Neutrino spectrum in $\mathbf{SU}(3)_\ell \times \mathbf{SU}(3)_E$ gauged lepton flavor model

The 24th Vietnam School of Physics (VSOP-24)

Wararat Treesukrat[1]

Advisers: Dr. Warintorn Sreethawong[1] and Dr. Patipan Uttayarat[2]

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[1]School of Physics, Suranaree University of Technology, Nakhon Ratchasima, Thailand.

[2]Department of Physics, Srinakharinwirot University, Bangkok, Thailand.

Outline

Shortcomings of the SM

Shortcomings of the SM:

- **Neutrino Oscillation** [Nobel2015]

  $\mapsto \nu$ change their types $\mapsto \nu$ have mass! (tiny mass)

Current neutrino data

- ν **oscillation**:

  Normal hierarchy, $m_{\nu_1} \lesssim m_{\nu_2} \ll m_{\nu_3}$:

$$m_{\nu_3}^2 - m_{\nu_1}^2 \in [2.45, 2.69] \times 10^{-3}\text{eV}^2$$
$$m_{\nu_2}^2 - m_{\nu_1}^2 \in [6.93, 7.96] \times 10^{-5}\text{eV}^2$$

  Inverted hierarchy, $m_{\nu_1} \ll m_{\nu_2} \lesssim m_{\nu_3}$:

$$m_{\nu_3}^2 - m_{\nu_1}^2 \in [2.42, 2.66] \times 10^{-3}\text{eV}^2$$
$$m_{\nu_3}^2 - m_{\nu_2}^2 \in [6.93, 7.96] \times 10^{-5}\text{eV}^2$$

- **cosmological observation**:

$$\sum m_{\nu_i} \leq 0.17 \text{ eV}$$

notice: the lightest neutrino mass can be zero.

$\mapsto$ Only experimental constraints on ν mass **cannot** determine the lower bound on the lightest ν mass.

▷ Let consider theoretical constraints which called **Partial Wave Unitary Constraints** (PWUC)

1. This technique decompose the scattering amplitude into the partial wave amplitude.
2. The coefficient of each partial mode is bounded by the unitary condition.

What we are doing?

We will combine experimental and partial wave unitary constraint to determine the lower bound on lightest ν mass.

Shortcomings of the SM:

- **Number of fermion generation**

- why are there three generations of quark or lepton?

Flavor Symmetry in the Standard Model

- $SU(3)_\ell$: $L_i \to U^i L_i \qquad L_i \equiv \left\{ \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix} \right\}$
- $SU(3)_E$: $e_{R,i} \to V^i e_{R,i} \qquad e_{R,i} \equiv \{e_R, \mu_R, \tau_R\}$

Lagrangian in lepton sector

$$\mathcal{L}_{SM} \supset i\overline{L_i}\not{D}L_i + i\overline{e_{R,i}}\not{D}e_{R,i} + y\overline{L_i}He_{R,i} + \text{h.c.}$$

This Yukawa terms, $y\overline{L_i}He^i_R$, breaks flavor symmetry

To restore flavor symmetry, **y is promoted to be field** which transform under flavor group, i.e., $y_{ij} \to U^i y_{ij} (V^j)^\dagger$.

There is one model that addresses these shortcomings of the SM (ν mass & number of fermion families).

Model with gauged lepton flavor $\mathbf{SU}(3)_\ell \times \mathbf{SU}(3)_E$

- ⋆ Tiny neutrino mass can be generated.
- Size of $SU(3)_\ell \times SU(3)_E$ representation = Number of fermion generations in the SM = 3.
- ▷ Three extra fermions are introduced for anomaly cancellation.
- ▷ Yukawa coupings are promoted to be fields.

$\mathbf{SU}(3)_\ell \times \mathbf{SU}(3)_E$ Model

Table 1: Transformation properties of SM, extra fermions (mirror fields) and flavon fields under the electroweak (the first two rows) and the lepton flavor (bottom two rows) gauge group.

	l_L	e_R	H	$\mathcal{E}_R$	$\mathcal{E}_L$	$\mathcal{N}_R$	$\mathcal{Y}_E$	$\mathcal{Y}_N$
$SU(2)_L$	2	1	2	1	1	1	1	1
$U(1)_Y$	-1/2	-1	1/2	-1	-1	0	0	0
$SU(3)_l$	3	1	1	3	1	3	$\bar{3}$	$\bar{6}$
$SU(3)_E$	1	3	1	1	3	1	3	1

We focus on $\mathcal{N}_R$, the right-handed neutrino, which lead to neutrino mass generation.

- Lagrangian

$$\mathcal{L} \supset i\bar{\psi}\not{D}\psi + \mathrm{Tr}\left[D_\mu \mathcal{Y}_N (D^\mu \mathcal{Y}_N)^\dagger\right] + \mathcal{L}_{Yuk} - V(H, \mathcal{Y}_N), \quad (1)$$

where ψ are the fermion fields and $\mathcal{Y}_N$ is the flavons.

- Covariant derivative

$$\begin{aligned} D_\mu \mathcal{N}_R &= (\partial_\mu + ig_\ell A^\ell_\mu)\mathcal{N}_R, \\ D_\mu \mathcal{Y}_N &= \partial_\mu \mathcal{Y}_N - ig_\ell (A^\ell_\mu)^T \mathcal{Y}_N - ig_\ell \mathcal{Y}_N A^\ell_\mu, \end{aligned} \quad (2)$$

where $A^\ell_\mu = \sum_{a=1}^{8} A^{\ell,a}_\mu T^a$ with T^a the SU(3) generator.

- The Yukawa interactions and fermion mass terms

$$\mathcal{L}_{Yuk} \supset \lambda_\nu \overline{\ell_L} \tilde{H} \mathcal{N}_R + \frac{\lambda_N}{2} \overline{\mathcal{N}_R^c} \mathcal{Y}_N \mathcal{N}_R + \text{h.c.}. \tag{3}$$

After spontaneous EW and flavor symmetries breaking,

$$H \equiv (v+h)/\sqrt{2}, \qquad \mathcal{Y}_N \equiv \langle \mathcal{Y}_N \rangle + \phi_N/\sqrt{2}. \tag{4}$$

The mass matrices neutral leptons in a see-saw form

$$\frac{1}{2} \begin{pmatrix} 0 & \lambda_\nu v/\sqrt{2} \\ \lambda_\nu v/\sqrt{2} & \lambda_N \mathcal{Y}_N \end{pmatrix} + \text{h.c.}, \tag{5}$$

assume $\mathcal{Y}_N \gg v$, [1]

$$M_N \simeq \lambda_N \mathcal{Y}_N, \quad m_\nu \simeq \frac{\lambda_\nu^2 v^2/2}{M_N}. \tag{6}$$

Note: M_N is the heavy leptons mass and
m_ν is the light leptons mass.

[1] for convenience, the symbol $\mathcal{Y}_N$ is used as VEV of Yukawa flavon.

- Working in the basis where $\mathcal{Y}_N$ is diagonal

$$\mathcal{Y}_N \simeq \frac{\lambda_\nu^2 v}{2\lambda_N}\text{diag}\left(\frac{v}{m_{\nu_1}}, \frac{v}{m_{\nu_2}}, \frac{v}{m_{\nu_3}}\right), \tag{7}$$

The mass matrix of the flavor gauge bosons

$$(M_{\ell\ell}^2)_{ab} \simeq g_\ell^2 \text{Tr}\left(\mathcal{Y}_N\{T^a, T^b\}\mathcal{Y}_N^\dagger\right). \tag{8}$$

Partial Wave Unitary Constraints

$$N_i\bar{N}_i \to A_L^a A_L^a$$

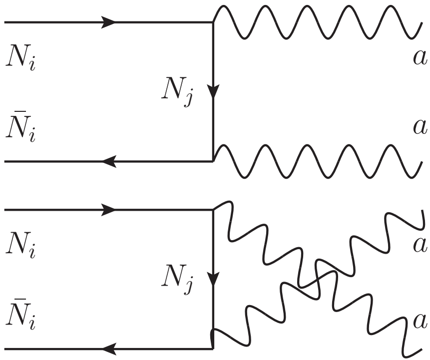


Figure 1: *t*- and *u*-channel Feynman diagrams contributing to the $N_i\bar{N}_i \to A_L^a A_L^a$.

$$A_L^a A_L^a \to A_L^b A_L^b$$

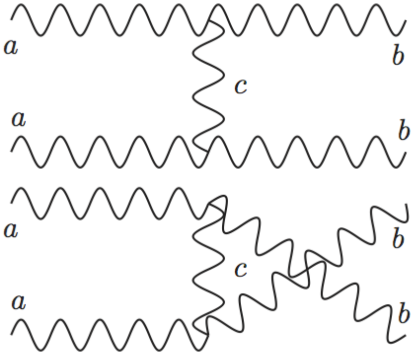


Figure 2: t- and u-channel Feynman diagrams contributing to the $A_L^a A_L^a \to A_L^b A_L^b$.

- **The asymptotic behavior amplitude at high energy of** $A_L^a A_L^a \to A_L^b A_L^b$

$$\mathcal{M} = g_\ell^2 \sum_c f^{abc}\Big(\frac{M_a^4}{M_b^2 M_c^2} + \frac{M_b^4}{M_a^2 M_c^2} + \frac{10M_a^2}{M_b^2} + \frac{2M_b^2}{M_a^2} - \frac{M_a^2}{M_c^2} - \frac{8M_a}{M_b} + \frac{8M_b}{M_a} - \frac{M_b^2}{M_c^2}\Big) \tag{9}$$

- **Partial wave unitary constraints** $A_L^7 A_L^7 \to A_L^1 A_L^1$
  $\longrightarrow$ the amplitude cannot be arbitrary large.

$$g_\ell^2 \sum_c f^{71c}\Big(\frac{M_7^4}{M_1^2 M_c^2} + \frac{M_1^4}{M_7^2 M_c^2} + \frac{10M_7^2}{M_1^2} + \frac{2M_1^2}{M_7^2} - \frac{M_7^2}{M_c^2} - \frac{8M_7}{M_1} + \frac{8M_1}{M_7} - \frac{M_1^2}{M_c^2}\Big) \leq 8\pi \tag{10}$$

Results: Viable Neutrino Spectrum

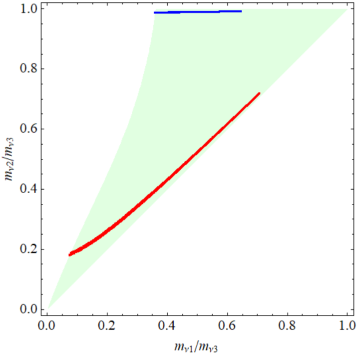


Figure 3: Viable neutrino spectrum for normal hierarchy (red) and inverted hierarchy (blue). Region compatible with perturbative unitarity is shown in green.

Results: the lower bound on the lightest neutrino mass

- **NH:** $m_{\nu_1} \geq 3.8 \times 10^{-3}$ eV
- **IH:** $m_{\nu_1} \geq 18.9 \times 10^{-3}$ eV

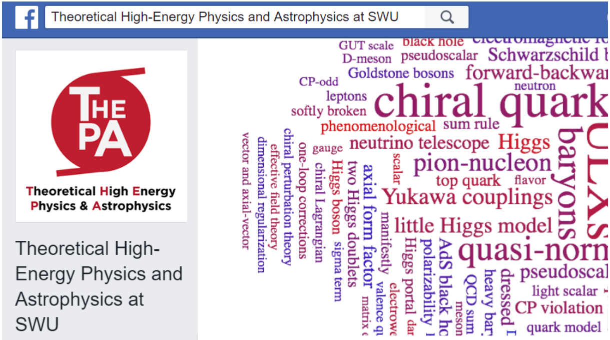
Take-home message:

- Experimental constraints + Partial wave unitary constraints $\rightarrow$ the lightest neutrino mass is not be zero

Future studies:

- Viable neutrino mixing angles
- Another process such as $N_i \bar{N}_i \rightarrow N_j \bar{N}_j$.

Thank you for your attention :)



Backup slides

Mass of gauge bosons (in the unit of $\frac{g_l^2 v^4 \lambda_\nu^4}{4\lambda_N^2 m_{\nu_3}^2}$)

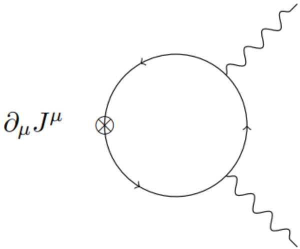
$$\begin{aligned}
\hat{M}^2_{A^1} &= M^2_{A^2} + 2xy = x^2 + xy + y^2, \\
\hat{M}^2_{A^4} &= M^2_{A^5} + 2x = x^2 + x + 1, \\
\hat{M}^2_{A^6} &= M^2_{A^7} + 2y = y^2 + y + 1, \\
\hat{M}^2_{A_\pm} &= x^2 + y^2 + 1 \pm \sqrt{x^4+y^4-x^2y^2-x^2-y^2+1},
\end{aligned} \tag{11}$$

- **Operators:**

$$T^\pm = -s_\alpha T^3 + c_\alpha T^8$$

$$s_\alpha = \sqrt{\frac{1}{2} + \frac{x^2+y^2-2}{4\sqrt{x^4+y^4-x^2y^2-x^2-y^2+1}}}.$$

Anomaly cancellation



- A triangle diagrams with the axial current and the two gauge currents at its vertices. $\rightarrow$ anomalous contribution.
- In QED, the anomalous terms violate Ward identity;

$$\text{Ward identity: } \partial_\mu < J^{\alpha 5} J^\mu J^\nu >= \partial_\nu < J^{\alpha 5} J^\mu J^\nu > = 0$$

$$\textbf{BUT} \text{ anomalies} \mapsto \partial_\alpha < J^{\alpha 5} J^\mu J^\nu > \neq 0$$