Some Aspects of Cosmology in Randall-Sundrum Model

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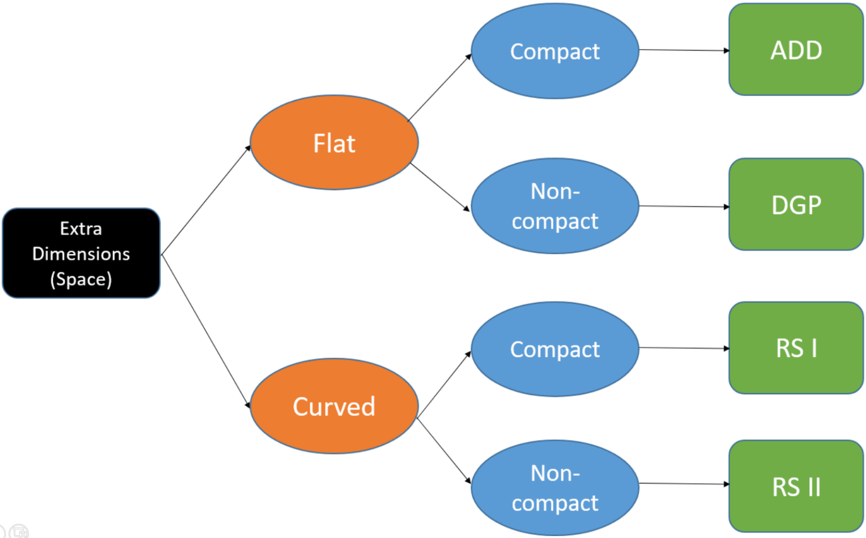
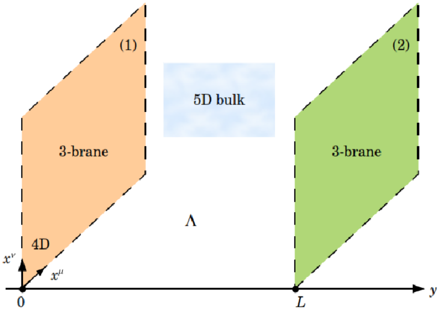
Introduction to RS Model





$$ds^2 = e^{-2\alpha y}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \quad (1)$$

The Age of the Universe in RS Model

$\boxed{\text{Robertson-Walker metric}} \rightarrow \boxed{\text{Einstein's equations}} \rightarrow \boxed{\text{Friedmann equation}}$

- Robertson-Walker metric in RS model

$$ds^2 = e^{-2\alpha y}(-dt^2 + a^2(t)\delta_{ij}dx^i dx^j) + dy^2 \tag{2}$$

- 5D Einstein's equations

$$R_{AB} - \frac{1}{2}RG_{AB} = 8\pi G_{(5)}(\hat{T}_{AB}|_{Bulk} + T_{AB}|_{brane}) \tag{3}$$

- Friedmann equation

$$H = \frac{8\pi G_{(5)}}{6}\rho_b = \frac{8\pi G_{(4)}}{3}A\rho_b \tag{4}$$

The Age of the Universe in RS Model

- Some backgrounds
  1. Definition of the redshift parameter

$$1 + z = \frac{\lambda_0}{\lambda_e} = \frac{a_0}{a} \tag{5}$$

  2. Matters are confined on "brane", so the fluid equation is the same

$$\rho = \frac{3H_0^2}{8\pi G_{(4)}} \sum_i \Omega_i^{(0)} (1+z)^{3(1+\omega_i)} \tag{6}$$

- The tricks
  1. At $z = 0$, $H = H_0 \rightarrow$ determine A ("normalization constant")
  2. Example: Matter only

$$H = \frac{8\pi G_{(4)}}{3} A\rho_b = A H_0^2 \Omega_m^{(0)} (1+z)^3 \rightarrow \boxed{A = (H_0 \Omega_m^{(0)})^{-1}} \tag{7}$$

The Age of the Universe in RS Model

$$\boxed{t_0 = \int_0^\infty \frac{dz}{H(1+z)}} \tag{8}$$

Types of Universe \ Models	FRW	RS
Matter	$H = H_0(1+z)^{3/2}$	$H = H_0(1+z)^3$
Matter + Dark Energy	$H = H_0[\Omega_m^{(0)}(1+z)^3 + \Omega_\Lambda^{(0)}]^{1/2}$	$H = H_0\left[\frac{\Omega_m^{(0)}(1+z)^3+\Omega_\Lambda^{(0)}}{\Omega_m^{(0)}+\Omega_\Lambda^{(0)}}\right]$

Table 1. Comparisons of the Friedmann equation between FRW and RS model.

Types of Universe \ Models	FRW	RS
Matter	9.6 Gyr	4.8 Gyr
Matter + Dark Energy	13.9 Gyr	8.3 Gyr

Table 2. Comparisons of the age of the universe between FRW and RS model.

Inflation in RS Model

- Slow-roll approximation is the must for inflation to happen

$$\dot{\phi}^2 << V(\phi) \rightarrow \epsilon << 1$$

- Procedures in an inflation problem

$$\boxed{\epsilon \approx 1} \rightarrow \boxed{\phi_f} \rightarrow \boxed{N} \rightarrow \boxed{\phi_i} \rightarrow \boxed{\epsilon, \eta} \rightarrow \boxed{n_s, r}$$

Inflation in RS Model

Quantities	Standard Inflation	Inflation on brane in RS
n_s	$1 - 6\epsilon + 2\eta$	$1 - 6\epsilon + 2\eta$
r	16ϵ	24ϵ

Table 3. Scalar spectral index and the tensor-to-scalar ratio in two models.

Quantities	Standard Inflation	Inflation on brane		Observations
	ϕ^2	ϕ^2	$exp(-\beta\phi)$	
n_s	0.967	0.959	0.934	0.9607 ± 0.0063
r	0.132	0.198	0.393	< 0.25

Table 4. Numerical values of the scalar spectral index and the tensor-to-scalar ratio in two models.

Inflation in RS Model

- Changes of the inflaton field

$$\begin{cases} \Delta\phi_{SI} \simeq 3M_4 \\ \Delta\phi_{RS} \simeq 500M_5 \end{cases} \tag{9}$$

- Bounds on 5D Planck scale

$$\text{(Newtonian limit) } 10^8\,GeV < M_5 < 10^{17}\,GeV \text{ (avoid quantum gravity)} \tag{10}$$

- Meanwhile,

$$M_4 = 1,22.10^{19}\,GeV \tag{11}$$

Summary

1. There are **2 main characteristics** of the extra spatial dimension(s) to classify various models: flat or curved and compact or non-compact.
2. The age of the universe in RS model is **too small** compared to the standard ΛCDM model. Therefore, we need further investigations about this model.
3. Inflaton **rolls more slowly** in RS model than in the standard 4D model. Therefore, it allows **a wider range** of the inflaton potential.
4. Inflation in RS model is **consistent** with observations, but because the inflaton potential is **arbitrary** so it is **not a persuasive evidence** for the existence of the extra dimension.

Thank you for watching!

"If I have ever made any valuable discoveries, it has been due more to patient attention than to any other talent."

Isaac Newton