QCD 2- and 3- point Green's functions:

From lattice results to phenomenology

In collaboration with:

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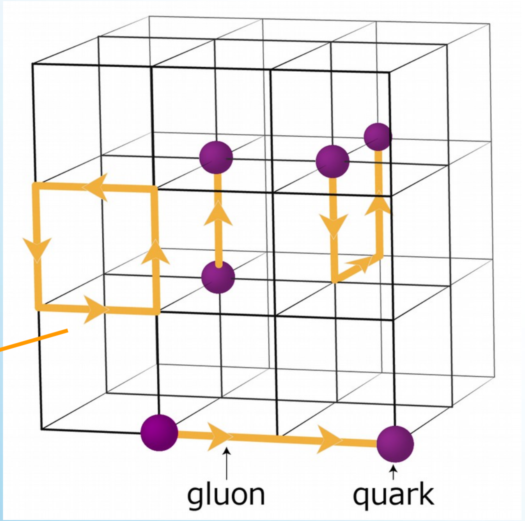
Lattice two- and three-point Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$



$$\widetilde{A}^a_\mu(q) = \frac{1}{2}\,\mathrm{Tr}\sum_x A_\mu(x+\hat{\mu}/2)\exp[iq\cdot(x+\hat{\mu}/2)]\lambda^a$$

$$A_\mu(x+\hat{\mu}/2) = \frac{U_\mu(x) - U^\dagger_\mu(x)}{2iag_0} - \frac{1}{3}\,\mathrm{Tr}\,\frac{U_\mu(x) - U^\dagger_\mu(x)}{2iag_0}$$

Tree-level Symanzik gauge action

$$S_g = \frac{\beta}{3}\sum_x\left\{b_0\sum_{\substack{\mu,\nu=1\\1\le\mu<\nu}}^{4}\left[1-\mathrm{Re\,Tr}\,(U^{1\times1}_{x,\mu,\nu})\right] + b_1\sum_{\substack{\mu,\nu=1\\\mu\ne\nu}}^{4}\left[1-\mathrm{Re\,Tr}(U^{1\times2}_{x,\mu,\nu})\right]\right\}$$

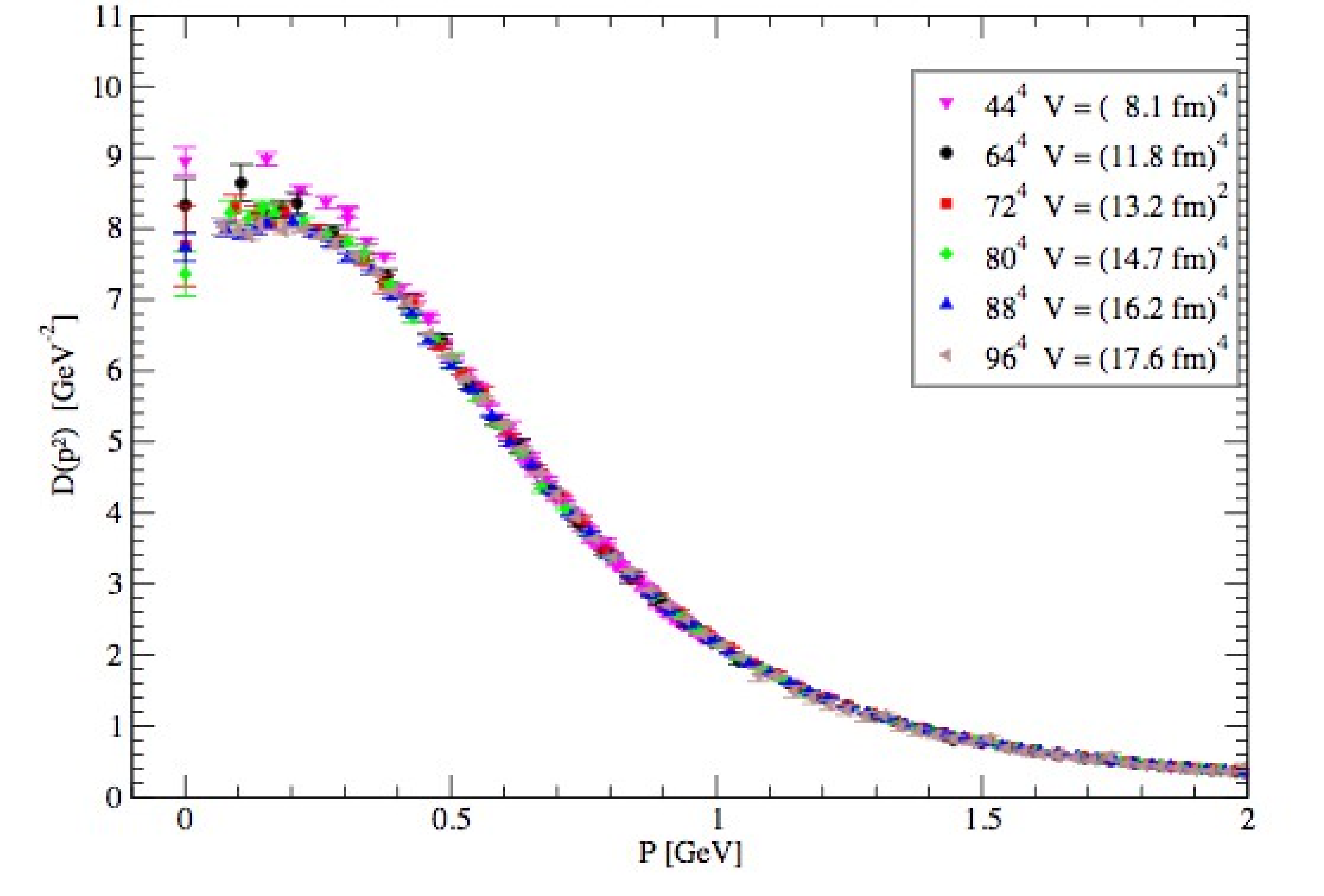
The gauge fields are to be nonperturbatively obtained from lattice QCD simulations and applied then to get the gluon Green's functions

The gluon propagator

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

Duarte, Oliveira, Silva
PRD94(2016)014502



Quenched lattice gluon propagators for different large volumes!

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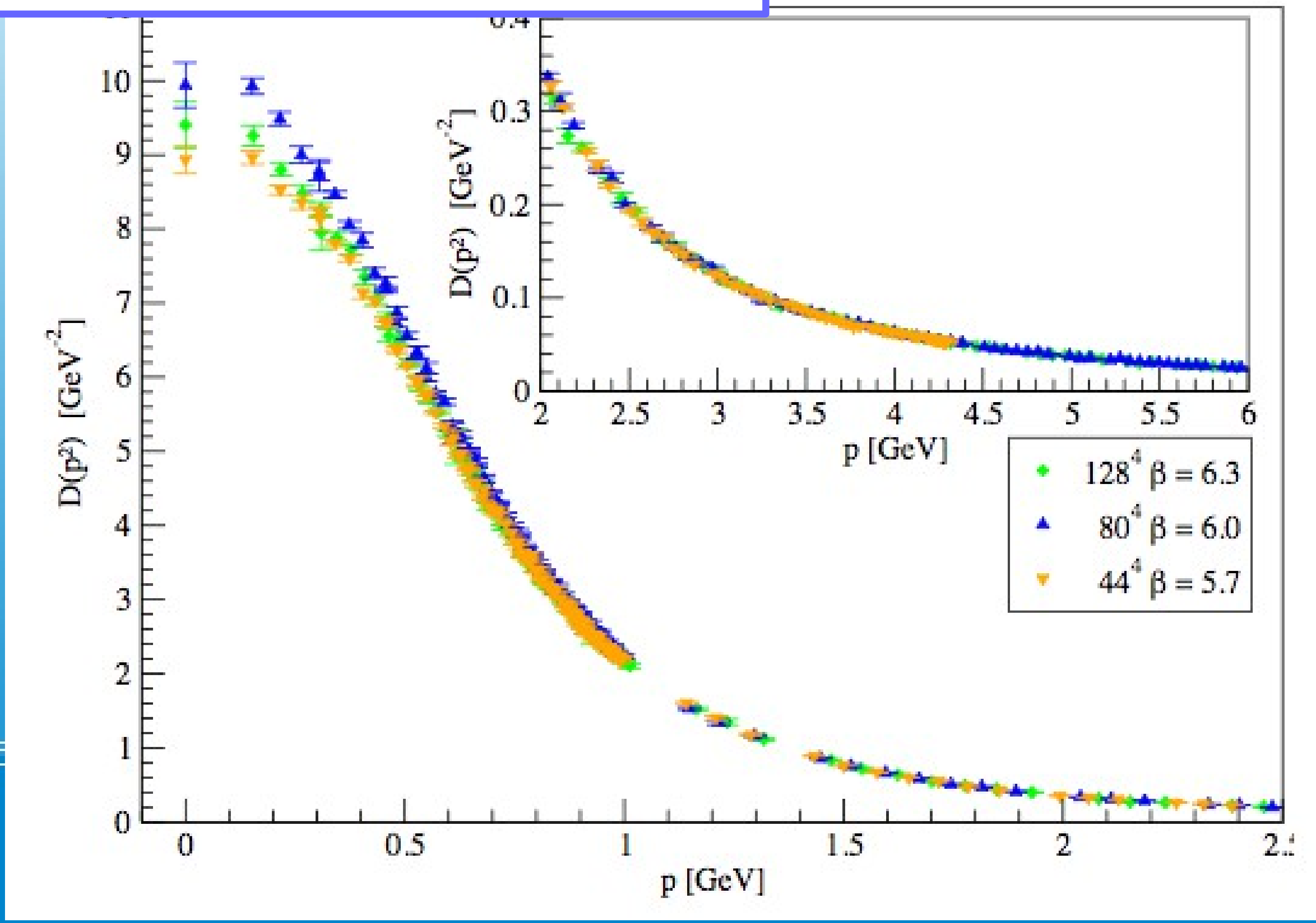

Quenched lattice gluon propagators for different beta and similar volume!

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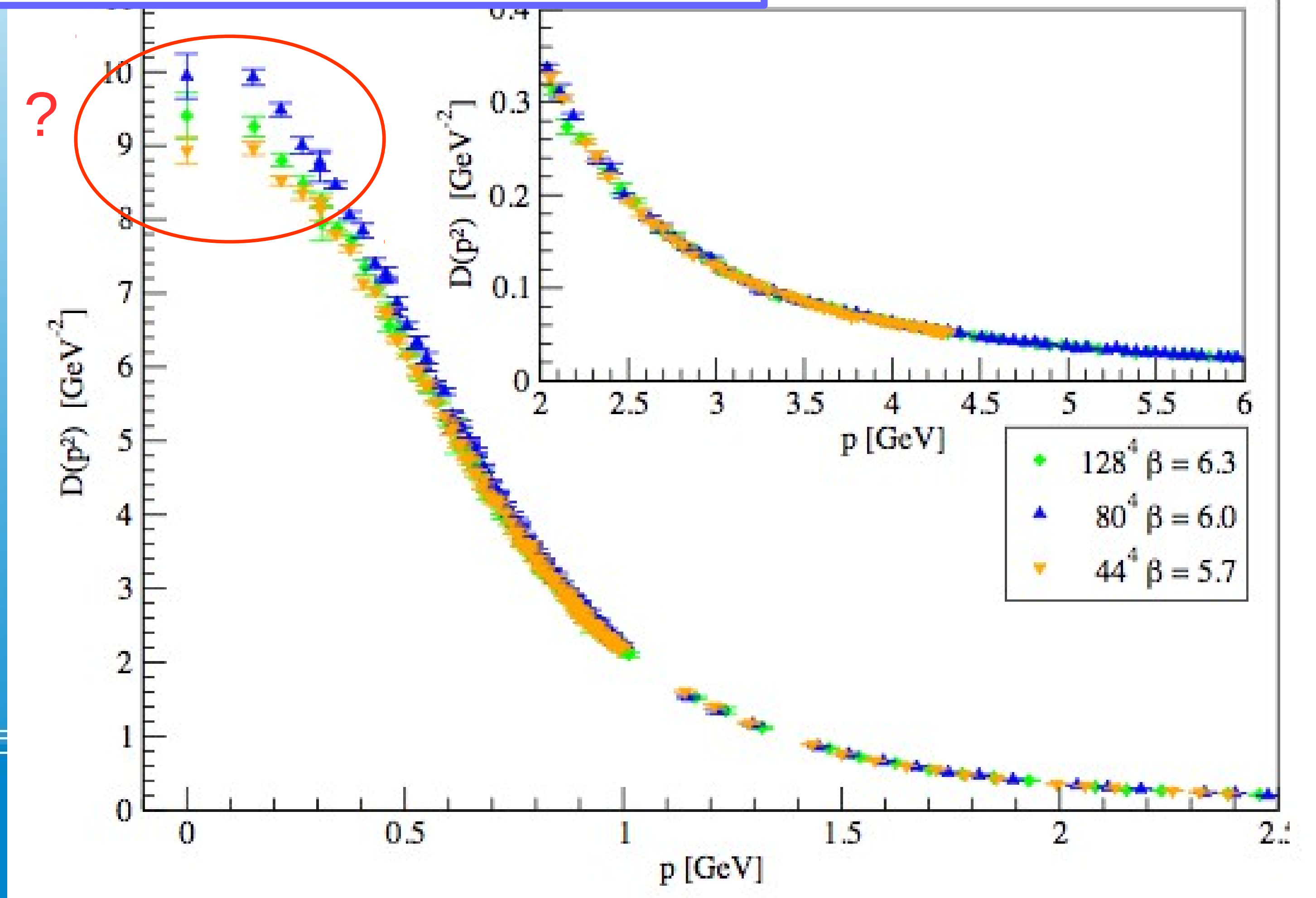

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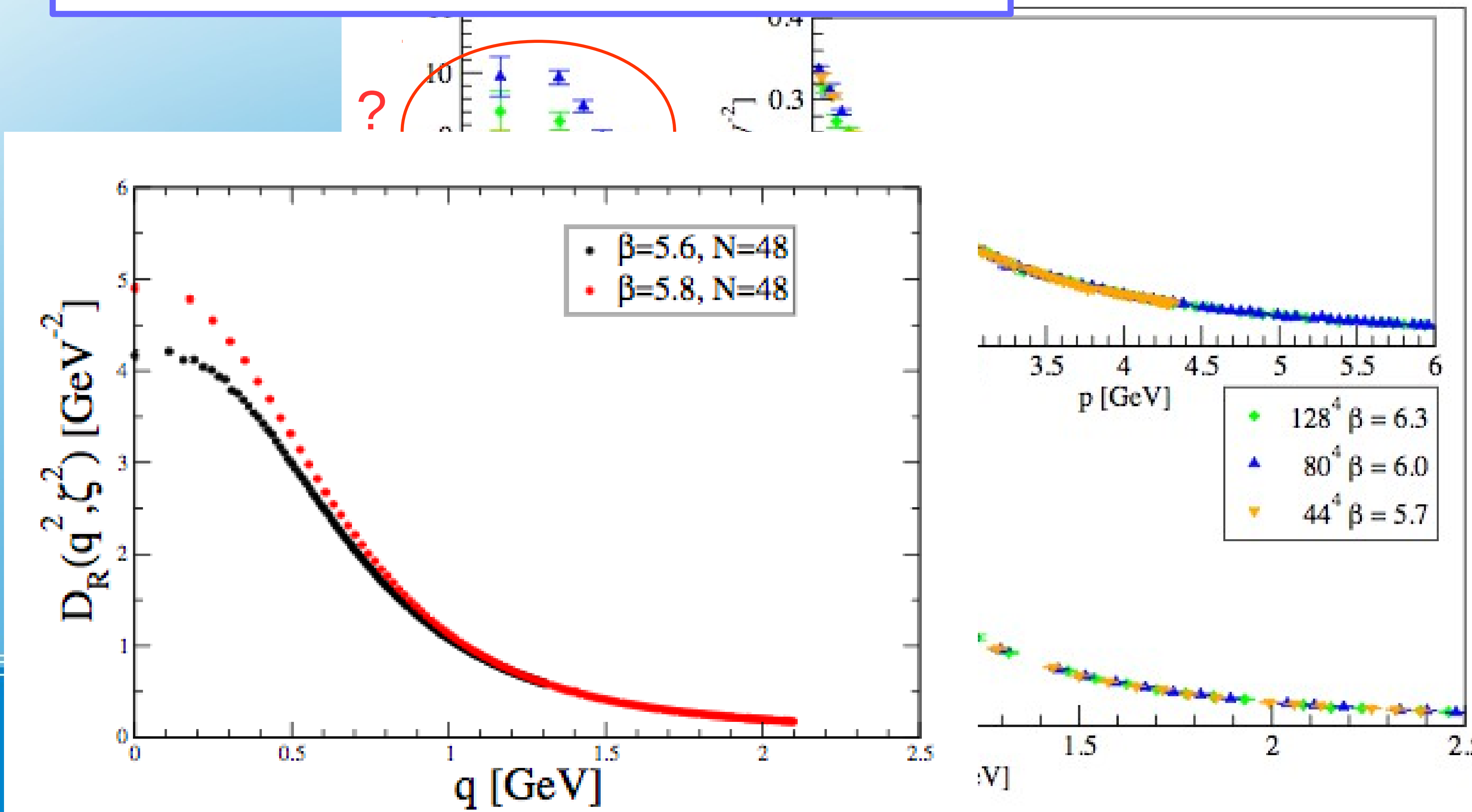

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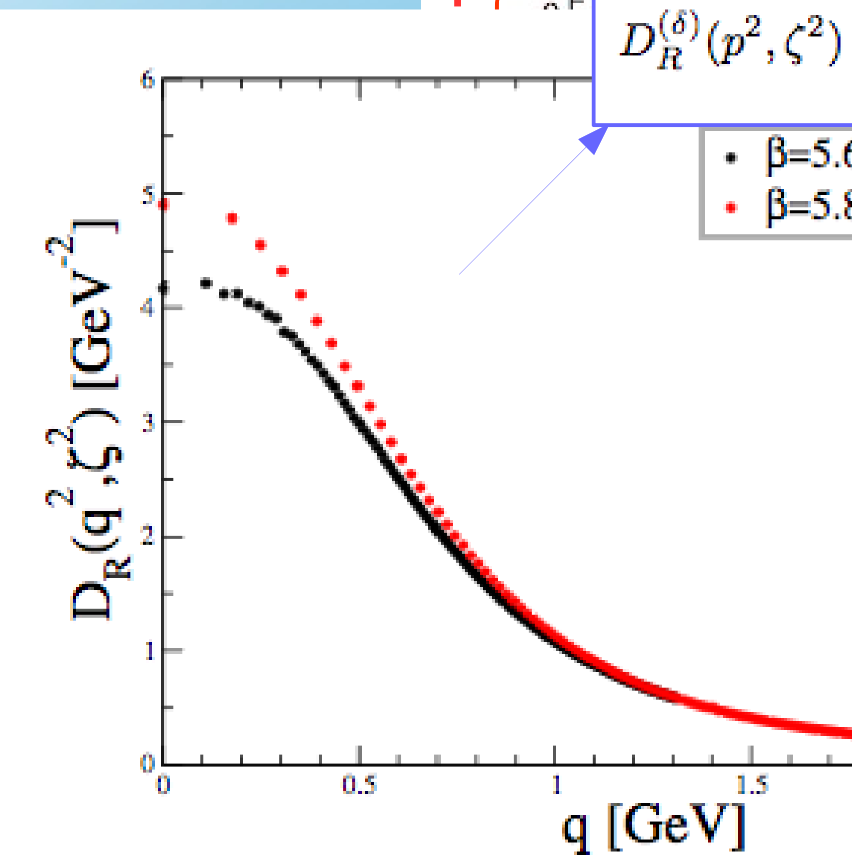
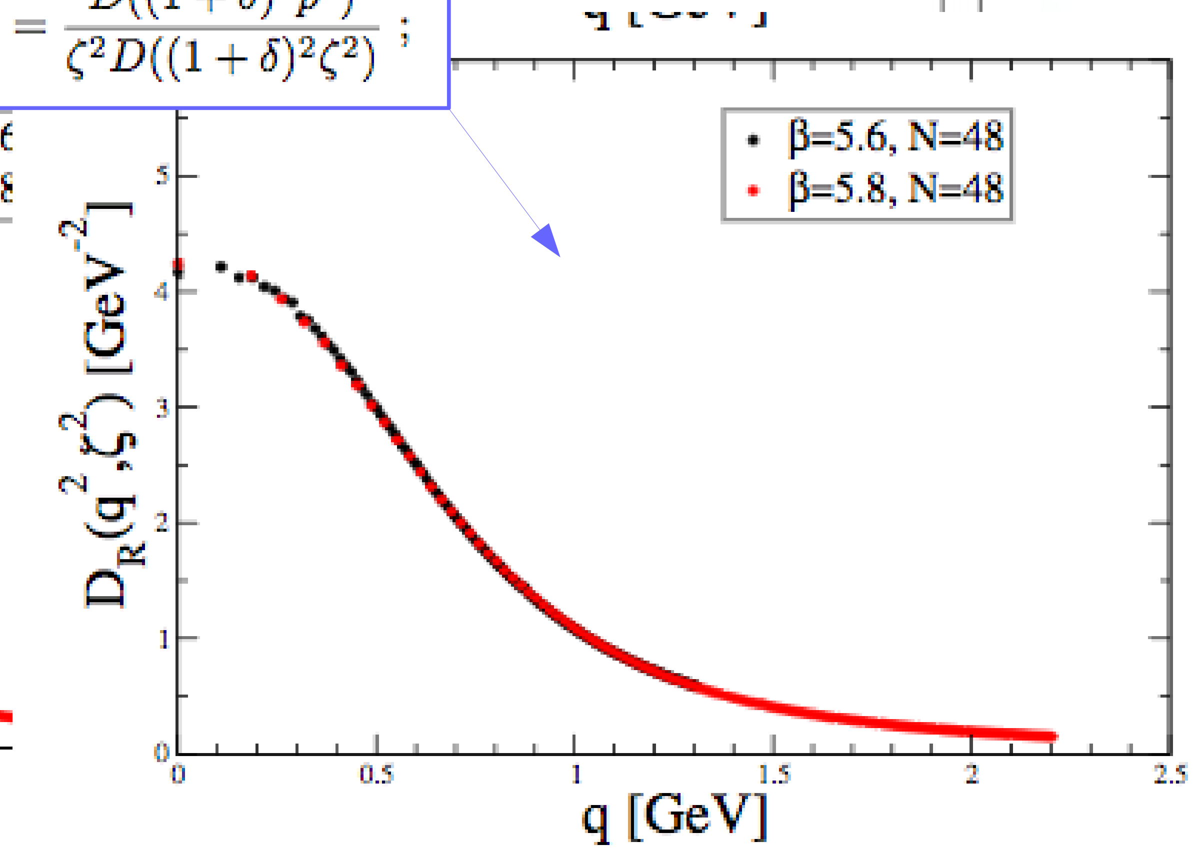
$$D_R^{(\delta)}(p^2,\zeta^2) = \frac{D((1+\delta)^2 p^2)}{\zeta^2 D((1+\delta)^2\zeta^2)} \; ;$$





ArXiv:1704.02053 (PRD): Essentially, a scale setting problem!!

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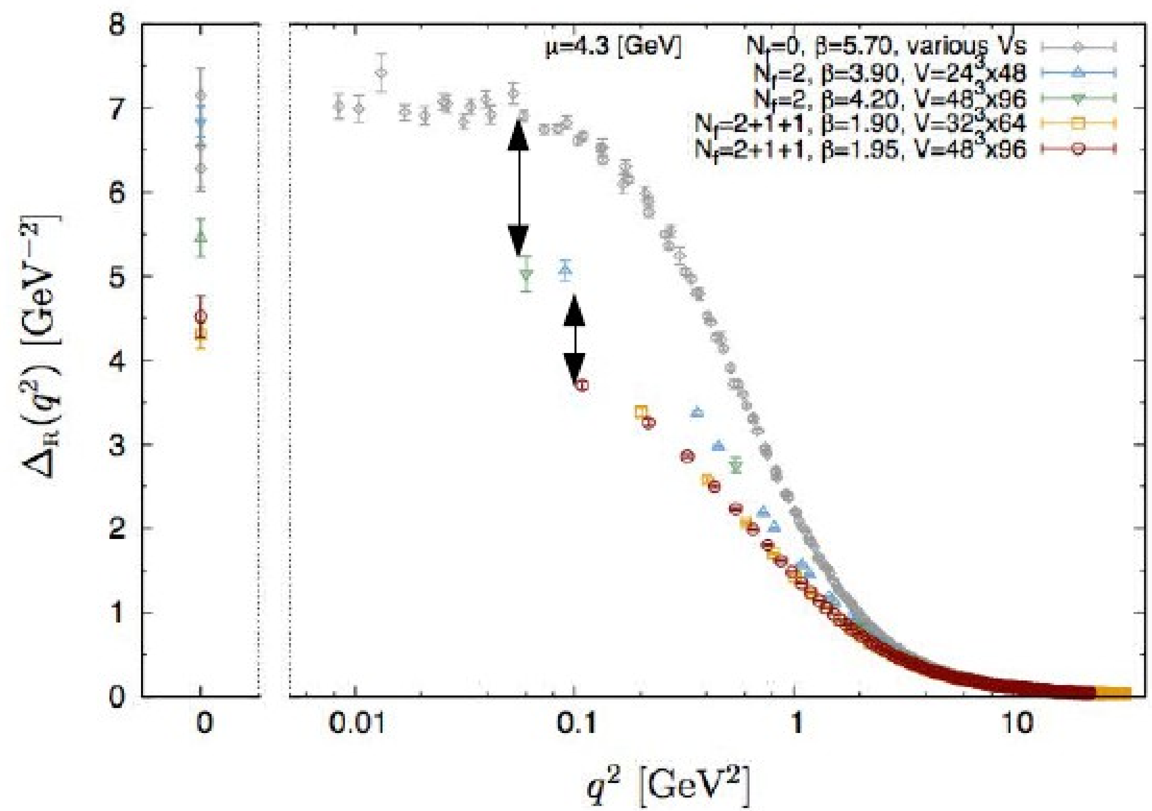
Ayala et al.
PRD86(2012)074512

- Effective gluon mass increases with the number of flavours



Unquenched lattice gluon propagators!

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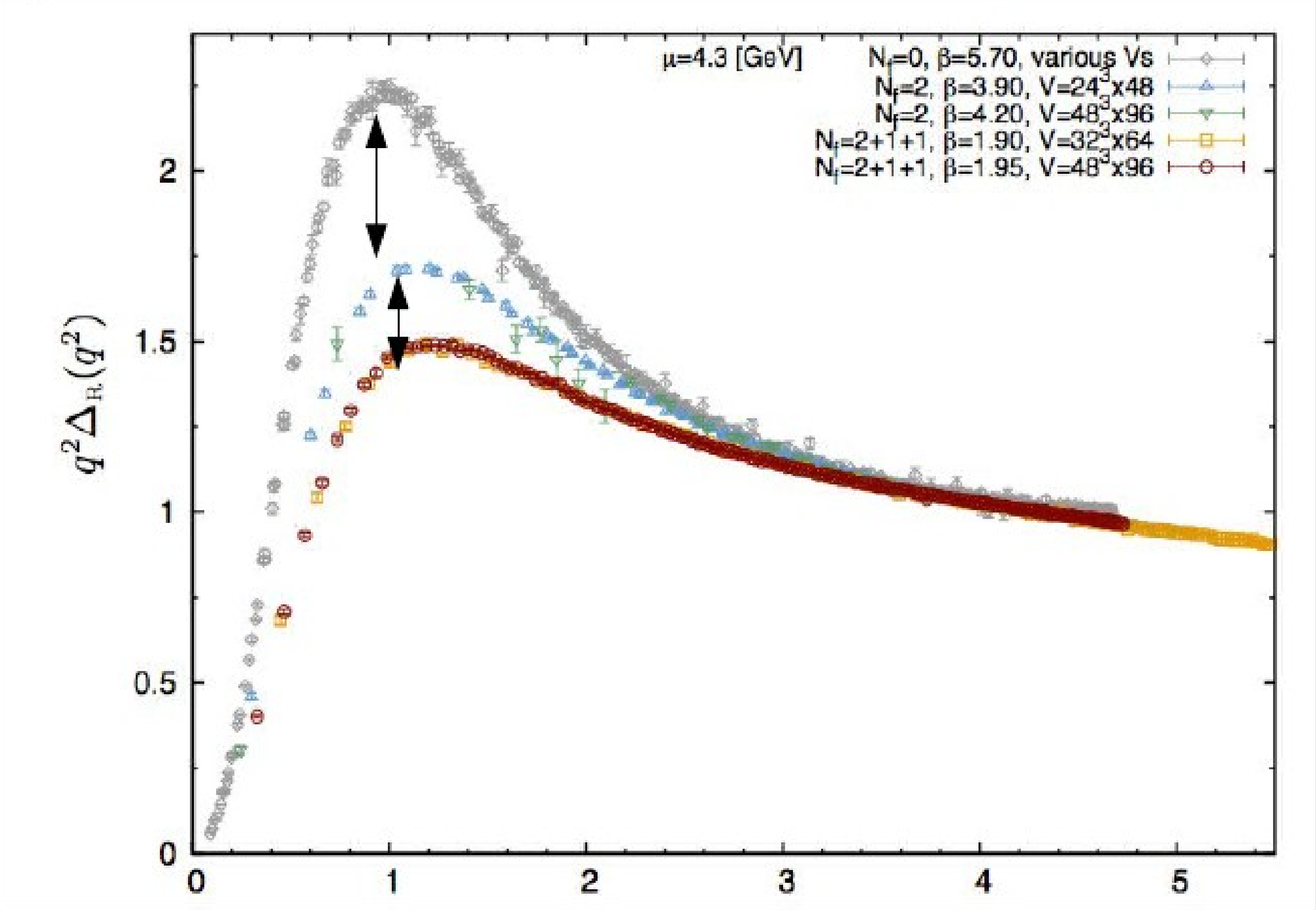

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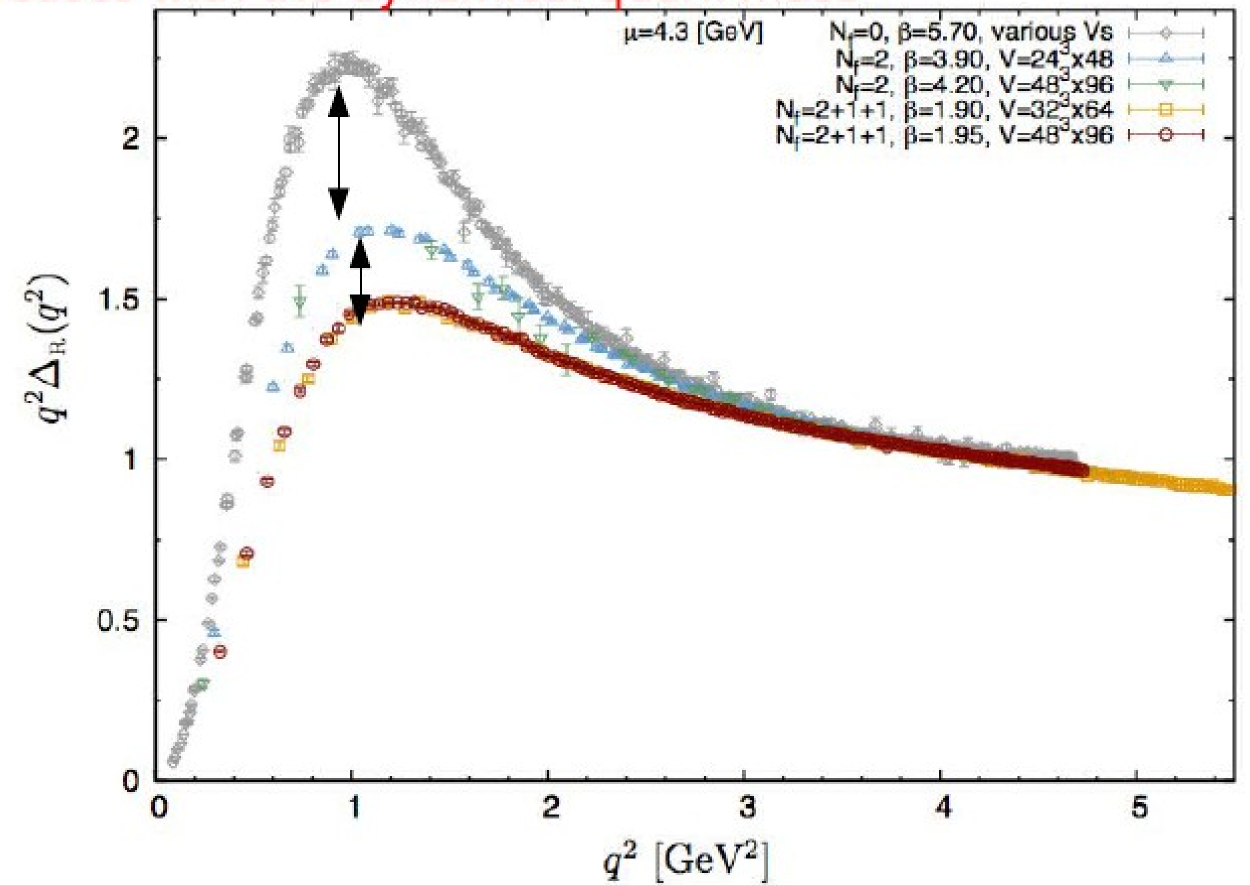
Ayala et al.
PRD86(2012)074512

• Effective gluon mass increases with the number of flavours
… and decreases with the dynamical quark mass



Unquenched lattice gluon propagators!

The ghost propagator

$$\left(F^{(2)}\right)^{ab}(x-y) \equiv \langle \left(M^{-1}\right)^{ab}_{xy} \rangle \,, \quad M(U) = -\frac{1}{N}\nabla \cdot \widetilde{D}(U)$$

$$\widetilde{D}(U)\eta(x) = \frac{1}{2}\left(U_\mu(x)\eta(x+\mu) - \eta(x)U_\mu(x) + \eta(x+\mu)U_\mu^\dagger - U_\mu^\dagger(x)\eta(x)\right)$$

Ayala et al.
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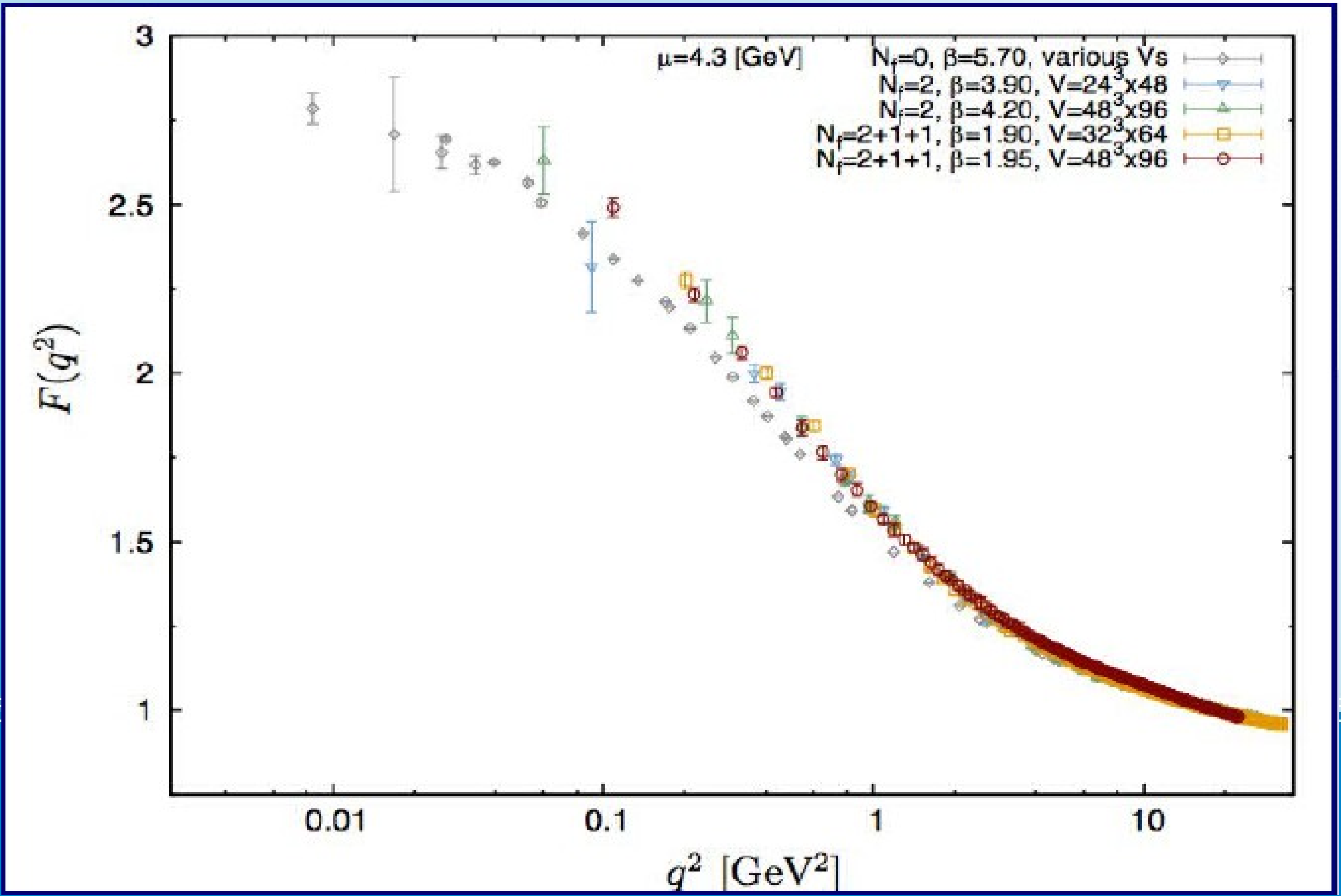

Unquenched lattice ghost propagators!

The vertex and the three-gluon Green's function

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Symmetric configuration:

$q^2 = r^2 = p^2$ and $q\cdot r = q\cdot p = r\cdot p = -q^2/2;$

$$\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\Gamma_{\alpha'\mu'\nu'}(q,r,p)\Delta_{\alpha'\alpha}(q)\Delta_{\mu'\mu}(r)\Delta_{\nu'\nu}(p),$$

$$G_{\alpha\mu\nu}(q,r,p) = T^{sym}(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + S^{sym}(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

$$\Gamma_{\alpha\mu\nu}(q,r,p) = \Gamma^{sym}_T(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + \Gamma^{sym}_S(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

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In Landau gauge and for particular kinematical configurations, transversality and Bose symmetry make possible a simple tensorial decomposition of the gluon Green's function

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$$W_{\alpha\mu\nu} = \lambda^{tree}_{\alpha\mu\nu} + \lambda^{S}_{\alpha\mu\nu}/2$$

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Asymmetric configuration:

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$$\Delta_R(q^2;\mu^2) = Z_A^{-1}(\mu^2)\,\Delta(q^2),$$
$$T_R^{\text{sym}}(q^2;\mu^2) = Z_A^{-3/2}(\mu^2) T^{\text{sym}}(q^2),$$

$$g^{\text{sym}}(q^2) = q^3 \frac{T^{\text{sym}}(q^2)}{[\Delta(q^2)]^{3/2}} = q^3 \frac{T_R^{\text{sym}}(q^2;\mu^2)}{[\Delta_R(q^2;\mu^2)]^{3/2}}.$$

MOM renormalization prescription:

$$\Delta_R(q^2;q^2) = Z_A^{-1}(q^2)\,\Delta(q^2) = 1/q^2,$$
$$T_R^{\text{sym}}(q^2;q^2) = Z_A^{-3/2}(q^2)\,T^{\text{sym}}(q^2) = g_R^{\text{sym}}(q^2)/q^6.$$

$$T^{\text{sym}}(q^2) = g\,\Gamma_T^{\text{sym}}(q^2)\,\Delta^3(q^2),$$

$$g^{sym}(\mu^2)\,\Gamma^{sym}_{T,R}(q^2;\mu^2) = \frac{g^{sym}(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

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After the required projection and the appropriate renormalization, one can define a QCD coupling from the Green's functions, and relate it to the 1PI vertex form factor, in both symmetric...

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$$T^{\rm asym}(r^2) = g\,\Gamma_T^{\rm asym}(r^2)\,\Delta(0)\,\Delta^2(r^2),$$

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After the required projection and the appropriate renormalization, one can define a QCD coupling from the Green's functions, and relate it to the 1PI vertex form factor, in both symmetric and asymmetric kinematical configurations.

Multi-instanton background

The classical gauge field solution from a multi-instanton ensemble can be cast as the so-called *ratio ansatz* [E.V. Shuryak; Nucl.Phys.B302(1988)574]

$$g_0 B^a_\mu(x) = \frac{2 \sum_{i=I,A} R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \rho_i^2 \frac{f(|y_i|)}{y_i^2}}{1 + \sum_{i=I,A} \rho_i^2 \frac{f(|y_i|)}{y_i^2}} ,$$

$y_i = x - z_i$

$$\sim 2 \sum_{i=I,A} R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \rho_i^2 \frac{f(|y_i|)}{y_i^2} \qquad y_i >> \rho_i \text{ for all } i,$$

$$\sim 2 \sum_{i=I,A} R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \frac{f(|y_i|)}{f(|y_i|) + \frac{y_i^2}{\rho_i^2}} \qquad y_j << \rho_j, \; y_i >> \rho_i \text{ for any } i \neq j,$$

$\overline{\eta}_{\mu\nu}$, $R^{a\alpha}_{(i)}$ `t Hooft symbols and color rotation matrices

ρ_i instanton radius

f(z) is a shape function [f(0)=1] that might be eventually obtained by minimization of the action per particle for some statistical ensemble of instantons (classical background).

Then:

$$\boxed{g_0 B^a_\mu(\mathbf{x}) = 2 \sum_i R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \phi_{\rho_i}\left(\frac{|y_i|}{\rho_i}\right)}$$

D. Diakonov, V. Petrov; Nucl.Phys.B45386(1992)236

Boucaud et al.; Phys.Rev.D70(2004)114503

$$\phi_\rho(z) = \begin{cases} \dfrac{f(\rho z)}{f(\rho z) + z^2} \simeq \dfrac{1}{1+z^2} & z \ll 1 \\[2ex] \dfrac{f(\rho z)}{z^2} & z \gg 1 \end{cases}$$

The classical gauge field can be effectively accounted for by an independent pseudo-particule sum ansatz approach in both large- and low-distance regimes.

Multi-instanton background

$$g_0^m G^{(m)}(k^2) = \frac{1}{N} W^{\mu_1 \dots \mu_m}_{a_1 \dots a_m} \langle g_0 A^{a_1}_{\mu_1}(k_1) \dots g_0 A^{a_m}_{\mu_m}(k_m) \rangle$$

$$G^{(2)}(k^2) = \Delta(k^2);\; G^{(3)}(k^2) = T^{sym}(k^2)$$

$$g_0 B^a_\mu(\mathbf{x}) = 2 \sum_i R^{a\alpha}_{(i)} \overline{\eta}^\alpha_{\mu\nu} \frac{y_i^\nu}{y_i^2} \phi_{\rho_i}\left(\frac{|y_i|}{\rho_i}\right)$$

$$\phi_\rho(z) = \begin{cases} \dfrac{f(\rho z)}{f(\rho z) + z^2} \simeq \dfrac{1}{1+z^2} & z \ll 1 \\ \dfrac{f(\rho z)}{z^2} & z \gg 1 \end{cases}$$

Multi-instanton background

Instanton density

$$g_0^m G^{(m)}(k^2) = \frac{1}{N} W^{\mu_1 \ldots \mu_m}_{a_1 \ldots a_m} \langle g_0 A^{a_1}_{\mu_1}(k_1) \ldots g_0 A^{a_m}_{\mu_m}(k_m) \rangle = \frac{k^{2-m}}{m 4^{m-1}} n \langle \rho^{3m} I^m(k\rho) \rangle$$

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$$I(s) = \frac{8\pi^2}{s} \int_0^\infty z dz\, J_2(sz) \phi(z)$$

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$$\Phi_\rho(0) = 1$$

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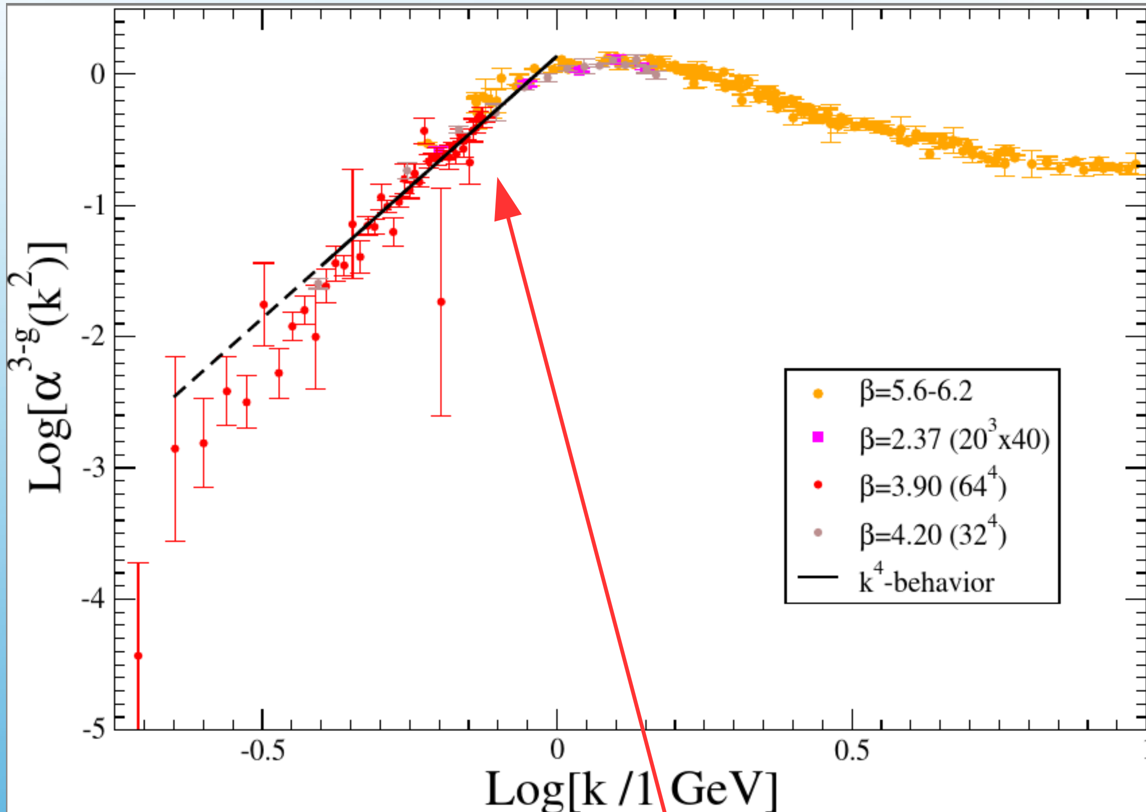
where $\bar{\rho} = \sqrt{\langle \rho^2 \rangle}$ and $\delta\rho^2 = \langle (\rho - \bar{\rho})^2 \rangle$

The asymptotic behavior at both the large- and low-momentum limits appears to be driven by the fourth power of the momentum, the result relying on a very general ground, irrespective of the details of the profile and its breaking of the scale independence.

Multi-instanton background



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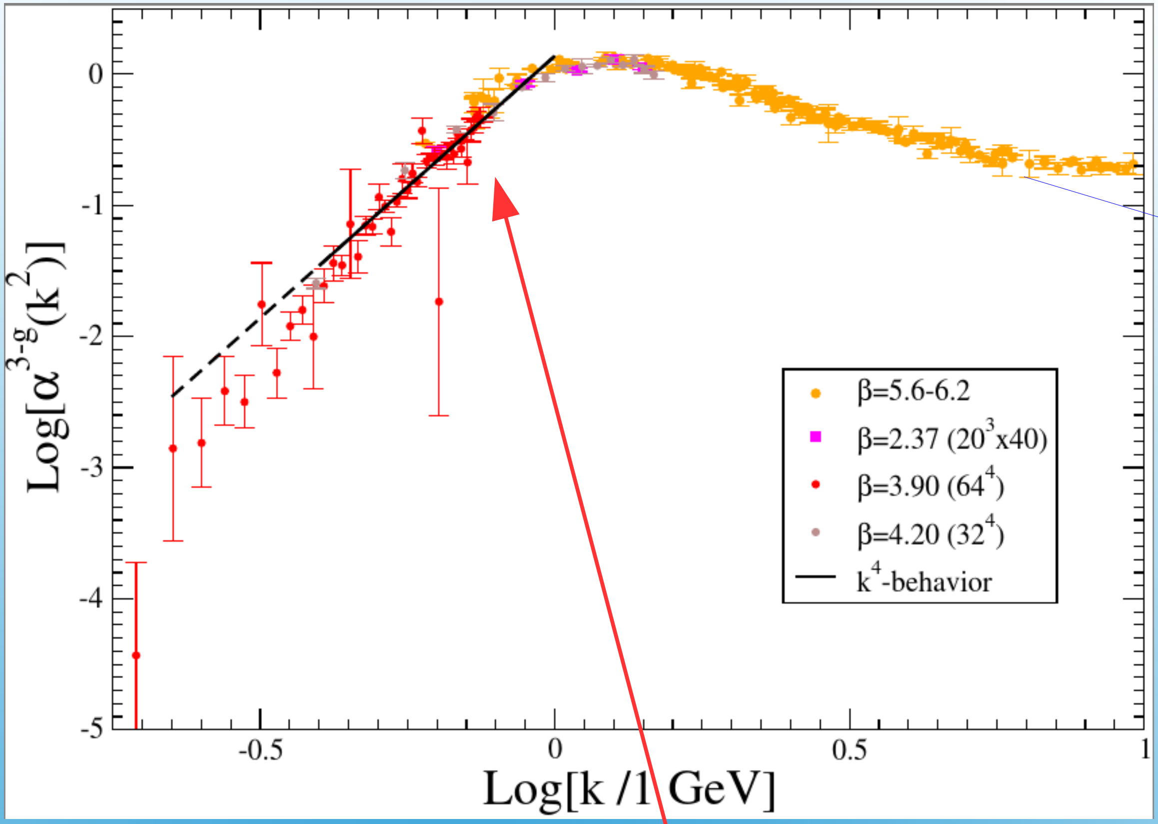

The large-momentum limit in the field of a multi-instanton solution appears here hidden by the quantum UV fluctuations!!!

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The Wilson flow smoothing procedure

The Wilson flow $B_\mu(t,x)$ of an SU(N) gauge field is defined by [M. Luescher; JHEP02(2010)071]

$$\partial_t B_\mu = D_\nu G_{\nu\mu}$$

where $t = a^2 \tau$ is the so-called flow time and

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$
$$D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

with the initial condition $B_\mu(0,x) = A_\mu(x)$.

Then, the expansion in terms of $A_\mu(x)$ gives at tree-level:

$$B_\mu(t,x) = \int d^4 y\, K(t; x-y)\, A_\mu(x)$$

$$K(t;x) = \frac{e^{-x^2/4t}}{(4\pi t)^2}$$

The Wilson flow has been proven to be an useful tool to deprive the lattice gauge fields from their short-distance (UV) quantum fluctuations.

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Table 1
Estimates for the densities, obtained as explained in the text, for the different flow times, also expressed in physical units. For this to be done, according to [27], we have defined $\sqrt{8t_0} = 0.3$ fm, whence $t_0 = a^2\tau_0 = 0.0113$ fm^2 and $t = \frac{\tau}{\tau_0} t_0$. At $\tau = 4$, in the unquenched case, the characteristic diffusion length is so small that quantum fluctuations have not been properly removed yet.

	τ	t/t_0	n (fm^{-4})
Quenched	4	6.84	
	8	13.7	
	15	25.6	
Unquenched	4	2.34	
	8	4.70	
	15	8.84	

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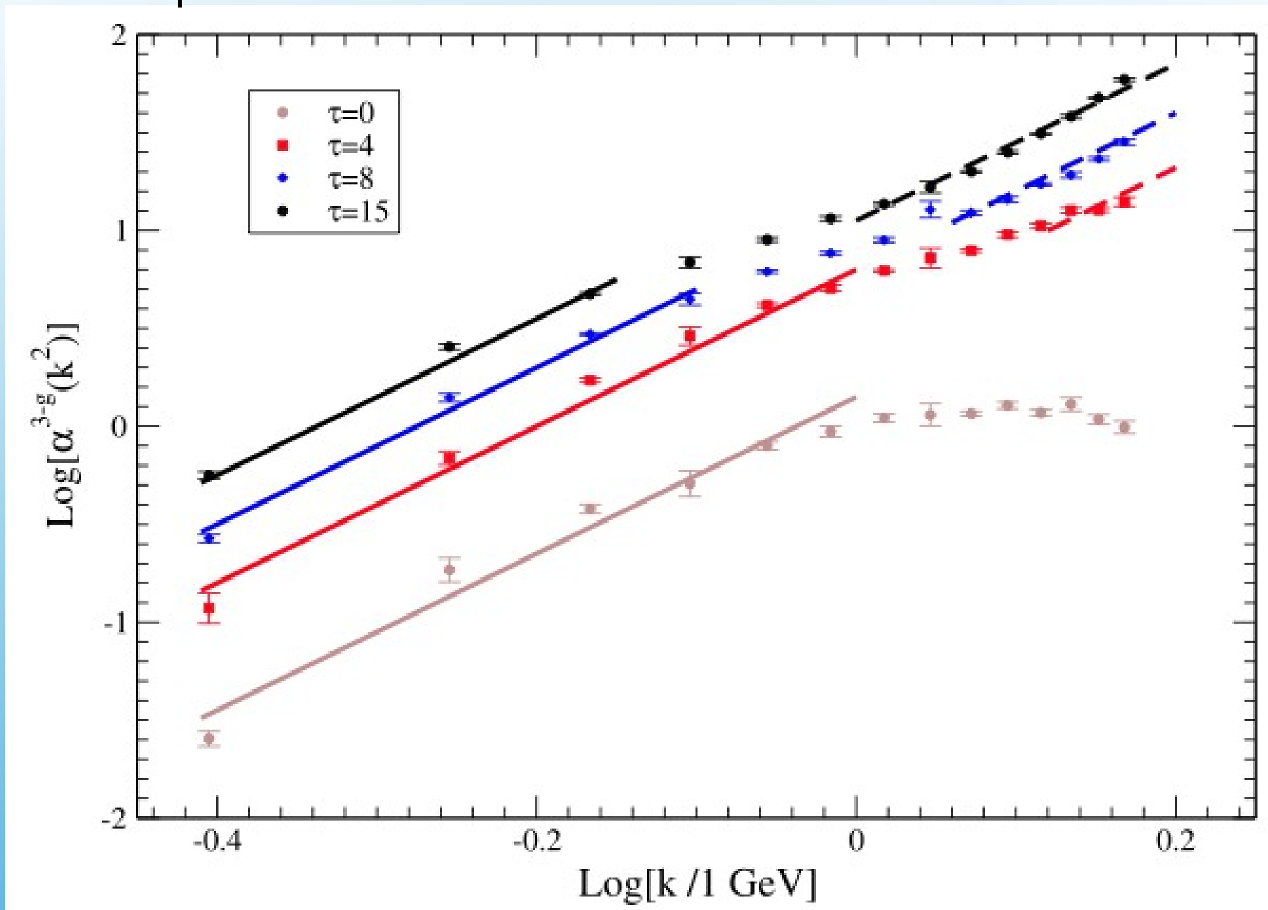
The Wilson flow smoothing procedure

$\beta = 4.20$



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	τ	t/t_0	n (fm^{-4})
Quenched	4	6.84	3.5(1)
	8	13.7	1.75(4)
	15	25.6	0.98(5)
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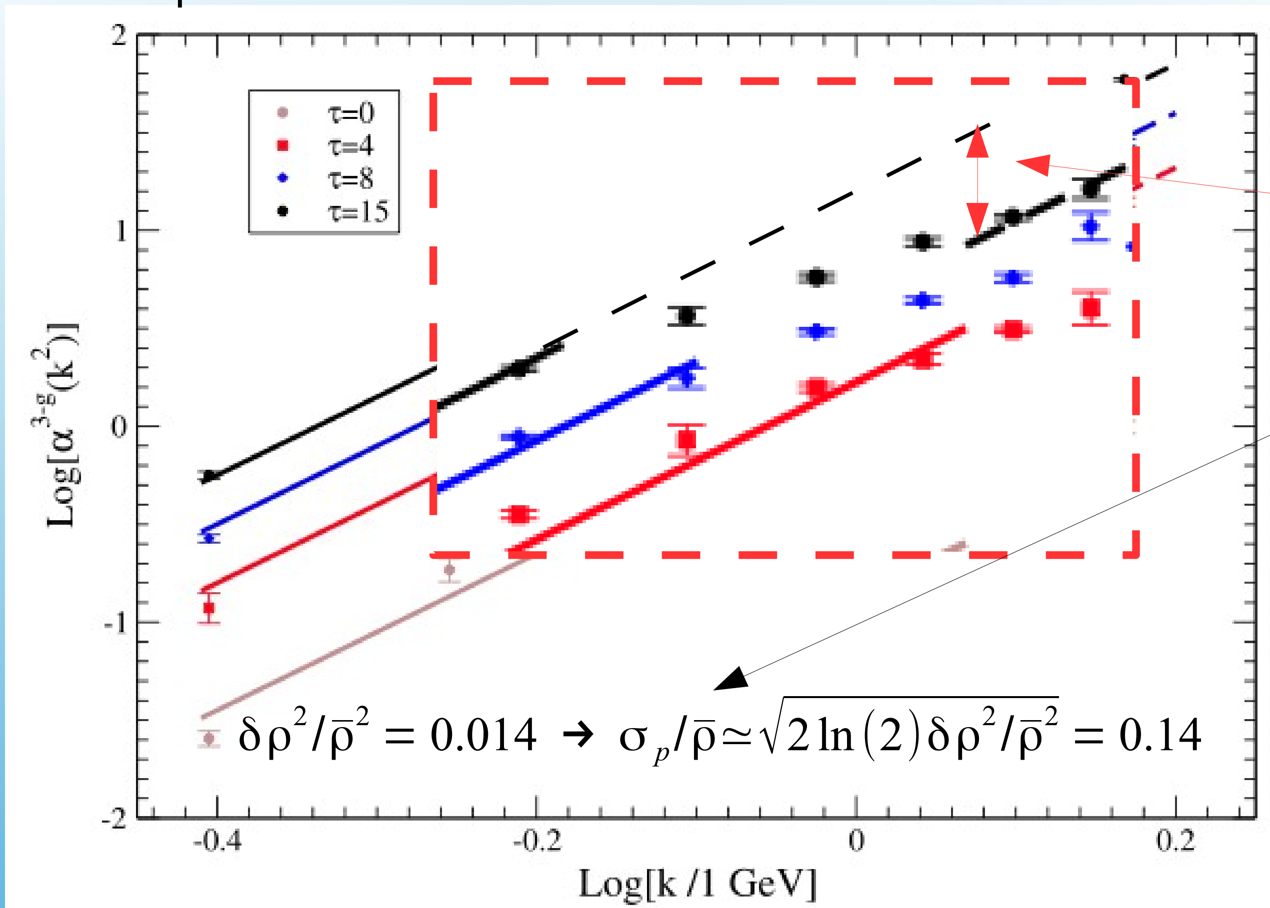

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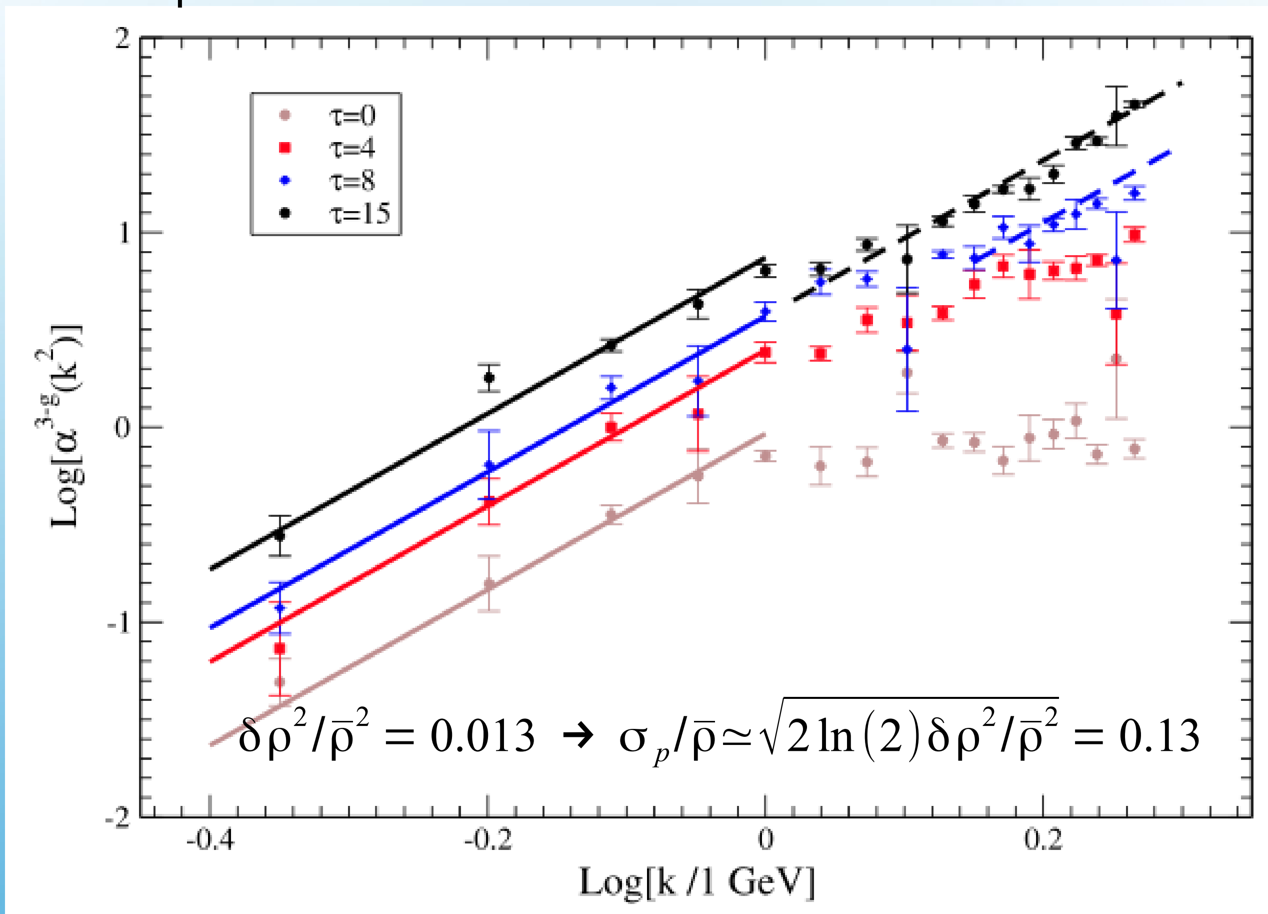
Fairly consistent with lattice estimates made by applying direct instanton detection: $\sigma_p/\bar{\rho} \sim 0.17 - 0.22$ [D.A. Smith, M.J. Teper; PRD58(1998)014505]

The Wilson flow has been proven to be an useful tool to deprive the lattice gauge fields from their short-distance (UV) quantum fluctuations.
The main features observed in the gluon correlations obtained with lattice flown gauge fields can be well described within the multi-instanton approach framework.

The Wilson flow smoothing procedure

$\beta = 1.95$



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Unquenched	4	2.34	–
	8	4.70	6.8(5)
	15	8.84	3.0(2)

The Wilson flow has been proven to be an useful tool to deprive the lattice gauge fields from their short-distance (UV) quantum fluctuations.
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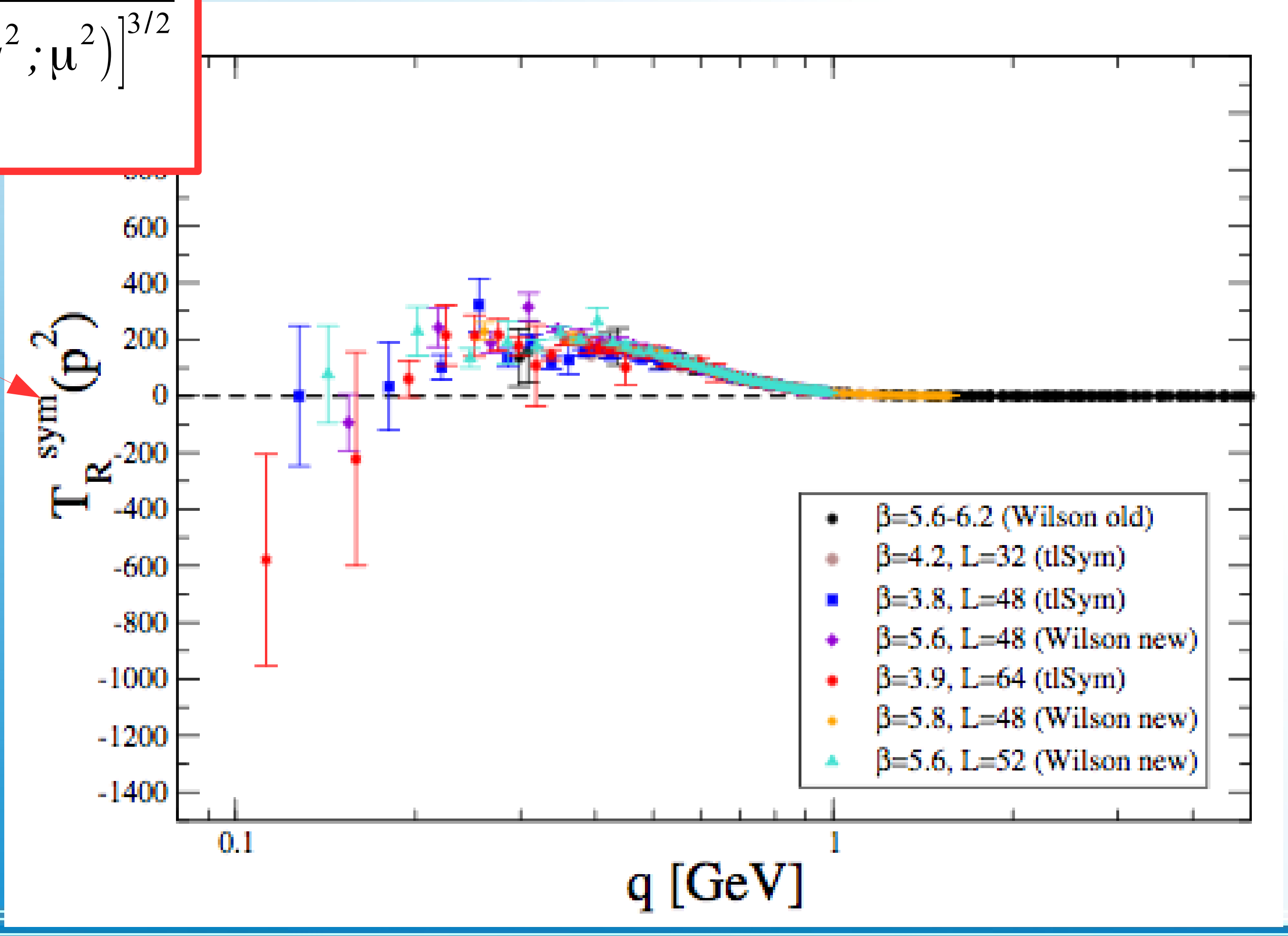
The zero-crossing of the three-gluon vertex

$$g^i(\mu^2)\Gamma^i_{T,R}(q^2;\mu^2) = \frac{g^i(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$$i = sym, asym.$$

$$g^{sym}(q^2) = q^3\frac{T^{sym}(q^2)}{[\Delta(q^2)]^{3/2}}$$

$$g^{asym}(q^2) = q^3\frac{T^{asym}(q^2)}{\Delta(0)[\Delta(q^2)]^{1/2}}$$



Let's then focus (again) on the symmetric case: the form factor appears to change its sign at very deep IR momenta and show then a zero-crossing. This appears to happen below ~0.2 GeV.

The zero-crossing of the three-gluon vertex

M. Tissier, N. Wschebor, PRD84(2011)045018
A.C Aguilar et al.; PRD89(2014)05008
A. Blum et al.; PRD89(2014)061703
G. Eichmann et al.; PRD89(2014)105014
A.K. Cyrol et al.; arXiv:1605.01856[hep-ph]
A. Cucchieri, A. Maas, T. Mendes;
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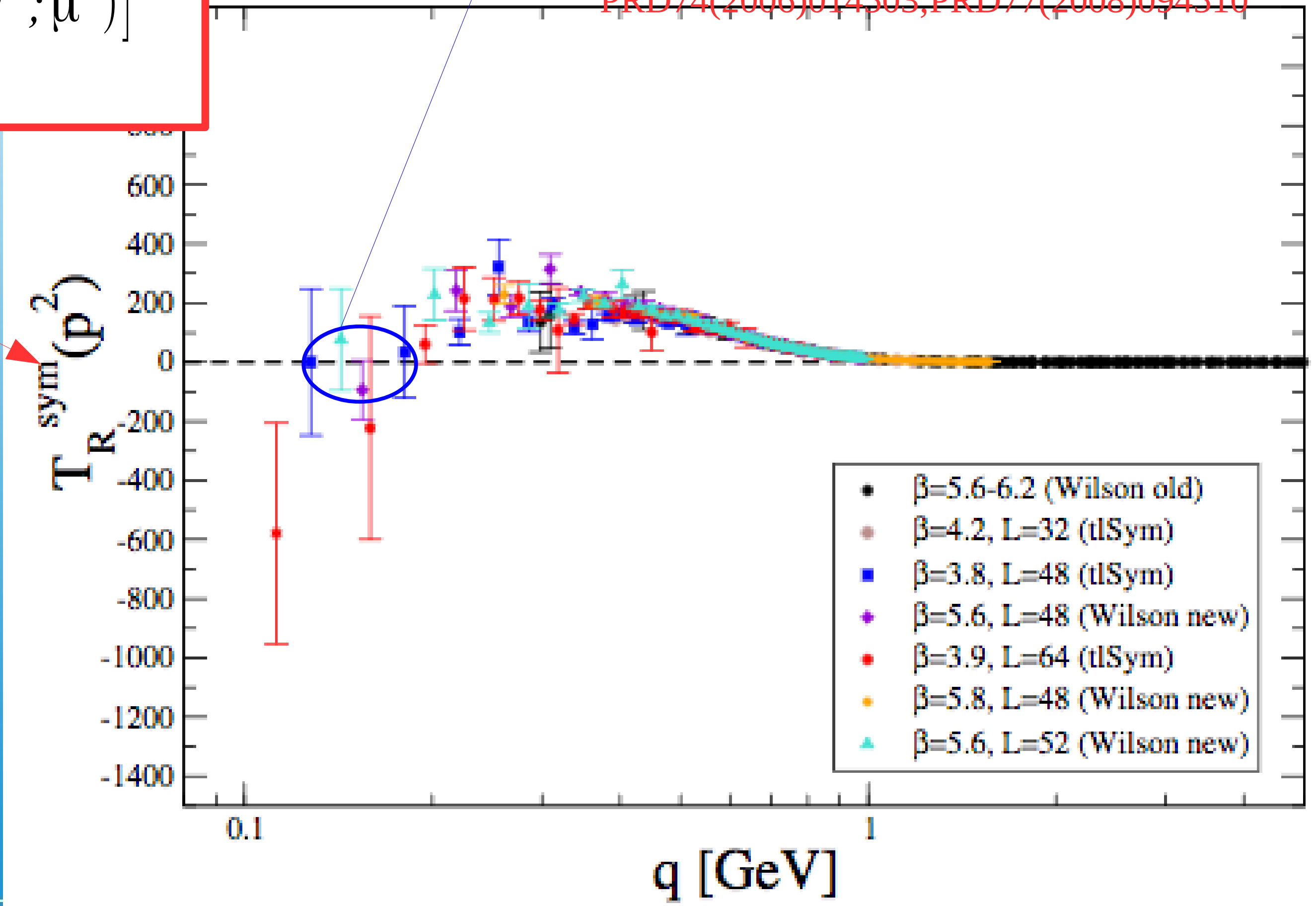
zero-crossing



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$$\boxed{g^i(\mu^2)\,\Gamma^i_{T,R}(q^2;\mu^2) = \frac{g^i(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}} \qquad i = sym, asym.}$$

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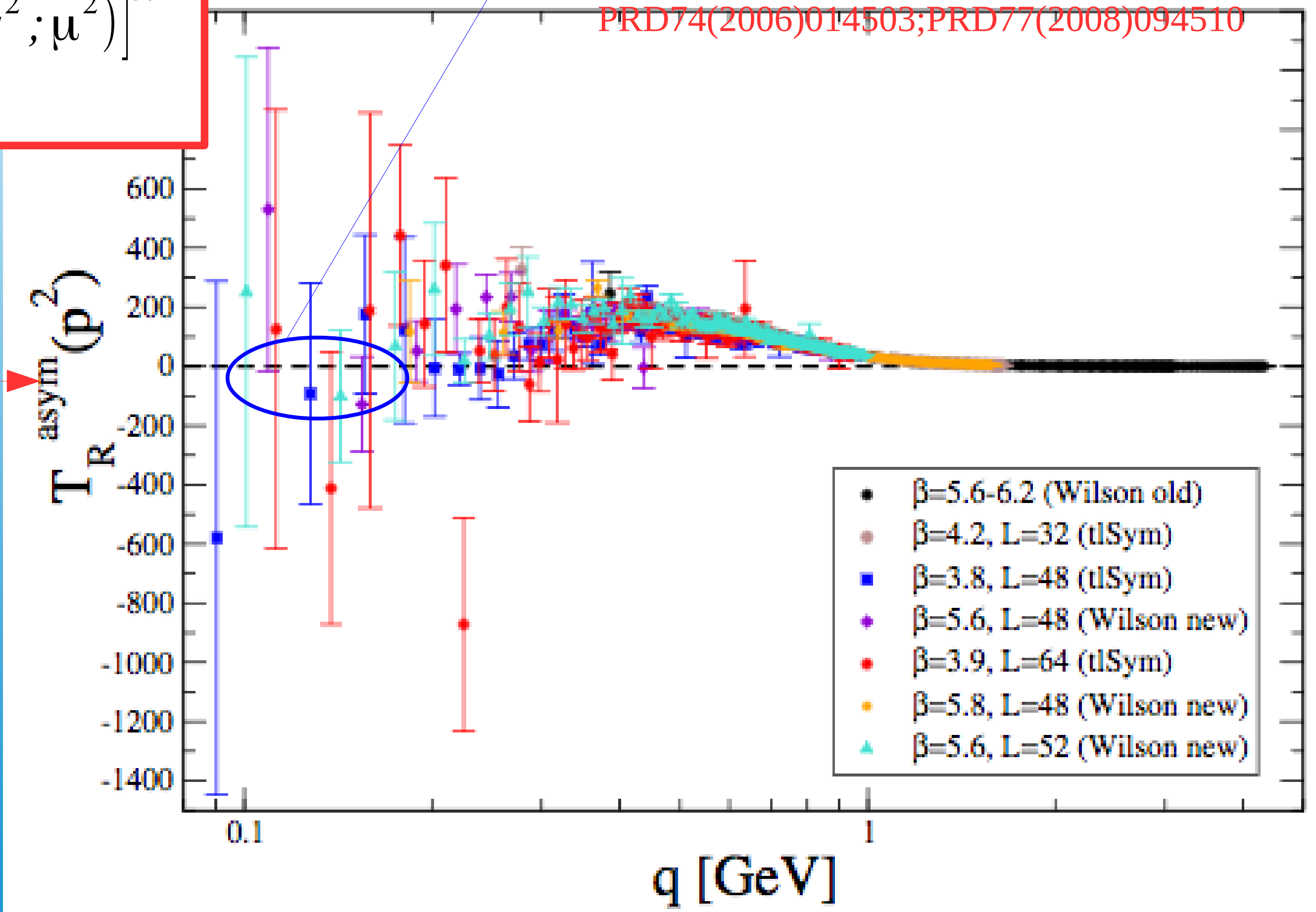
Zero-crossing?

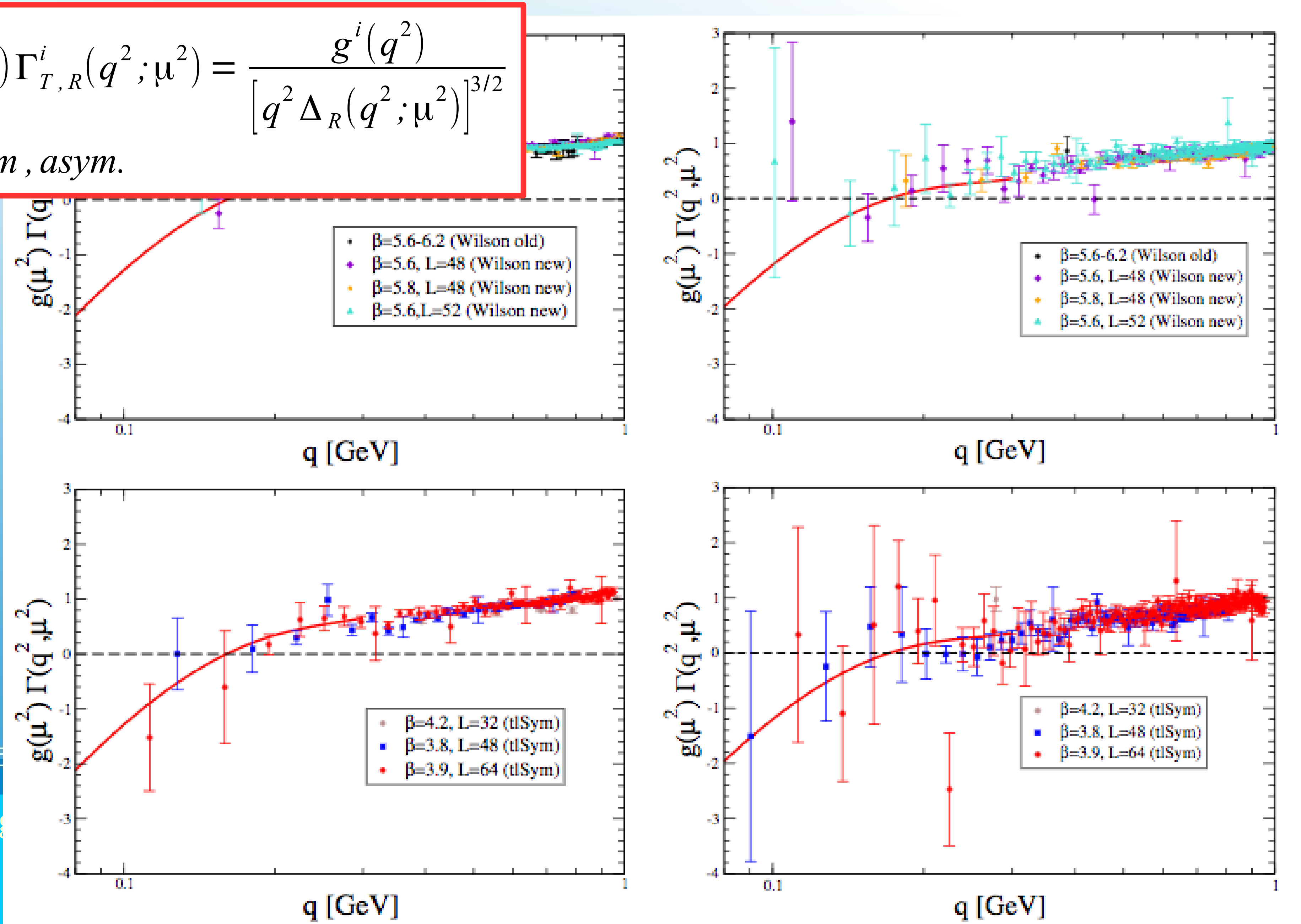


Let's consider now the asymmetric case: the results are much noisier (surely because of the zero-momentum gluon field in the correlation function), although there appear to be strong indications for the happening of the zero-crossing.

The zero-crossing of the three-gluon vertex

$$g^{i}(\mu^{2})\,\Gamma^{i}_{T,R}(q^{2};\mu^{2})=\frac{g^{i}(q^{2})}{\left[q^{2}\Delta_{R}(q^{2};\mu^{2})\right]^{3/2}}$$

$i = sym, asym.$

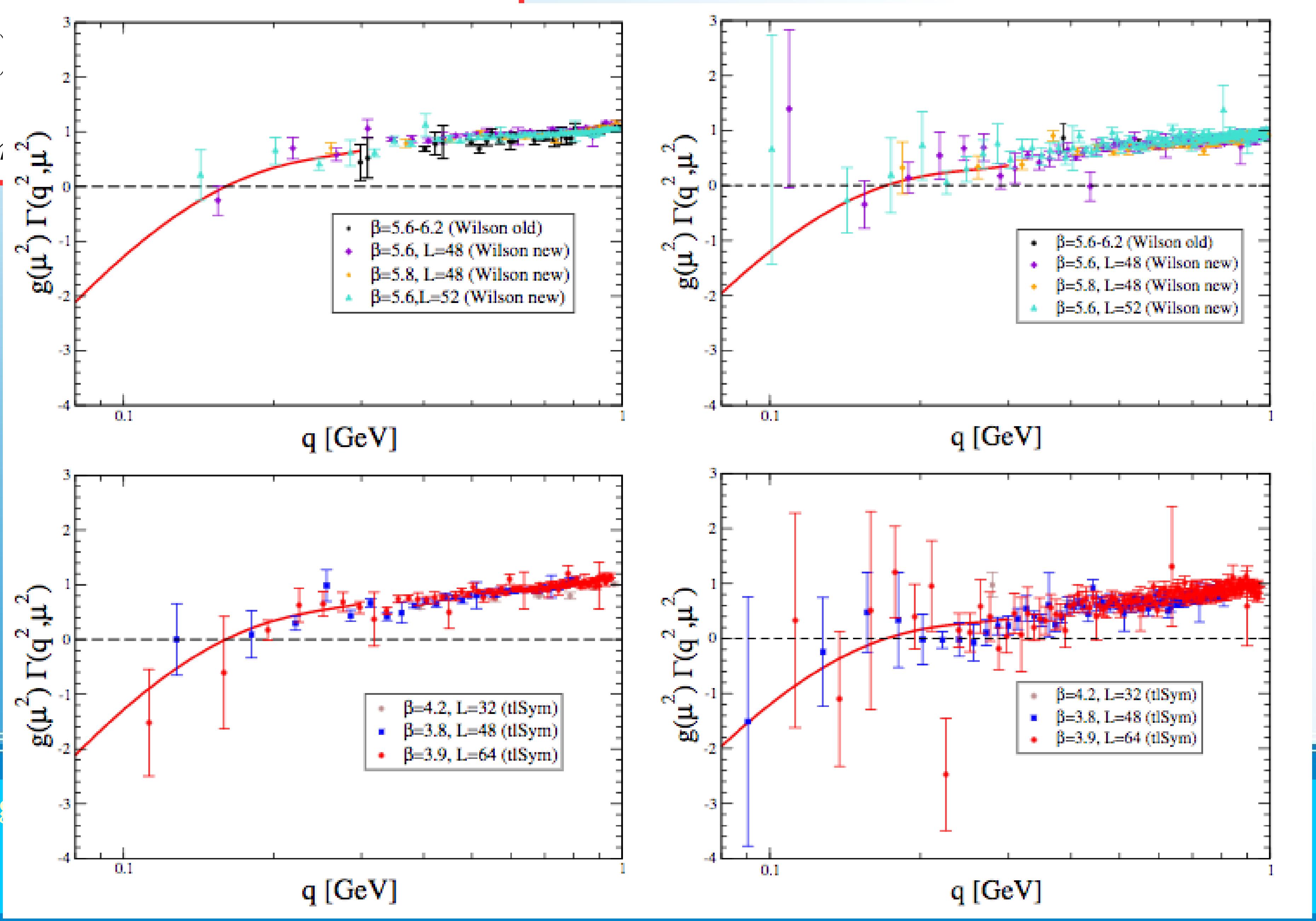


After le
trend is

The zero-crossing of the three-gluon vertex

$g^i(\mu^2$

$i = sym$

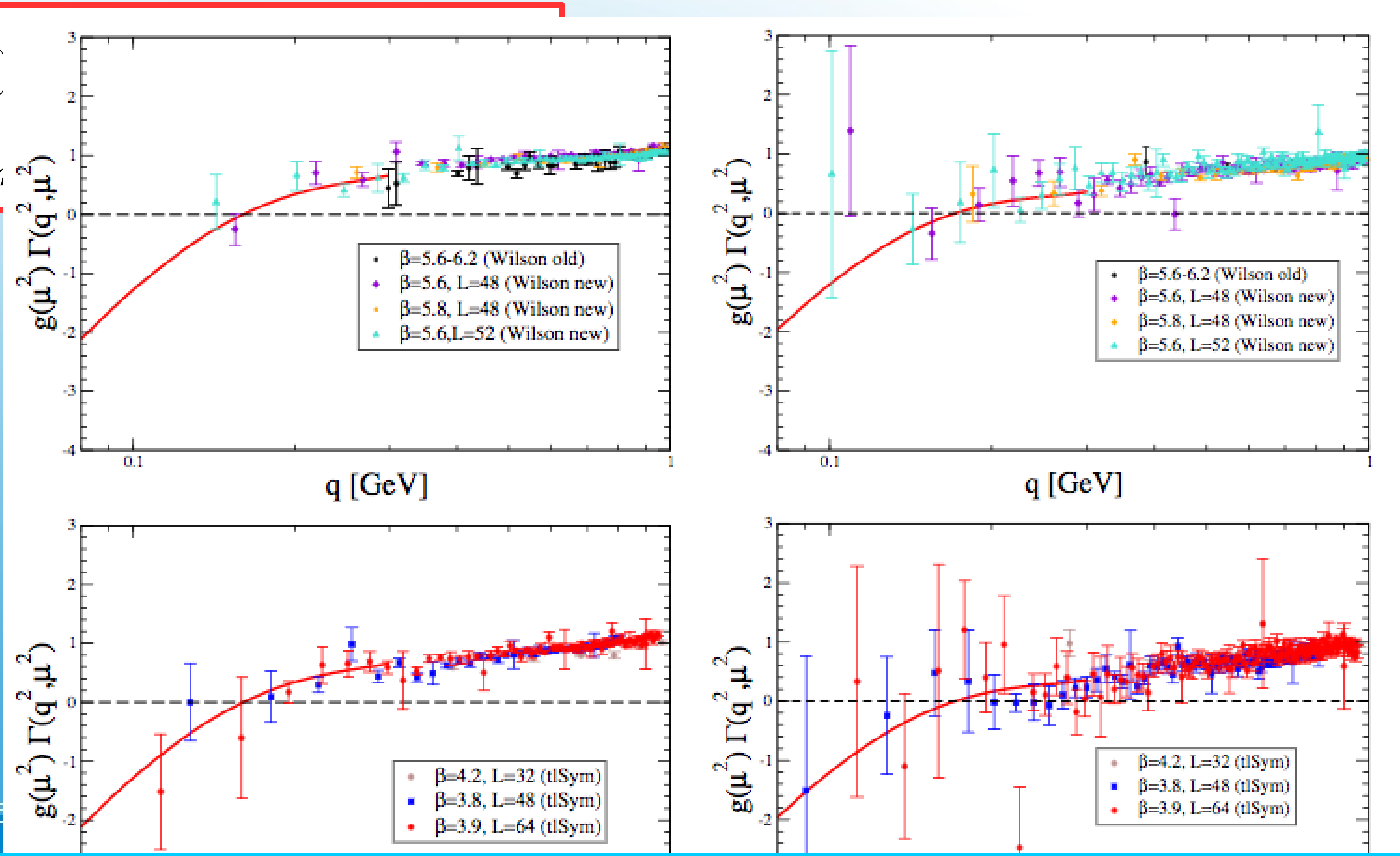


After leg
trend is

The zero-crossing of the three-gluon vertex

$g^i(\mu^2$

$i = sy$

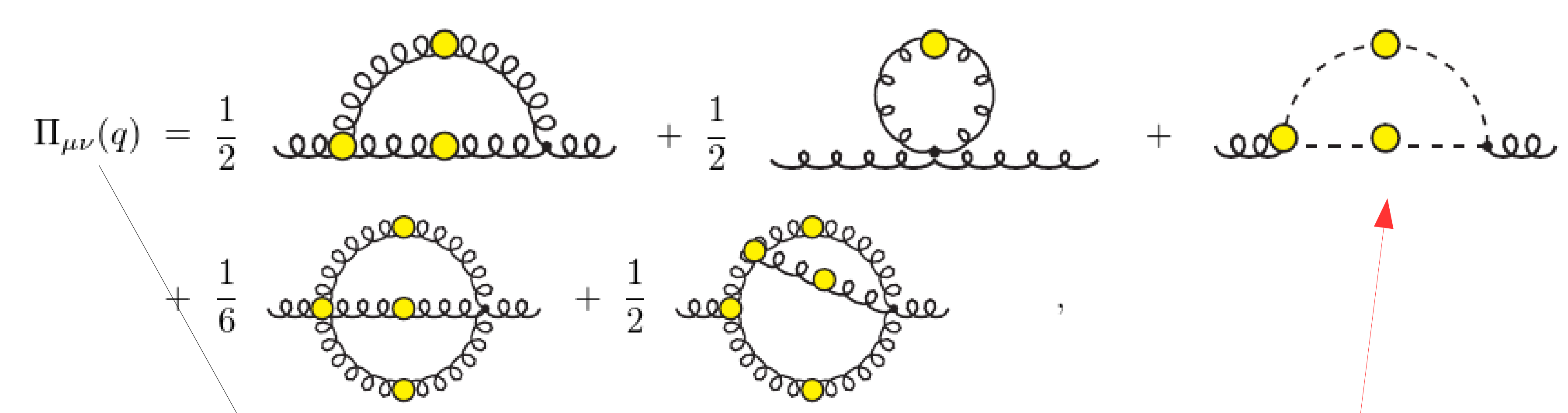


After leg amputation, the 1PI form factor for the tree-level tensor shows clearly the zero-crossing. Tthe trend is the same for both Wilson and tlSym actions and symmetric and asymmetric configurations.

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008

DSE-based explanation:



$$\Delta_R^{-1}(q^2;\mu^2) \underset{q^2\to 0}{=} q^2\left[a + b\log\frac{q^2+m^2}{\mu^2} + c\log\frac{q^2}{\mu^2}\right] + m^2,$$

$$\Pi_c(q^2) = \frac{g^2 C_A}{6} q^2 F(q^2) \int_k \frac{F(k^2)}{k^2(k+q)^2},$$

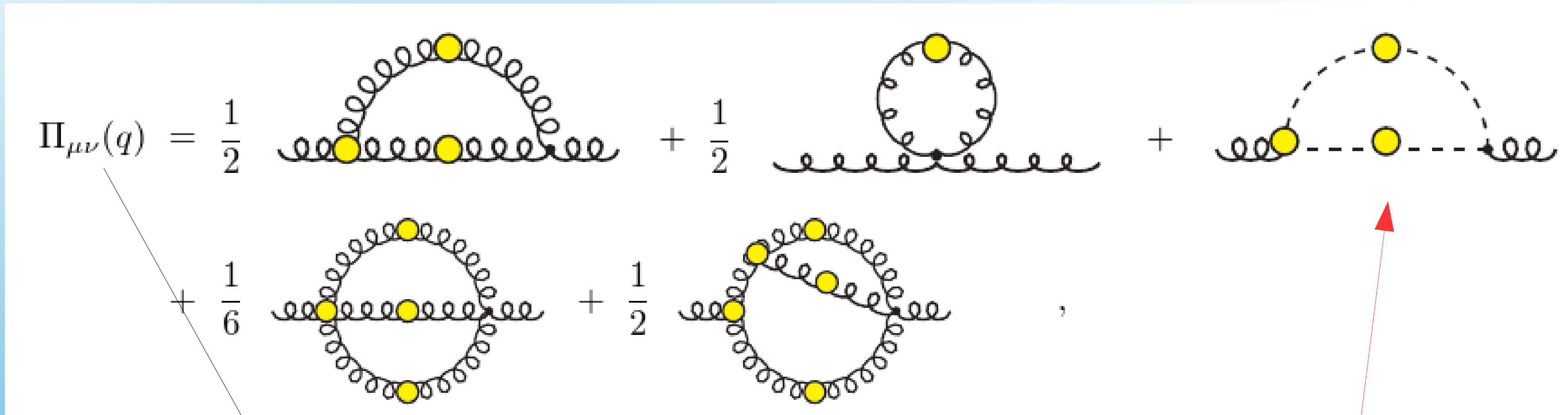
In PT-BFM truncation:

$$\Gamma_{T,R}^{i,(B)}(p^2;\mu^2) \underset{p^2/\mu^2 \ll 1}{\simeq} F_R(0;\mu^2)\frac{\partial}{\partial p^2}\Delta_R^{-1}(p^2;\mu^2) \; + \; \ldots$$

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008
Ph. Boucaud et al.; PRD95(2017)114503

DSE-based explanation:



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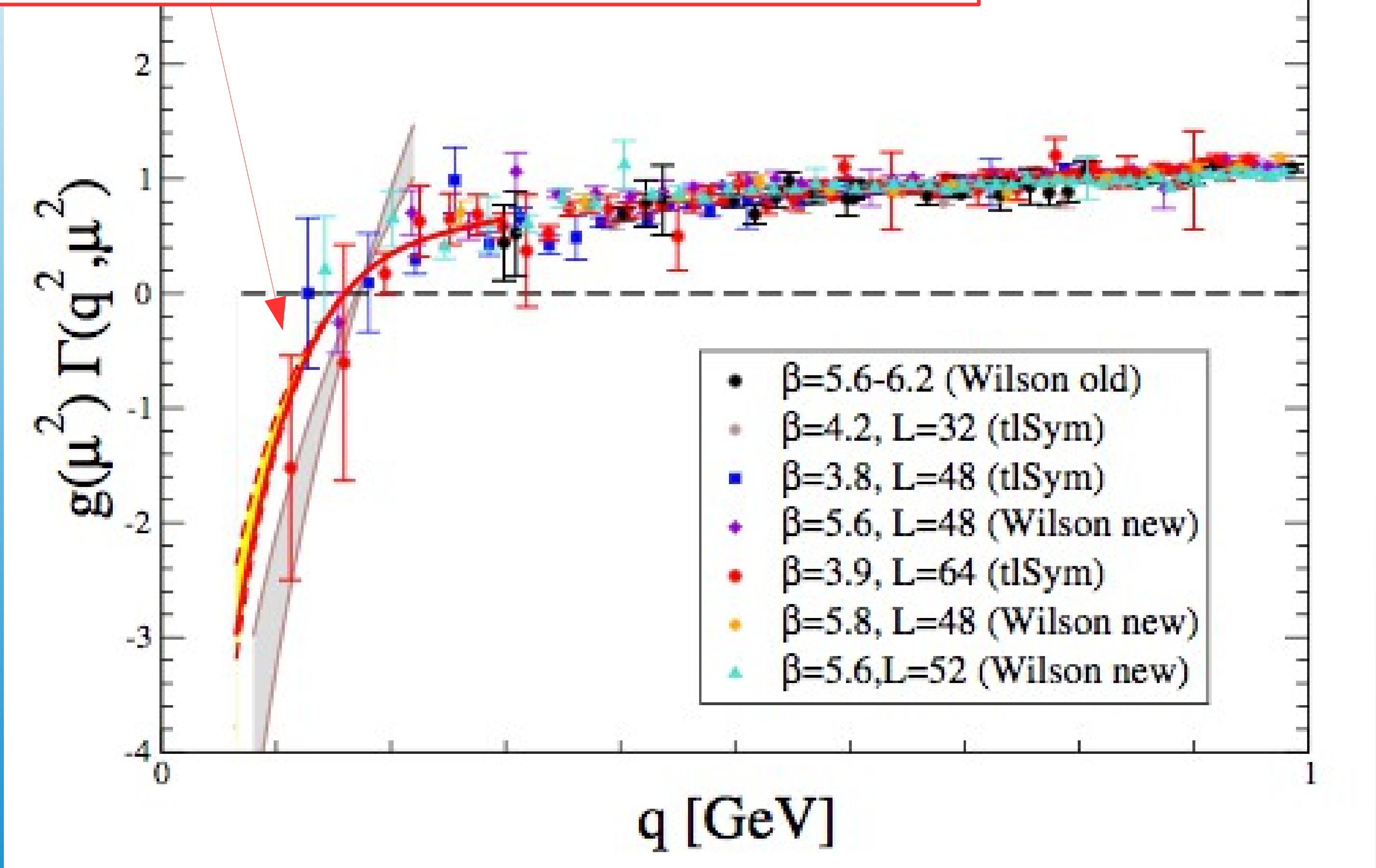
A logarithmic divergent contribution at vanishing momentum, pulling down the 1PI form factor and generating a zero crossing, can be understood with a DSE analysis.

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008
Ph. Boucaud et al.; PRD95(2017)114503

$$g_R^i(\mu^2)\Gamma_R^i(p^2;\mu^2) \;=\; a_{ln}^i(\mu^2)\ln\frac{p^2}{\mu^2} \;+\; a_0^i(\mu^2) \;+\; a_2^i(\mu^2)\;p^2\ln\frac{p^2}{M^2} \;+\; o(p^2)$$

i = symmetric



We can thus perform a fit, only over a deep IR domain, of our data to the DSE-based formula and describe the behaviour of the 1PI form factor.

The zero-crossing of the three-gluon vertex

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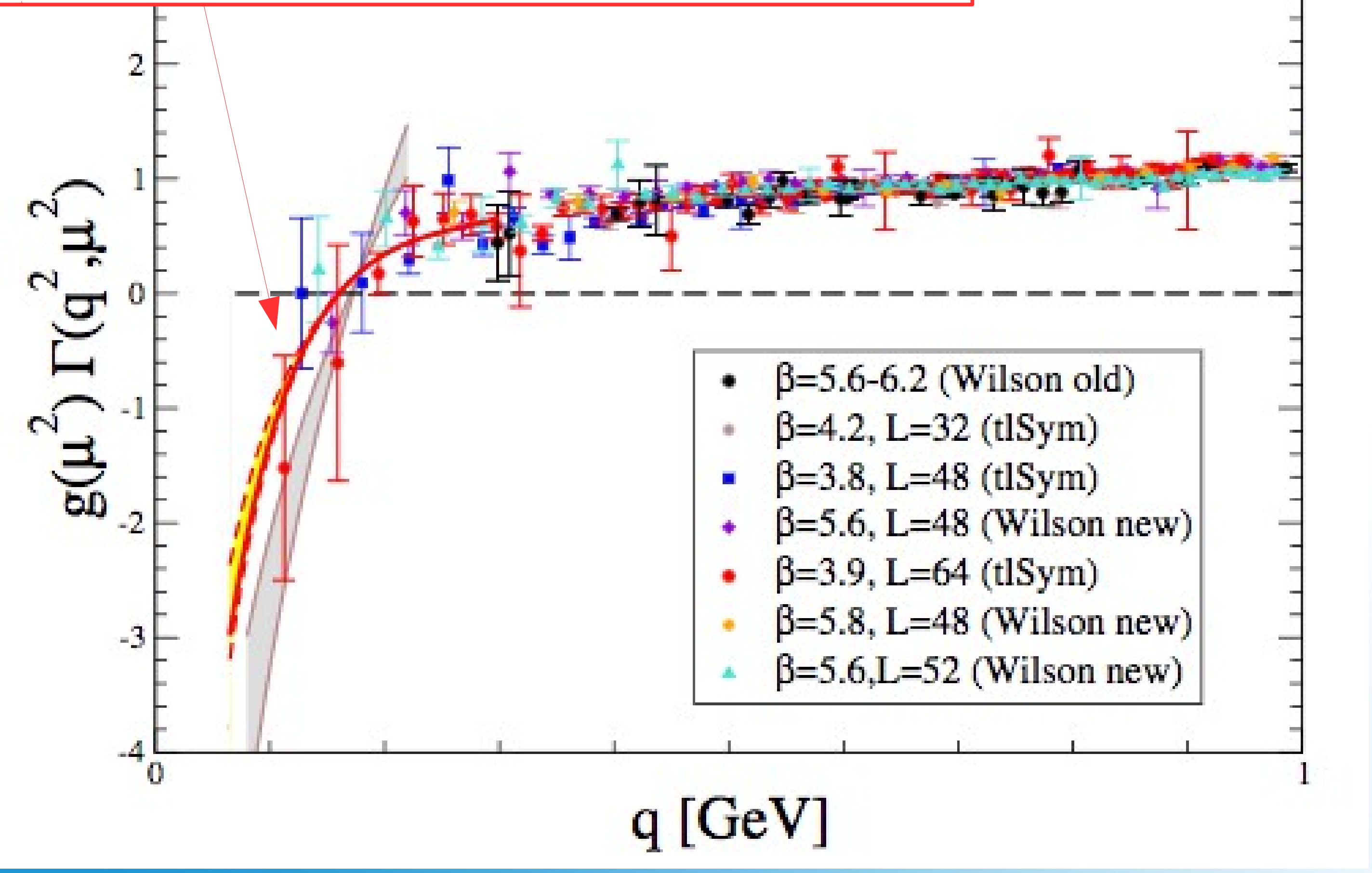
$$g_R^i(\mu^2)\,c\,F_R(0,\mu^2)$$

Consistent with direct large-volume lattice evaluations of the gluon and ghost two-point Green functions.



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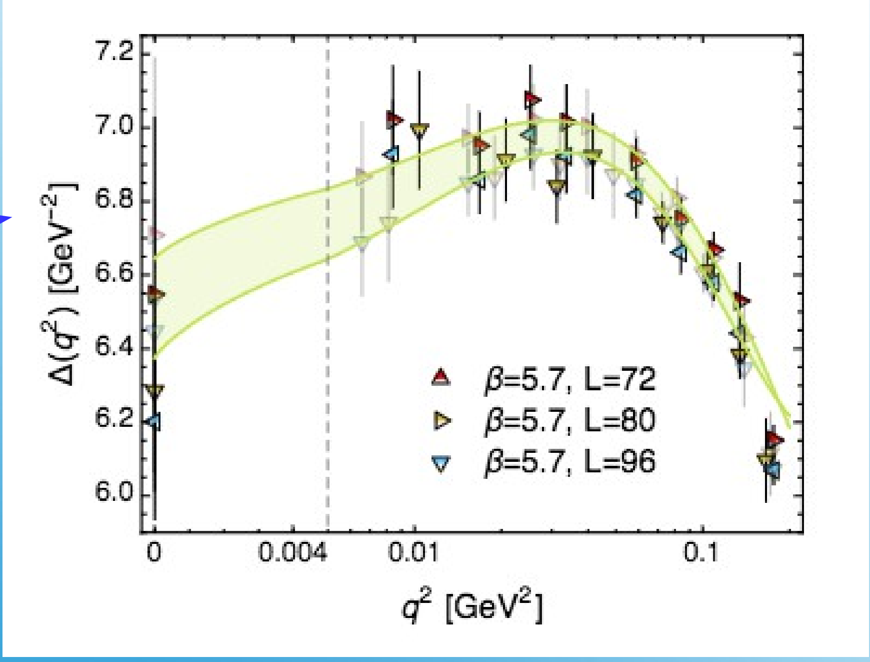

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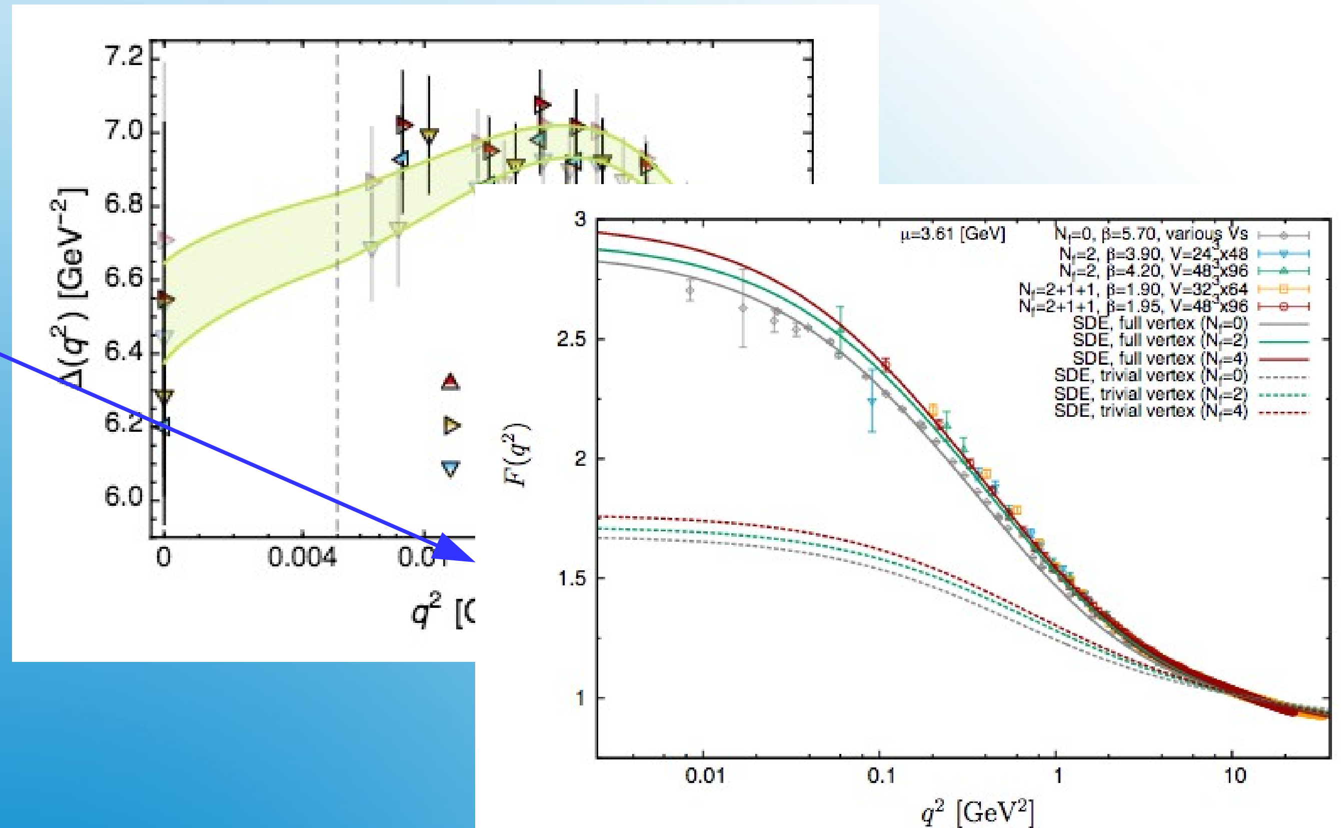

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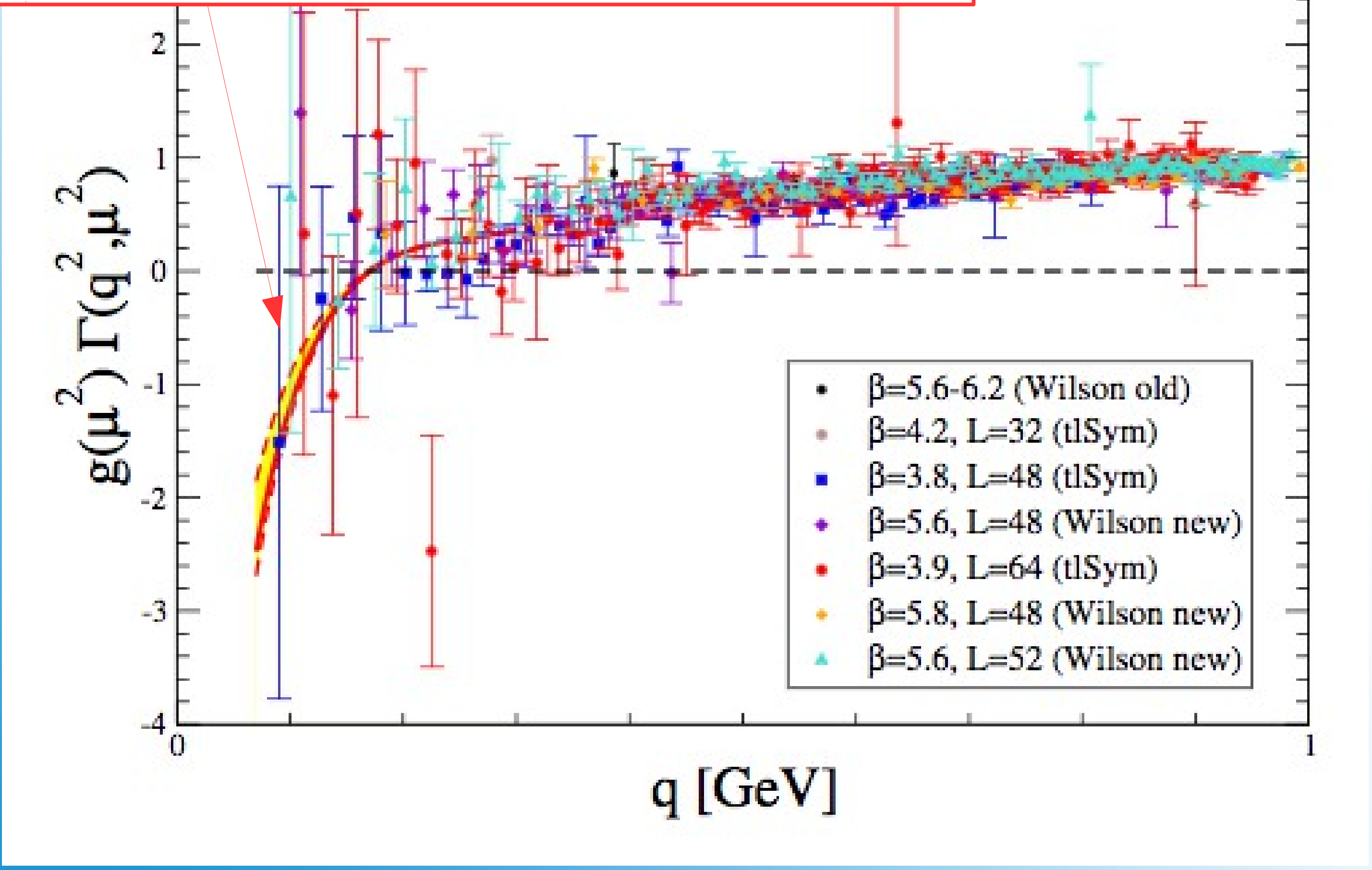
i = asymmetric

$$g_R^i(\mu^2)\, c\, F_R(0,\mu^2)$$

Consistent with direct large-volume lattice evaluations of the gluon and ghost two-point Green functions.



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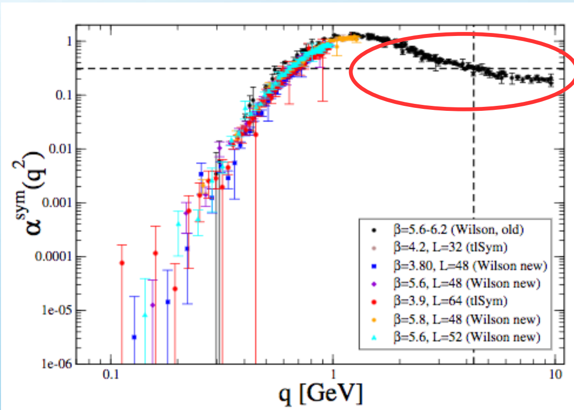
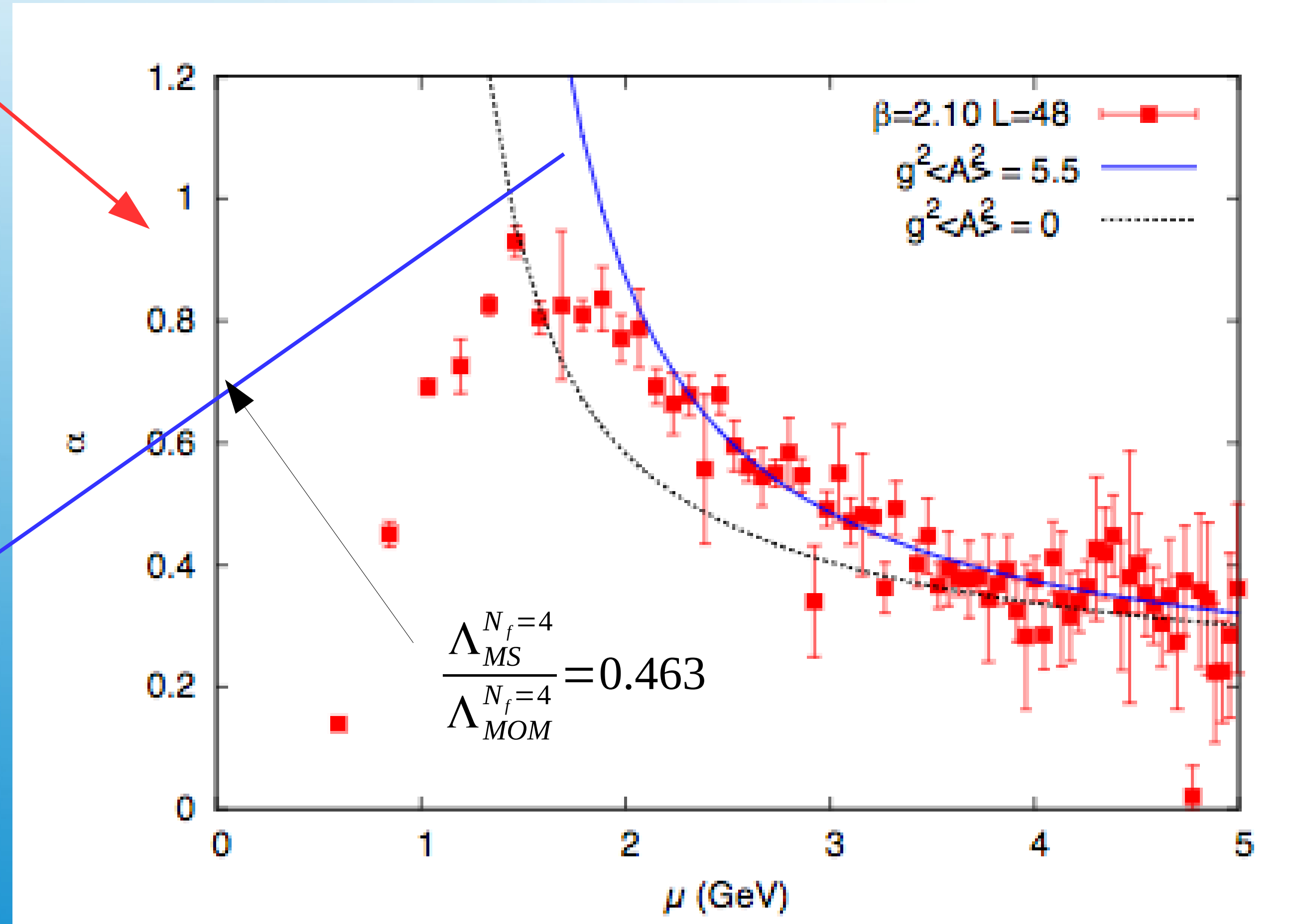
The three-gluon running coupling:

ETMC Nf=2+1+1





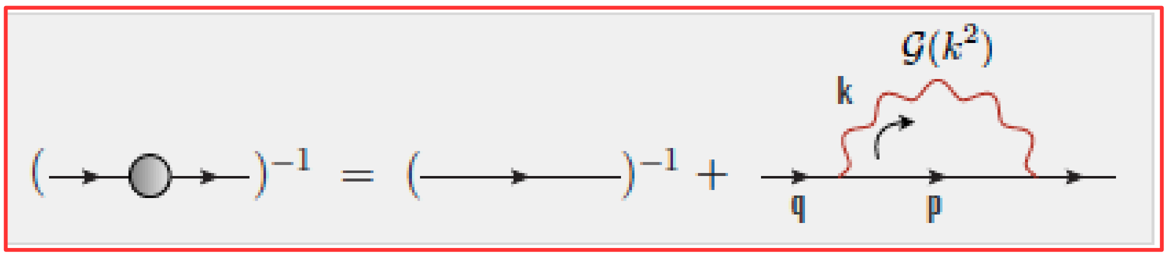
$$\Lambda_{MS}^{N_f=4}=314\,MeV$$

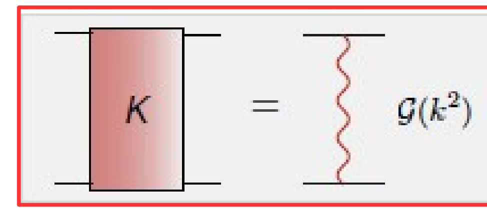
A final remark on some work in progress: the UV domain gives direct access to the strong running coupling in a particular scheme that can be properly translated to MS. combining different Green's functions, a reliable prediction can be obtained!!!

Quark's gap equation

Use Rainbow-Ladder truncation:

One-gluon exchange effective kernel + Tree-level quark-gluon vertex





$$S^{-1}(p) = Z_2\,(i\gamma\cdot p + m^{\mathrm{bm}}) + \Sigma(p)\,,$$

$$\Sigma(p) = Z_1 \int_{dq}^{\Lambda} g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\Gamma_\nu(q,p),$$

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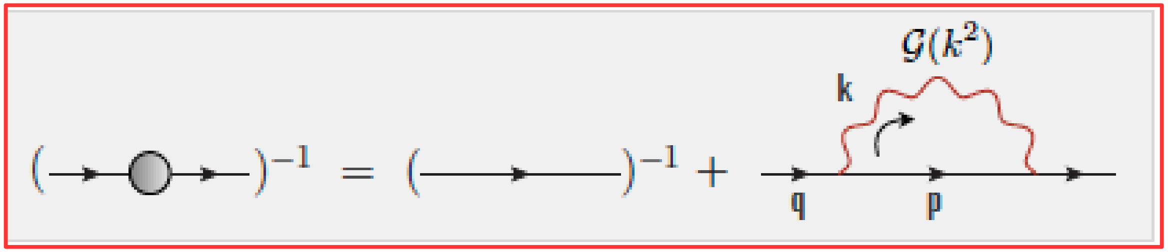

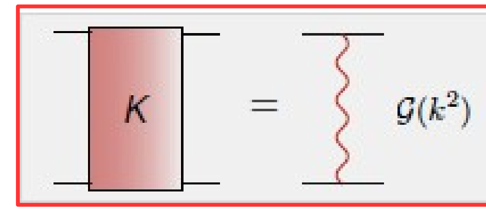

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$$4\pi\; I(k^2)\,\frac{1}{k^2}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)$$

$$\Gamma_\mu = \gamma_\mu$$

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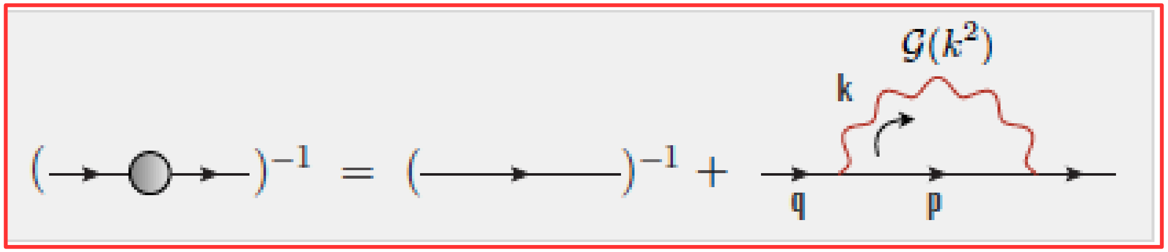

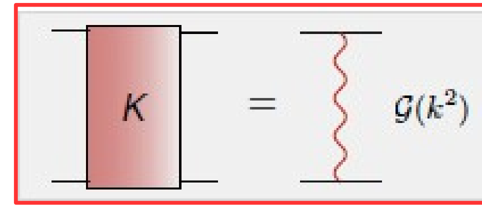

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Model parameters:

$\Lambda = 0.234\ GeV$

$\zeta = 0.87\ GeV$

$\omega \in [0.4, 0.6]\ GeV$

Quark's gap equation

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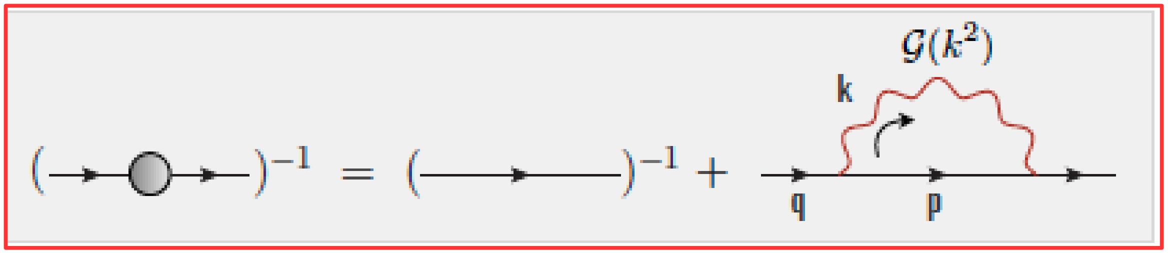

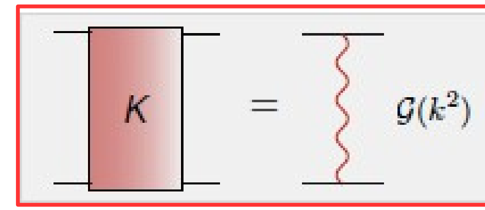

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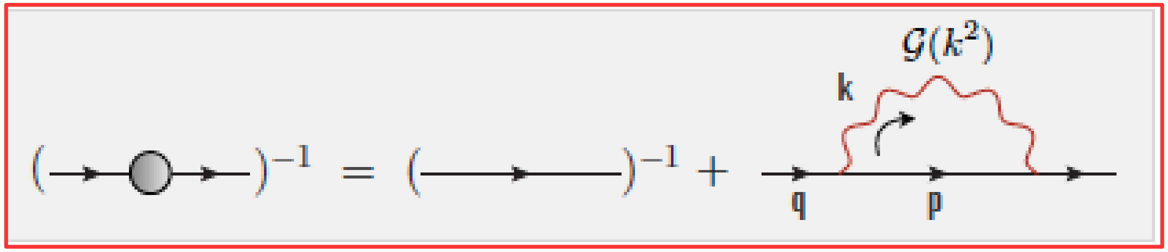
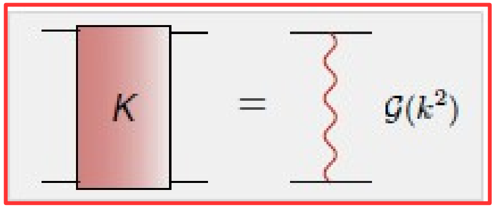
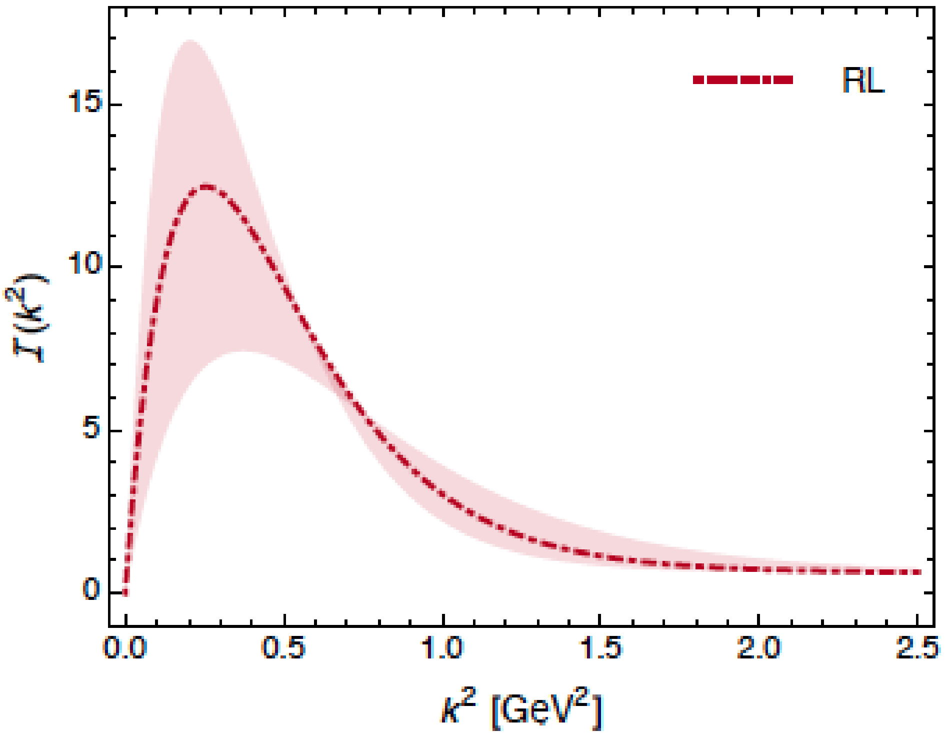


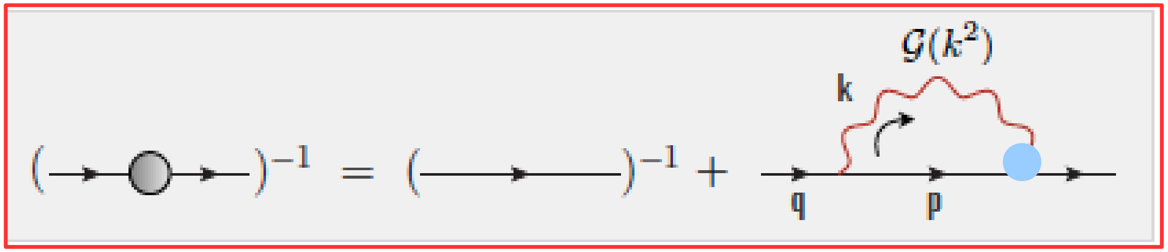


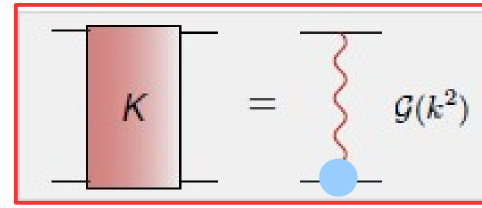


Quark's gap equation

Beyond Rainbow-Ladder truncation:

One-gluon exchange effective kernel + ~~Tree-level~~ quark-gluon vertex





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- Ball-Chiu vertex [PRD(22)1980]
- Anomalous Chromomagnetic vertex

Consistent with both linear and transverse STI

$$\Gamma_\mu = \Gamma_\mu^{BC} + \Gamma_\mu^{ACM}$$

Model parameters:

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$\varsigma = 0.55\ GeV$

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Quark's gap equation

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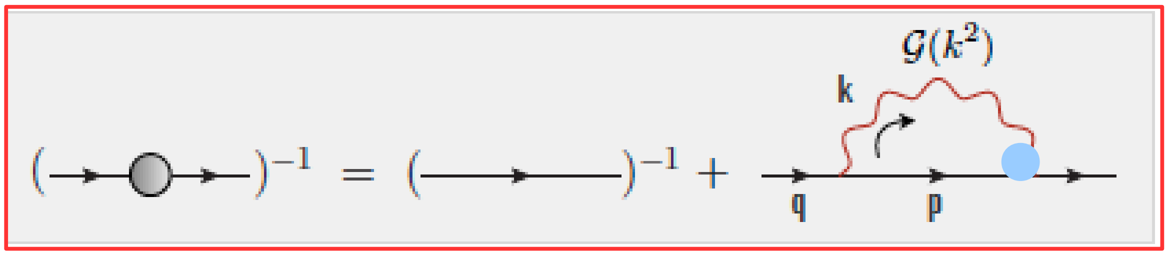

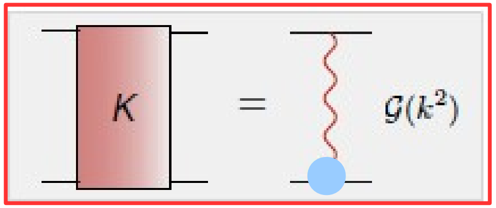

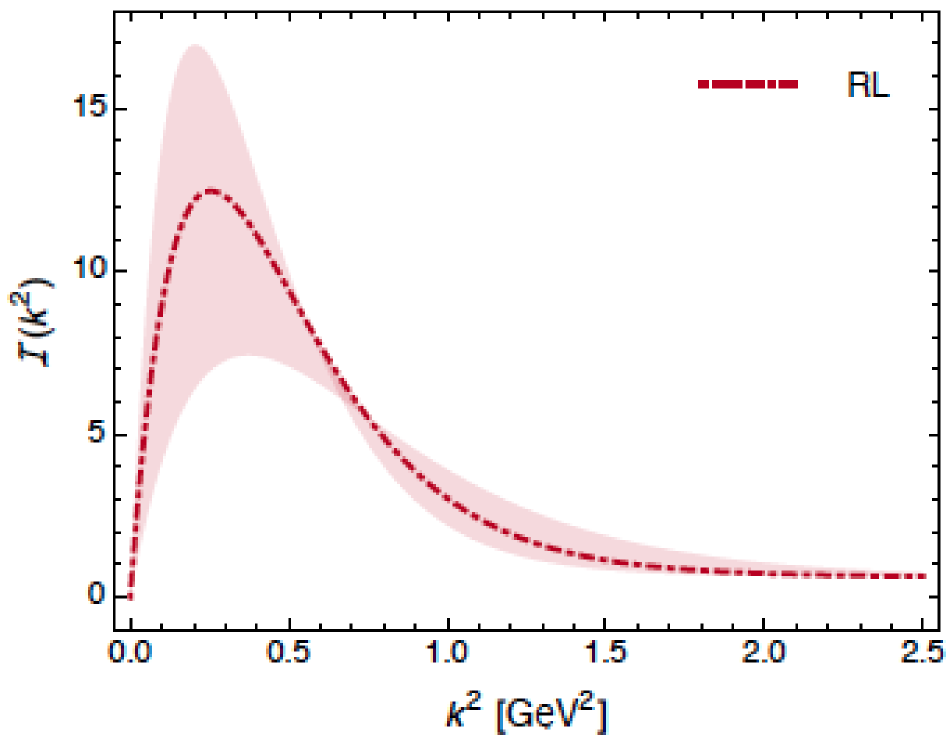

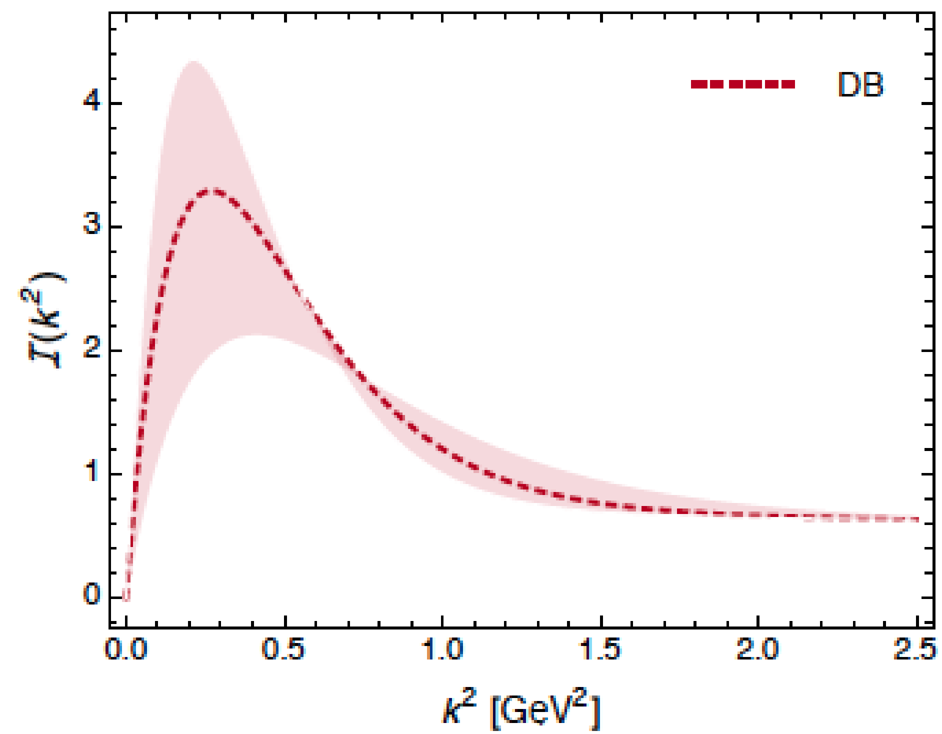

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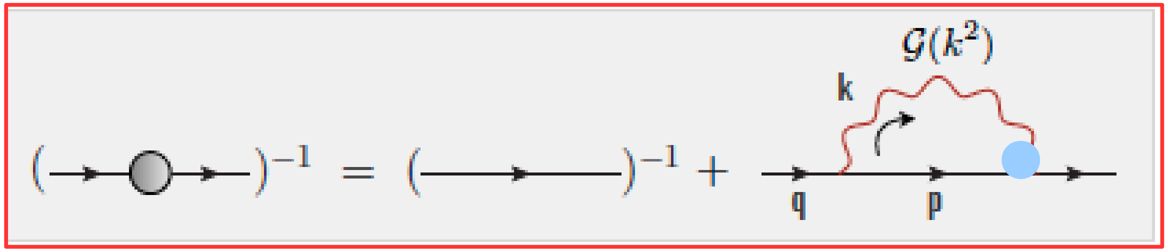
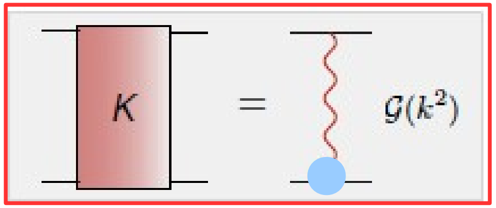
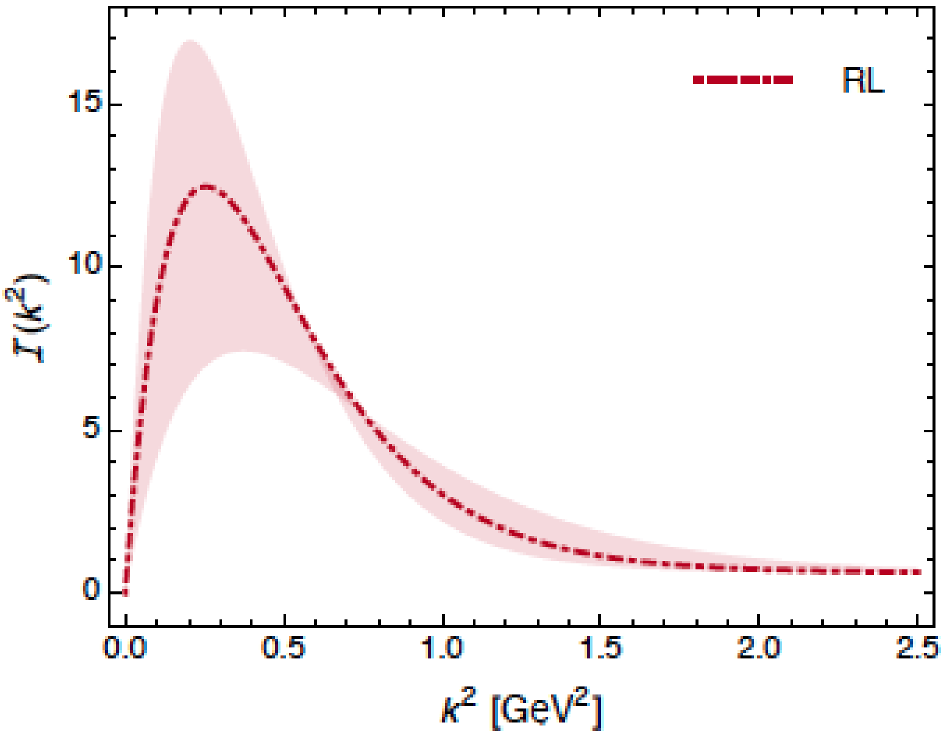
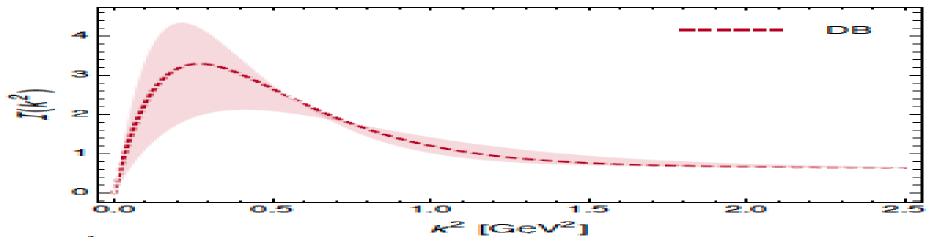


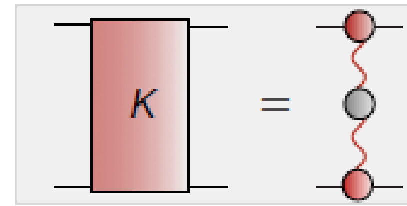






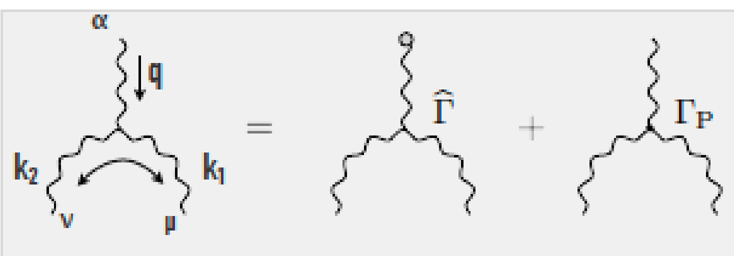
Quark's gap equation: RGI interaction

Universal (process-independent) contribution:
originates entirely from the gauge sector



Fundamental quantities: PT-BFM propagators/vertices
satisfy Abelian-like Slavnov-Taylor (ST) identities

How to get them?
use the PT algorithm
Cornwall, Papavassiliou, PRD 40 (1989)

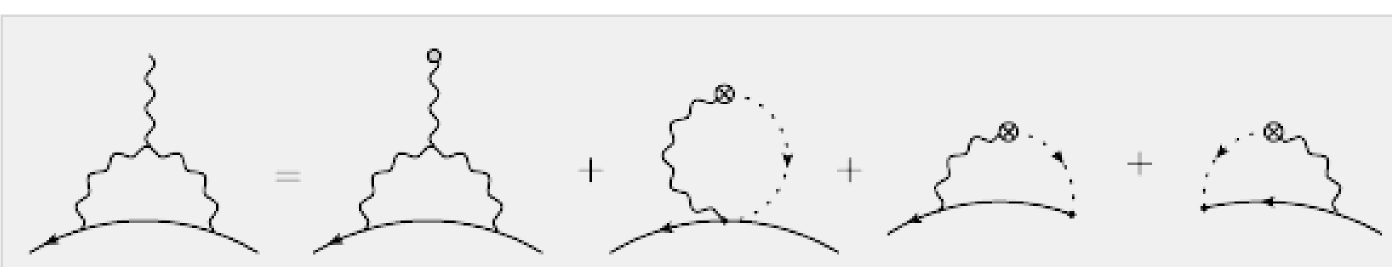


$$\widehat{\Gamma}^{\alpha\mu\nu} = (k_2 - k_1)^\alpha g^{\mu\nu} + 2q^\nu g^{\alpha\mu} - 2q^\mu g^{\alpha\nu}$$

$$\Gamma_{\rm P}^{\alpha\mu\nu} = k_1^\mu g^{\alpha\nu} - k_2^\nu g^{\alpha\mu}$$

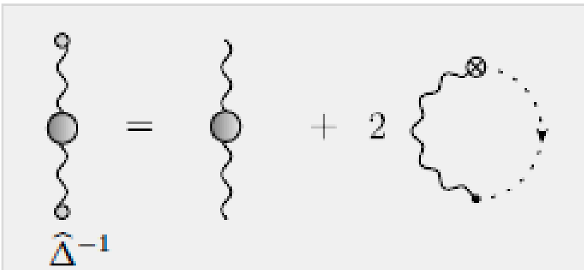
- **longitudinal momenta**
  trigger elementary Ward identities

Apply the PT to the quark-gluon vertex
one loop result:

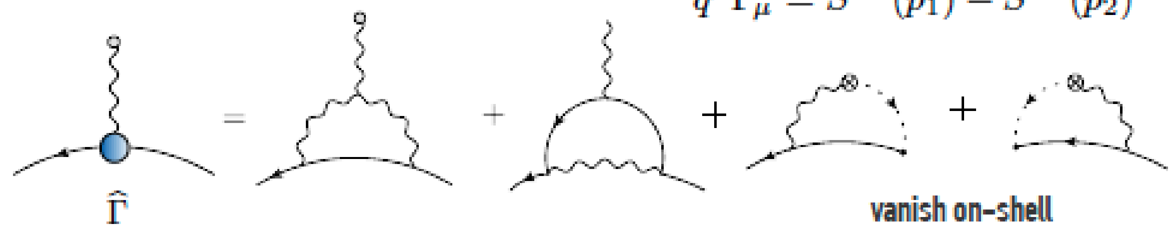


Quark's gap equation: RGI interaction

Allot pieces to different Green's functions
construct $\widehat{\Delta}$ and $\widehat{\Gamma}_\mu$



$$q^\mu \widehat{\Gamma}_\mu = S^{-1}(p_1) - S^{-1}(p_2)$$



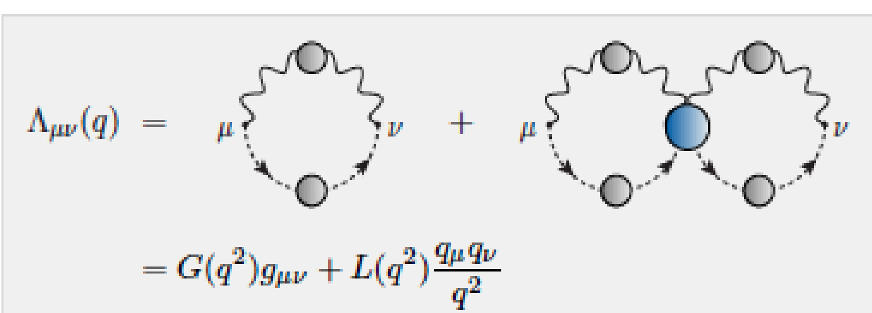
Crucial all-order equivalence: PT=BFM
yields Feynman rules for systematic calculation

$$\widehat{\Delta} \sim \frac{1}{q^2[1 + bg^2 \log q^2/\mu^2]}; \qquad b = 11C_A/48\pi^2$$

- **Absorbs all the RG logs** as the photon in QED
- **Renormalizes as** Z_g^{-2}

An additional equivalence holds: antiBRST+BRST=BFM
plethora of symmetry identities, in particular BQ identities
D. Binosi, Quandri, PRD88(2013)

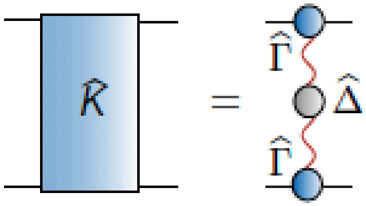
$$\Delta(q^2) = [1 + G(q^2)]^2 \widehat{\Delta}(q^2)$$



- ***G* special PT-BFM function:** determined by ghost-gluon dynamics
- **Combination 1+*G* appears in *all* BQIs** fundamental non-Abelian quantity
- ***G* is related (Landau gauge) to the ghost dressing:** use ghost gap equation to constrain *1+G, L*

$$F^{-1}(q^2) = 1 + G(q^2) + L(q^2)$$

Quark's gap equation: RGI interaction



Convert vertices/propagators into PT-BFM ones
new RG invariant combination appears

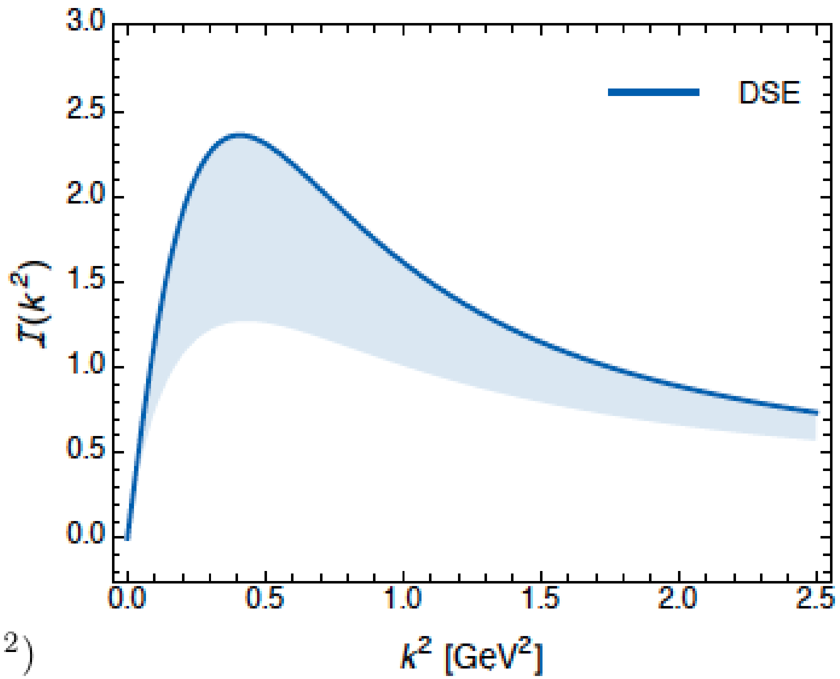
$$\widehat{d}(k^2) = \alpha(\mu^2)\widehat{\Delta}(k^2;\mu^2)$$

Use symmetry identity
to identify the interaction strength

A.C Aguilar, D. Binosi, J. Papavassiliou, J. R-Q, PRD90(2009)
D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, PLb742(2015)

$$\mathcal{I}(k^2) = k^2\widehat{d}(k^2) = \left[\frac{1}{1-L(q^2)F(q^2)}\right]^2 \alpha_T(q^2)\ .$$

$$\widehat{d}(k^2) = \frac{\alpha(\mu^2)\Delta(k^2;\mu^2)}{[1+G(k^2;\mu^2)]^2}$$



1+*G* and *L* determined by their own SDEs
under simplifying assumptions:

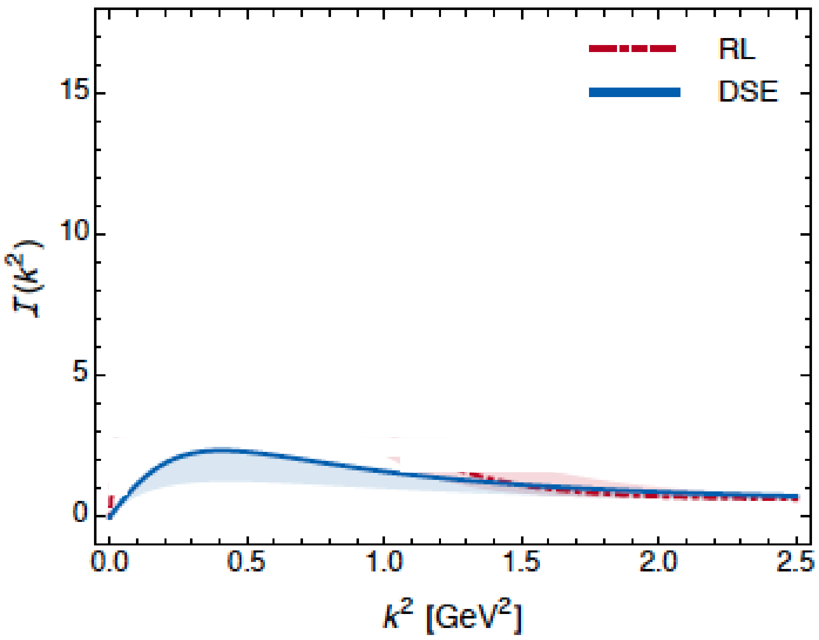
$$1+G(p^2) = Z_c - g^2\int_k \left[2+\frac{(k\cdot p)^2}{k^2p^2}\right] B_1(k)\Delta(k)\frac{F((k+p)^2)}{(k+p)^2},$$

$$L(p^2) = -g^2\int_k \left[1-4\frac{(k\cdot p)^2}{k^2p^2}\right] B_1(k)\Delta(k)\frac{F((k+p)^2)}{(k+p)^2}.$$

$$F^{-1}(q^2) \ = \ Z_c - 3\, g^2\int_k \left[1-\frac{(k\cdot p)^2}{k^2p^2}\right] B_1(k)\Delta(k)\frac{F((k+p)^2)}{(k+p)^2}$$

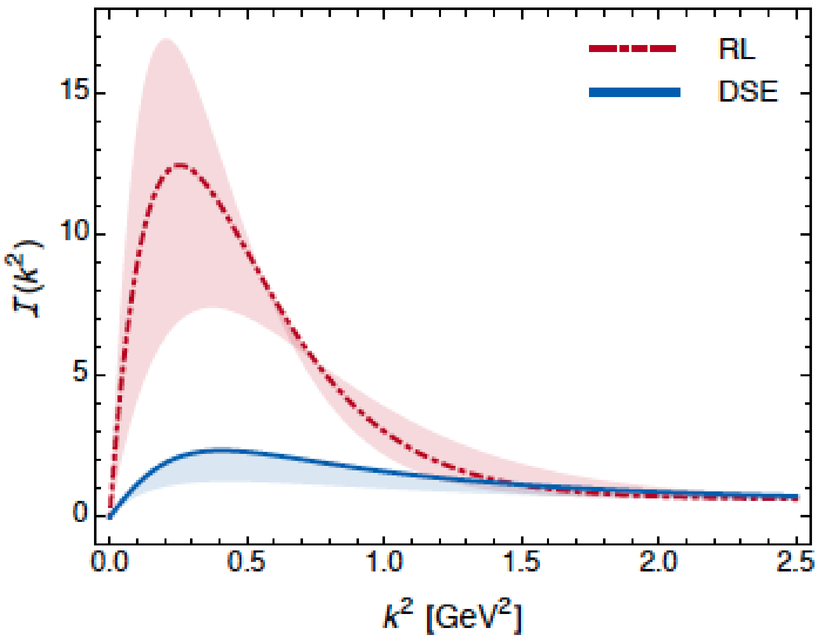
- **Main source of uncertainties:**
  needs assumptions on ghost vertex behavior
- **Parametrized by δ∈[0,1]**
  lower bound (δ=0): *1/F=1+G*

Top-down vs. Bottom-up approaches



D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, PLB 742 (2015)

Top-down vs. Bottom-up approaches



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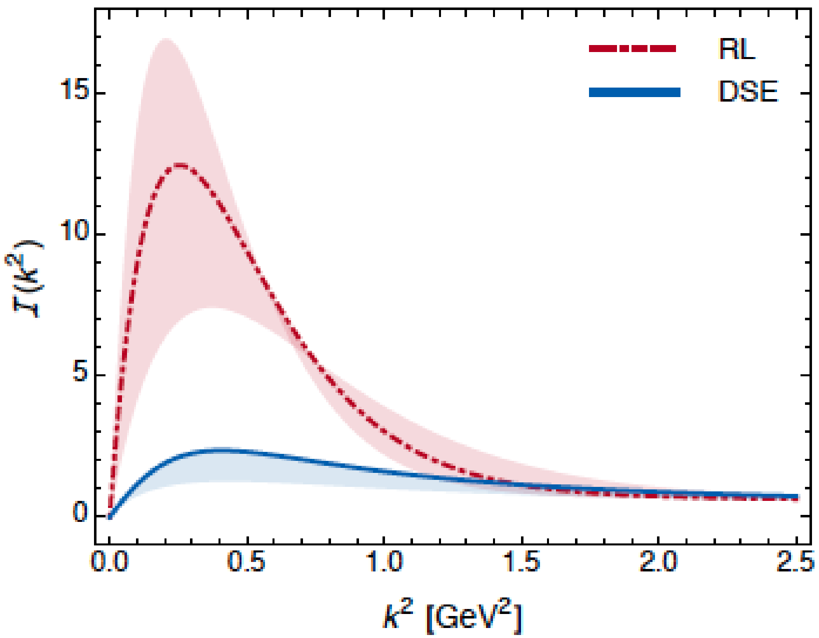
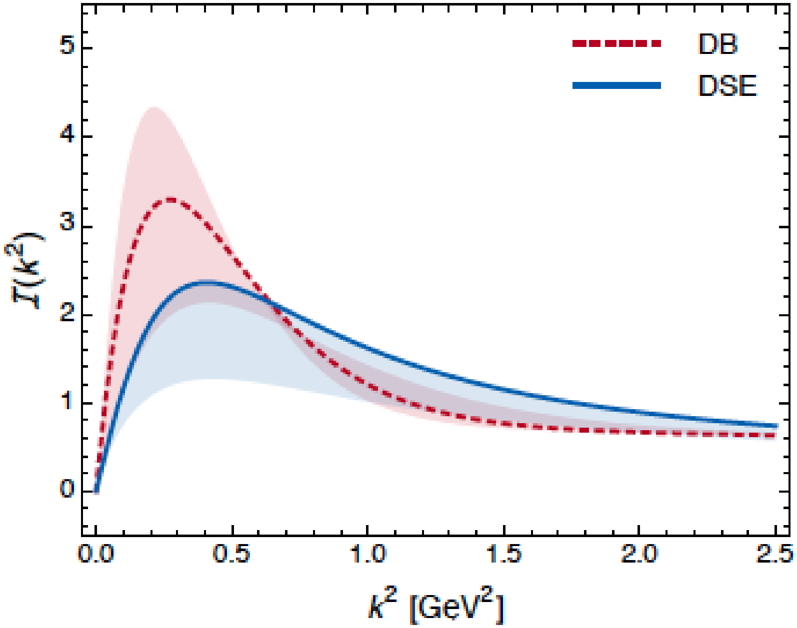
Top-down vs. Bottom-up approaches





D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, PLB 742 (2015)

QCD effective charge

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A running strong coupling in a particular scheme (Taylor), well-known in perturbation

$$\boxed{\alpha_T(k^2) = \lim_{a \to 0} g^2(a) k^2 \Delta(k^2; a) F^2(k^2; a)}$$

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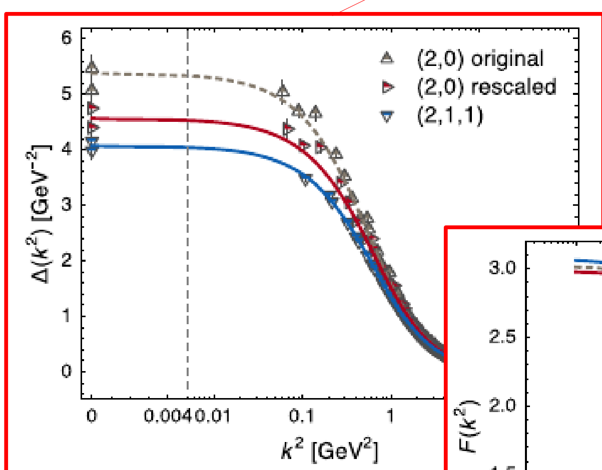

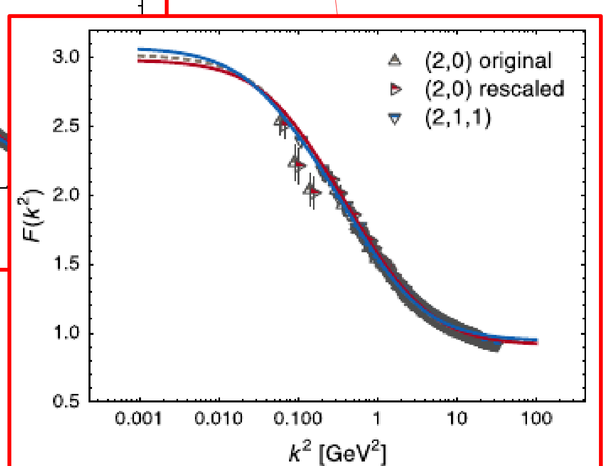

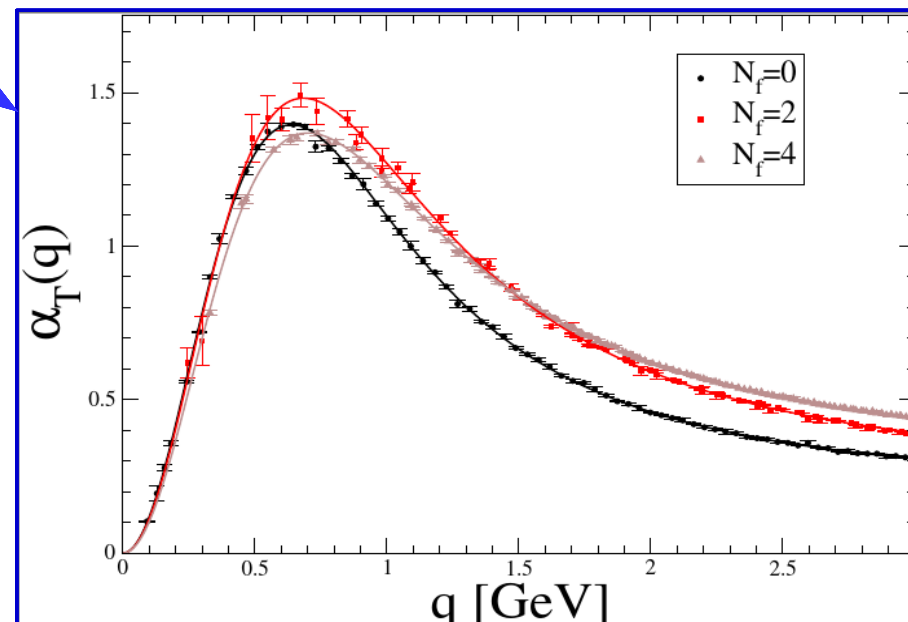

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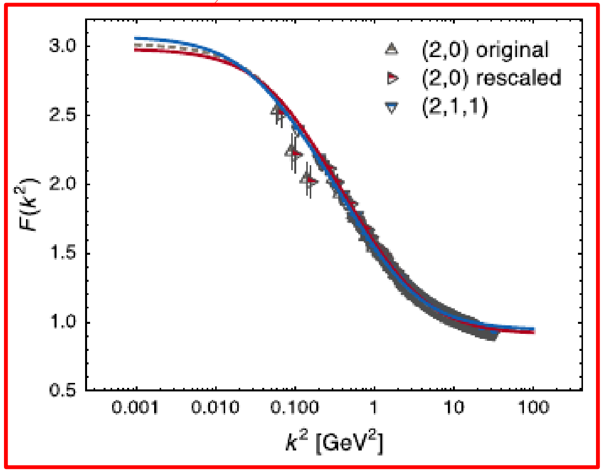

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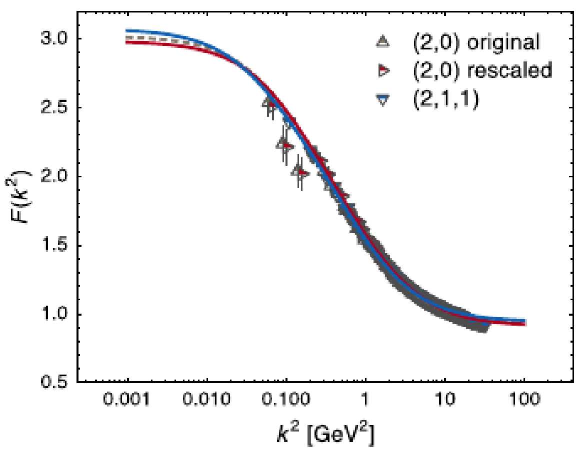

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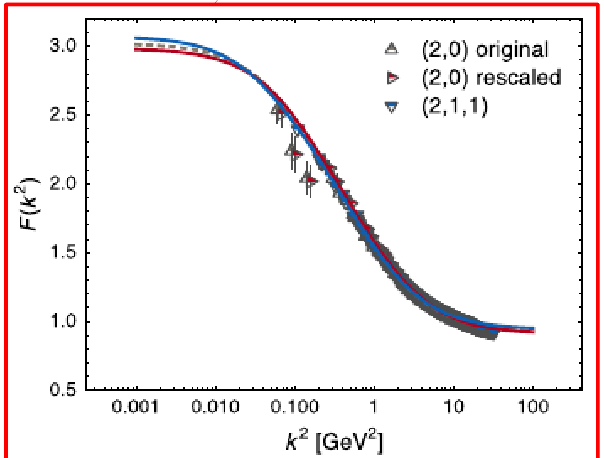

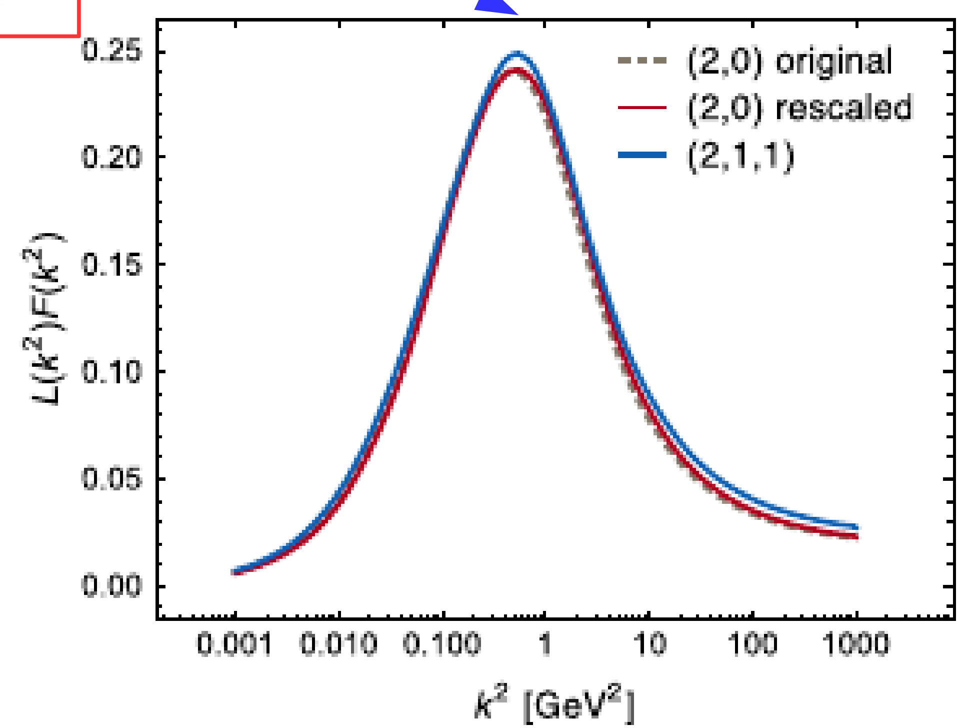

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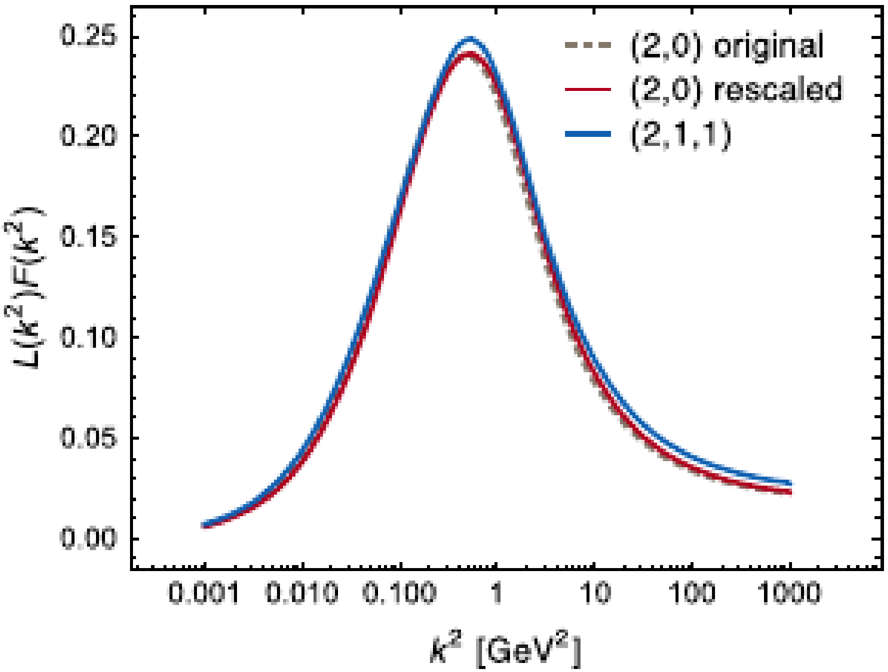

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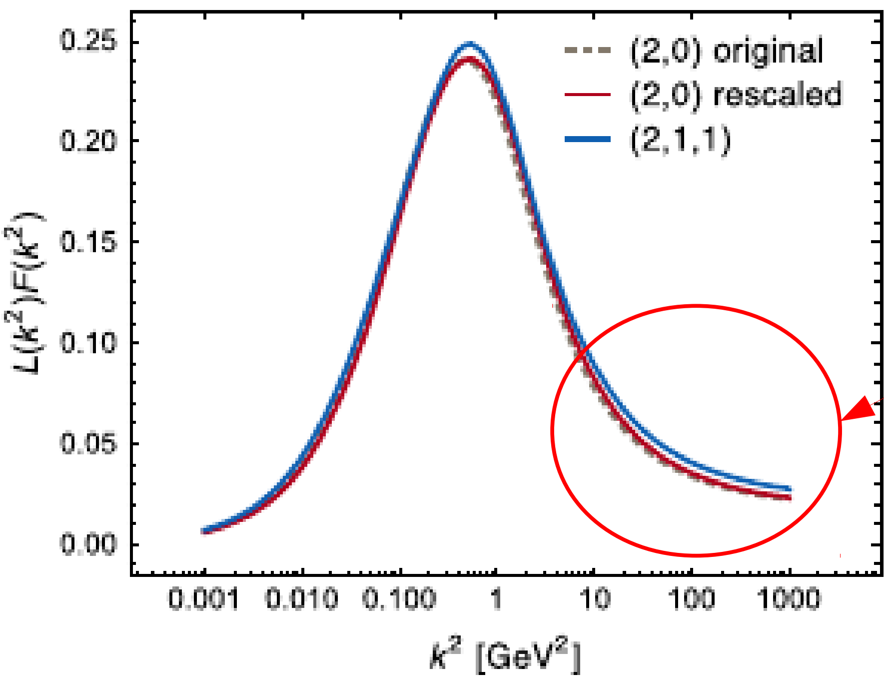

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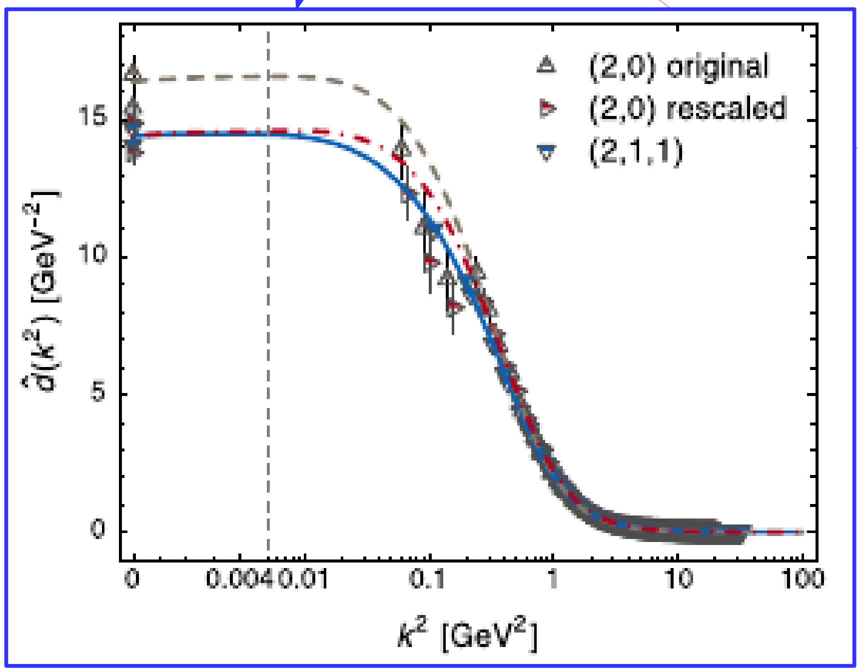

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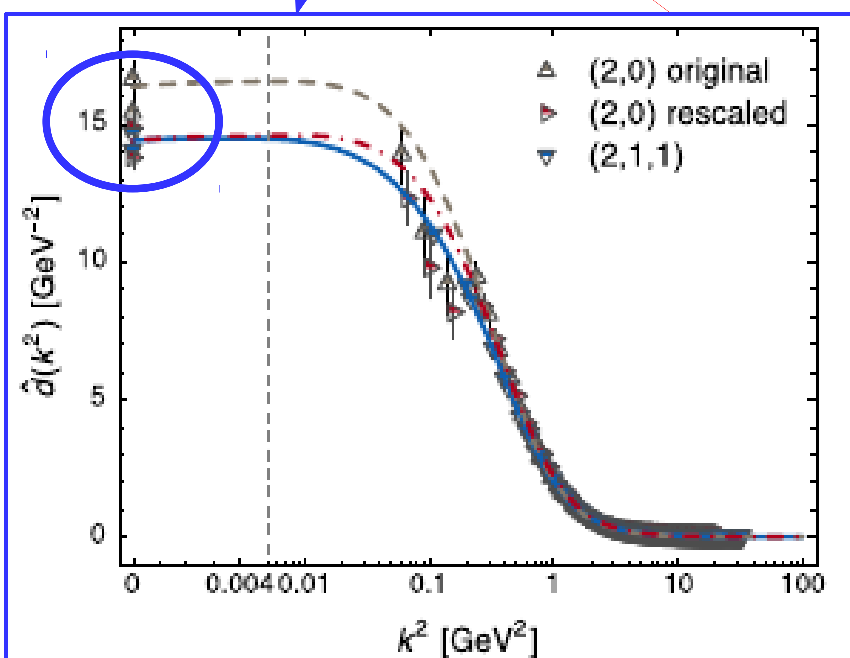

Zero-momentum freezing!

QCD effective charge

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$$\lim_{k^2 \to 0} \frac{I_{n_f}(k^2)}{k^2} = \lim_{k^2 \to 0} \frac{I_{N_f}(k^2)}{k^2} \quad \Leftrightarrow \quad \widehat{d}_{n_f}(0) = \widehat{d}_{N_f}(0).$$

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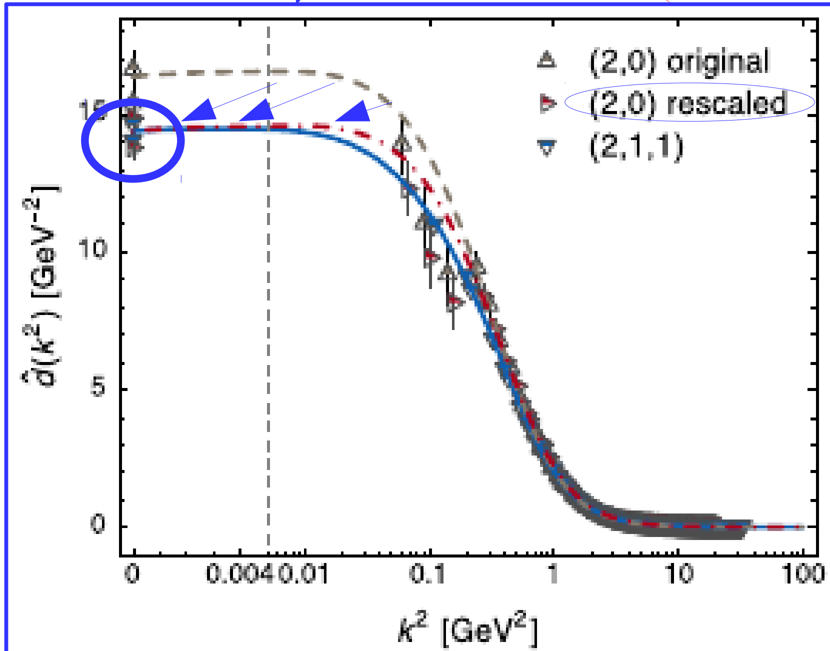

Zero-momentum freezing!
Re-setting of the physical scale for lQCD

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A divergent ghost-loop contribution to the gluon vacuum polarization in its DSE

A.C. Aguilar et al,. PRD89(2014)05008

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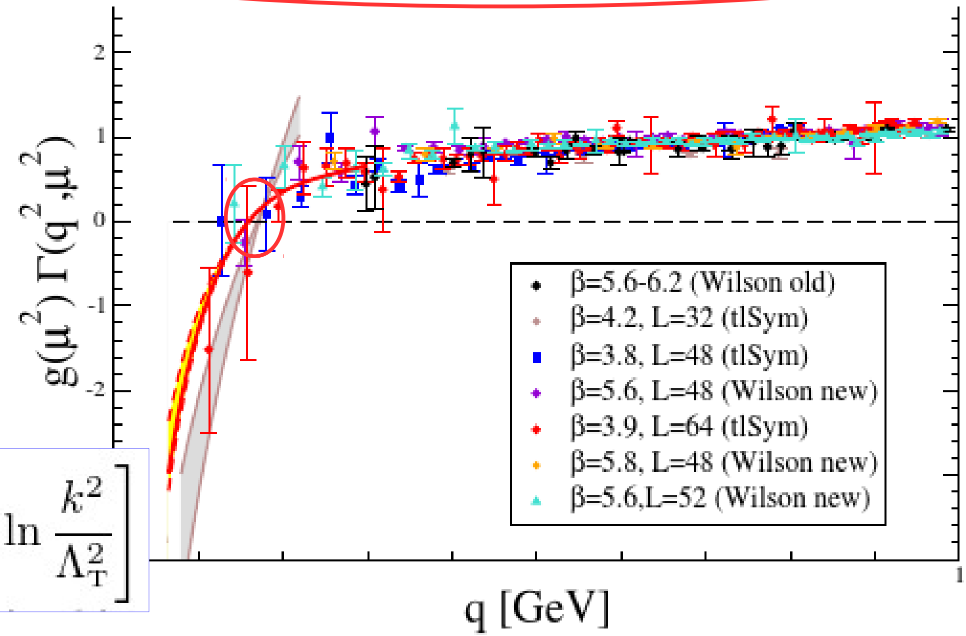
$$\Gamma^{i,(B)}_{T,R}(p^2;\mu^2) \underset{p^2/\mu^2 \ll 1}{\simeq} F_R(0;\mu^2)\frac{\partial}{\partial p^2}\Delta_R^{-1}(p^2;\mu^2) + \ldots$$



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A divergent ghost-loop contribution to the gluon vacuum polarization in its DSE, which can be manifestly traced back from the zero-crossing of the three-gluon vertex

A.C. Aguilar et al,. PRD89(2014)05008
A.K. Cyrol et al. PRD94(2016)054005
Ph. Boucaud et al,. PRD95(2017)114503

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Remarkable QCD feature: saturation of the RG key ingredient $\hat{d}(0)$

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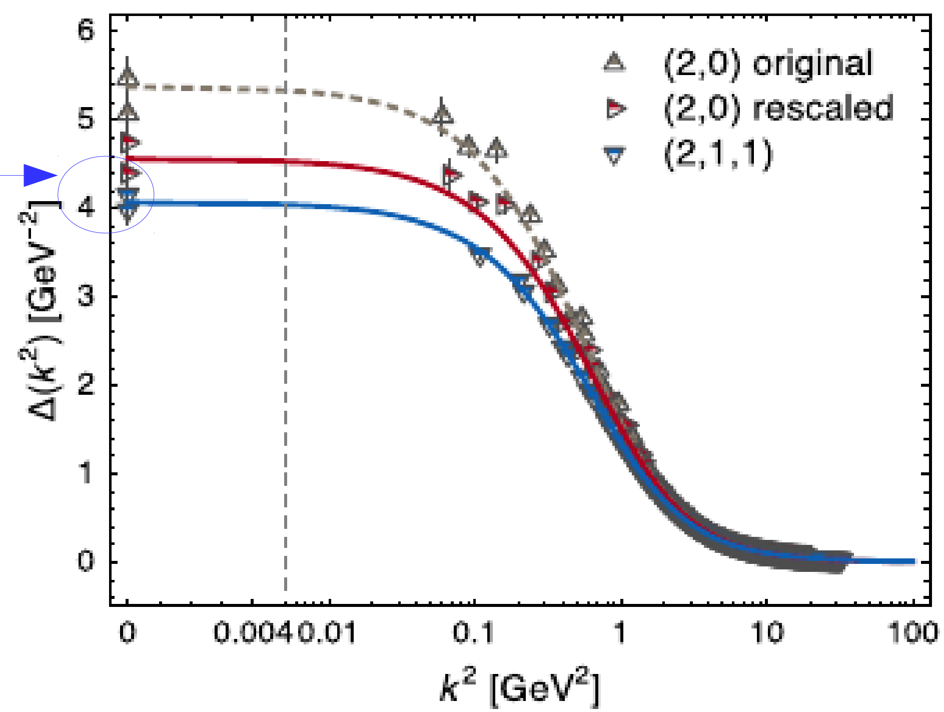
D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, PLB 742 (2015)

$m_0 \sim m_P/2$

$z(\zeta^2) \sim 1$

$$\Delta(0, \zeta_0^2) = \frac{z(\zeta_0^2)}{m_0^2}$$



The mass scale m_0 comes out from the gluon propagator, which acquires a dynamical mass through the Schwinger mechanism.

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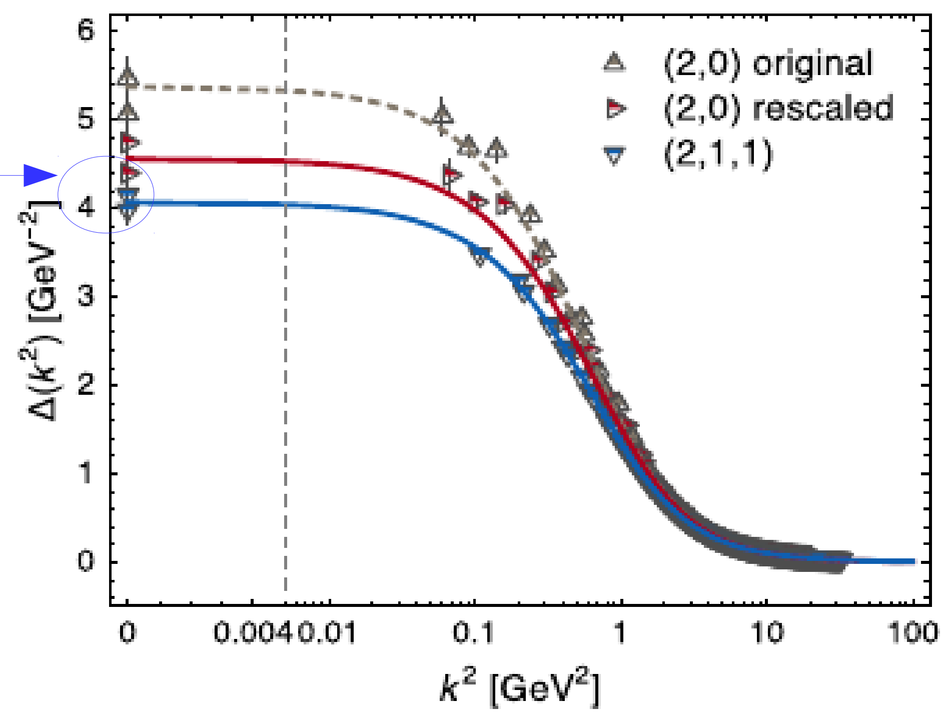

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Define then the RGI invariant function:

$$\boxed{\mathcal{D}(k^2) = \frac{\Delta(k^2;\mu^2)}{\Delta(0;\mu^2) m_0^2}} = \begin{cases} 1/m_0^2 & k^2 \ll m_0^2 \\ 1/k^2 & k^2 \gg m_0^2 \end{cases}$$

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D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

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D. Binosi, C. Mezrag, J. Papavassiliou, J.R-Q, C.D. Roberts, arXiv:1612.04835

QCD effective charge

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D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

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Extract the (process-independent) coupling

Using the quark gap equation

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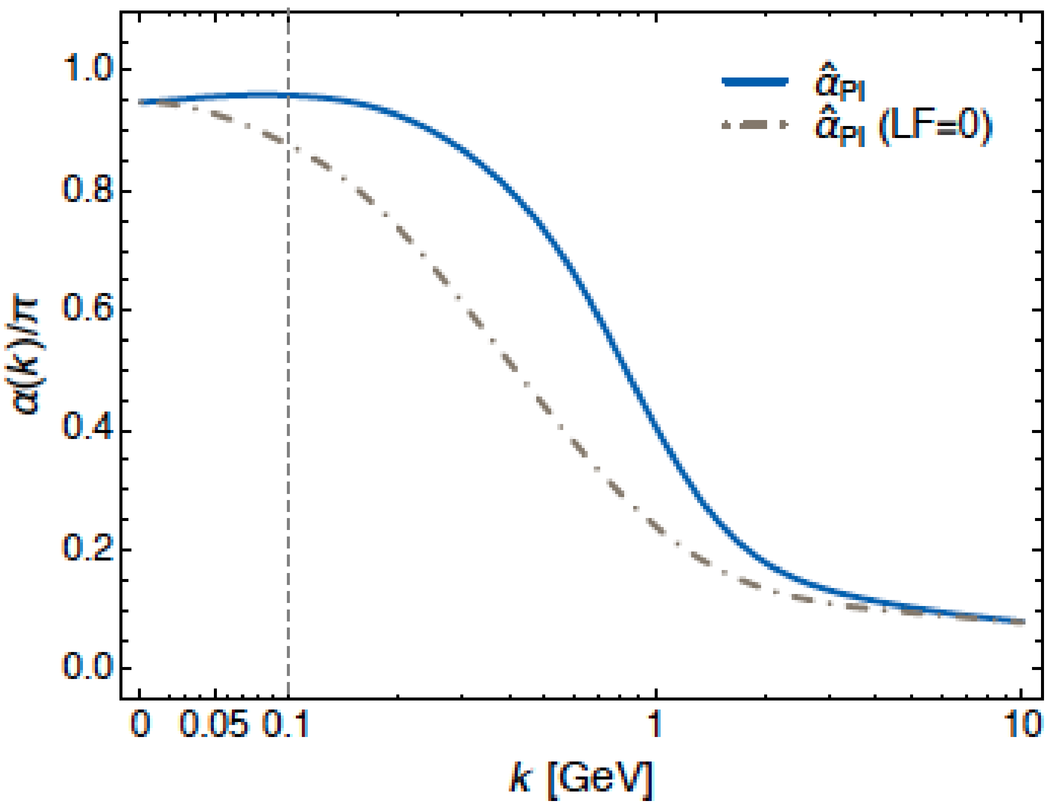

•Parameter free
completely determined from 2-points sector

•No Landau pole
physical coupling showing an IR fixed point

•Smoothly connects IR and UV domains
no explicit matching procedure

•Essentially non-perturbative result
continuum/lattice results plus setting of single mass scale (from the gluon)

•Ghost gluon dynamics critical
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  - **Tight sum rules constraints on the integral**
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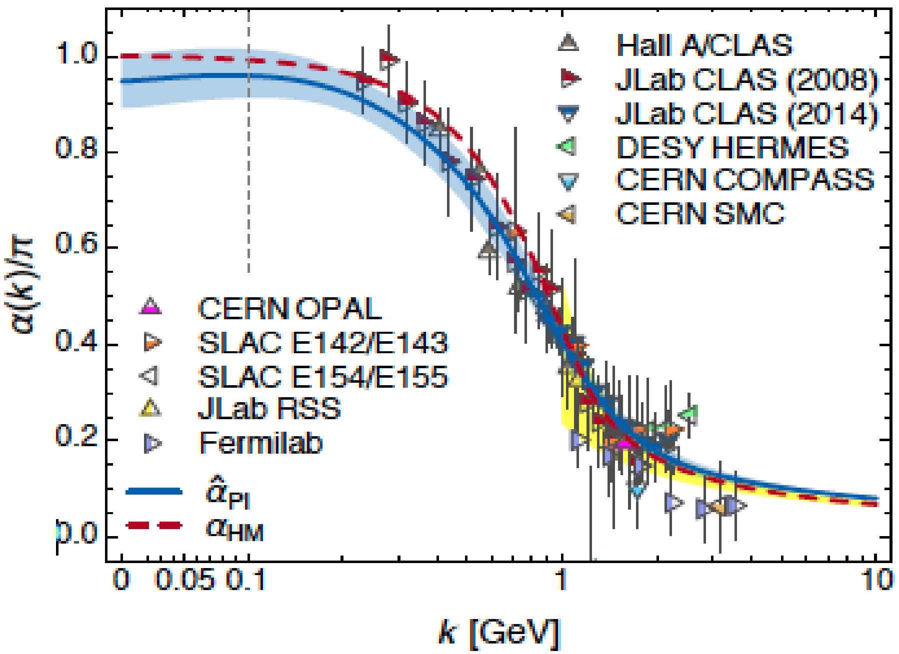

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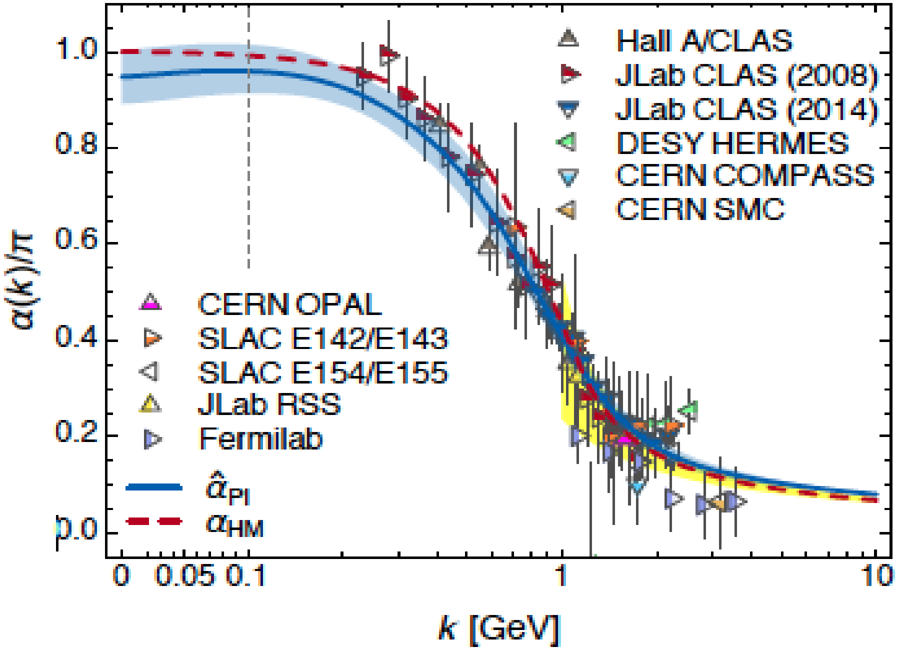

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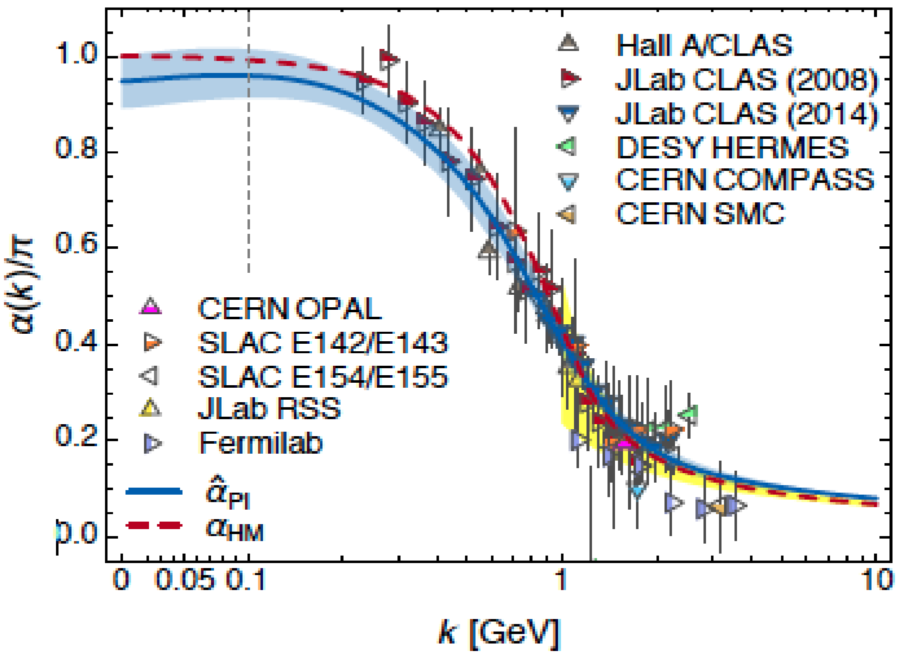

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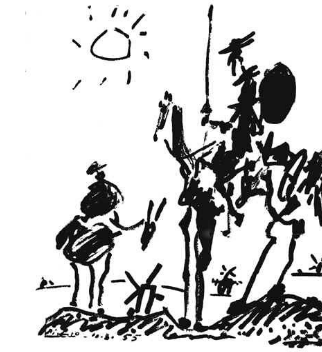

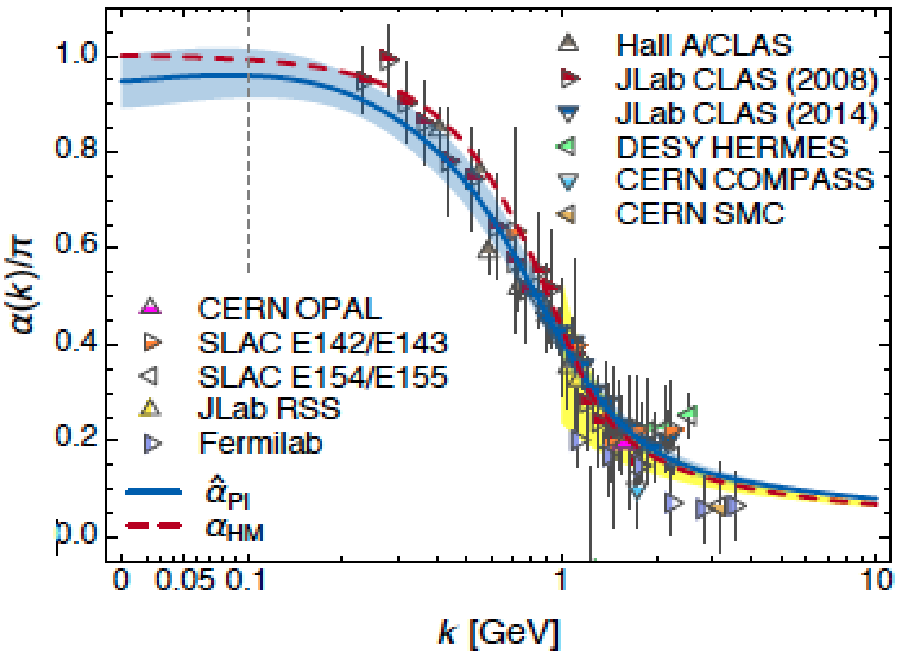

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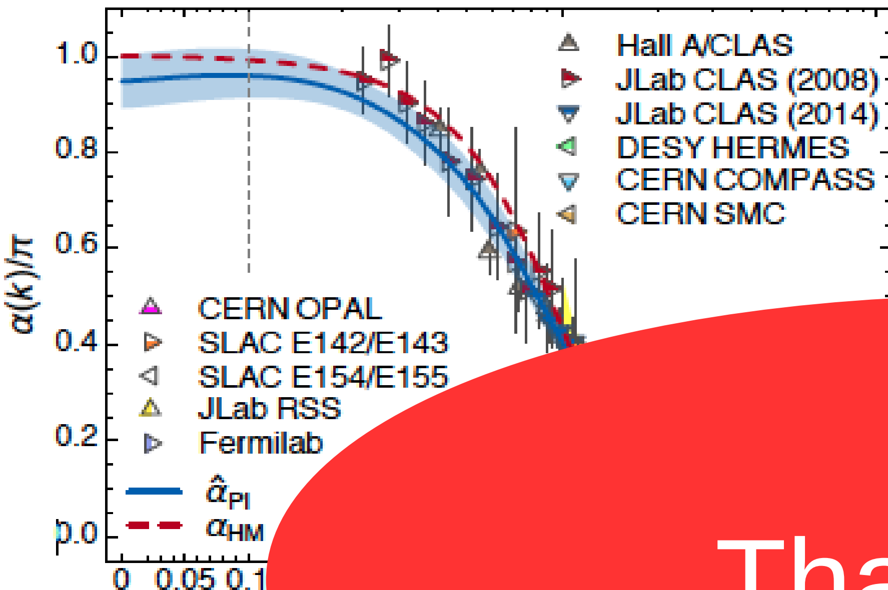

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