Landau-gauge Yang-Mills correlation functions from the functional renormalization group

Mario Mitter

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APC, Paris Diderot University, November 2017

fQCD collaboration - QCD (phase diagram) with FRG:

J. Braun, L. Corell, A. K. Cyrol, W. J. Fu, M. Leonhardt, MM,
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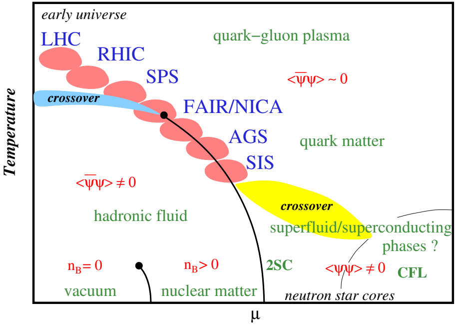

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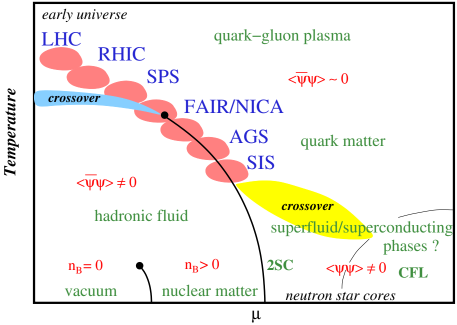

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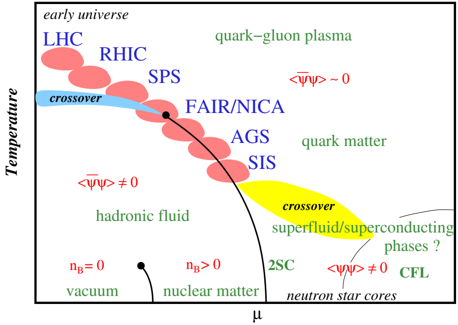

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why?

QCD from the effective action (gauge fixing necessary)

$$\Gamma[\Phi] = \sum_n \int_{\{p_i\}} \Gamma^{(n)}_{\Phi_1 \cdots \Phi_n}(p_1, \ldots, p_{n-1}) \Phi^1(p_1) \cdots \Phi^n(-p_1 - \cdots - p_{n-1})$$

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  - form factors: photon-particle correlators

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  - thermodynamic quantities: $\Gamma[\Phi] \propto$ grand potential
    - ⋆ equation of state e.g. [Herbst, MM, Pawlowski, Schaefer, Stiele, '13]
    - ⋆ fluctuations of conserved charges e.g. [Fu, Rennecke, Pawlowski, Schaefer, '16]

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  - further quantities: $\Gamma[\Phi] \propto$ eff. potential, propagators, 't Hooft determinant
    - ⋆ chiral condensate(s)/$\langle\sigma\rangle$

      e.g. [Schaefer, Wambach '04], [Fischer, Luecker, Mueller '11], [MM, Schaefer, '13]
    - ⋆ (dressed) Polyakov loop

      e.g. [Fischer, '09], [Braun, Haas, Marhauser, Pawlowski, '09], [MM, et al., '17]
    - ⋆ axial anomaly e.g.[Grahl, Rischke, '13], [MM, Schaefer, '13], [Fejos, '15], [Heller, MM, '15]
    - ⋆ spectral functions e.g. [Tripolt, Strodthoff, Smekal, Wambach, '14]

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crawling towards QCD at finite density:

- quenched matter part [MM, Strodthoff, Pawlowski, 2014]
- pure $SU(N)$ YM-theory [Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]
- $N_f = 2$ QCD [Cyrol, MM, Strodthoff, Pawlowski, 2017]
- YM-theory at finite temperature $T > 0$ [Cyrol, MM, Strodthoff, Pawlowski, 2017]

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- use results from lattice gauge theory to check truncation:

  what do we need from the lattice?

Wetterich Equation

- mass-like IR regulator:
  - $S[\Phi] \to S[\Phi] + \langle \Phi, R_k \Phi \rangle$
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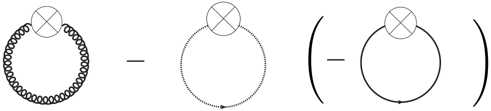
- $\partial_k \Leftrightarrow$ integration of momentum shells: [Wetterich '93]

$$\partial_k \Gamma_k[A, \bar{c}, c\,(, \bar{q}, q)] = \frac{1}{2}$$



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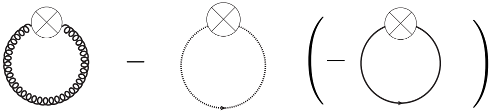

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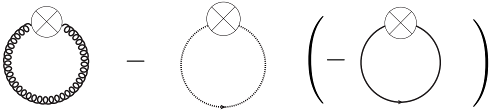

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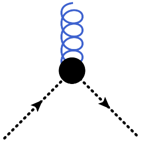
- Landau gauge:
  - ghosts appear
  - gauge symmetry $\to$ BRST-symmetry

Truncation for $SU(N)$ YM-theory [Cyrol, Fister, MM, Pawlowski, Strodthoff, '16]

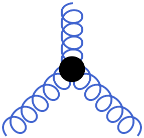
classical tensors with momentum dependent-dressings:

$1/Z_A(p)$

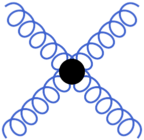
$1/Z_c(p)$



$\lambda_{\bar{c}cA}(p,q,z)$



$\lambda_{A^3}(p,q,z)$



$\lambda_{A^4}(\bar{p})/\lambda_{A^4}(p,q,z)$

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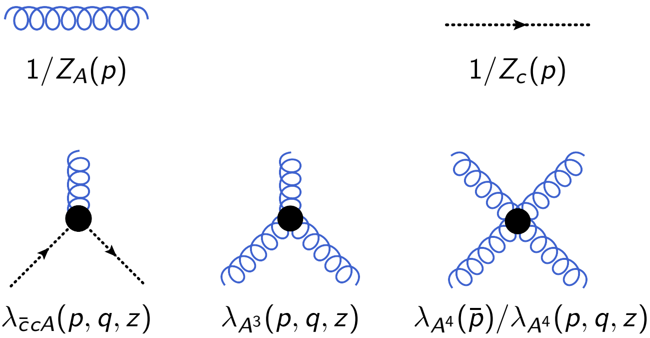

aim for "apparent convergence" of $\Gamma[\Phi] = \lim_{k\to 0} \Gamma_k[\Phi]$

(Euclidean) Correlation functions with the FRG

[Cyrol, Fister, MM, Pawlowski, Strodthoff, '16]

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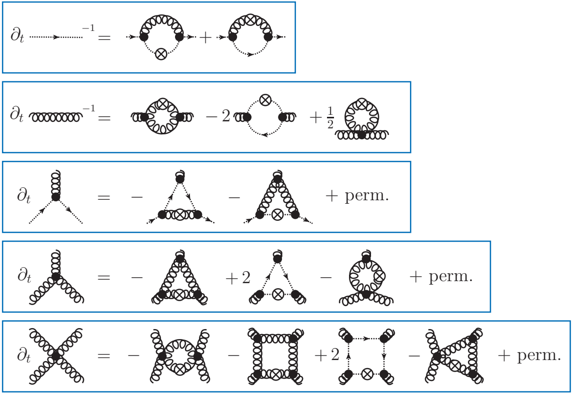


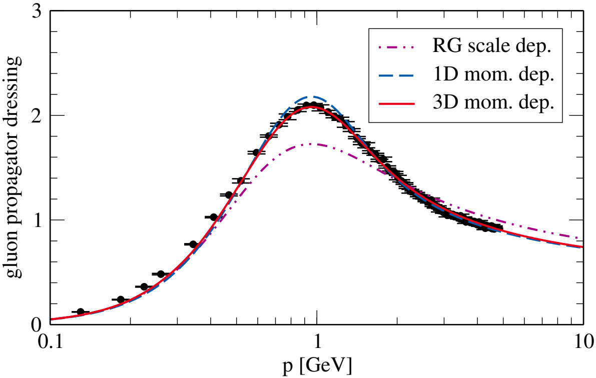
- set of coupled equations: cf. DoFun [Huber, Braun, '11], FormTracer [Cyrol, MM, Strodthoff, '16]

  all propagators/vertices dressed and momentum-dependent

Intermission: “apparent convergence”

[Cyrol, Fister, MM, Pawlowski, Strodthoff, ’16]

- different approximations for vertex dressing functions:



- RG scale dep.: $\lambda_X(k;p,q,z) \equiv \lambda_X(k)$
- 1D mom dep.: $\lambda_X(k;p,q,z) \equiv \lambda_X(k;\bar{p})$
- 3D mom dep.: $\lambda_{\bar{c}cA/A^3}(k;p,q,z)$ and $\lambda_{A^4}(k;\bar{p})/\lambda_{A^4}(k;p,q,z)$

Landau gauge, BRST symmetry and the Functional RG I

- Landau gauge: gauge symmetry $\rightarrow$ BRST-symmetry:

$$s\Phi = s(A_\mu, \bar{c}, c) = (D_\mu c, 0, igc^2)$$

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  - massless (longitudinal) gluon
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- perturbation theory with e.g. sharp cutoff, ...
  - mass counter term for gluon required
  - nontrivial counterterms for vertices to recover degenerate α's

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[Ellwanger '94]

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- e.g. QED: photon mass term fixed by massless photon at $k \to 0$

Vertex counter terms

[Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]

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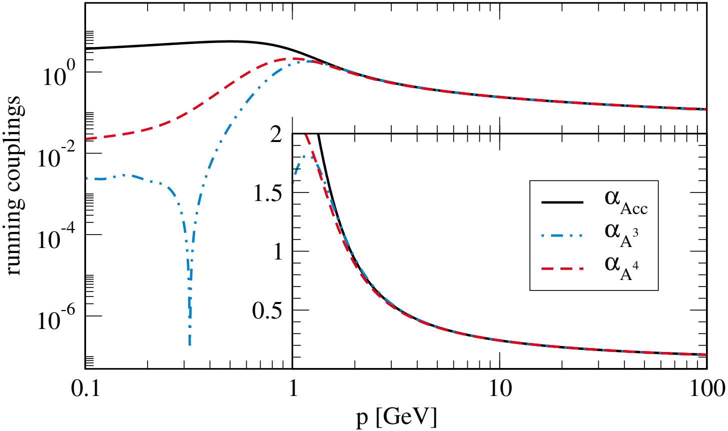

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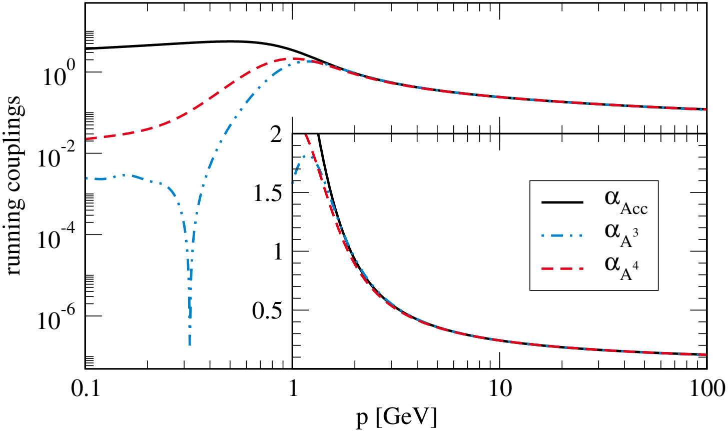

- degeneracy @ $p \neq \mu$: nontrivial check of STI/BRST-symmetry

Gluon mass term

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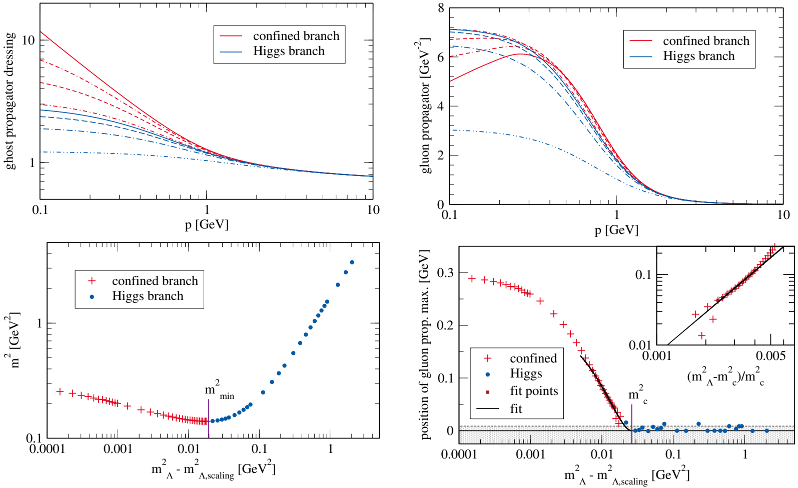

Mass gap and irregularities

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- regular vertices $\Rightarrow$ no gluon mass gap @ $\Phi = 0$

cf. [Cornwall '82]

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$\Longrightarrow \exists$ *irregular vertices* or *STIs not valid @ small* p

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[Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]

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- my guess: "spont. breaking" $\Phi \neq 0$ but scaling fixes m_Λ^2

cf. $T > 0$ talk of Jan

Numerical propagators

[Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]

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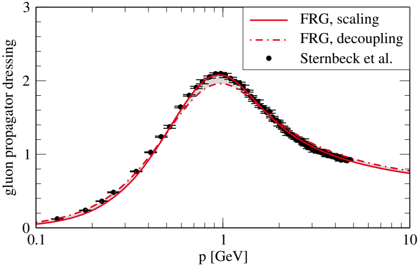

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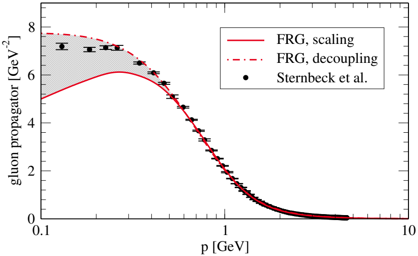

lattice data: A. Sternbeck, E. M. Ilgenfritz, M. Muller-Preussker, A. Schiller, and I. L. Bogolubsky, PoS LAT2006, 076.

Numerical correlators I

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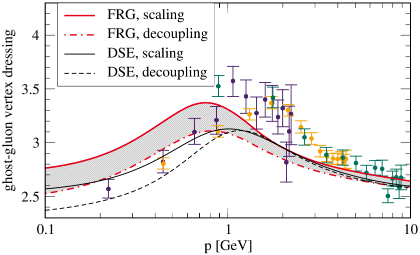
- $\Gamma^{(3)}_{A\bar{c}c}(\bar{p})$



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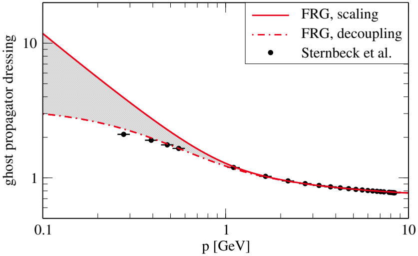

vertex lattice data: [A. Cucchieri, A. Maas, T. Mendes '06 '08], [A. Maas, in preparation]

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DSE data: [M. Huber, L. v. Smekal, '13], [M. Huber, private communications]

Numerical correlators II

[Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]

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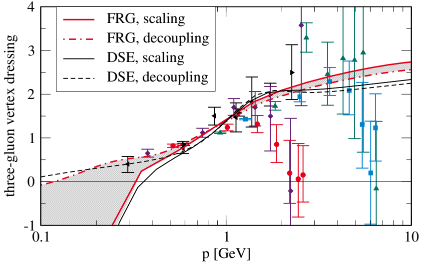
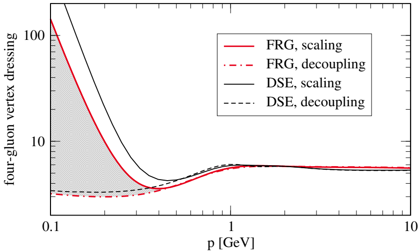


- $\Gamma^{(4)}_{AAAA}(\bar{p})$



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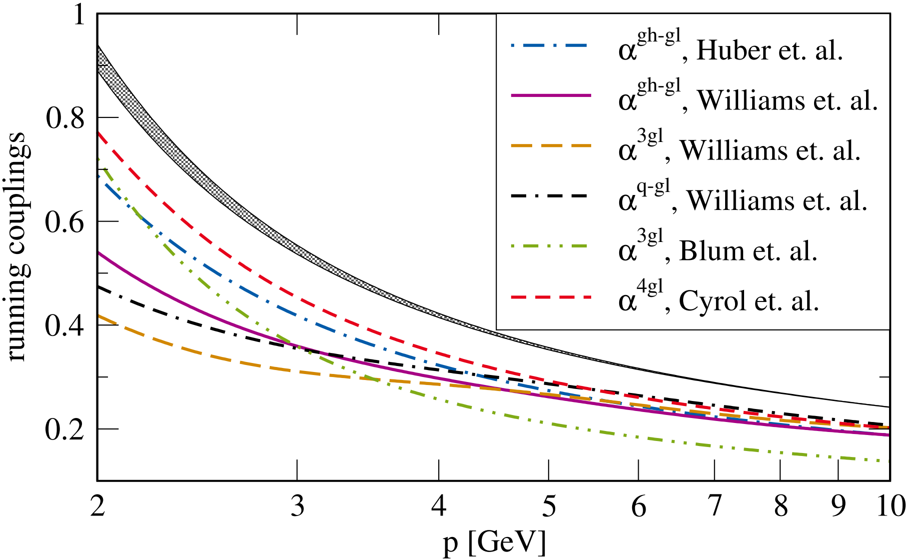
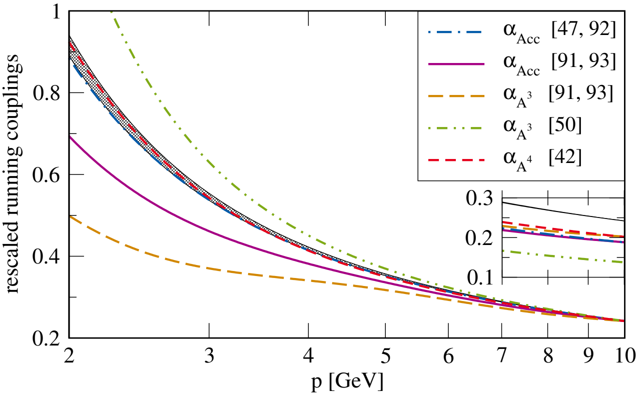
Running couplings: FRG vs. DSE [Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]



Running couplings: FRG vs. DSE (rescaled)

[Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]



[42] [A. Cyrol, M. Huber, L. v. Smekal, '15],

[47] [M. Huber, L. v. Smekal, '13],, [92] [M. Huber, private communications]

[50] [A. Blum, M. Huber, MM, L. v. Smekal, '14]

[91] [R. Williams, '14], [93] [R. Williams, private communications]

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- further applications:
  - QCD phase structure
  - other strongly-interacting theories