

$K^0 - \bar{K}^0$ mixing in the minimal flavor violating two-Higgs doublet models

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Outline

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2 Theoretical Framework

3 Analytic Results

4 Numerical Results

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Motivation

Standard Model (SM) & New Physics (NP)

The Standard Model of Particle Physics

Advantages

- Covers almost every particle.
- Predictions agree well with experiments.

Disadvantages

- Gravity is not included.
- Not enough CP violation sources.

Motivation

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Two-Higgs Doublet Models (2HDM)

Baryon asymmetry of the Universe (BAU)

Supersymmetry

- Minimal Supersymmetric Standard Model (MSSM)

Motivation

2HDM

Flavor Changing Neutral Current (FCNC)

- Natural Flavor Conservation (NFC)
 - ▶ At most one scalar doublet can be coupled to right-handed fermion which can be done by applying discrete \mathbb{Z}_2 symmetry on two scalar doublets differently.
- Minimal Flavor Violation (MFV)
 - ▶ The source of flavor-violating interactions is the same as the SM case, via Cabibbo-Kobayashi-Maskawa (CKM) matrix.

Motivation

Kaon mixing

CP violation (CPV)

$$(\Delta m_K)_{exp} = 3.4839(59) \times 10^{-15} \text{ GeV}$$

$$(\epsilon_K)_{exp} = 2.228(11) \times 10^{-3}$$

Theoretical Framework

Yukawa Section

Yukawa interaction

$$\begin{aligned} -\mathcal{L}_Y = & \bar{q}_L^0 \tilde{\Phi}_1 Y^U u_R^0 + \bar{q}_L^0 \Phi_1 Y^D d_R^0 \\ & + \bar{q}_L^0 \tilde{\Phi}_2^{(a)} T^{(a)} \bar{Y}^U u_R^0 + \bar{q}_L^0 \Phi_2^{(a)} T^{(a)} \bar{Y}^D d_R^0 + \text{h.c.} \end{aligned} \quad (1)$$

$$\tilde{\Phi}_j = i\sigma_2 \Phi_j^* \quad j = 1, 2 \quad (2)$$

MFV hypothesis

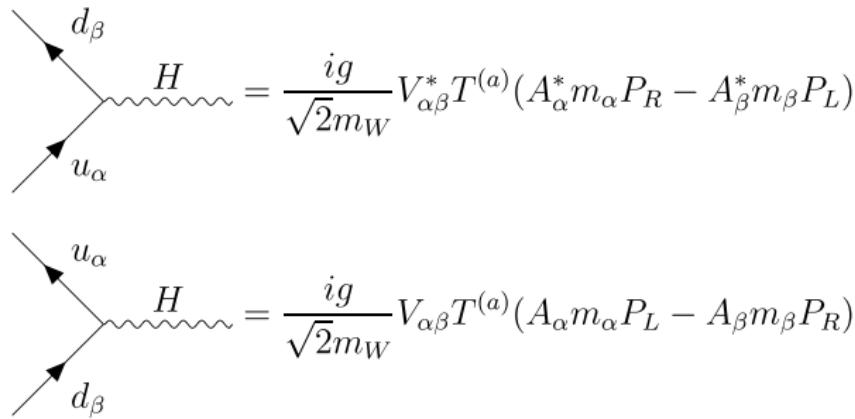
$$\begin{aligned} \bar{Y}^U &= A_u^* (1 + \epsilon_u^* Y^U Y^{U\dagger} + \dots) Y^U \\ \bar{Y}^D &= A_d (1 + \epsilon_d Y^U Y^{U\dagger} + \dots) Y^D \end{aligned} \quad (3)$$

Theoretical Framework

Charged Higgs Feynman rules

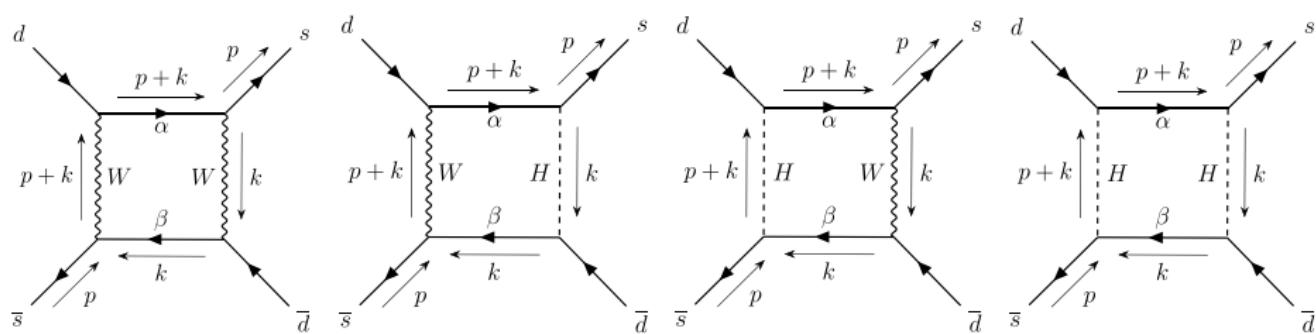
$$\mathcal{L}_{H^+} = \frac{g}{\sqrt{2}m_W} \sum_{i,j=1}^3 \bar{u}_i T^{(a)} (A_u^i m_{u_i} P_L - A_d^j m_{d_j} P_R) V_{ij} d_j H_{(a)}^+ + \text{h.c.} \quad (4)$$

$$A_{u,d}^i = A_{u,d} \left(1 + \epsilon_{u,d} \frac{m_t^2}{v^2} \delta_{i3} \right) \approx A_{u,d} \quad (5)$$



Theoretical Framework

$K^0 - \bar{K}^0$ mixing (1)



$$\mathcal{H}_{\text{eff}}^{\Delta F=2} = \frac{G_F^2 m_W^2}{4\pi^2} \left[C^{VLL}(\mu) Q^{VLL} + C^{SLL}(\mu) Q^{SLL} + C^{TLL}(\mu) Q^{TLL} \right] \quad (6)$$

$$Q^{VLL} = \bar{s}^\alpha \gamma_\mu (1 - \gamma_5) d^\alpha \bar{s}^\beta \gamma^\mu (1 - \gamma_5) d^\beta$$

$$Q^{SLL} = \bar{s}^\alpha (1 - \gamma_5) d^\alpha \bar{s}^\beta (1 - \gamma_5) d^\beta \quad (7)$$

$$Q^{TLL} = \bar{s}^\alpha \sigma_{\mu\nu} (1 - \gamma_5) d^\alpha \bar{s}^\beta \sigma^{\mu\nu} (1 - \gamma_5) d^\beta$$

Theoretical Framework

$K^0 - \bar{K}^0$ mixing (2)

$$\Delta m_K = 2\text{Re} \langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle \quad (8)$$

$$\epsilon_K = \frac{\kappa_e e^{i\phi_\epsilon}}{\sqrt{2}} \frac{\text{Im} \langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle}{\Delta m_K^{\text{exp}}} \quad (9)$$

$$\langle \bar{K}^0 | Q^{VLL} | K^0 \rangle = \frac{1}{3} m_K F_K^2 B_1^{VLL}(\mu)$$

$$\langle \bar{K}^0 | Q^{SLL} | K^0 \rangle = -\frac{5}{24} R(\mu) m_K F_K^2 B_1^{SLL}(\mu) \quad R(\mu) = \left(\frac{m_K}{m_s(\mu) + m_d(\mu)} \right)^2$$

$$\langle \bar{K}^0 | Q^{TLL} | K^0 \rangle = -\frac{1}{2} R(\mu) m_K F_K^2 B_2^{SLL}(\mu)$$

Theoretical Framework

Renormalization Group Evolution

RGE of Wilson Coefficients

$$C^{VLL}(3 \text{ GeV}) = [\eta(3 \text{ GeV})]_{VLL} C^{VLL}(\mu) \quad (10)$$

$$\begin{pmatrix} C^{SLL}(3 \text{ GeV}) \\ C^{TLL}(3 \text{ GeV}) \end{pmatrix} = \begin{pmatrix} [\eta_{11}(3 \text{ GeV})]_{SLL} & [\eta_{12}(3 \text{ GeV})]_{SLL} \\ [\eta_{21}(3 \text{ GeV})]_{SLL} & [\eta_{22}(3 \text{ GeV})]_{SLL} \end{pmatrix} \begin{pmatrix} C^{SLL}(\mu) \\ C^{TLL}(\mu) \end{pmatrix} \quad (11)$$

P factors

$$P^{VLL} = [\eta(\mu_L)]_{VLL} B_1(\mu_L) \quad (12)$$

$$P^{SLL} = -\frac{5}{8}[\eta_{11}(\mu_L)]_{SLL}[B_2(\mu_L)]_{\text{eff}} - \frac{3}{2}[\eta_{21}(\mu_L)]_{SLL}[B_3(\mu_L)]_{\text{eff}} \quad (13)$$

$$P^{TLL} = -\frac{5}{8}[\eta_{12}(\mu_L)]_{SLL}[B_2(\mu_L)]_{\text{eff}} - \frac{3}{2}[\eta_{22}(\mu_L)]_{SLL}[B_3(\mu_L)]_{\text{eff}} \quad (14)$$

Analytic Results

Standard Model (SM)

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle_{SM}^{VLL} = \zeta \left[\hat{B} C_{SM, x_s=0}^{VLL}(\text{SI}) + P_{SM}^{VLL} x_{s, \mu_W} C_{SM, x_s}^{VLL}(\mu_W) \right] \quad (15)$$

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle_{SM}^{SLL} = \zeta \left[P_{SM}^{SLL} C_{SM}^{SLL}(\mu_W) \right] \quad (16) \quad \zeta = \frac{G_F^2 m_W^2 m_K F_K^2}{12\pi^2}$$

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle_{SM}^{TLL} = \zeta \left[P_{SM}^{TLL} C_{SM}^{TLL}(\mu_W) \right] \quad (17) \quad x_i = \left(\frac{m_i}{m_W} \right)^2$$

where

$$\begin{aligned} C_{SM, x_s=0}^{VLL} &= \lambda_c^2 \eta_{cc} S_0(\bar{x}_c) + \lambda_t^2 \eta_{tt} S_0(\bar{x}_t) + 2\lambda_c \lambda_t \eta_{ct} S_0(\bar{x}_c, \bar{x}_t) \\ C_{SM, x_s}^{VLL} &= \lambda_c^2 f_1(x_{c, \mu_W}) + \lambda_t^2 f_1(x_{t, \mu_W}) + 2\lambda_c \lambda_t f_1(x_{c, \mu_W}, x_{t, \mu_W}) \end{aligned} \quad (18)$$

Analytic Results

2HDM

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle_{III}^{VLL} = \zeta \left[P_{NP}^{VLL} C_{III}^{VLL}(\mu_t) \right] \quad (19)$$

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle_{III}^{SLL} = \zeta \left[P_{NP}^{SLL} C_{III}^{SLL}(\mu_t) \right] \quad (20)$$

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle_{III}^{TLL} = 0 \quad (21)$$

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle_C^{VLL} = \zeta \left[P_{NP}^{VLL} C_C^{VLL}(\mu_t) \right] \quad (22)$$

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle_C^{SLL} = \zeta \left[P_{NP}^{SLL} C_C^{SLL}(\mu_t) \right] \quad (23)$$

$$\langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle_C^{TLL} = \zeta \left[P_{NP}^{TLL} C_C^{TLL}(\mu_t) \right] \quad (24)$$

Numerical Results

Input parameters

General \diamond

$$m_W = 80.385(15) \text{ GeV}$$

$$m_Z = 91.1876(21) \text{ GeV}$$

$$\mu_W = 80.385 \text{ GeV}$$

$$\mu_t = 163.427 \text{ GeV}^*$$

$$G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$$

QCD coupling constant

$$\alpha_s(\mu_t) = 0.1086(10)^*$$

$$\alpha_s(m_Z) = 0.1182(12)^\diamond$$

$$\alpha_s(\mu_W) = 0.1205(12)^*$$

$$\alpha_s(\mu_b) = 0.2243(45)^*$$

$$\alpha_s(3 \text{ GeV}) = 0.2521^{+0.0058}_{-0.0057} *$$

CKM \diamond

$$\lambda = 0.22506(50)$$

$$A = 0.811(26)$$

$$\bar{\rho} = 0.124^{+0.019}_{-0.018}$$

$$\bar{\eta} = 0.356(11)$$

$$\rho = 0.127^{+0.019}_{-0.018}$$

$$\eta = 0.365(11)$$

Quark masses

$$m_d(2 \text{ GeV}) = 0.0047^{+0.0005}_{-0.0004} \text{ GeV}^\diamond$$

$$m_d(3 \text{ GeV}) = 0.0043^{+0.0005}_{-0.0004} \text{ GeV}^*$$

$$m_s(2 \text{ GeV}) = 0.096^{+0.008}_{-0.004} \text{ GeV}^\diamond$$

$$m_s(3 \text{ GeV}) = 0.087^{+0.007}_{-0.004} \text{ GeV}^*$$

$$m_s(\mu_W) = 0.057^{+0.005}_{-0.002} \text{ GeV}^*$$

$$m_s(\mu_t) = 0.054^{+0.005}_{-0.002} \text{ GeV}^*$$

$$m_c(m_c) = 1.27(3) \text{ GeV}^\diamond$$

$$m_c(\mu_W) = 0.660(21) \text{ GeV}^*$$

$$m_c(\mu_t) = 0.623(20) \text{ GeV}^*$$

$$M_t = 173.21(87) \text{ GeV}^\diamond$$

$$m_t(m_t) = 163.427^{+0.828}_{-0.829} \text{ GeV}^*$$

$$m_t(\mu_W) = 173.276^{+1.590}_{-1.586} \text{ GeV}^*$$

Kaon

$$m_K = 0.497611(13) \text{ GeV}^\diamond$$

$$F_K = 0.1562(9) \text{ GeV}^\clubsuit$$

$$\hat{B}_K = 0.7625(97) \clubsuit$$

$$B_1(3 \text{ GeV}) = 0.519(26) \spadesuit$$

$$B_2(3 \text{ GeV}) = 0.525(23) \spadesuit$$

$$B_3(3 \text{ GeV}) = 0.360(16) \spadesuit$$

$$B_4(3 \text{ GeV}) = 0.981(62) \spadesuit$$

$$B_5(3 \text{ GeV}) = 0.751(68) \spadesuit$$

$$\eta_{cc} = 1.87(76) \heartsuit$$

$$\eta_{tt} = 0.5765(65) \heartsuit$$

$$\eta_{ct} = 0.496(47) \heartsuit$$

Experimental Value \diamond

$$(\Delta m_K)_{\text{exp}} = 3.4839(59) \times 10^{-15} \text{ GeV}$$

$$(\epsilon_K)_{\text{exp}} = 2.228(11) \times 10^{-3}$$

Ref:

\diamond : C. Patrignani et al.
[Particle Data Group],
Chin. Phys. C 40 (2016)
100001.

\clubsuit : S. Aoki et al. [FLAG
Working Group],
arXiv:1607.00299[hep-lat].

\spadesuit : B. J. Choi et al.
[SWME Collaboration],
Phys. Rev. D 93 (2016)
014511.

\heartsuit : C. Bobeth, A. J. Buras,
A. Celis and M. Jung,
arXiv:1609.04783[hep-ph].

*: RunDec package.

Numerical Results

SM: Δm_K

Table: Theoretical prediction of Δm_K in the SM without any correction.

Correction	None	Ref[1108.2036]	Ref[0406094]
$(\Delta m_K)_{\text{SM}} (\times 10^{-15} \text{ GeV})$	3.109(1.258)	3.1(1.2)	3.1(0.5)
$\frac{(\Delta m_K)_{\text{SM}}}{(\Delta m_K)_{\text{exp}}}$	89.24%	88.98%	88.98%

Table: Theoretical prediction of Δm_K in the SM with the corrections.

Correction	LD	x_s	LD, x_s
$(\Delta m_K)_{\text{SM}} (\times 10^{-15} \text{ GeV})$	3.458(1.258)	3.321(1.258)	3.670(1.258)
$\frac{(\Delta m_K)_{\text{SM}}}{(\Delta m_K)_{\text{exp}}}$	99.24%	95.34%	105.34%

Numerical Results

SM: ϵ_K

Table: Theoretical prediction of ϵ_K in the SM with LD correction.

Correction	LD	Ref[1108.2036]	Ref[1007.0684]
$(\epsilon_K)_{\text{SM}} (\times 10^{-3})$	$2.086^{+0.294}_{-0.280}$	1.81(28)	1.90(26)
$\frac{(\epsilon_K)_{\text{SM}}}{(\epsilon_K)_{\text{exp}}}$	93.63%	81.24%	85.28%

Table: Other scenarios for theoretical prediction of ϵ_K in the SM.

Correction	None	x_s	LD, x_s
$(\epsilon_K)_{\text{SM}} (\times 10^{-3})$	$2.219^{+0.309}_{-0.294}$	$2.218^{+0.309}_{-0.294}$	$2.085^{+0.294}_{-0.280}$
$\frac{(\epsilon_K)_{\text{SM}}}{(\epsilon_K)_{\text{exp}}}$	99.61%	99.55%	93.58%

Numerical Results

2HDM type-III: real coupling case

$$(\Delta m_K)_{III}^{VLL} \times 10^{19} \approx 31.70 A_u^2 + 5.35 A_u^4 + 4.10 \times 10^{-4} A_u A_d \quad (25)$$

$$\begin{aligned} (\Delta m_K)_{III}^{SLL} \times 10^{23} \approx & -1.03 A_u^2 + 0.03 A_u^4 + 96.81 A_u A_d \\ & - 0.14 A_u^3 A_d + 0.14 A_u^2 A_d^2 \end{aligned} \quad (26)$$

$$(\epsilon_K)_{III}^{VLL} \times 10^4 \approx -3.09 A_u^2 - 0.53 A_u^4 - 3.37 \times 10^{-7} A_u A_d \quad (27)$$

$$\begin{aligned} (\epsilon_K)_{III}^{SLL} \times 10^9 \approx & 0.98 A_u^2 - 0.03 A_u^4 - 7.55 A_u A_d \\ & + 0.14 A_u^3 A_d - 0.14 A_u^2 A_d^2 \end{aligned} \quad (28)$$

Numerical Results

2HDM type-C: real coupling case

$$(\Delta m_K)_C^{VLL} \times 10^{19} \approx 10.56 A_u^2 + 3.27 A_u^4 + 1.36 \times 10^{-4} A_u A_d \quad (29)$$

$$\begin{aligned} (\Delta m_K)_C^{SLL} \times 10^{23} \approx & 0.43 A_u^2 - 0.01 A_u^4 - 30.34 A_u A_d \\ & + 0.04 A_u^3 A_d - 0.04 A_u^2 A_d^2 \end{aligned} \quad (30)$$

$$\begin{aligned} (\Delta m_K)_C^{TLL} \times 10^{23} \approx & -0.11 A_u^2 + 3.49 \times 10^{-3} A_u^4 + 10.73 A_u A_d \\ & - 0.02 A_u^3 A_d + 0.02 A_u^2 A_d^2 \end{aligned} \quad (31)$$

$$(\epsilon_K)_C^{VLL} \times 10^4 \approx -1.03 A_u^2 - 0.32 A_u^4 - 1.12 \times 10^{-7} A_u A_d \quad (32)$$

$$\begin{aligned} (\epsilon_K)_C^{SLL} \times 10^9 \approx & -0.41 A_u^2 + 0.01 A_u^4 + 3.15 A_u A_d \\ & - 0.04 A_u^3 A_d + 0.04 A_u^2 A_d^2 \end{aligned} \quad (33)$$

$$\begin{aligned} (\epsilon_K)_C^{TLL} \times 10^9 \approx & 0.11 A_u^2 - 3.41 \times 10^{-3} A_u^4 - 0.84 A_u A_d \\ & + 0.02 A_u^3 A_d - 0.02 A_u^2 A_d^2 \end{aligned} \quad (34)$$

Numerical Results

2HDM: complex coupling case

Type-III

$$(\Delta m_K)_{III}^{VLL} \times 10^{19} \approx 31.70 |A_u|^2 + 5.35 |A_u|^4 + 4.10 \times 10^{-4} |A_u| |A_d| \cos(\theta) - 3.53 \times 10^{-6} |A_u| |A_d| \sin(\theta)$$

$$(\epsilon_K)_{III}^{VLL} \times 10^4 \approx -3.09 |A_u|^2 - 0.53 |A_u|^4 - 3.37 \times 10^{-7} |A_u| |A_d| \cos(\theta) - 3.91 \times 10^{-5} |A_u| |A_d| \sin(\theta)$$

Type-C

$$(\Delta m_K)_C^{VLL} \times 10^{19} \approx 10.56 |A_u|^2 + 3.27 |A_u|^4 + 1.36 \times 10^{-4} |A_u| |A_d| \cos(\theta) - 1.18 \times 10^{-6} |A_u| |A_d| \sin(\theta)$$

$$(\epsilon_K)_C^{VLL} \times 10^4 \approx -1.03 |A_u|^2 - 0.32 |A_u|^4 - 1.12 \times 10^{-7} |A_u| |A_d| \cos(\theta) - 1.30 \times 10^{-5} |A_u| |A_d| \sin(\theta)$$

Numerical Results

Conditions for parameter spaces

Parameter range

$$|A_u| \in [0, 3], \quad |A_d| \in [0, 500], \quad \theta \in [-\pi, \pi], \quad m_{H^\pm} = 100, 250, 500 \text{ GeV}$$

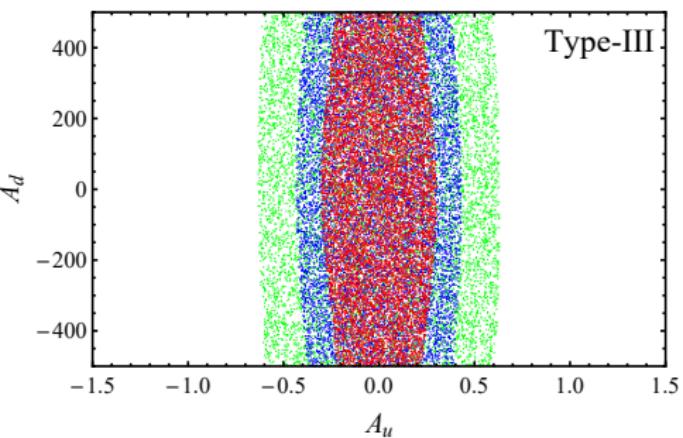
Condition for allowed values

$$R_{m_K} = \frac{(\Delta m_K)_{\text{exp}}}{(\Delta m_K)_{\text{SM}}} = 1.049^{+0.640}_{-0.288}, \quad R_{\epsilon_K} = \frac{(\epsilon)_{\text{exp}}}{(\epsilon)_{\text{SM}}} = 1.069^{+0.166}_{-0.132}$$

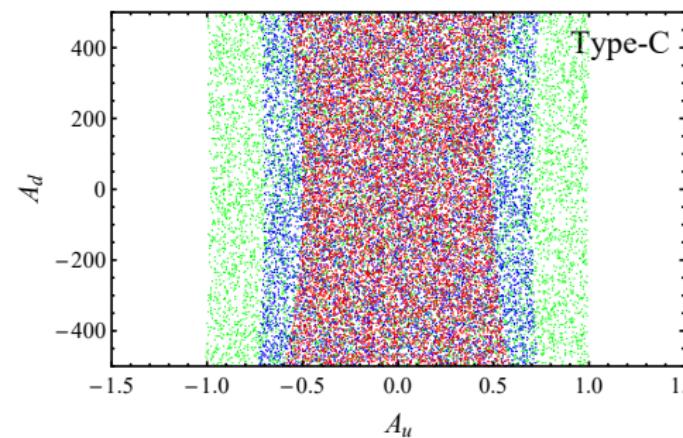
$$R_i - \sigma_i^- < 1 + \frac{(\mathcal{O}_i)_{\text{NP}}}{(\mathcal{O}_i)_{\text{SM}}} < R_i + \sigma_i^+$$

Numerical Results

Allowed parameter spaces: real coupling



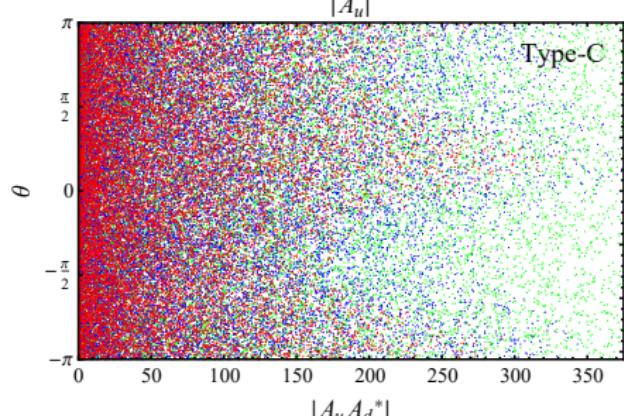
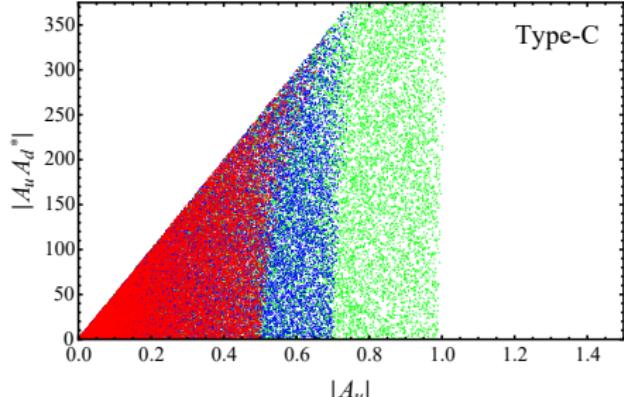
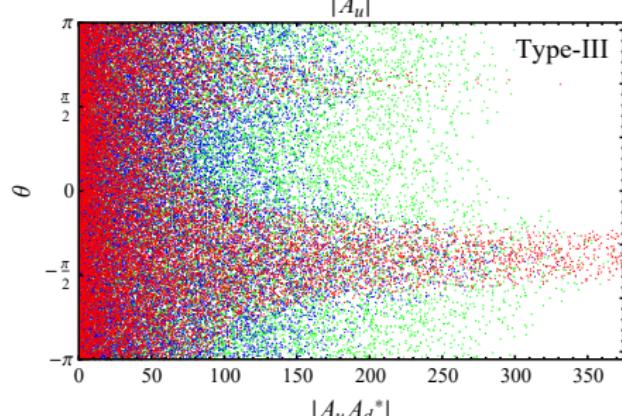
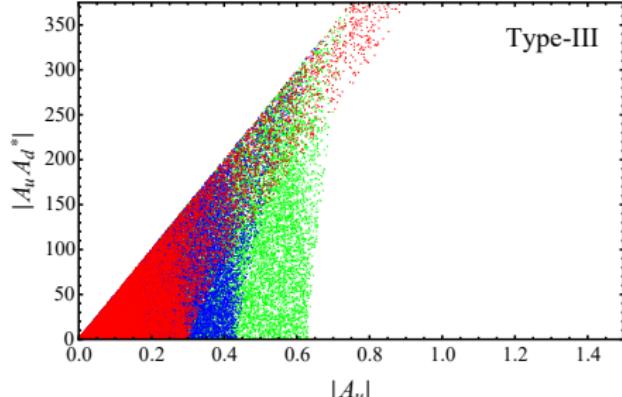
Type-III



Type-C

Numerical Results

Allowed parameter spaces: complex coupling



Conclusion

- ① The similar results between the type-III and the type-C have been obtained with the additional factors and operator, Q^{TLL} , for the type-C due to the color factors.

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- ① The similar results between the type-III and the type-C have been obtained with the additional factors and operator, Q^{TLL} , for the type-C due to the color factors.
- ② The allowed parameter spaces for A_u and A_d in the case of real coupling are similar for both types with the wider range in the type-C. If we extend A_d range, however, the allowed region for A_u will be smaller in the type-III and larger in the type-C, like “convex lens” and “concave lens” respectively.

Conclusion

- ➊ The similar results between the type-III and the type-C have been obtained with the additional factors and operator, Q^{TLL} , for the type-C due to the color factors.
- ➋ The allowed parameter spaces for A_u and A_d in the case of real coupling are similar for both types with the wider range in the type-C. If we extend A_d range, however, the allowed region for A_u will be smaller in the type-III and larger in the type-C, like “convex lens” and “concave lens” respectively.
- ➌ In the case of complex coupling, the strong correlation between $|A_u|$ and $|A_u A_d^*|$ is obtained especially for $m_{H^\pm} = 100$ GeV in the type-III. The phase between $|A_u|$ and $|A_d|$, θ , allows the large values of $|A_u A_d^*|$ at $\theta \approx \pm\pi/2$ in the type-III and $\theta \approx 0$ and $\pm\pi$ in the type-C. This is the result of the cancellation effects.

Thank you for your attention

CP violation in Kaon

$$CP|K^0\rangle = -|\bar{K}^0\rangle, \quad CP|\bar{K}^0\rangle = -|K^0\rangle. \quad (35)$$

$$|K_1\rangle = \frac{1}{\sqrt{2}} (K^0 - \bar{K}^0) \quad \text{with} \quad CP|K_1\rangle = |K_1\rangle, \quad (36)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}} (K^0 + \bar{K}^0) \quad \text{with} \quad CP|K_2\rangle = -|K_2\rangle. \quad (37)$$

$$|K_{S(L)}\rangle = \frac{1}{\sqrt{1 + |\bar{\varepsilon}|^2}} (|K_{1(2)}\rangle + \bar{\varepsilon}|K_{2(1)}\rangle) \quad (38)$$

If CP is conserved, $\bar{\varepsilon}$ vanishes and these two representations are identical,
 $|K_{S(L)}\rangle \equiv |K_{1(2)}\rangle$.

x_s correction term

Wilson coefficient for x_s -independent terms:

$$0.000466665\lambda_c^2 + 0.00219197\lambda_c\lambda_t + 1.3375\lambda_t^2$$

Wilson coefficient for x_s -dependent terms (TLL operator):

$$-5.028 \times 10^{-7}(1.87858\lambda_c^2 + 3.47969\lambda_c\lambda_t + 0.321934\lambda_t^2)$$

After rescale

x_s -independent terms: 1.72389×10^{-5}

x_s -dependent terms (TLL operator): -1.29582×10^{-6}

LD contribution

Δm_K

$$\Delta m_K = \Delta m_K^{SD} + \Delta m_K^{LD}|_{\pi\pi} + \Delta m_K^{LD}|_{\eta'}, \quad (39)$$

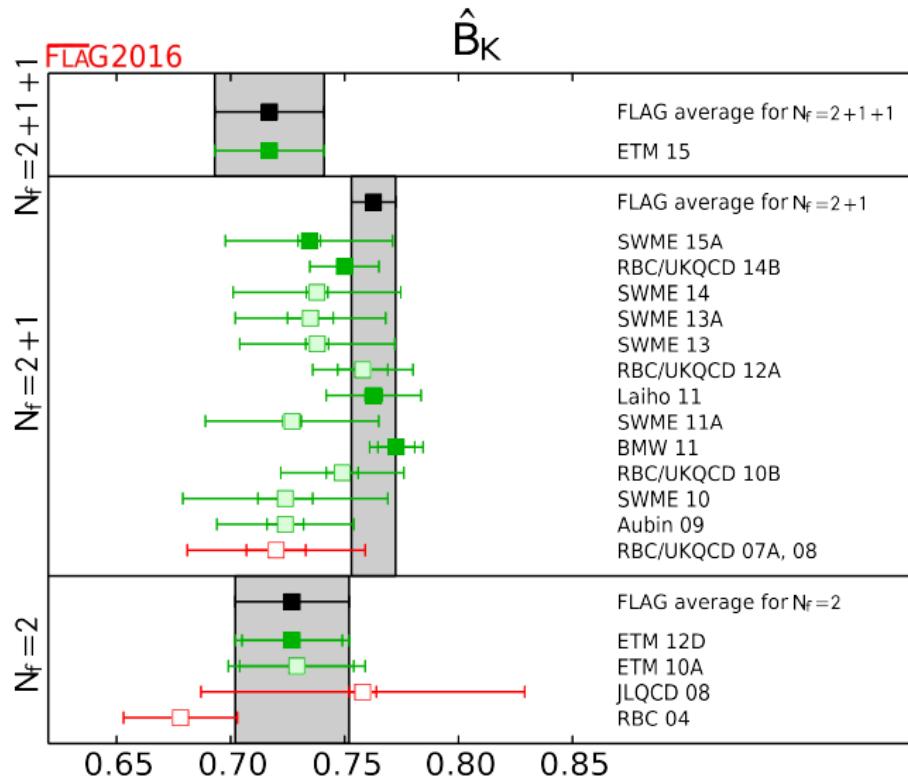
$$\Delta m_K^{LD}|_{\pi\pi} = 0.4 \Delta m_K^{\text{exp}}, \quad \Delta m_K^{LD}|_{\eta'} = -0.3 \Delta m_K^{\text{exp}}. \quad (40)$$

ϵ_K

$$\epsilon_K = \frac{\kappa_\epsilon e^{i\phi_\epsilon}}{\sqrt{2}} \frac{\text{Im } M_{12}^{SD}}{\Delta m_K^{\text{exp}}}, \quad (41)$$

where $\kappa_\epsilon = 0.94(2)$, $\phi_\epsilon = 43.52(5)^\circ$

Compare different number of effective flavors



Fierz Transformation

Q^{VLL}

$$(\bar{s}_a \gamma_\mu P_L d_b)(\bar{s}_b \gamma^\mu P_L d_a) \rightarrow (\bar{s}_a \gamma_\mu P_L d_a)(\bar{s}_b \gamma^\mu P_L d_b)$$

$$\tilde{Q}^{VLL} \rightarrow Q^{VLL}$$

Q^{SLL}

$$(\bar{s}_a P_L d_b)(\bar{s}_b P_L d_a) \rightarrow -\frac{1}{2}(\bar{s}_a P_L d_a)(\bar{s}_b P_L d_b) + \frac{1}{8}(\bar{s}_a \sigma_{\mu\nu} P_L d_a)(\bar{s}_b \sigma^{\mu\nu} P_L d_b)$$

$$\tilde{Q}^{SLL} \rightarrow -\frac{1}{2}Q^{SLL} + \frac{1}{8}Q^{TLL}$$

Coefficient for $K^0 - \bar{K}^0$ mixing

$$S_0(x_c, x_t) = x_c x_t \left[\frac{(x_c^2 - 8x_c + 4) \ln(x_c)}{4(x_c - 1)^2 (x_c - x_t)} - \frac{(x_t^2 - 8x_t + 4) \ln(x_t)}{4(x_t - 1)^2 (x_c - x_t)} - \frac{3}{4(x_c - 1)(x_t - 1)} \right]$$

Coefficient for $K^0 - \bar{K}^0$ mixing

$$\begin{aligned}
f_1(x_c, x_t) = & \frac{\ln(x_t)}{12(x_t - 1)^4(x_c - x_t)^3} \left[x_c^3 (x_t^4 - 9x_t^3 + 36x_t^2 - 42x_t + 12) \right. \\
& + x_c^2 x_t (-3x_t^4 + 22x_t^3 - 87x_t^2 + 108x_t - 36) + x_c x_t^3 (15x_t^2 - 23x_t + 6) \Big] \\
& + \frac{\ln(x_c)}{12(x_c - 1)^4(x_t - x_c)^3} \left[x_t^3 (x_c^4 - 9x_c^3 + 36x_c^2 - 42x_c + 12) \right. \\
& + x_t^2 x_c (-3x_c^4 + 22x_c^3 - 87x_c^2 + 108x_c - 36) + x_t x_c^3 (15x_c^2 - 23x_c + 6) \Big] \\
& - \frac{1}{72(x_c - 1)^3(x_t - 1)^3(x_c - x_t)^2} \left[x_c^5 (65x_t^3 - 130x_t^2 + 113x_t - 60) \right. \\
& + x_c^4 (-118x_t^4 + 34x_t^3 + 250x_t^2 - 298x_t + 180) \\
& + x_c^3 (65x_t^5 + 34x_t^4 + 66x_t^3 - 386x_t^2 + 329x_t - 180) \\
& - 2x_c^2 (65x_t^5 - 125x_t^4 + 193x_t^3 - 217x_t^2 + 90x_t - 30) \\
& + x_c x_t (113x_t^4 - 298x_t^3 + 329x_t^2 - 180x_t + 24) - 60(x_t - 1)^3 x_t^2 \Big]
\end{aligned}$$

Coefficient for $K^0 - \bar{K}^0$ mixing

$$f_2(x_c, x_t) = \frac{x_t}{36(x_c - 1)^3 (x_t - 1)^3 (x_c - x_t)^2} \left[x_c^5 (5x_t^2 - 22x_t + 5) \right.$$
$$+ 2x_c^4 (x_t^3 - x_t^2 + 35x_t - 11) + x_c^3 (5x_t^4 - 2x_t^3 - 78x_t^2 - 2x_t + 5)$$
$$- 2x_c^2 x_t (11x_t^3 - 35x_t^2 + x_t - 1) + x_c (5x_t^4 - 22x_t^3 + 5x_t^2) \left. \right]$$
$$- \frac{x_c^3 x_t \ln(x_c)}{6(x_c - 1)^4 (x_c - x_t)^3} \left[3x_c^2 (x_t + 1) - x_c (x_t^2 + 10x_t + 1) + 3x_t (x_t + 1) \right]$$
$$- \frac{x_t^3 x_c \ln(x_t)}{6(x_t - 1)^4 (x_t - x_c)^3} \left[3x_t^2 (x_c + 1) - x_t (x_c^2 + 10x_c + 1) + 3x_c (x_c + 1) \right]$$

Coefficient for $K^0 - \bar{K}^0$ mixing

$$f_3(x_c, x_t) = -\frac{1}{36(x_c - 1)^3(x_t - 1)^3(x_c - x_t)^2} \left[x_c^5 (10x_t^3 - 25x_t^2 + 8x_t - 5) \right.$$
$$+ x_c^4 (-20x_t^4 + 25x_t^3 + 49x_t^2 - 21x_t + 15)$$
$$+ x_c^3 (10x_t^5 + 25x_t^4 - 102x_t^3 + 2x_t^2 + 8x_t - 15)$$
$$+ x_c^2 (-25x_t^5 + 49x_t^4 + 2x_t^3 + 26x_t^2 - 9x_t + 5)$$
$$+ x_c (8x_t^5 - 21x_t^4 + 8x_t^3 - 9x_t^2 + 2x_t) - 5(x_t - 1)^3 x_t^2 \left. \right]$$
$$- \frac{\ln(x_t)}{6(x_t - 1)^4(x_c - x_t)^3} \left[x_c^3 (3x_t - 1) - x_c^2 (x_t^3 + 6x_t^2 - 3x_t) + x_c (3x_t^4 - x_t^3) \right]$$
$$- \frac{\ln(x_c)}{6(x_c - 1)^4(x_t - x_c)^3} \left[x_t^3 (3x_c - 1) - x_t^2 (x_c^3 + 6x_c^2 - 3x_c) + x_t (3x_c^4 - x_c^3) \right]$$