Fourier domain gravitational waveforms for eccentric precessing systems

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Paris, Oct. 12 2017

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Outline

Introduction

Several black hole binary formation models have been proposed, both for supermassive and stellar mass black holes.

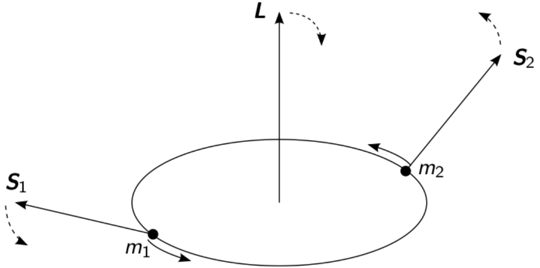
Two of the key distinguishing features of these formation models are their predictions for the eccentricity and spin alignment distributions for the binaries.

In order to verify the relative importance of different formation channels, it is important to be able to simultaneously measure the eccentricity and spin distributions of those binaries.

Moderate eccentricities, if present, might lead to a significant bias in parameter recovery.

Precession

Waveforms for precessing compact object binaries vary on four different timescales: orbital, spin-precessional, periastron-precessional, and radiation reaction.



The presence of these separate timescales is a challenge for the construction of accurate waveforms in the Fourier domain.

Quasi-Keplerian parametrization

At 3PN, one can find a quasi-Keplerian parametrization of the orbit in the form

$$
\begin{aligned}
r &= a(1 - e_r \cos u) + f_r(v), \\
\phi &= (1 + k)v + f_\phi(v), \\
\tan \frac{v}{2} &= \sqrt{\frac{1 + e_\phi}{1 - e_\phi}} \tan \frac{u}{2}, \\
l &= u - e_t \sin u + f_t(v), \\
\dot{l} &= n + f_n(v),
\end{aligned}
$$

valid if the orbital plane is slowly varying.

Evolution equations

The quantities varying on the radiation reaction timescale in the Kepler equations can be expressed as a function of some PN parameter, e.g. $x = \omega^{1/3}$ and an eccentricity parameter, e.g. $e = e_t$.

The evolution equations are known to 3PN for the nonspinning term, and 2PN for the spin terms

$$\frac{dx}{dt} = \frac{x^9}{(1-e^2)^{7/2}} \sum_n a_n(e) x^n,$$
$$\frac{de^2}{dt} = -\frac{x^8}{(1-e^2)^{5/2}} \sum_n b_n(e) x^n.$$

The presence of spins induces a residual eccentricity when the binaries have fully circularized, as $de^2/dt > 0$ as $e \to 0$.

Eccentric waveform

We can express the waveform emitted by an eccentric system as

$$h_{+,\times}(t) = \sum_n h^{(n)}_{+,\times}(u) e^{-in\phi}.$$

In order to compute its Fourier transform, we need to express the phase as a linearly growing quantity with some corrections, and to separate terms that evolve on the spin-precession timescale.

Linearly growing phase

Using

$$\phi = (1+k)v + f_\phi(v),$$
$$\dot{l} = n + f_n(v),$$

we can express $\phi = \lambda + W_\phi$ and $l = \bar{l} + W_l$ as linearly growing phases with a modification on the orbital timescale.

$$\dot{\lambda} = (1+k)n,$$
$$\dot{\bar{l}} = n,$$
$$W_l = \int f_n(v) dt,$$
$$W_\phi = (1+k)(v - l + W_l) + f_\phi(v).$$

Linearly growing phase

The eccentric anomaly can be expressed as a function of the mean anomaly using

$$\tan\frac{v}{2} = \sqrt{\frac{1+e_\phi}{1-e_\phi}}\tan\frac{u}{2},$$
$$l = u - e_t \sin u + f_t(v).$$

We can invert the latter equation as

$$u = \bar{l} + \sum_n A_n \sin n\bar{l}.$$

The Kepler equations ensure that $A_n = \mathcal{O}\left(e^n\right)$, so that as we expand this relation for small eccentricities, only a small numer of terms are needed in the inversion.

Eccentric waveform

Similarly, we can express W_l, W_ϕ, and v as Fourier series in $\bar{l}$, and the waveform can then be expressed as

$$\begin{aligned} h_{+,\times}(t) &= \sum_n h^{(n)}_{+,\times}(u) e^{-in\phi} \\ &= \sum_{p,n} H^{(p,n)}_{+,\times} e^{-i(n\lambda + p\bar{l})}. \end{aligned}$$

Eccentric waveform

In order to isolate terms evolving on the periastron precession timescale, we express

$$\bar{l} = \lambda + \delta\lambda,$$

with $\delta\dot{\lambda}/\dot{\lambda} = \mathcal{O}\left(x^2\right)$, and the waveform becomes

$$h_{+,\times}(t) = \sum_{m,n} H_{+,\times}^{(m,n)} e^{-i(n\lambda + m\delta\lambda)}.$$

Eccentric waveform

We can then use the SUA and express the Fourier transform

$$\tilde{h}(f) = \sum_{m,n} \sqrt{2\pi}\, T_{m,n} e^{i[2\pi f t_{m,n} - n\lambda - m\delta\lambda - \pi/4]}$$
$$\times \left[\sum_{k=-k_{\max}}^{k_{\max}} H_{+,\times}^{(m,n)}(t_{m,n} + k T_{m,n}) \right],$$
$$2\pi f = n\dot{\lambda}(t_{m,n}) + m\delta\dot{\lambda}(t_{m,n}),$$
$$T_{m,n} = \left(n\ddot{\lambda}(t_{m,n}) + m\delta\ddot{\lambda}(t_{m,n}) \right)^{-1/2}.$$

In this waveform, the phase is only PN expanded, and the amplitude is also expanded for small eccentricities.

Eccentric waveform

By expanding the time-frequency relation and the Fourier phase at M^{th} order in m, we can rewrite

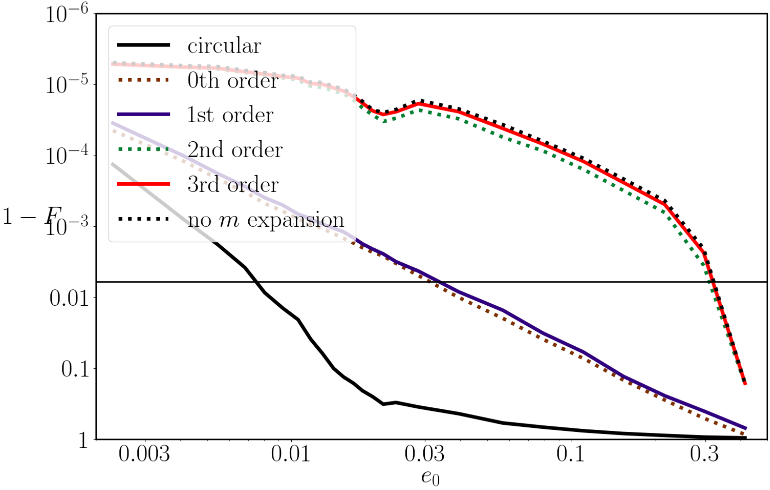
$$\tilde{h}(f) = \sum_n \sqrt{2\pi} T_n e^{i[2\pi f t_n - n\lambda(t_n) - \pi/4]} \times \sum_m \Delta H_m \left[\sum_{k=-k_{\text{max}}}^{k_{\text{max}}} G^{(n)}_{+,\times}(t_n + kT_n) \right],$$

$$2\pi f = n\dot{\lambda}(t_n),$$

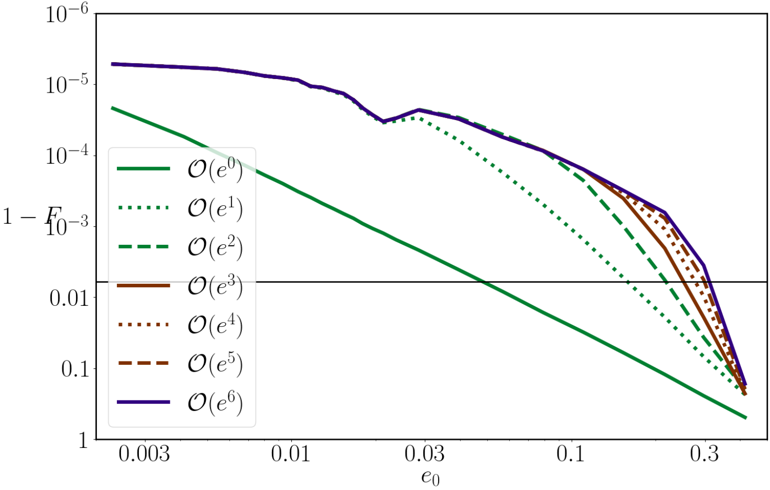
$$T_n = \left(n\ddot{\lambda}(t_n) \right)^{-1/2}.$$

We thus have a family of eccentric waveforms, characterized by the order M of the m expansion of the Fourier transform, and the order $\mathcal{O}(e^p)$ of the expansion of the amplitudes.

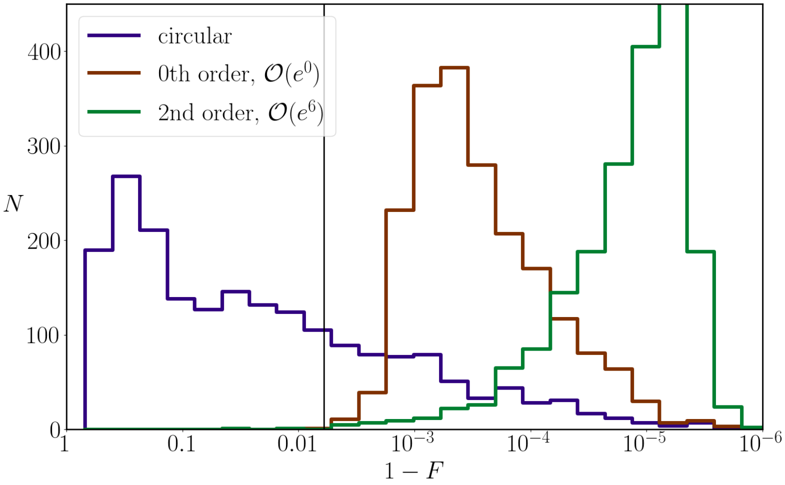
Faithfulness vs. e_0, amplitudes $\mathcal{O}(e^6)$



Faithfulness vs. e_0, 2nd order



Fully circularized binaries



Outlook

- A fast, Fourier-domain eccentric and spin-precessing waveform allows to simultaneously measure the eccentricity and the spin alignment.
- With large spins, the residual eccentricity may cause large biases in parameter recovery even for fully circularized binaries.
- One of the simplifications made is the approximation that the orbital plane is slowly varying. This needs to be investigated.
- An extension for merger/ringdown will be eventually needed.