

Scattering process $e^+ + e^- \rightarrow t + \bar{t}$ and fully polarized top quark decays in Standard Model Effective Field Theory

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Introduction

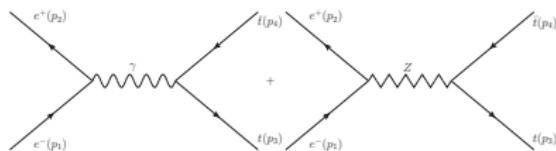
Main purpose of this thesis

- Provide an overall perspective for SMEFT framework.
- Study the effects of several dimension-six (D6) operators.
- Improve the sensitivity for top-quark new physics.

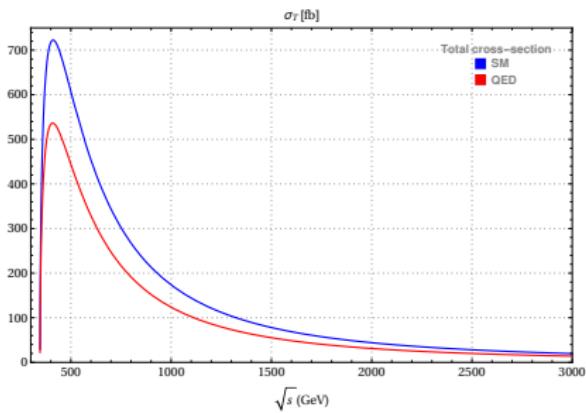
Outline

- Scattering process $e^+ + e^- \rightarrow t + \bar{t}$ in QED and SM.
- Scattering process $e^+ + e^- \rightarrow t + \bar{t}$ in SMEFT.
- Fully polarized top-quark decays in SMEFT.
- Summary and outlook.

Process $e^+ + e^- \rightarrow t + \bar{t}$ in QED and SM



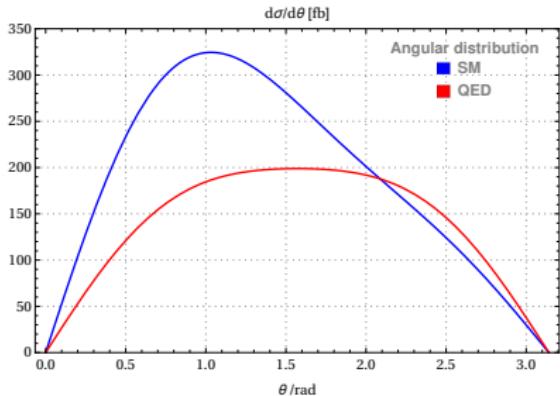
(a)



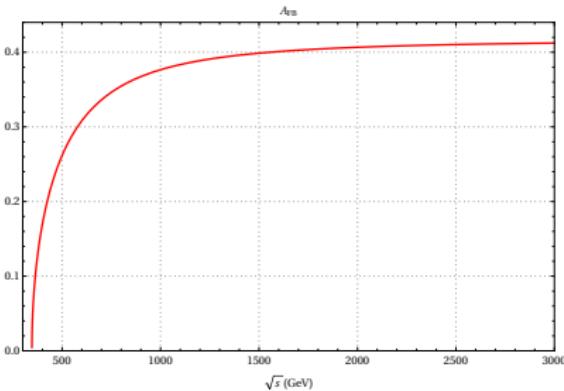
(b)

Figure: Feynman diagrams of process $e^+ + e^- \rightarrow t + \bar{t}$ (a),
 The total cross-section of process $e^+ + e^- \rightarrow t + \bar{t}$ in QED and SM (b)

Process $e^+ + e^- \rightarrow t + \bar{t}$ in QED and SM



(a)



(b)

Figure: Angular distribution for $\sqrt{s} = 500$ GeV (a), forward-backward asymmetry (b).

- Forward-backward asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{\sigma_F - \sigma_B}{\sigma_T} \quad (1)$$

An introduction to SMEFT

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_i C_i^{(5)} Q_i^{(5)} + \frac{1}{\Lambda^2} \sum_i C_i^{(6)} Q_i^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right) \quad (2)$$

Main procedure in SMEFT

- We followed the "Warsaw basis"¹, then we perform the SSB mechanism and further field, coupling rescalings when moving from weak to mass eigenstates basis³
⇒ All bilinear terms of gauge, Higgs, fermion fields are canonical².
- We added R_ξ -gauge fixing terms² in the Lagrangian
⇒ In mass basis of SMEFT, all propagators become SM-like.
- We derive new Feynman rules³ ⇒ Calculate physical quantities.

¹JHEP **10** (2010) 085, arXiv: 1008.4884

²arXiv: 1704.0388

³Further informations see backup slide

Square amplitude calculation with FORM

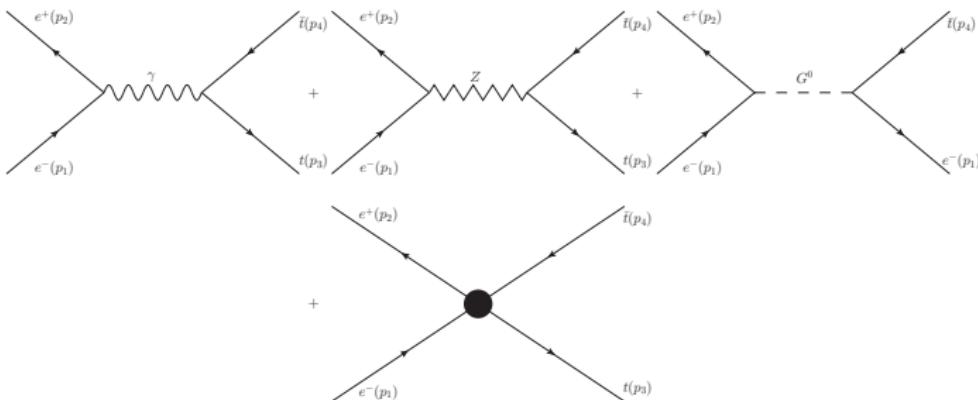


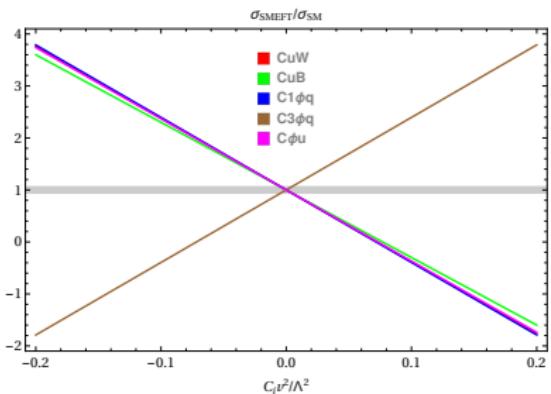
Figure: The Feynman diagrams of process $e^- + e^+ \rightarrow t + \bar{t}$ in SMEFT.

$$|\mathcal{M}|^2 = \frac{3}{4} \sum_{spins} (\mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_{4-f}) (\mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_{4-f})^\dagger \quad (3)$$

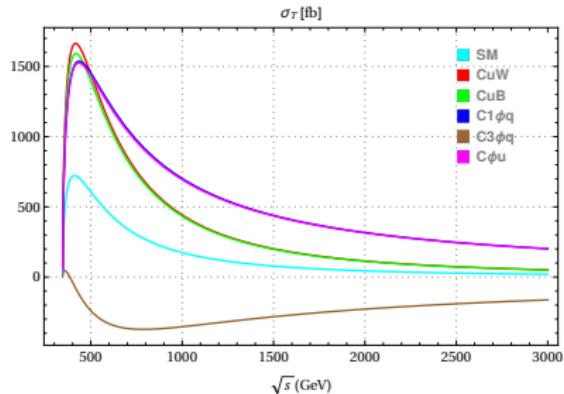
The unpolarized squared amplitude calculation is done using FORM.

Top-quark electroweak couplings

- Setting $C_{\phi B}$, $C_{\phi W}$, $C_{\phi D}$, $C_{\phi WB}$ and C_ϕ to zero
 \Rightarrow Using the parameters in SM.
- Top-quark electroweak couplings: C_{uW} , C_{uB} , $C_{\phi q}^1$, $C_{\phi q}^3$, and $C_{\phi u}$.

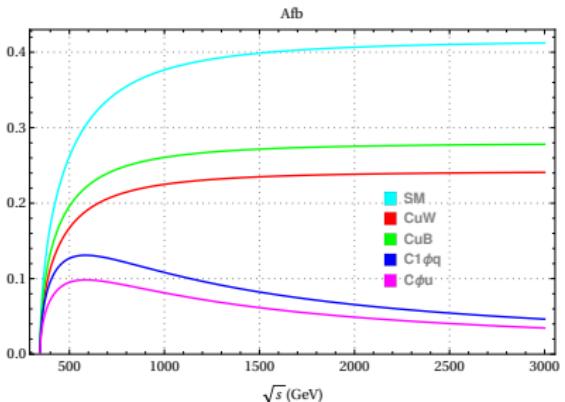
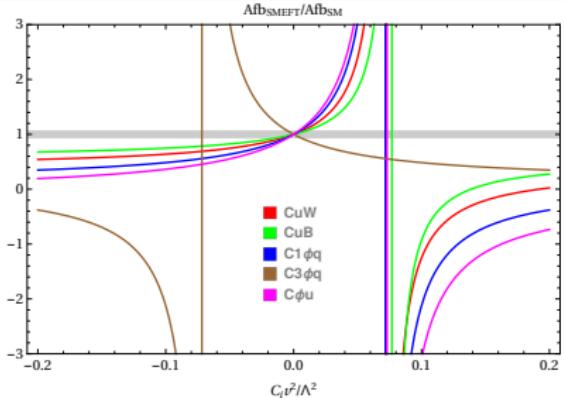


(a) $\sqrt{s} = 500$ (GeV)



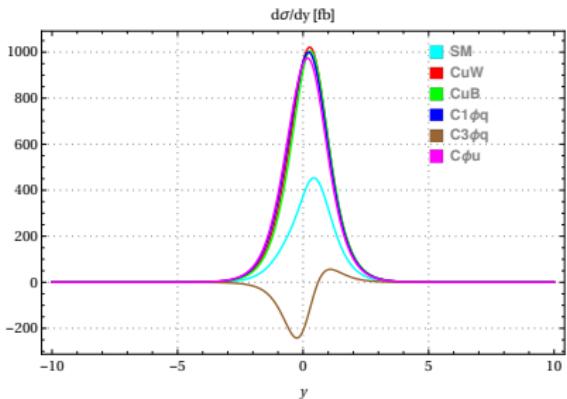
(b) $\bar{C}_i = -0.1$

Figure: The dependence of total cross-section on D6 operators for $\sqrt{s} = 500$ GeV (a). The effect of D6 operators on total cross-section at $\bar{C}_i = C_i v^2 / \Lambda^2 = -0.1$ (b).



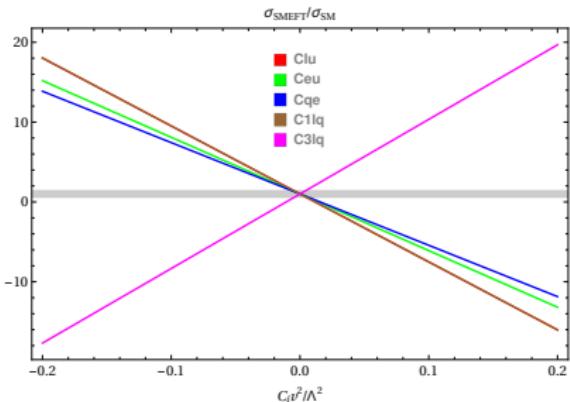
Follow a counter-clockwise:

- Ratio of Afb_{SMEFT}/Afb_{SM} for $\sqrt{s} = 500$ (GeV).
- The effects of D6 operators on FB asymmetry, rapidity distribution for $\sqrt{s} = 500$ (GeV) at $\bar{C}_i = -0.1$.

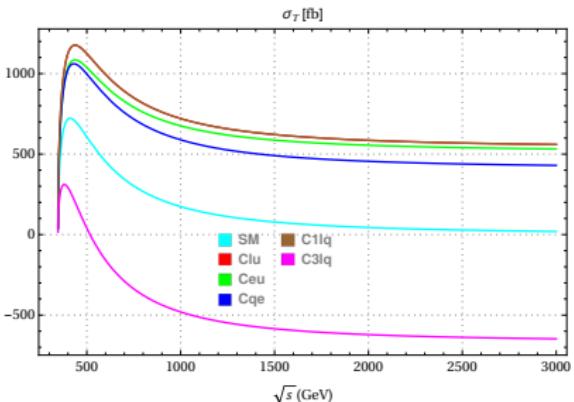


Four-fermion interactions

- Four-fermion couplings: $C_{lu}, C_{eu}, C_{qe}, C_{lq}^1$, and C_{lq}^3 .

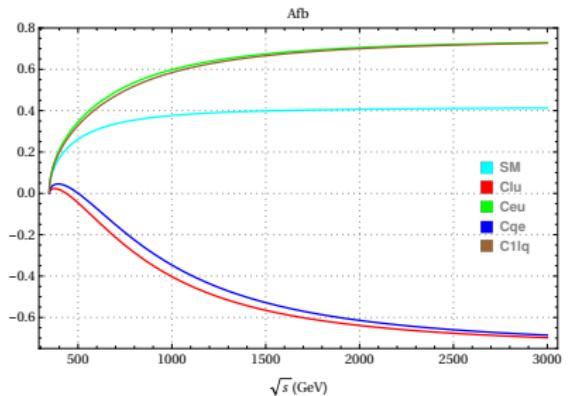
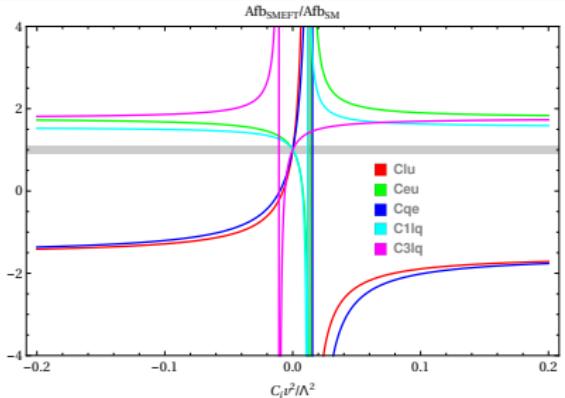


(a) $\sqrt{s} = 500$ (GeV)



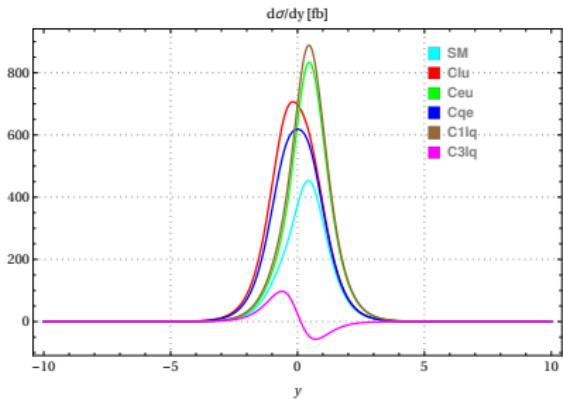
(b) $\bar{C}_i = -0.01$

Figure: The dependence of total cross-section on Four-fermion operators for $\sqrt{s} = 500$ GeV (a), the effects of Four-fermion operators with $\bar{C}_i = -0.01$ (b).



Follow a counter-clockwise:

- Ratio of Af_{bSMEFT}/Af_{bSM} for $\sqrt{s} = 500$ (GeV).
- The effects of four-fermion operators on FB asymmetry, rapidity distribution for $\sqrt{s} = 500$ (GeV) at $\bar{C}_i = -0.1$.



Spin density matrix methodology

Main purpose

- We re-construct a direct methodology for connecting the decay distributions to the spin observables of W-boson¹
- We apply this method to the decay of top-quark with an anomalous Wtb coupling and cross-check the results with previous papers¹²

W-boson spin observables

Being spin-1 particle, W spin state can be described by 3×3 matrix with

- hermitian with unit trace,
- positive semidefinite.

⇒ Need 8 parameters (spin observables) to describe the spin state of W-boson.

¹Phys. Rev. **D93** (2016) 011301, arXiv: 1508.04592.

²Nucl. Phys. **B840** (2010) 349–378, arXiv: 1005.5382.

W-boson density matrix

$$\rho = \frac{1}{3} \mathbb{1} + \frac{1}{2} \sum_{M=-1}^1 \langle S_M \rangle^* S_M + \sum_{M=-2}^2 \langle T_M \rangle^* T_M \quad (4)$$

W-boson density matrix elements¹

$$\Rightarrow \begin{cases} \rho_{\pm 1 \pm 1} &= \frac{1}{3} \pm \frac{1}{2} \langle S_z \rangle + \frac{1}{\sqrt{6}} \langle T_0 \rangle, & \rho_{00} = \frac{1}{3} - \frac{2}{\sqrt{6}} \langle T_0 \rangle, \\ \rho_{\pm 10} &= \frac{1}{2\sqrt{2}} [\langle S_x \rangle \mp i \langle S_y \rangle] \mp \frac{1}{\sqrt{2}} [\langle A_1 \rangle \mp i \langle A_2 \rangle], \\ \rho_{1-1} &= \langle B_1 \rangle - i \langle B_2 \rangle, \end{cases} \quad (5)$$

¹See backup slide for conventions and notations

Normalized distribution of W^+ boson

- Fully differential W decay width

$$\frac{d\Gamma}{d\cos\theta d\phi} = C \sum_{m,m'} \rho_{mm'} e^{i(m-m')\phi} d_{m\Lambda}^1(\theta) d_{m'\Lambda}^1(\theta) \quad (6)$$

- Normalized distribution of W^+ boson¹

$$\begin{aligned} \frac{1}{\Gamma} \cdot \frac{d\Gamma}{d\cos\theta d\phi} &= \frac{3}{8\pi} \left[\frac{1}{2} (1 + \cos^2\theta) + \left[\frac{1}{6} - \frac{1}{\sqrt{6}} \langle T_0 \rangle \right] (1 - 3\cos^2\theta) \right. \\ &\quad + \langle S_z \rangle \cos\theta + \langle S_x \rangle \cos\phi \sin\theta + \langle S_y \rangle \sin\phi \sin\theta \\ &\quad - \langle A_1 \rangle \cos\phi \sin 2\theta - \langle A_2 \rangle \sin\phi \sin 2\theta \\ &\quad \left. + \langle B_1 \rangle \cos 2\phi \sin^2\theta + \langle B_2 \rangle \sin 2\phi \sin^2\theta \right]. \end{aligned} \quad (7)$$

¹See backup slide for connection of spin-observables to angular distributions and asymmetries.

Fully polarized top-quark decay

Anomalous Wtb coupling

$$\begin{aligned} \mathcal{L}_{Wtb} = & -\frac{\bar{g}_2}{\sqrt{2}} \bar{b} \gamma^\mu [V_L P_L + V_R P_R] t W_\mu^- - \frac{\bar{g}_2}{\sqrt{2}} \bar{b} \frac{i \sigma^{\mu\nu} q_\nu}{M_W} [g_L P_L + g_R P_R] t W_\mu^- \\ & - \frac{\bar{g}_2}{\sqrt{2}} \bar{t} \gamma^\mu [V_L^* P_L + V_R^* P_R] b W_\mu^+ + \frac{\bar{g}_2}{\sqrt{2}} \bar{t} \frac{i \sigma^{\mu\nu} q_\nu}{M_W} [g_L^* P_R + g_R^* P_L] b W_\mu^+ \end{aligned} \quad (8)$$

whereas the parameters in Lagrangian (8) are

$$\begin{aligned} V_L = V_{tb}, & \qquad \qquad \qquad V_R = \frac{1}{2} C_{\phi ud}^{3*} \frac{v^2}{\Lambda^2} \\ g_L = \sqrt{2} C_{dW}^{3*} \frac{v^2}{\Lambda^2}, & \qquad \qquad \qquad g_R = \sqrt{2} C_{uW}^3 \frac{v^2}{\Lambda^2}. \end{aligned} \quad (9)$$

Polarized square amplitude

- We choose top-quark rest frame and setting the positive of z -axis in the direction of W-boson momentum \vec{q} . The spin four-vector of polarized top-quark defined as

$$s_\mu^t = (0, \sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta). \quad (10)$$

- Polarized squared amplitude¹:

$$\mathcal{M}_{ij} = 3 \cdot \frac{1}{3} \sum_{\lambda_b} \mathcal{M}_i(t \rightarrow W_i b) \mathcal{M}_j^*(t \rightarrow W_j b) = \frac{\bar{g}_2^2}{4} m_t^2 \rho_{ij} \quad (11)$$

- Preserve $m_b \neq 0$ and calculate each matrix elements with fully higher order of $x_b = m_b/m_t$.

¹Each matrix elements are calculated by program FORM

Fully polarized top-quark decay

W-boson density matrix elements¹

$$\begin{aligned}\rho_{00} &= A_0 + 2 \frac{|\vec{q}|}{m_t} A_1 \cos \theta, \\ \rho_{\pm 1 \pm 1} &= B_0(1 \pm \cos \theta) \pm 2 \frac{|\vec{q}|}{m_t} B_1(1 \pm \cos \theta), \\ \rho_{\pm 1 0} = \rho_{0 \pm 1}^* &= \left[\frac{m_t}{\sqrt{2} M_W} (C_0 + iD_0) \pm \frac{|\vec{q}|}{\sqrt{2} M_W} (C_1 + iD_1) \right] \sin \theta e^{\mp i\phi}, \\ \rho_{\pm 1 \mp 1} &= 0.\end{aligned}\tag{12}$$

¹Good agreement with *Nucl. Phys.* **B840** (2010) 349–378 up to $\mathcal{O}(x_b^2)$

		$\langle S_1 \rangle$	$\langle S_2 \rangle$	$\langle S_3 \rangle$	$\langle T_0 \rangle$	$\langle A_1 \rangle$	$\langle A_2 \rangle$
SM	Ref. ¹	0.510	0	-0.302	-0.445	0.255	0
	Results 1	0.510	0	-0.303	-0.445	0.255	0
	$\delta[\%]$	0%	0%	0.3%	0%	0%	0%
$g_R = 0.03$	Ref. ¹	0.500	0	-0.278	-0.472	0.249	0
	Results 1	0.499	0	-0.281	-0.472	0.249	0
	$\delta[\%]$	0.2%	0%	1.08%	0%	0%	0%
$g_R = 0.10i$	Ref. ¹	0.507	-0.084	-0.284	-0.434	0.253	-0.042
	Results 1	0.508	-0.084	-0.312	-0.434	0.254	-0.042
	$\delta[\%]$	0.2%	0%	9.85%	0%	0.3%	0%

Table: Cross-check with Ref. 1 for W spin observables in polarized top-quark decays.

¹Phys. Rev. D93 (2016) 011301, arXiv: 1508.04592.

Summary and outlook

- Calculated the total cross-section for process $e^+ + e^- \rightarrow t + \bar{t}$ in SMEFT and effects of D6 operators relevant to top-quark physics (NEW).
- Several distributions and FB asymmetry are sensitive to Wilson coefficients of D6 operators (NEW).
- Re-constructed the density matrix of W-boson and identified eight spin-observables of W-boson by the angular distributions and asymmetries.
- Calculated analytically density matrix elements of W-boson resulting from polarized top-quark decays with fully higher order of x_b (NEW).
- Re-produced W spin observables in fully polarized top-quark decays with the presence of Wtb anomalous couplings.

Outlook

- Fitting Willision coefficients by using experimental data.
- Re-calculate process $e^+ + e^- \rightarrow t + \bar{t}$ in polarized case.

Backup slide (Higgs sector)

The full Lagrangian of Higgs field including dimension-six operators is

$$\begin{aligned} \mathcal{L}_{Higgs} = & (D_\mu \phi)^\dagger (D^\mu \phi) - \left[-\mu^2 (\phi^\dagger \phi) + \frac{\lambda}{2} (\phi^\dagger \phi)^2 \right] \\ & + C^\phi (\phi^\dagger \phi)^3 + C^{\phi \square} (\phi^\dagger \phi) \square (\phi^\dagger \phi) + C^{\phi D} (\phi^\dagger D_\mu \phi)^* (\phi^\dagger D^\mu \phi). \end{aligned} \quad (13)$$

Minimization of the potential one obtains the vacuum state with a correction of D6 operators, which reads

$$\Rightarrow \begin{cases} \langle \phi \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \\ \text{with } v &= \sqrt{\frac{2\mu^2}{\lambda}} + \frac{3\mu^3}{\sqrt{2}\lambda^{5/2}} C^\phi, \end{cases} \quad (14)$$

notice that v is called the vacuum expectation value (vev). From now on, we just only use this vev for all expressions and Feynman rules.

Backup slide (Higgs sector)

The Lagrangian bilinear terms of the scalar fields are given by,

$$\begin{aligned} \mathcal{L}_{Higgs} = & \frac{1}{2} \left[1 + \frac{1}{2} C_{\phi D} v^2 - 2 C_{\phi \square} v^2 \right] (\partial_\mu H)^2 + \left[\frac{1}{2} \mu^2 - \frac{3}{4} \lambda v^2 + \frac{15}{8} v^2 C_\phi \right] H^2 \\ & + \frac{1}{2} \left[1 + \frac{1}{2} C_{\phi D} v^2 \right] (\partial_\mu \Phi^0)^2 + (\partial_\mu \Phi^-) (\partial^\mu \Phi^+). \end{aligned} \quad (15)$$

In order to obtain the canonically normalized kinetic terms one needs field rescalings as follows:

$$h \equiv \left(1 + \frac{1}{4} C_{\phi D} v^2 - C_{\phi \square} v^2 \right) H, \quad G^0 \equiv \left(1 + \frac{1}{4} C_{\phi D} v^2 \right) \Phi^0, \quad G^\pm \equiv \Phi^\pm. \quad (16)$$

The squared mass of the normalized Higgs field h is

$$M_H^2 = \lambda v^2 - \left[3C_\phi - 2\lambda C_{\phi \square} + \frac{\lambda}{2} C_{\phi D} \right] v^4. \quad (17)$$

Backup slide (Gauge sector)

Within SMEFT framework, the gauge sector now reads,

$$\begin{aligned} \mathcal{L}_{gauge}^{full} = & -\frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4}W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) \\ & + C_{\phi G} (\phi^\dagger \phi) G_{\mu\nu}^A G^{A\mu\nu} \\ & + C_{\phi W} (\phi^\dagger \phi) W_{\mu\nu}^I W^{I\mu\nu} + C_{\phi B} (\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu} + C_{\phi WB} (\phi^\dagger \tau^I \phi) W_{\mu\nu}^I B^{\mu\nu} \\ & + C_{\phi D} (\phi^\dagger D_\mu \phi)^* (\phi^\dagger D^\mu \phi). \end{aligned} \quad (18)$$

To obtain the normalized kinetic term of gauge fields we rescaled the fields as

$$\begin{cases} \bar{W}_\mu^I &= (1 - C_{\phi W} v^2) W_\mu^I \\ \bar{B}_\mu &= (1 - C_{\phi B} v^2) B_\mu \\ \bar{G}_\mu^A &= (1 - C_{\phi G} v^2) G_\mu^A. \end{cases} \quad (19)$$

Backup slide (Mass of W and Z boson)

The bilinear part of the Lagrangian for electroweak gauge bosons is

$$\begin{aligned} \mathcal{L}_{EW}^{full} = & -\frac{1}{4} (\bar{W}_{\mu\nu}^1 \bar{W}^{1\mu\nu} + \bar{W}_{\mu\nu}^2 \bar{W}^{2\mu\nu}) + \frac{1}{2} \cdot \frac{\bar{g}_2^2 v^2}{4} (\bar{W}_{\mu\nu}^1 \bar{W}^{1\mu\nu} + \bar{W}_{\mu\nu}^2 \bar{W}^{2\mu\nu}) \\ & - \frac{1}{4} (\bar{W}_{\mu\nu}^3 \quad \bar{B}^{\mu\nu}) \begin{bmatrix} 1 & C_{\phi WB} v^2 \\ C_{\phi WB} v^2 & 1 \end{bmatrix} \begin{pmatrix} \bar{W}^{3\mu\nu} \\ \bar{B}^{\mu\nu} \end{pmatrix} \\ & - \frac{1}{2} \cdot \frac{v^2}{4} \left[1 + \frac{1}{2} C_{\phi D} v^2 \right] (\bar{W}_{\mu}^3 \quad \bar{B}_{\mu}) \begin{bmatrix} \bar{g}_2^2 & -\bar{g}_1 \bar{g}_2 \\ -\bar{g}_1 \bar{g}_2 & \bar{g}_1^2 \end{bmatrix} \begin{pmatrix} \bar{W}^{3\mu} \\ \bar{B}^{\mu} \end{pmatrix}. \quad (20) \end{aligned}$$

To normalize canonically kinetic terms and diagonal masses simultaneously, we introduced a rotation which reads,

$$\begin{cases} W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (\bar{W}_{\mu}^1 \mp i \bar{W}_{\mu}^2) \\ \begin{pmatrix} \bar{W}_{\mu}^3 \\ \bar{B}_{\mu} \end{pmatrix} = \begin{pmatrix} 1 & -\epsilon/2 \\ -\epsilon/2 & 1 \end{pmatrix} \begin{pmatrix} \cos \bar{\theta} & \sin \bar{\theta} \\ -\sin \bar{\theta} & \cos \bar{\theta} \end{pmatrix} \begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix}. \end{cases} \quad (21)$$

Backup slide (Mass of W and Z boson)

Chronologically, the masses of W^\pm boson, Z boson, and photon corrected by dimension-six operators are presented as follow:

$$\begin{cases} M_W &= \frac{\bar{g}_2 v}{2} \\ M_Z &= \frac{1}{2} \sqrt{\bar{g}_1^2 + \bar{g}_2^2} v \left[1 + \frac{1}{4} C_{\phi D} v^2 \right] \left(1 + \epsilon \frac{\bar{g}_2 \bar{g}_1}{\bar{g}_2^2 + \bar{g}_1^2} \right), \\ M_A &= 0 \end{cases} \quad (22)$$

whereas

$$\epsilon \equiv C_{\phi WB} v^2. \quad (23)$$

Backup slide

Cancellation of gauge-fixing parameters

- Z-boson propagator in R_ξ – gauge:

$$\begin{aligned}\Delta_{\mu\nu}(q) &= \frac{-i}{q^2 - m_Z^2} \left[g_{\mu\nu} - (1 - \xi_Z) \frac{q_\mu q_\nu}{q^2 - \xi_Z m_Z^2} \right] \\ &= \frac{-i}{q^2 - m_Z^2} \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{m_Z^2} \right] - i \frac{q_\mu q_\nu}{m_Z^2 (q^2 - \xi_Z m_Z^2)} \quad (24)\end{aligned}$$

→ The first term of (24) is independent with gauge-fixing parameter, the second term will cancel the Goldstone-boson diagram G^0 .

Backup slide (Conventions in spin density matrix)

In Cartesian coordinates:

$$S_{\pm 1} = \mp \frac{1}{\sqrt{2}} (S_x \pm i S_y), \quad S_0 = S_z, \quad T_{\pm 2} = S_{\pm 1}^2$$

$$T_0 = \frac{1}{\sqrt{6}} [S_{+1}S_{-1} + S_{-1}S_{+1} + 2S_0^2], \quad T_{\pm 1} = \frac{1}{\sqrt{2}} [S_{\pm 1}S_0 + S_0S_{\pm 1}] \quad (25)$$

whereas the explicit form of S_x, S_y, S_z are

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{1}{\sqrt{2}i} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

For convenience we used the following operators in the Cartesian basis:

$$\begin{aligned} A_1 &= \frac{1}{2} (T_1 - T_{-1}), & A_2 &= \frac{1}{2i} (T_1 + T_{-1}), \\ B_1 &= \frac{1}{2} (T_2 + T_{-2}), & B_2 &= \frac{1}{2i} (T_2 - T_{-2}). \end{aligned} \quad (26)$$

Eight spin-observable of W^+ boson

$$A_{FB} = \frac{1}{\Gamma} [\Gamma(\cos \theta > 0) - \Gamma(\cos \theta < 0)] = \frac{3}{4} \langle S_z \rangle \quad (27)$$

$$A_{EC} = \frac{1}{\Gamma} \left[\Gamma \left(|\cos \theta| > \frac{1}{2} \right) - \Gamma \left(|\cos \theta| < \frac{1}{2} \right) \right] = \frac{3}{8} \sqrt{\frac{3}{2}} \langle T_0 \rangle \quad (28)$$

$$\frac{\delta_1 \Gamma}{\Gamma} = \frac{1}{\Gamma} [\Gamma(\cos \phi > 0) - \Gamma(\cos \phi < 0)] = \frac{3}{4} \langle S_x \rangle \quad (29)$$

$$\frac{\delta_2 \Gamma}{\Gamma} = \frac{1}{\Gamma} [\Gamma(\sin \phi > 0) - \Gamma(\sin \phi < 0)] = \frac{3}{4} \langle S_y \rangle \quad (30)$$

$$A_{FB}^1 = \frac{1}{\Gamma} [\Gamma(\cos \phi \cos \theta > 0) - \Gamma(\cos \phi \cos \theta < 0)] = -\frac{2}{\pi} \langle A_1 \rangle \quad (31)$$

$$A_{FB}^2 = \frac{1}{\Gamma} [\Gamma(\sin \phi \cos \theta > 0) - \Gamma(\sin \phi \cos \theta < 0)] = -\frac{2}{\pi} \langle A_2 \rangle \quad (32)$$

$$A_\phi^1 = \frac{1}{\Gamma} [\Gamma(\cos 2\phi > 0) - \Gamma(\cos 2\phi < 0)] = \frac{2}{\pi} \langle B_1 \rangle \quad (33)$$

$$A_\phi^2 = \frac{1}{\Gamma} [\Gamma(\sin 2\phi > 0) - \Gamma(\sin 2\phi < 0)] = \frac{2}{\pi} \langle B_2 \rangle \quad (34)$$