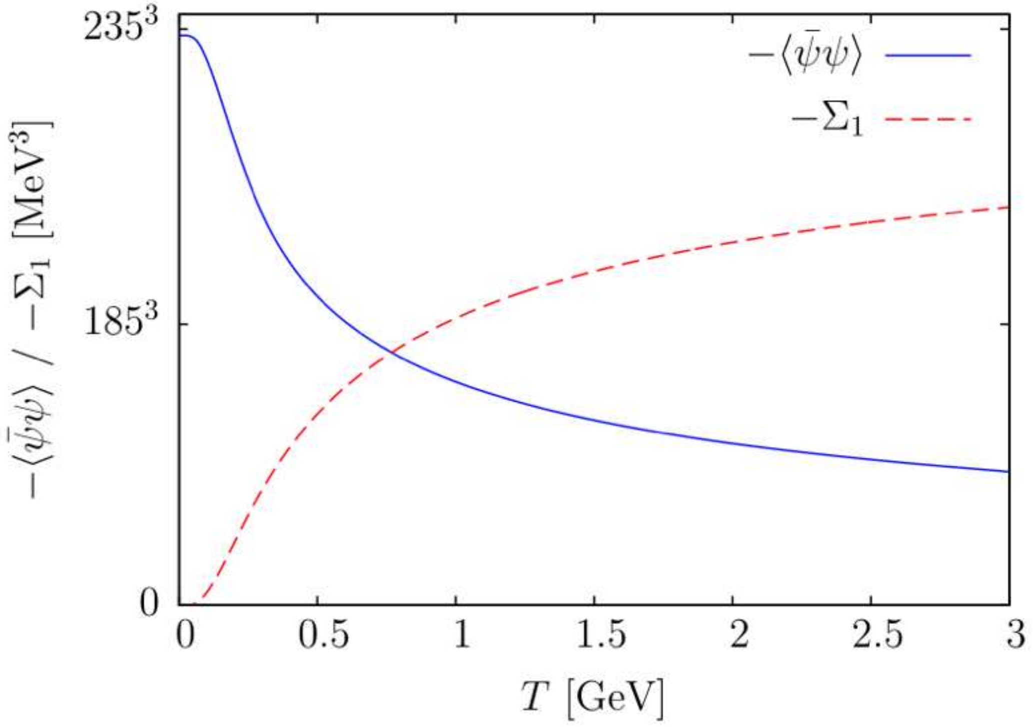


FIG. 7. Chiral (full curve) compared to the dual quark condensate (dashed curve) as a function of the temperature for $SU(3)$ for a quark-gluon coupling constant of $g \simeq 2.1$.

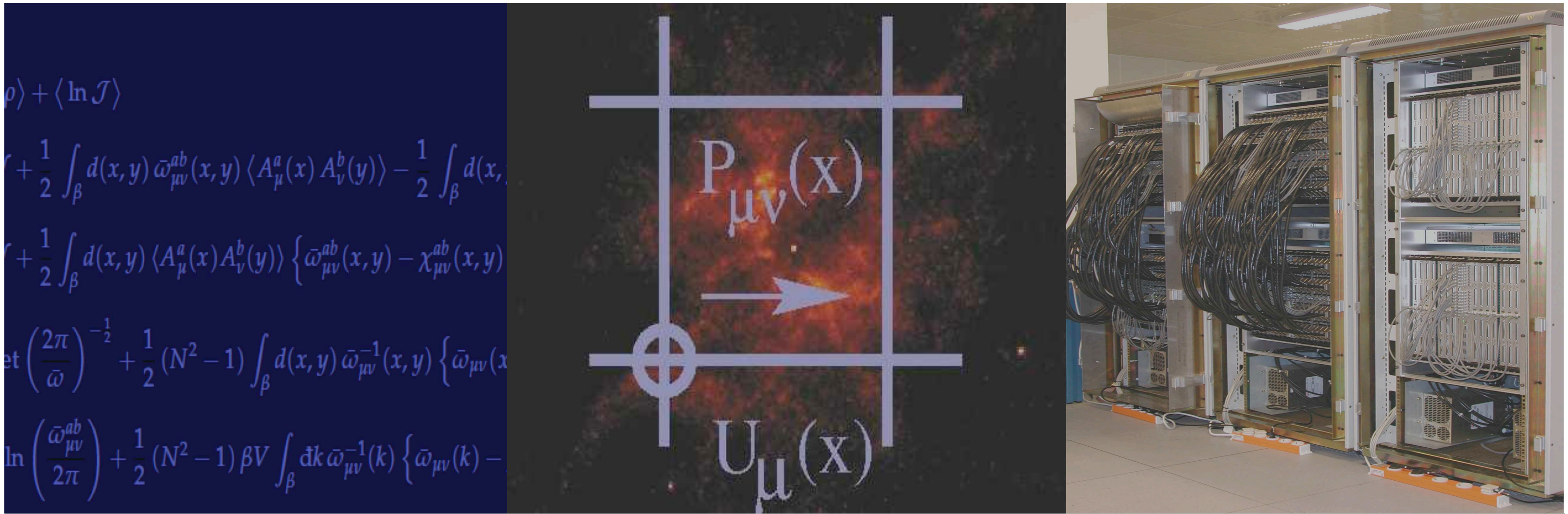
$\sigma_C = 2.5\sigma$

$T_\chi \simeq 170\,MeV$

$lattice: 155\,MeV$

$T_C \simeq 198\,MeV$

$lattice: 165\,MeV$



Sumary and Outlook

Summary

- Covariant variational approach (in Landau gauge)
  - optimized DSE approach
  - Gaussian trial action (DSE not needed):
    - decent description of propagators at T=0 and T>0
    - effective potential of Polyakov loop
    - pressure & energy density
- Hamiltonian variational approach
  - 3-gluon+ghost-gluon vertex
  - chiral symmetry breaking
  - deconfinement transition:Polyakov loop, dual condensate

Outlook

- beyond the Gaussian ansatz (covariant approach)
- Fermions with *real* chemical potentials

Thank you