

Results for Yang-Mills vacuum correlation functions in the Landau gauge from equations of motion



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Infrared QCD Workshop, Paris, France

Nov. 8, 2017



Motivation: Where Yang-Mills theory is important in QCD

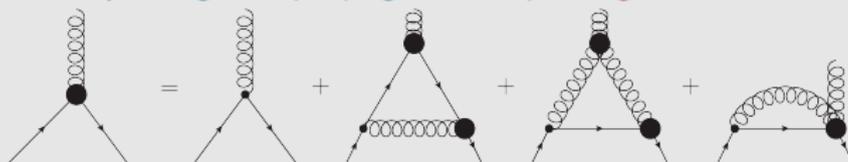
Widely used truncation: Rainbow + ladder + variant of Maris-Tandy interaction

The top equation shows the Dyson-Schwinger equation for the quark-gluon vertex $\Gamma_{[a]}(q, P)$. It is equal to the tree-level vertex $\Gamma_{[a]}(k, P)$ plus a loop diagram where a quark line with momentum k_+ and a quark line with momentum k_- are connected by a gluon loop with momentum q . The loop is enclosed in a shaded box representing the kernel $K(k, q, P)$.

The bottom equation shows the Dyson-Schwinger equation for the inverse quark propagator $S(p)^{-1}$. It is equal to the tree-level propagator $S_0(p)^{-1}$ plus a loop diagram where a quark line with momentum p and a quark line with momentum q are connected by a gluon loop with momentum $p-q$. The loop is enclosed in a shaded box representing the kernel $K(p, q, p)$.

How to reduce model dependence

- Improve kernel K
- Use explicit **gluon propagator** + **quark-gluon vertex**



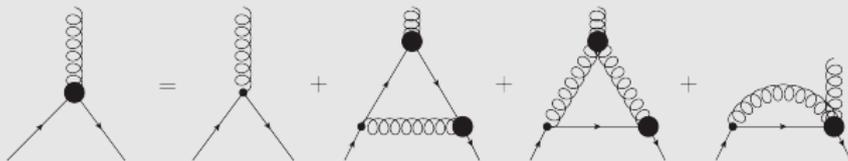
Motivation: Where Yang-Mills theory is important in QCD

Widely used truncation: Rainbow + ladder + variant of Maris-Tandy interaction

The diagram shows two equations. The first equation is for the quark self-energy $\Gamma_{[q]}(q, P)$, which is equal to a sum of a rainbow diagram (a quark line with a gluon loop) and a ladder diagram (a quark line with a gluon exchange between two quark lines). The kernel $K(k, q, P)$ is shown in a shaded box. The second equation is for the inverse quark propagator $S(p)^{-1}$, which is equal to the inverse of the bare propagator $S_0(p)^{-1}$ plus a diagram representing the Maris-Tandy interaction: a quark line with a gluon loop and a ghost loop, with vertices γ_μ and $\Gamma_\mu(p, q)$.

How to reduce model dependence

- Improve kernel K
- Use explicit **gluon propagator** + **quark-gluon vertex**



→ We need full control over the gluonic sector for self-contained calculations.

- Gluon propagator
- Three-gluon vertex
- ...?
- **Glueballs**

Overview

- Some details on Dyson-Schwinger equations:
 - Renormalization
 - Resummed perturbation theory
- Testing truncations:
 - **Hierarchy** of diagrams (testing in 3 dimensions)
 - **Extensions** of truncations in 4 dimensions:
 - Two-loop terms
 - Non-primitively divergent correlation functions

Landau gauge QCD

$$\mathcal{L} = \bar{\mathbf{q}}(-\not{D} + m)\mathbf{q} + \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

$$F_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + i g [\mathbf{A}_\mu, \mathbf{A}_\nu]$$



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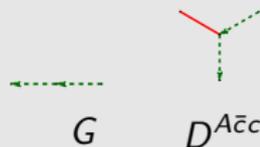


Landau gauge

- simplest one for functional equations

- $\partial_\mu \mathbf{A}_\mu = 0$: $\mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu \mathbf{A}_\mu)^2$, $\xi \rightarrow 0$

- requires ghost fields: $\mathcal{L}_{gh} = \bar{\mathbf{c}}(-\square + g \mathbf{A} \times) \mathbf{c}$

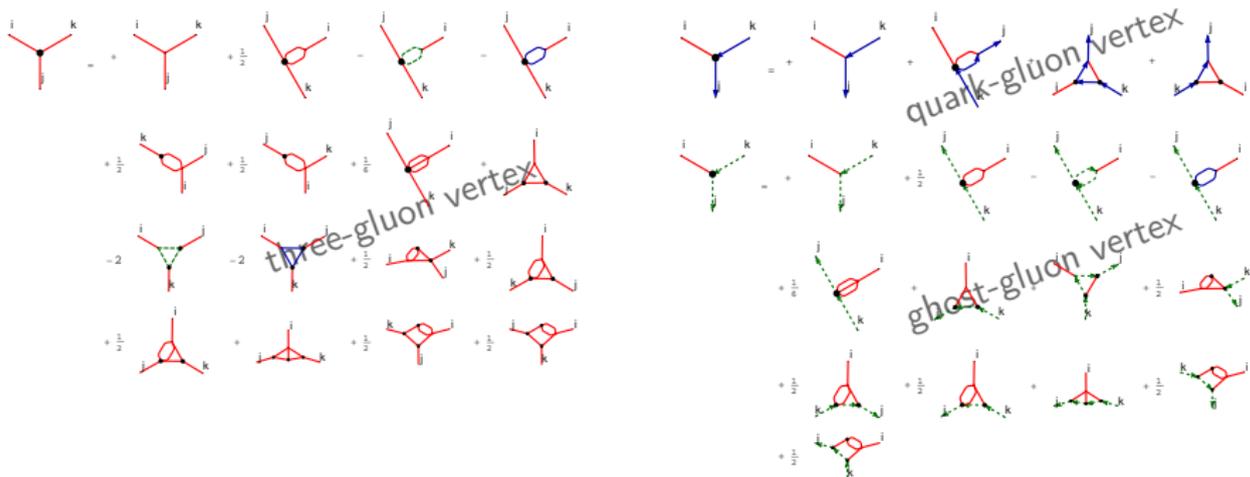


The tower of DSEs

$$\begin{aligned}
 & \text{red line with dot} \stackrel{-1}{=} + \text{red line} \stackrel{-1}{=} - \frac{1}{2} \text{red line with self-energy loop} - \frac{1}{2} \text{red line with ghost loop} + \text{red line with ghost loop} \\
 & \quad + \text{red line with gluon loop} - \frac{1}{6} \text{red line with gluon loop} - \frac{1}{2} \text{red line with ghost loop} \quad \text{gluon propagator} \\
 & \text{dashed green line with dot} \stackrel{-1}{=} + \text{dashed green line} \stackrel{-1}{=} - \text{dashed green line with self-energy loop} \quad \text{ghost propagator} \\
 & \text{blue line with dot} \stackrel{-1}{=} + \text{blue line} \stackrel{-1}{=} - \text{blue line with self-energy loop} \quad \text{quark propagator}
 \end{aligned}$$

The tower of DSEs

$$\begin{aligned}
 \text{quark propagator} &= \text{tree} + \text{self-energy} - \frac{1}{2} \text{gluon loop} - \frac{1}{2} \text{ghost loop} + \text{ghost-gluon loop} \\
 \text{ghost propagator} &= \text{tree} - \text{ghost loop} \\
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Infinitely many equations. In QCD, every n -point function depends on $(n + 1)$ - and possibly $(n + 2)$ -point functions.



The tower of DSEs

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Infinitely many equations. In QCD, every n -point function depends on $(n + 1)$ - and possibly $(n + 2)$ -point functions.

Is it possible to find and solve a truncation with all **relevant contributions**?



Questions about truncations

- Influence of higher correlation functions?

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- Influence of **higher correlation functions**?
- Hierarchy of diagrams/correlation functions?
- **Model** dependence \leftrightarrow **Self-contained** truncation?
- How to realize **resummation**?
- Equivalence between different functional methods?

UV behavior of the gluon propagator

Resummed **one-loop** order: anomalous dimension $\gamma = -13/22$

One-loop truncation:

$$\text{Gluon Propagator}^{-1} = \left(\text{Gluon Propagator}^{-1} - \frac{1}{2} \text{Gluon Loop}^{-1} - \frac{1}{2} \text{Ghost Loop}^{-1} - \frac{1}{2} \text{Ghost Loop}^{-1} \right)$$

UV behavior of the gluon propagator

Resummed **one-loop** order: anomalous dimension $\gamma = -13/22$

One-loop truncation:

$$\text{Gluon line with vertex}^{-1} = \text{Gluon line with vertex}^{-1} \left[\text{ghost loop}^{-1/2} + \text{gluon loop}^{-1/2} + \text{ghost-gluon loop}^{-1/2} \right]$$

Self-consistent solution puts constraints on UV behavior of vertices [von Smekal, Hauck, Alkofer '97]:

- Ghost-gluon vertex: $\sim \text{const.} \rightarrow \checkmark$

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Self-consistent solution puts constraints on UV behavior of vertices [von Smekal, Hauck, Alkofer '97]:

- Ghost-gluon vertex: $\sim \text{const.} \rightarrow \checkmark$
- Three-gluon vertex: $\propto (\log p)^{17/22}$
Anomalous dimension $\gamma_{3g} = 17/44 \rightarrow \ominus$

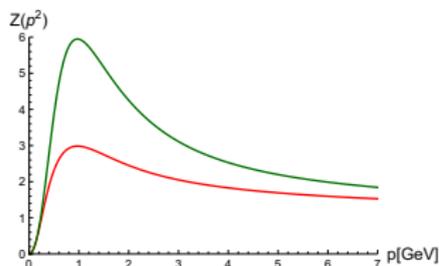
Solutions: $Z_1 \rightarrow Z_1(p^2) \leftrightarrow$ modified three-gluon vertex model [von Smekal, Hauck, Alkofer '97; Fischer, Alkofer '02]

Truncation artifact!

Resummed behavior

- Resolving the UV behavior within this truncation leads to an additional parameter dependence \rightarrow part of the **model**

Extreme example:



- Study of such effects for three-gluon vertex:
[Eichmann, Williams, Alkofer, Vujanovic '14]
- However, **correct UV behavior** is required for **self-consistency**.

\rightarrow How to get resummed one-loop behavior?

One-loop resummation

One-loop **anomalous dimension**

Origin in resummation of higher order diagrams.

$$\left(1 + \frac{\alpha(s)11N_c}{12\pi} \ln \frac{p^2}{s}\right)^\gamma$$

One-loop resummation

One-loop **anomalous dimension**

Origin in resummation of higher order diagrams.

$$\left(1 + \frac{\alpha(s)11N_c}{12\pi} \ln \frac{p^2}{s}\right)^\gamma = 1 + c_1 g^2 \ln p^2 + c_2 g^4 \ln^2 p^2 + \mathcal{O}(g^6)$$

- $\mathcal{O}(g^2)$: One-loop diagrams
- $\mathcal{O}(g^4)$: Iterated one-loop diagrams, **squint** (*not* sunset)

Resummed behavior

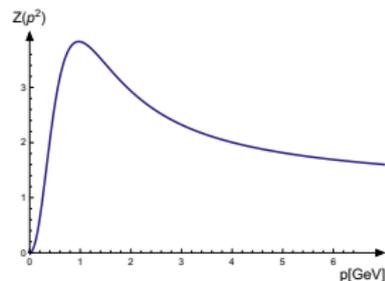
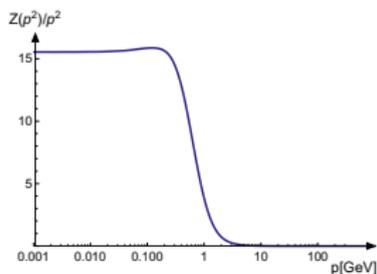
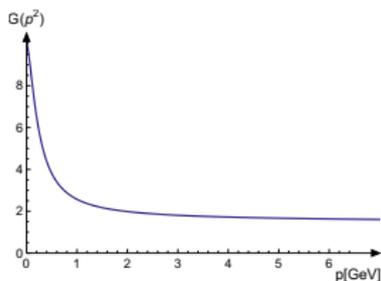
Minimal requirements to obtain one-loop resummed behavior:

- Squint diagram
- Correct anomalous dimensions of three-point functions
- Correct renormalization (constants)

Resummed behavior

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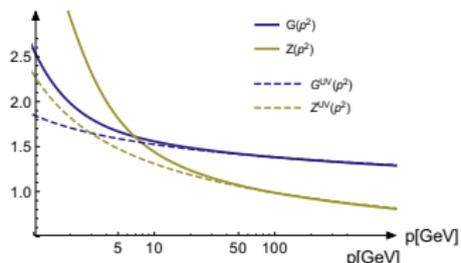
- Squint diagram
- Correct anomalous dimensions of three-point functions
- Correct renormalization (constants)



[propagator+ghost-gluon eqs. full, 3-gluon vertex model, bare 4-gluon vertex]

- Resummed behavior is recovered

[MQH, EPJC (2017)].



Renormalization of gluon propagator ($d=4$)

- 1 'Physics': Logarithmic divergences handled by subtraction at p_0 .
- 2 Breaking of gauge covariance by cutoff regularization (also in perturbation theory):
Quadratic divergences subtracted, coefficient C_{sub} .

$$Z(p^2)^{-1} := Z_\Lambda(p^2)^{-1} - C_{\text{sub}}(\Lambda) \left(\frac{1}{p^2} - \frac{1}{p_0^2} \right)$$

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One-loop diagrams with model vertices: C_{sub} can be calculated analytically, since it is **purely perturbative** [MQH, von Smekal '14].

Dynamic vertices? Two-loop diagrams?

left-hand side

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~~Quadratic~~ **Linear and logarithmic** divergences subtracted.

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Calculation of C_{sub} in $d = 4$

Example: Ghost loop

$$I_{gh}^{spur}(p^2) \propto \frac{1}{p^2} \int_{p_0^2}^{\Lambda^2} dq^2 G_{UV}^2(q^2)$$

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What about the finite part [MQH, von Smekal '14]?

- Perturbatively: **no mass term** generated?
- Independent of UV parametrization.
- Dim. reg. calculation yields the same finite part.

Calculation of C_{sub} in $d = 3$

Example: Ghost loop

Note: $[g] = [mass]^{\frac{1}{2}}$

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Form of spurious divergences (analytic):

$$C_{sub} = a \Lambda + b \ln \Lambda$$

Dynamic vertices? Two-loop diagrams?

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Dynamic vertices? Two-loop diagrams?

→ Fit possible, since the functional form is the same [MQH '16].

Testing truncations in $d = 3$:
Vary equations and systems of equations.

$d = 3$ Yang-Mills theory as testing ground

Advantages:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle
- UV behavior 'easier': $\propto \frac{g^2}{p}$ instead of resummed logarithm

→ Many complications from $d = 4$ absent.

→ Disentanglement of UV easier.

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Historically interesting because cheaper on the lattice → easier to reach the IR.

Numerically not cheaper for functional equations of 2- and 3-point functions.

Continuum results:

- Coupled propagator DSEs: [Maas, Wambach, Grüter, Alkofer '04]
- (R)GZ: [Dudal, Gracey, Sorella, Vandersickel, Verschelde '08]
- DSEs of PT-BFM: [Aguilar, Binosi, Papavassiliou '10]
- YM + mass term: [Tissier, Wschebor '10, '11]

Dyson-Schwinger equations: Truncation

Untruncated propagators:

$$\text{---} \bullet \text{---}^{-1} = \text{---} \text{---}^{-1} - \text{---} \text{---} \text{---}^{-1}$$

$$\text{---} \bullet \text{---}^{-1} = \text{---} \text{---}^{-1} - \frac{1}{2} \text{---} \text{---} \text{---}^{-1} - \frac{1}{2} \text{---} \text{---} \text{---}^{-1} + \text{---} \text{---} \text{---}^{-1} - \frac{1}{4} \text{---} \text{---} \text{---}^{-1} - \frac{1}{2} \text{---} \text{---} \text{---}^{-1} - \frac{1}{2} \text{---} \text{---} \text{---}^{-1}$$

Truncated vertices:

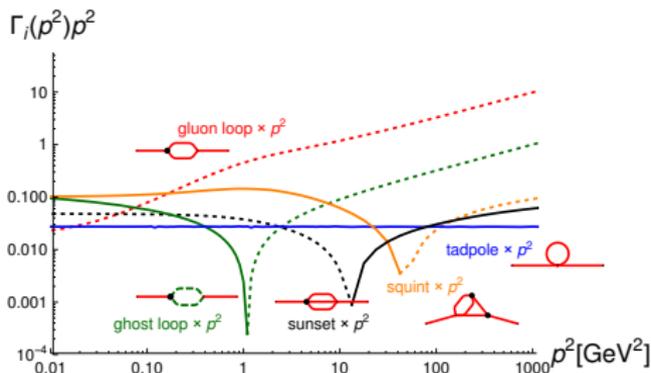
$$\text{---} \bullet \text{---} = \text{---} \text{---} - 2 \text{---} \text{---} + \text{---} \text{---} + \frac{1}{2} \text{---} \text{---} + \frac{1}{2} \text{---} \text{---} + \frac{1}{2} \text{---} \text{---}$$

$$\text{---} \bullet \text{---} = \text{---} \text{---} + \text{---} \text{---} + \text{---} \text{---}$$

$$\text{---} \bullet \text{---} = \text{---} \text{---} - \text{---} \text{---} + \text{perm.}$$

Gluon propagator: Single diagrams

$$\begin{array}{c} \text{gluon line} \end{array}^{-1} = \begin{array}{c} \text{gluon line} \end{array}^{-1} - \frac{1}{2} \begin{array}{c} \text{gluon loop} \end{array} \begin{array}{c} \text{gluon line} \end{array}^{-1} - \frac{1}{2} \begin{array}{c} \text{ghost loop} \end{array} \begin{array}{c} \text{gluon line} \end{array}^{-1} + \begin{array}{c} \text{tadpole} \end{array} \begin{array}{c} \text{gluon line} \end{array}^{-1} - \frac{1}{6} \begin{array}{c} \text{sunset} \end{array} \begin{array}{c} \text{gluon line} \end{array}^{-1} - \frac{1}{2} \begin{array}{c} \text{squint} \end{array} \begin{array}{c} \text{gluon line} \end{array}^{-1}$$

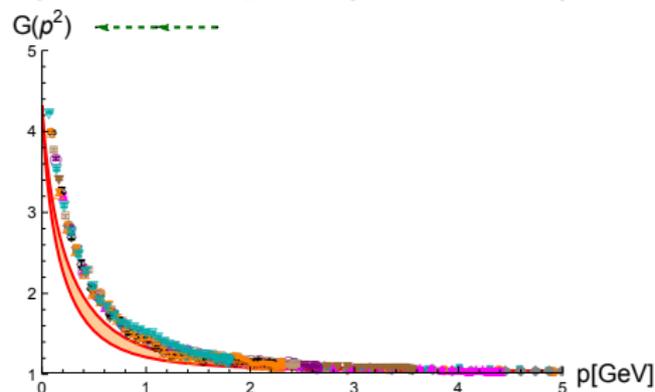
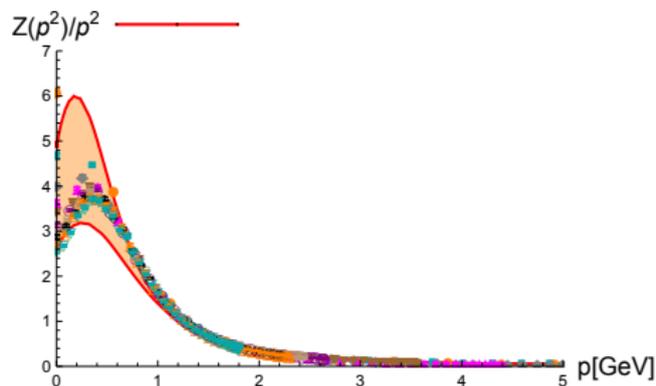
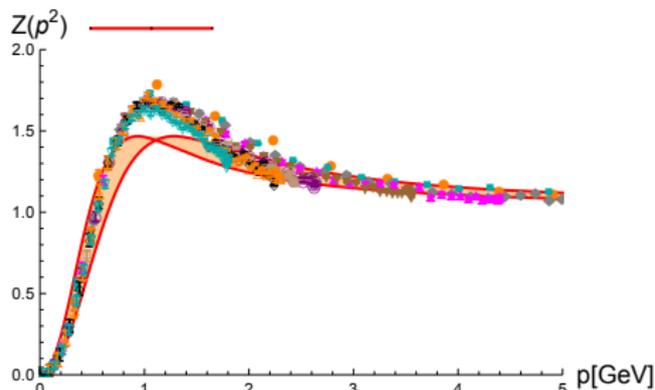


[MQH '16]

Clear **hierarchies** identified:

- UV: as expected perturbatively
- non-perturbative: squint important, sunset small
($d=4$: [Mader, Alkofer '13; Meyers, Swanson '14])

Results: Propagators

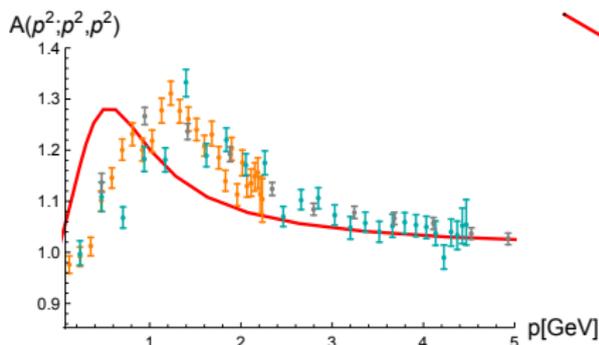


Bands from uncertainty in setting the physical scale.

[MQH '16; lattice: Maas '14]

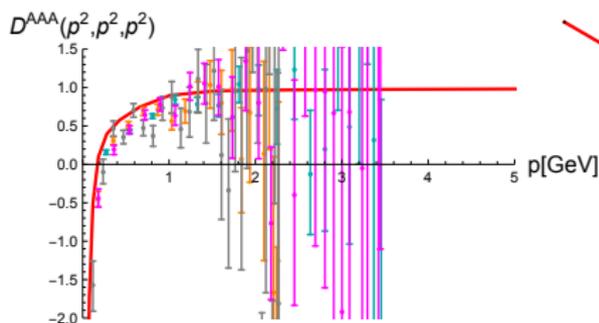
Comparison of three-point functions with lattice results

Dressings:



[MQH '16; lattice: Maas, unpublished]

- Deviation from tree-level in midmomentum 'small'
- Maximum position shifted.
- Bump height ok.

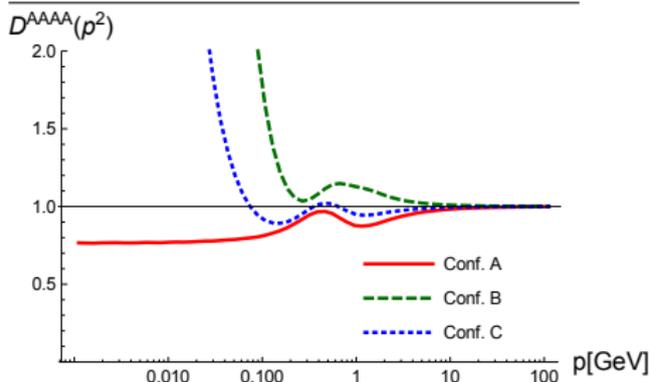


[MQH '16; lattice: Cucchieri, Maas, Mendes '08]

- Close to tree-level above 1 GeV
- Good agreement with lattice data.
- Linear IR divergence [Pelaez, Tissier, Wschebor '13; Aguilar et al. '14]

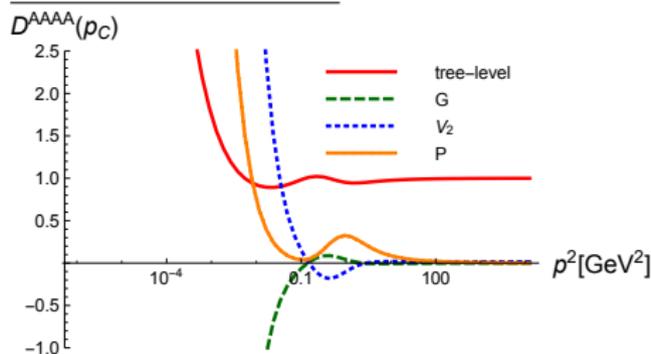
Four-gluon vertex

Different momentum configurations:



[MQH '16]

Different projections:



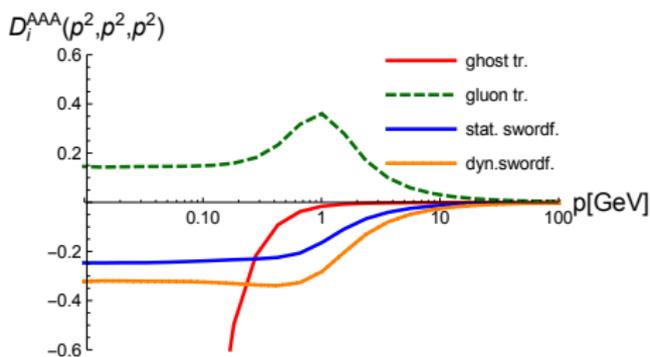
Four-gluon vertex:

- Close to tree-level down to 1 GeV

→ Corrections small individually?

Cancellations in gluonic vertices

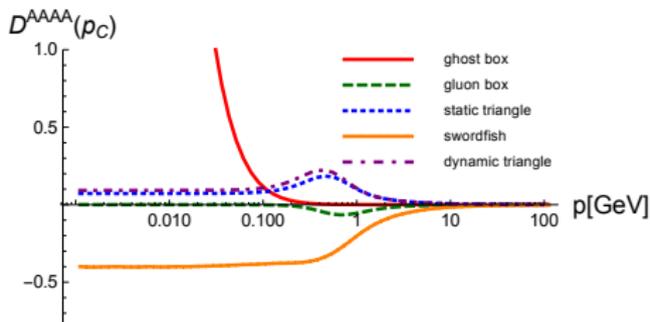
Three-gluon vertex:



- Individual contributions large.
- **Sum is small!**

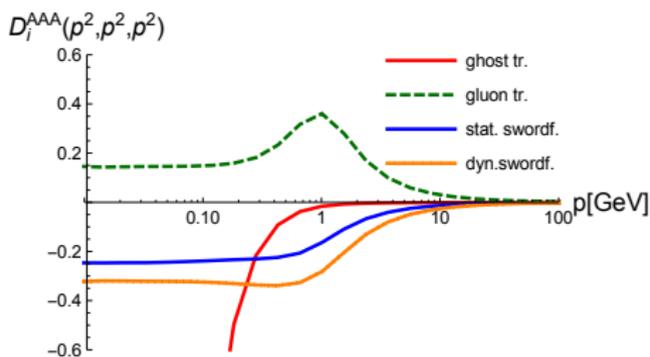
[MQH '16]

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Cancellations in gluonic vertices

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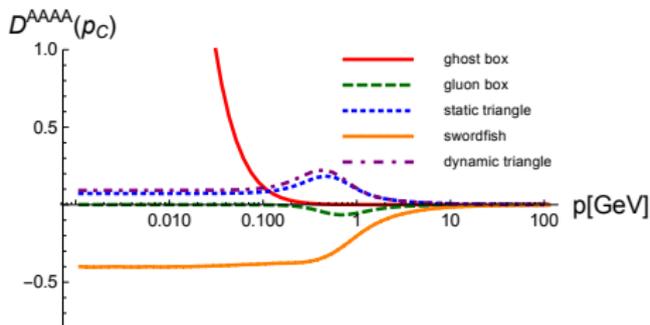


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[MQH '16]

Four-gluon vertex:



Higher contributions:

- Higher vertices close to 'tree-level'?
→ Small.
- If pattern changes (higher vertices large): cancellations required.

Solution from the 3PI effective action

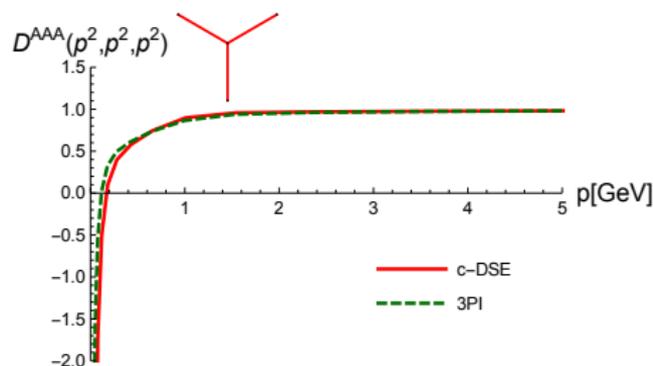
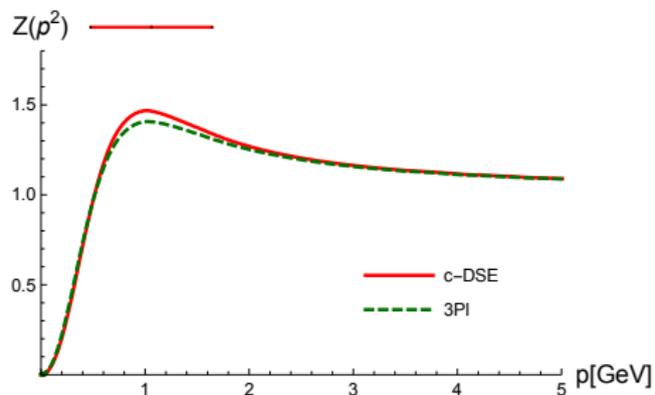
Different set of functional equations:

equations of motion from 3PI effective action (at three-loop level)

Solution from the 3PI effective action

Different set of functional equations:

equations of motion from 3PI effective action (at three-loop level)



→ Very similar results.

[MQH '16]

Summary about three dimensions

- **Hierarchy** of correlation functions and diagrams
- **Cancellations**
- Some degree of **stability** (but no complete list of checks done) when
 - varying *system* of equations.
 - varying *equations* of system.
- Discrepancies with lattice results:
 - Nonperturbative gauge fixing?
 - Lattice systematics?
 - Missing diagrams for vertices?
 - Incomplete tensor bases for some vertices?

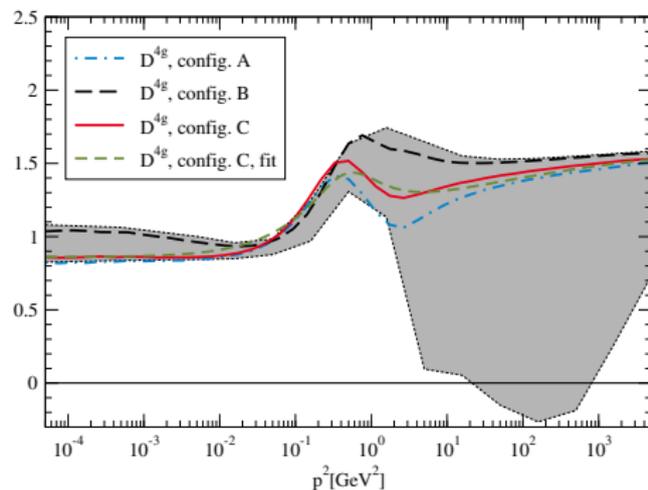
Extending truncations in four dimensions:
Include four-point functions.

Four-gluon vertex

Full calculation with fixed input:

[Cyrol, MQH, von Smekal '14]

Computationally expensive!

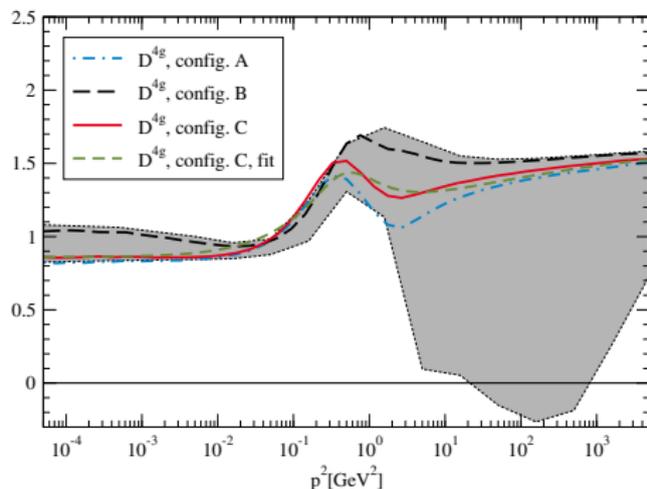
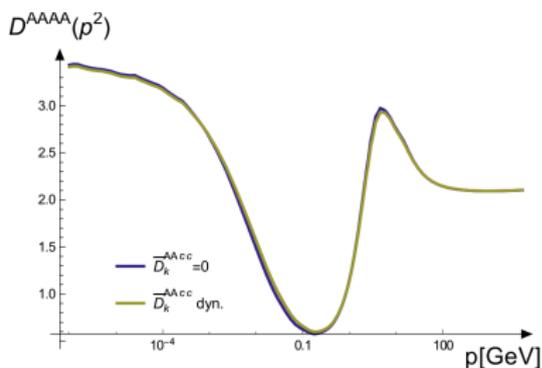


Four-gluon vertex

Full calculation with fixed input:

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Three-gluon vertex has small angle dependence.

→ For dynamic inclusion: Resort to a **one-momentum approximation** (symmetric point); see also FRG calculations by fQCD collaboration.

Effect of four-gluon vertex

In **three-gluon vertex DSE**:

Important for convergence within current truncations in $d = 4$

[Blum, MQH, Mitter, von Smekal '14;

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→ Related to renormalization.



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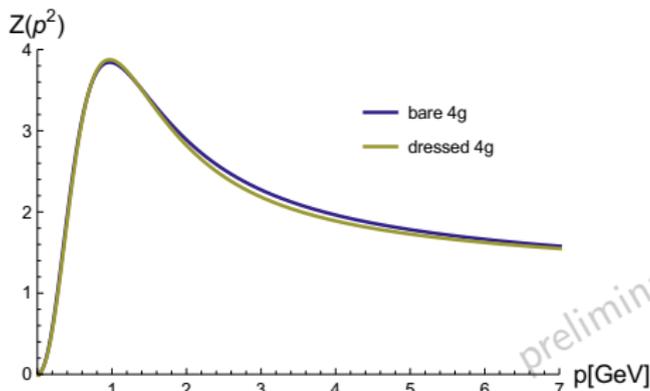
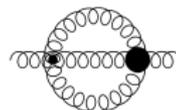
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In **gluon propagator**: Via sunset diagram, small contribution of tree-level dressing; model studies: [Mader, Alkofer '13; Meyers, Swanson '14]



preliminary

preliminary

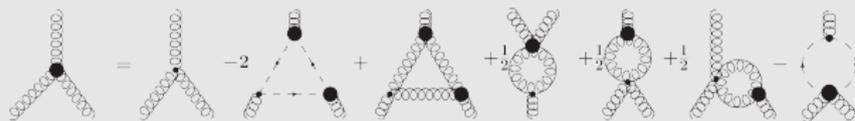
Extending truncations of three-point functions

Extend truncations of equations of three-point functions by adding the two-ghost-two-gluon vertex:

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Three-gluon vertex

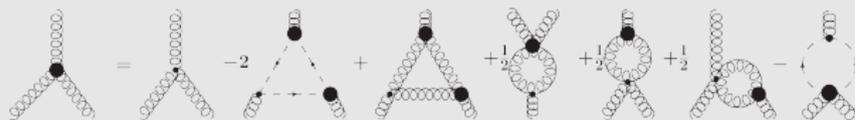


→ One-loop complete equation.

Extending truncations of three-point functions

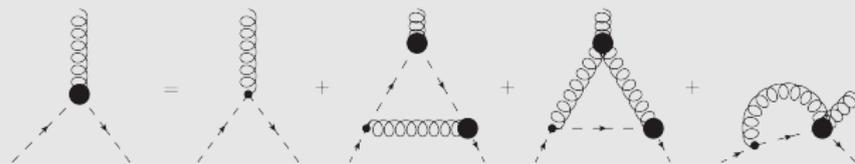
Extend truncations of equations of three-point functions by adding the two-ghost-two-gluon vertex:

Three-gluon vertex



→ One-loop complete equation.

Ghost-gluon vertex



→ Complete equation.

Four-ghost vertex:

In alternative ghost-gluon vertex DSE and in four-point functions.

Four-point functions: Color space

15 possibilities:

$\delta\delta$: 3 combinations

ff : 3 combinations

dd : 3 combinations

df : 6 combinations

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9/8/3 linearly independent in $SU(N/3/2)$, $N > 3$
 [Pascual, Tarrach '80].

$SU(3)$: $\{\sigma_1, \dots, \sigma_8\}$ chosen with these symmetries:

| | σ_1 | σ_2 | σ_3 | σ_4 | σ_5 | σ_6 | σ_7 | σ_8 |
|-----------------------|------------|------------|------------|------------|------------|------------|------------|------------|
| $a \leftrightarrow b$ | + | + | + | - | - | - | - | + |
| $c \leftrightarrow d$ | + | + | + | - | - | + | - | - |

$\{\sigma_1, \dots, \sigma_5\}$ orthogonal to $\{\sigma_6, \sigma_7, \sigma_8\}$. $\rightarrow \{\sigma_6, \sigma_7, \sigma_8\}$ decouple.

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Four-ghost vertex



$$\Gamma^{\bar{c}\bar{c}cc,abcd}(p, q, r, s) = g^4 \sum_{k=1}^8 \sigma^{k,abcd} E_k^{\bar{c}\bar{c}cc}(p, q, r, s).$$

The two-ghost-two-gluon vertex: Lorentz space

Non-primitively divergent correlation function \rightarrow no guide from tree-level tensor
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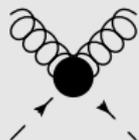
Lorentz basis transverse wrt gluon legs \rightarrow 5 tensors $\tau_{\mu\nu}^i(p, q; r, s)$,
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Two-ghost-two-gluon vertex



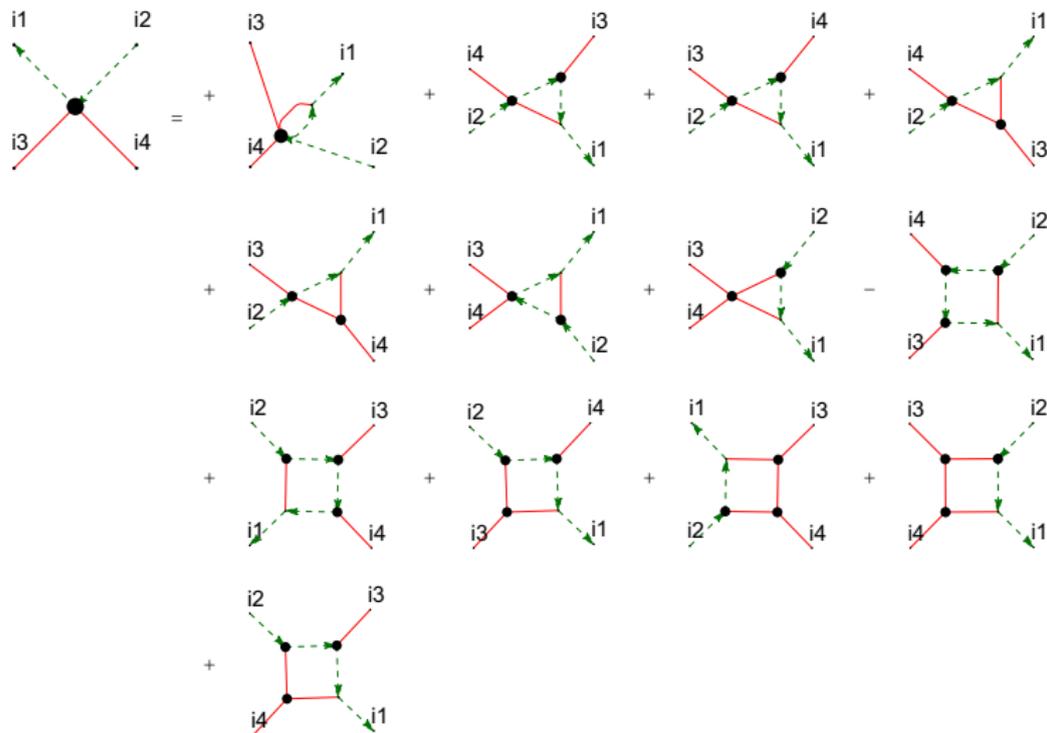
$$\Gamma_{\mu\nu}^{AA\bar{c}c,abcd}(p, q; r, s) = g^4 \sum_{k=1}^{40} \rho_{\mu\nu}^{k,abcd} D_{k(i,j)}^{AA\bar{c}c}(p, q; r, s)$$

with

$$\rho_{\mu\nu}^{k,abcd} = \sigma_i^{abcd} \tau_{\mu\nu}^j, \quad k = k(i, j) = 5(i - 1) + j$$

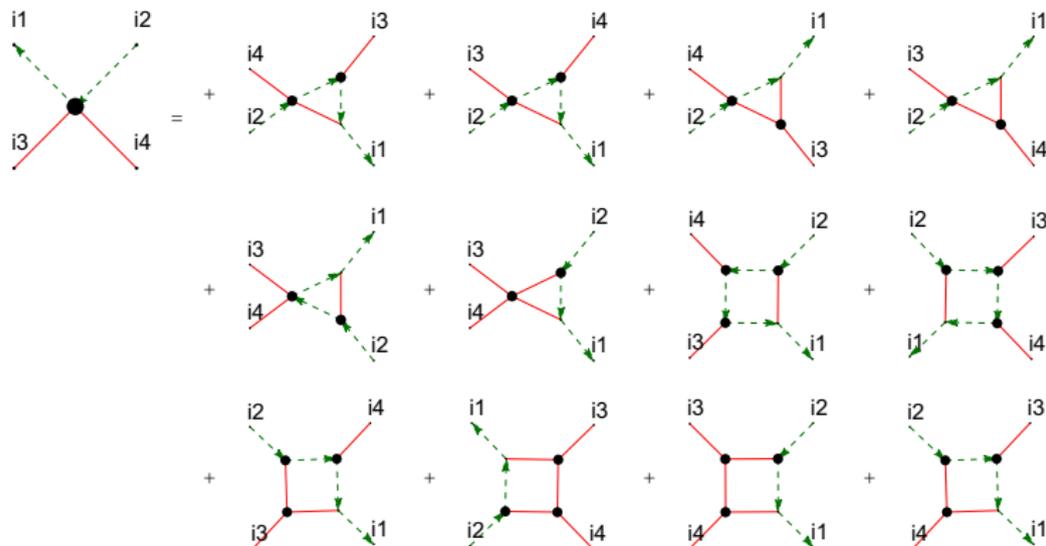
The two-ghost-two-gluon vertex DSE

2 DSEs, choose the one with the ghost leg attached to the bare vertex
 → Truncation discards only one diagram.



The two-ghost-two-gluon vertex DSE

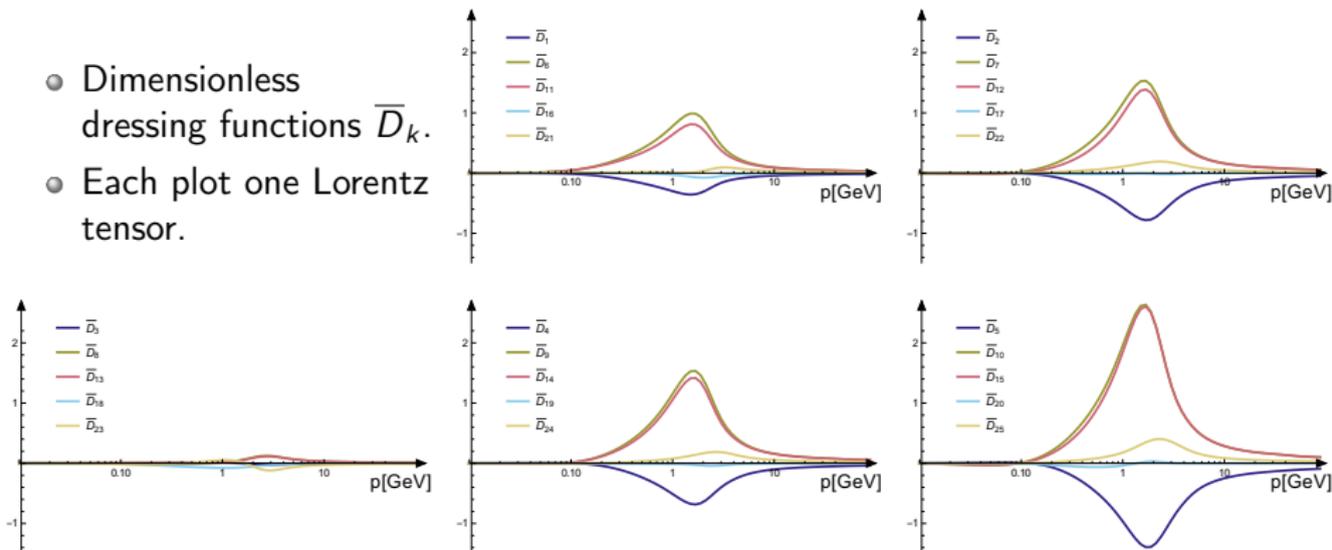
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Results for the two-ghost-two-gluon vertex

Kinematic approximation: one-momentum configuration

- Dimensionless dressing functions \bar{D}_k .
- Each plot one Lorentz tensor.



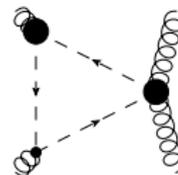
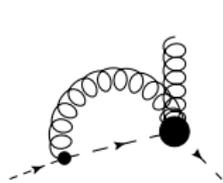
→ Two classes of dressings: 13 very small, 12 not small

→ No nonzero solution for $\{\sigma_6, \sigma_7, \sigma_8\}$ found.

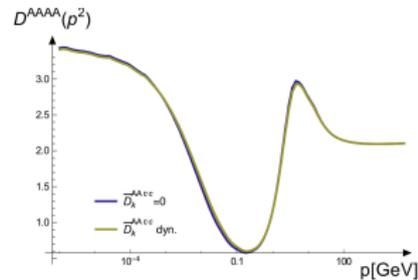
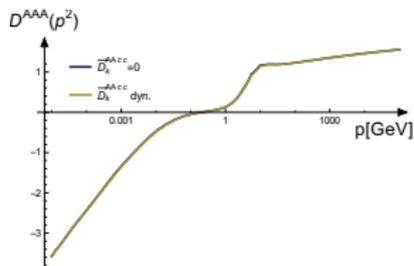
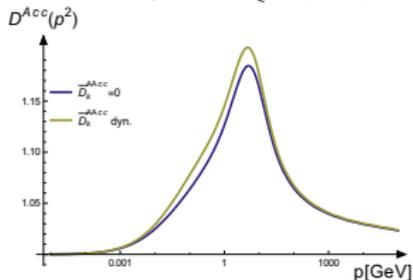
[MQH, EPJC (2017)]

Influence of two-ghost-two-gluon vertex

Coupled system of ghost-gluon, three-gluon and four-gluon vertices **with and without** two-ghost-two-gluon vertex:



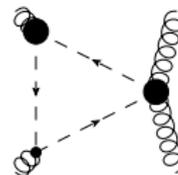
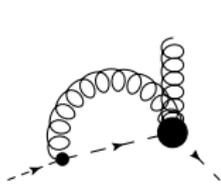
Color space: $\{\sigma_6, \sigma_7, \sigma_8\}$ do not appear here!



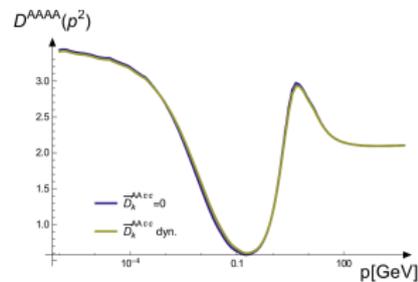
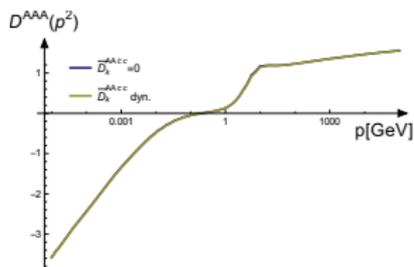
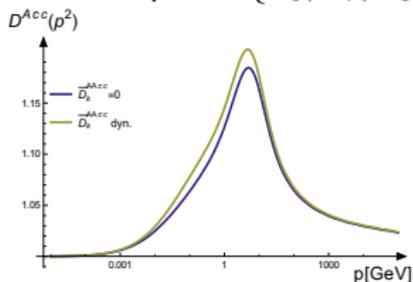
- **Small** influence on ghost-gluon vertex ($< 1.7\%$)
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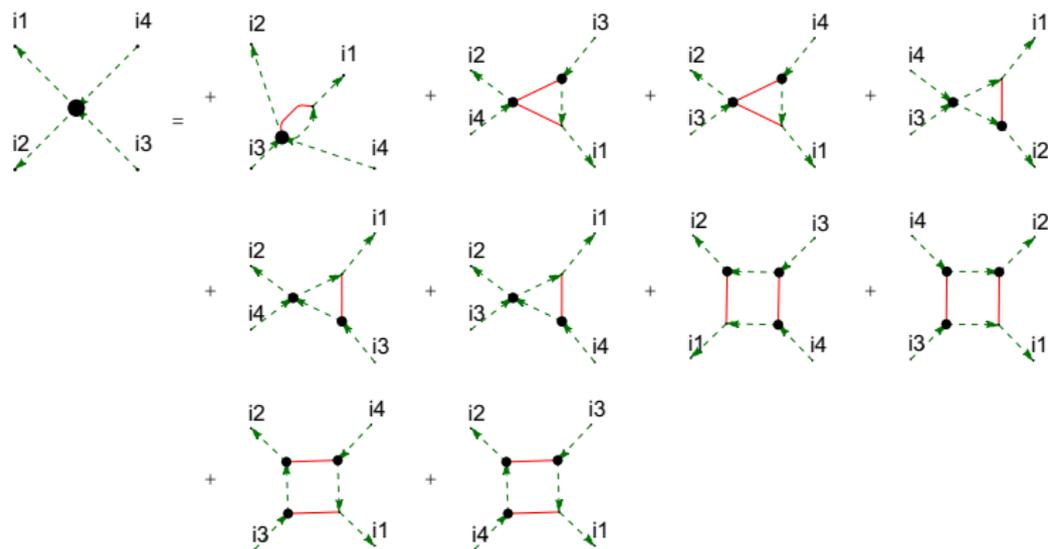


- **Small** influence on ghost-gluon vertex ($< 1.7\%$)
- **Negligible** influence on three- and four-gluon vertices.
(Color space: Only small dressings couple to three-gluon vertex.)

[MQH, EPJC (2017)]

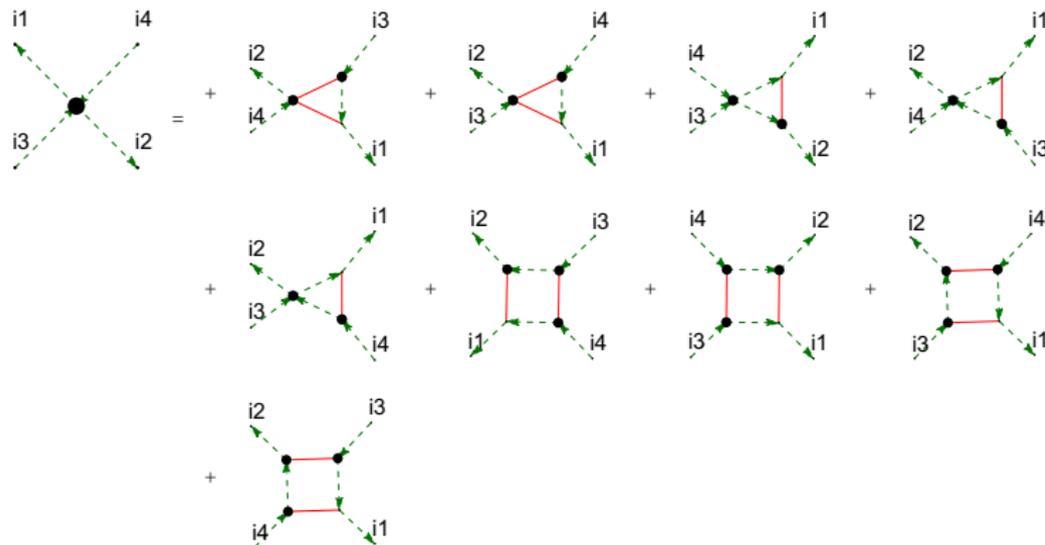
The four-ghost vertex DSE

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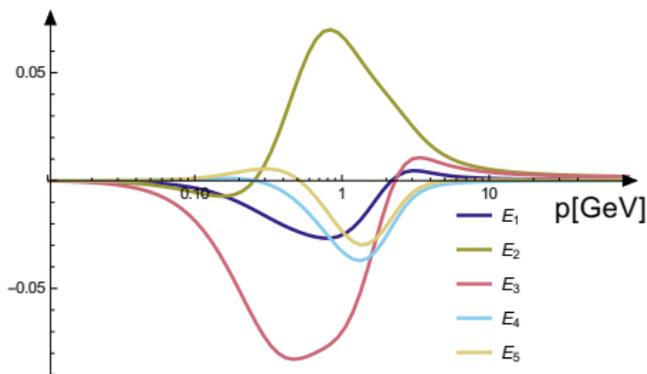
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Results for the four-ghost vertex

Kinematic approximation: one-momentum configuration



→ All dressings very small.

[MQH, EPJC (2017)]

$E_6, E_7, E_8 (\{\sigma_6, \sigma_7, \sigma_8\})$

Decouple into a homogeneous, linear equation. → Trivial solution always exists.
Nontrivial one? → None found.

(Same applies to two-ghost-two-gluon vertex.)

Summary and conclusions

Based on

- tests in $d = 3$ including comparison with 3PI calculations
- analysis of one-loop resummation
- testing non-primitively divergent correlation functions

a **non-perturbative hierarchy** of correlations functions and diagrams can be identified.

Three- and four-gluon vertices:

- ① **Cancellations** between diagrams
- ② **Negligible diagrams**

Two-loop diagrams in propagators:

Required quantitatively and for self-consistency.

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- 1 Cancellations between diagrams
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Remaining caveats:

- Three- and four-gluon vertex restricted to tree-level dressings here.
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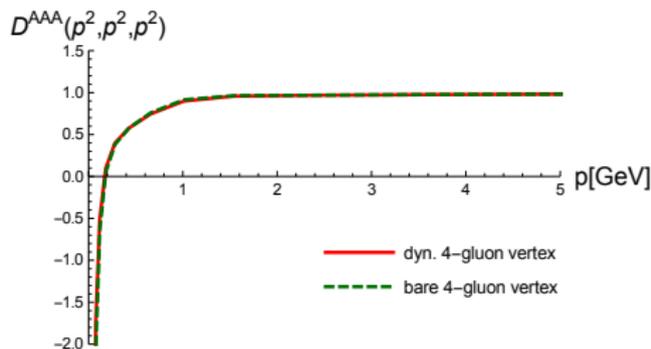
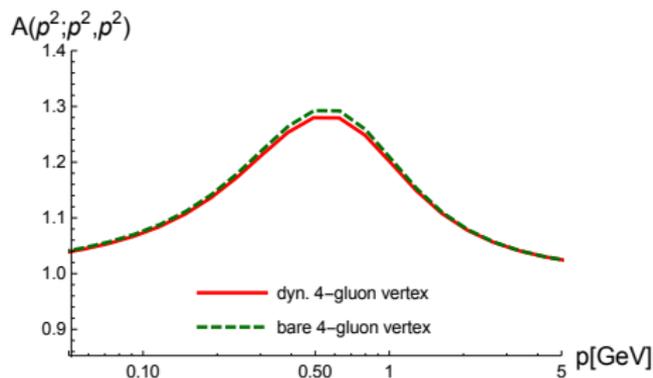
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Thank you for your attention!

Influence of four-gluon vertex on three-point functions



[MQH '16]

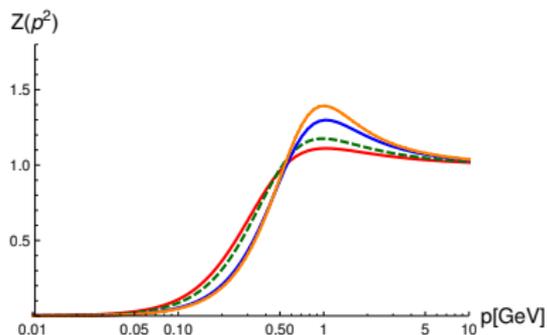
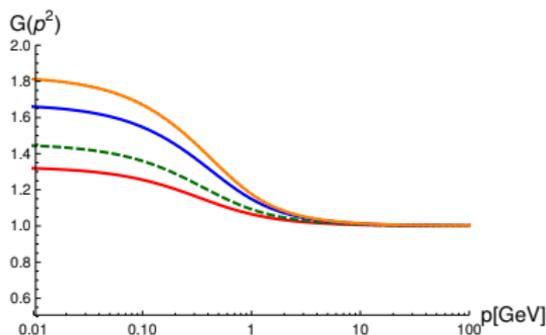
- Influence of four-gluon vertex small.

Family of solutions in three dimensions

Cf. FRG results: Bare mass parameter from modified STIs [Cyrol, Fister, Mitter, Pawłowski, Strodthoff '16].

DSEs: Enforce family of solutions by fixing the gluon propagator at $p^2 = 0$.

Simple toy system with bare vertices [MQH, 1606.02068]:



⇒ Possibility of family of solutions.

NB: Effect overestimated here since vertices are fixed.