Electroweak Phase Transition in $\mathrm{SU}(2)_1 \otimes \mathrm{SU}(2)_2 \otimes \mathrm{U}(1)_Y$ Model

Nguyen Minh Anh

UNIVERSITY OF SCIENCE, HCM National University

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Overview

Baryon Asymmetry of Universe (BAU)

BAU can be explained by Electroweak Baryogenesis (EWBG).

A first-order electroweak phase transition (EWPT) is an crurial role.

Three Sakharov conditions

- Baryon number (B) violation.
- C and CP violations.
- Deviation from thermal equibrium.

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Not a strong first-order phase transition unless the mass of Higgs boson is below 70 GeV.

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First and second-order phase transition

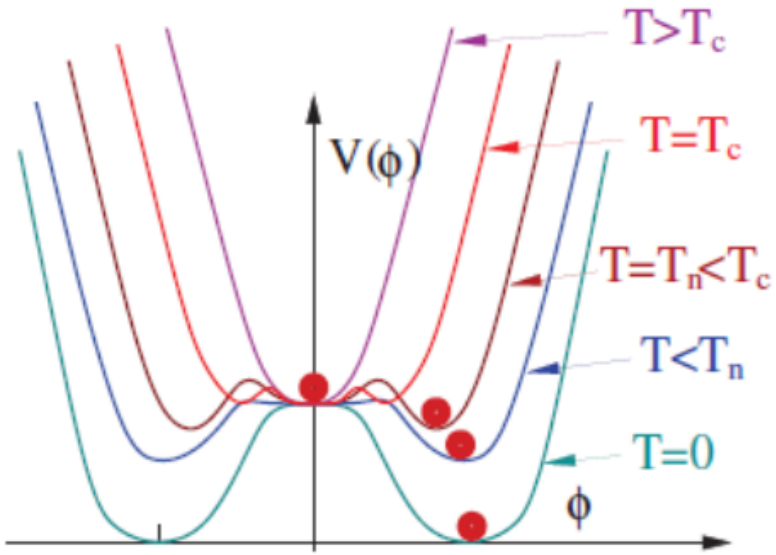

Figure: First-order phase transition

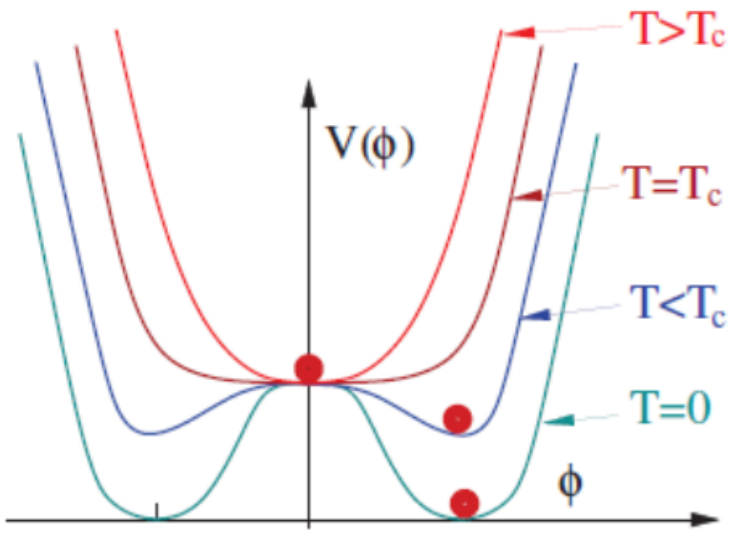

Figure: Second-order phase transition

Main aim

2-2-1 model

- New heavy gauge bosons and exotic quarks.
- New frame of Higgs potential.
- A good candidate for strengthening the EWPT.

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- Check if 2-2-1 model has a first-order EWPT.
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In this model, Fermion sectors are like SM,

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}; \begin{pmatrix} c_L \\ s_L \end{pmatrix}; \begin{pmatrix} t_L \\ b_L \end{pmatrix},$$
$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}; \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}; \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix},$$
$$u_R; c_R; t_R,$$
$$d_R; s_R; b_R,$$
$$e_R; \mu_R; \tau_R.$$

The electric charges of particles are defined as follows

$$Q_{em} = T_3^{(1)} + T_3^{(2)} + Y, \tag{1}$$

where $T_3^{(1,2)} = \dfrac{\sigma_3}{2}$ and σ_3 is the third Pauli matrix.
Besides, in order to minimize the number of the particles and increase the decays of the heavy scalar boson of H_2, a doublet quark $Q'^T = (U', D')$ is introduced.

Higgs Potential

The Higgs potential is given by:

$$\begin{aligned} V(H_1, H_2, S') = &\sum_{i=1,2} [\mu_1^2 H_i^\dagger H_i + \lambda_i (H_i^\dagger H_i)^2] + \mu_s^2 S'^2 + \lambda_S S'^4 \\ &+ \mu_3 S'^3 + S'(\mu_{1S} H_1^\dagger H_1 + \mu_{2S} H_2^\dagger H_2) \\ &+ \lambda_{12} H_1^\dagger H_1 H_2^\dagger H_2 + \lambda_{1S} S'^2 H_1^\dagger H_1 \\ &+ \lambda_{2S} S'^2 H_2^\dagger H_2. \end{aligned} \tag{2}$$

The scalar fields can be expressed as:

$$H_i = \begin{pmatrix} G_i^\dagger \\ (v_i + h_i + iG_i^0)/\sqrt{2} \end{pmatrix}, \qquad S' = (v_S + S)/\sqrt{2} \tag{3}$$

After averaging over all space, we get:

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \langle H_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \qquad \langle S' \rangle = \frac{1}{\sqrt{2}} v_S \tag{4}$$

Mass formation of new particles

Table: Masses of new bosons and fermions in 2-2-1 model

Particles	$m^2(v, v_2, v_S)$	Degree of freedom
$W'^{\pm}$	$\frac{g_2^2 v_2^2}{4}$	6
Z'	$\frac{1}{4}\frac{g'^4 v^2 + g_2^4 v_2^2}{g_2^2 - g'^2}$	3
H	$2\lambda_2 v_2^2$	1
H_S	$2\lambda_S v_S^2 + \frac{3\mu_S v_S}{2\sqrt{2}} - \frac{\mu_{1S} v^2 + \mu_{2S} v_2^2}{2\sqrt{2} v_S}$	1
Q	$(m_\psi + \frac{y_F}{\sqrt{2}} v_S)^2$	-24

Two symmetry breaking scenarios in 2-2-1 model

First: 3 phase transitions

2-2-1 model: $SU(2)_1 \otimes SU(2)_2 \otimes U(1)_Y$

$$\Big\Downarrow v_s \; \mathbf{H_S}, \mathbf{Q}$$

$SU(2)_2 \otimes U(1)_Y$

$$\Big\Downarrow v_2 \; \mathbf{H}, \mathbf{H_S}, \mathbf{W}', \mathbf{Z}'$$

SM model: $SU(2)_L \otimes U(1)_Y$

$$\Big\Downarrow v \; \mathbf{h}, \mathbf{H_S}, \mathbf{W}, \mathbf{Z}', \mathbf{Z}, \mathbf{t}$$

QED: $U(1)_Q$

Second: 2 phase transitions

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The symmetry breaking scale v_{S0} is about a few TeV. The effective potential can be rewritten as:

$$V_{eff}(v_s) = \frac{\lambda_S}{4} v_s^4 - \frac{\theta_S}{3} T v_s^3 + \frac{\gamma_S (T^2 - T_{S0}^2)}{2} v_s^2 + \Lambda_S, \tag{5}$$

which satisfies the minimum conditions:

$$V_{eff}(v_{S0}, 0) = 0, \qquad \frac{\partial V_{eff}(v_s)}{\partial v_s}(v_{S0}, 0) = 0, \qquad \frac{\partial^2 V_{eff}(v_s)}{\partial v_s^2}(v_{S0}, 0) = \mathbf{m_{H_s}}^2(v_s)\Big|_{v_s = v_{S0}}, \tag{6}$$

where

$$\begin{aligned}
\lambda_S &= \frac{m_{H_s}^2(v_{S0})}{2v_{S0}^2} + \frac{1}{16\pi^2 v_{S0}^4}\left[m_{H_s}^4(v_{S0}) \ln \frac{bT^2}{m_{H_s}^2(v_{S0})} - 24 m_Q^4(v_{S0}) \ln \frac{b_F T^2}{m_Q^2(v_{S0})}\right], \\
\theta_S &= \frac{1}{4\pi v_{S0}^3} m_{H_s}^3(v_{S0}), \\
\gamma_S &= \frac{1}{12 v_{S0}^2}\left[m_{H_s}^2(v_{S0}) + 12 m_Q^2(v_{S0})\right], \\
T_{S0}^2 &= \frac{1}{2\gamma_S}\left\{m_{H_s}^2(v_{S0}) - \frac{1}{8\pi^2 v_{S0}^2}\left[m_{H_s}^4(v_{S0}) - 24 m_Q^4(v_{S0})\right]\right\}.
\end{aligned}$$

This phase transition is just a mediate stage where the exotic quarks can get their masses by interacting with the heavy Higgs field. Therefore, we will not consider this transition.

The symmetry breaking scale is v_{20} . The effective potential which depends on the vev v_2 can be rewritten as:

$$V_{eff}(v_2) = \frac{\lambda_T}{4} v_2^4 - \frac{\theta}{3} T v_2^3 + \frac{\gamma(T^2 - T_0^2)}{2} v_2^2. \tag{7}$$

This potential satisfies the minimum conditions:

$$V_{eff}(v_{20}, 0) = 0, \qquad \frac{\partial V_{eff}(v_2)}{\partial v_2}(v_{20}, 0) = 0, \qquad \frac{\partial^2 V_{eff}(v_2)}{\partial v_2^2}(v_{20}, 0) = \mathbf{m_H}^2(v_2) + \mathbf{m_{H_s}}^2(v_2)\Big|_{v_2 = v_{20}} \tag{8}$$

where

$$\lambda_T = \frac{m_H^2(v_{20}) + m_{H_s}^2(v_{20})}{2v_{20}^2} + \frac{1}{16\pi^2 v_{20}^4}\Bigg[6m_{W'^\pm}^4(v_{20}) \ln \frac{bT^2}{m_{W'^\pm}^2(v_{20})} + 3m_{Z'}^4(v_{20}) \ln \frac{bT^2}{m_{Z'}^2(v_{20})}$$
$$+ m_H^4(v_{20}) \ln \frac{bT^2}{m_H^2(v_{20})} + m_{H_s}^4(v_{20}) \ln \frac{bT^2}{m_{H_s}^2(v_{20})}\Bigg],$$

$$\theta = \frac{1}{4\pi v_{20}^3}\Big[6m_{W'^\pm}^3(v_{20}) + 3m_{Z'}^3,(v_{20}) + m_H^3(v_{20}) + m_{H_s}^3(v_{20})\Big],$$

$$\gamma = \frac{1}{12v_{20}^2}\Big[6m_{W'^\pm}^2(v_{20}) + 3m_{Z'}^2(v_2) + m_H^2(v_{20}) + m_{H_s}^2(v_{20})\Big],$$

$$T_0^2 = \frac{1}{2\gamma}\Big\{m_H^2(v_{20}) + m_{H_s}^2(v_{20}) - \frac{1}{8\pi^2 v_{20}^2}\Big[6m_{W'^\pm}^4(v_{20}) + 3m_{Z'}^4(v_{20}) + m_H^4(v_{20}) + m_{H_s}^4(v_{20})\Big]\Big\}.$$

Since the two new Higgs bosons contribute to the effective potential of this phase transition, the minimum conditions are:

$$V_{eff}(v_{20},0) = 0, \qquad \frac{\partial V_{eff}(v_2)}{\partial v_2}(v_{20},0) = 0, \qquad \frac{\partial^2 V_{eff}(v_2)}{\partial v_2^2}(v_{20},0) = \mathbf{m_H}^2(v_2) + \mathbf{m_{H_s}}^2(v_2)\Big|_{v_2=v_{20}} \tag{9}$$

The effective potential has two minima, the first one is $v = 0$, the other is:

$$v_C = \frac{2\theta T_C}{\lambda_{T_C}}, \tag{10}$$

where $T_C = \dfrac{T_0}{\sqrt{1 - \frac{2\theta^2}{9\gamma\lambda_{T_C}}}}$ is the critical temperature.

Condition to have a first-order phase transition

$$S = \frac{v_C}{T_C} = \frac{2\theta}{\lambda_{T_C}} \geq 1 \tag{11}$$

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Range of masses of new particles

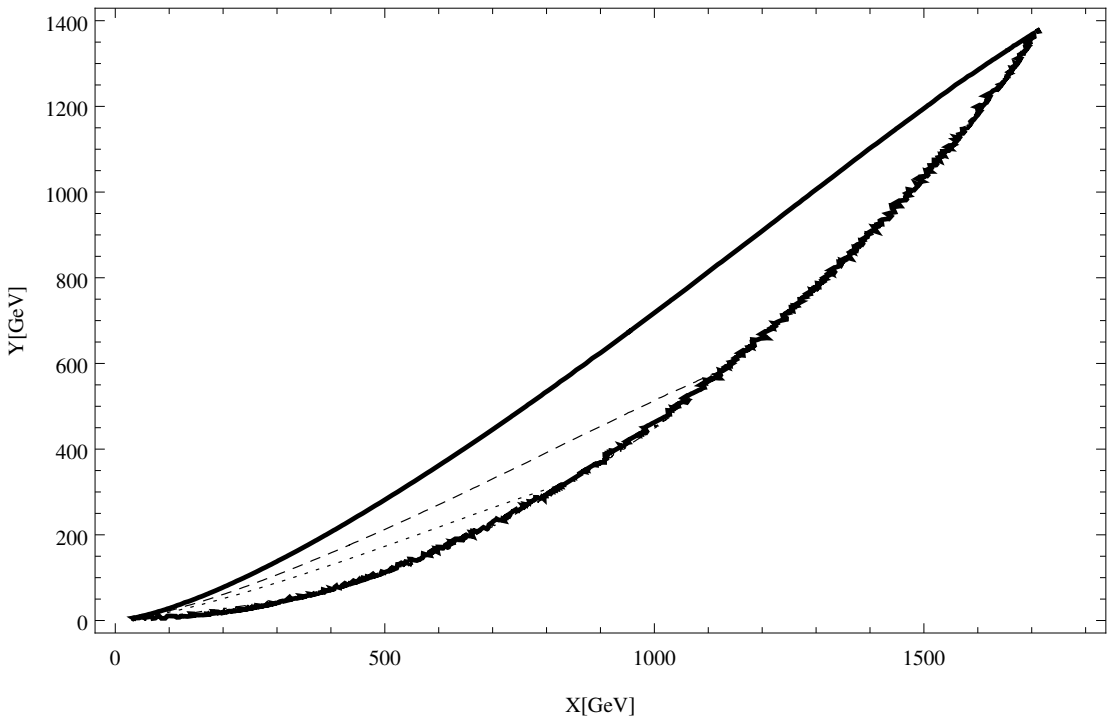

Figure: VEV $v_{20} = 535$GeV. Thick contour $S = 1$, dashed contour $S = 1.5$, dotted contour $S = 2$, dash-dotted contour $S_{max} = 5$

The range of unknown masses can be derived as:

$$0 \text{ GeV} < X = m_{W'}(v_{20}) = m_{Z'}(v_{20}) < 1700 \text{ GeV}, \qquad 0 \text{ GeV} < Y = m_H(v_{20}) = m_{H_S}(v_{20}) < 1370 \text{ GeV}$$

The symmetry breaking scale is $v_0 = 246$ GeV. The high-temperature effective potential has the form:

$$V_{eff}(v) = \frac{\lambda'_T}{4} v^4 - \frac{\theta'}{3} T v^3 + \frac{\gamma'(T^2 - T_0'^2)}{2} v^2, \tag{12}$$

which satisfies the minimum conditions:

$$V_{eff}(v_0, 0) = 0, \qquad \frac{\partial V_{eff}(v)}{\partial v}(v_0, 0) = 0, \qquad \frac{\partial^2 V_{eff}(v)}{\partial v^2}(v_0, 0) = \mathbf{m_h}^2(v)\big|_{v=v_0} \tag{13}$$

where

$$\lambda'_T = \frac{m_h^2(v)}{2v_0^2} + \frac{1}{16\pi^2 v_0^4}\Big[6m_{W^\pm}^4(v_0)\ln\frac{bT^2}{m_{W^\pm}^2(v_0)} + 3m_Z^4(v_0)\ln\frac{bT^2}{m_Z^2(v_0)} + 3m_{Z'}^4(v_0)\ln\frac{bT^2}{m_{Z'}^2(v_0)}$$
$$+ m_h^4(v_0)\ln\frac{bT^2}{m_h^2(v_0)} + m_{H_s}^4(v_0)\ln\frac{bT^2}{m_{H_s}^2(v_0)} - 12m_t^4(v_0)\ln\frac{b_F T^2}{m_t^2(v_0)}\Big],$$

$$\theta' = \frac{1}{4\pi v_0^3}\Big[6m_{W^\pm}^3(v_0) + 3m_Z^3(v_0) + 3m_{Z'}^3(v_0) + m_h^3(v_0) + m_{H_s}^3(v_0)\Big],$$

$$\gamma' = \frac{1}{12v_0^2}\Big[6m_{W^\pm}^2(v_0) + 3m_Z^2(v_0) + 3m_{Z'}^2(v_0) + m_h^2(v_0) + m_{H_s}^2(v_0) + 6m_t^2(v_0)\Big],$$

$$T_0'^2 = \frac{1}{2\gamma'}\Big\{m_h^2(v_0) - \frac{1}{8\pi^2 v_0^2}\Big[6m_{W^\pm}^4(v_0) + 3m_Z^4(v_0) + 3m_{Z'}^4(v_0) + m_h^4(v_0) + m_{H_s}^4(v_0) - 12m_t^4(v_0)\Big]\Big\}$$

Range of masses of new particles

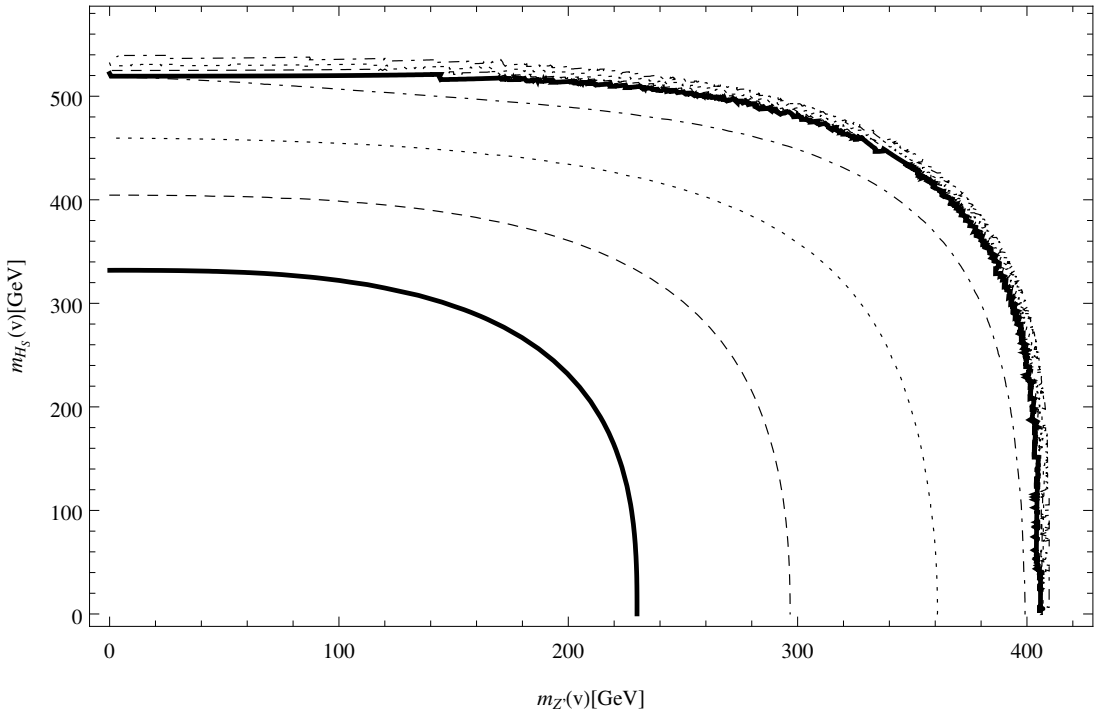

Figure: Thick contour $S = 1$, dashed contour $S = 1.2$, dotted contour $S = 1.5$, dash-dotted contour $S_{max} = 2.7$

The range of unknown masses can be derived as:

$$0 < m_{Z'}(v_0) < 408 \text{ GeV}, \qquad 95 < m_{H_s}(v_0) < 524 \text{ GeV}$$

Choose v_{20} as the symmetry breaking scale of this phase transition, then we can rewrite the effective potential as:

$$V_{eff}(v_2) = \frac{\lambda_T}{4} v_2^4 - \frac{\theta}{3} T v_2^3 + \frac{\gamma(T^2 - T_0^2)}{2} v_2^2, \tag{14}$$

which satisfies the minimum conditions:

$$V_{eff}(v_{20}, 0) = 0, \qquad \frac{\partial V_{eff}(v_2)}{\partial v_2}(v_{20}, 0) = 0, \qquad \frac{\partial^2 V_{eff}(v_2)}{\partial v_2^2}(v_{20}, 0) = \mathbf{m_H}^2(v_2) + \mathbf{m_{H_s}}^2(v_2)\Big|_{v_2 = v_{20}} \tag{15}$$

where

$$\lambda_T = \frac{m_H^2(v_{20}) + m_{H_s}^2(v_{20})}{2v_{20}^2} + \frac{1}{16\pi^2 v_{20}^4}\Bigg[6m_{W'^\pm}^4(v_{20}) \ln \frac{bT^2}{m_{W'^\pm}^2(v_{20})} + 3m_{Z'}^4(v_{20}) \ln \frac{bT^2}{m_{Z'}^2(v_{20})}$$
$$+ m_H^4(v_{20}) \ln \frac{bT^2}{m_H^2(v_{20})} + m_{H_s}^4(v_{20}) \ln \frac{bT^2}{m_{H_s}^2(v_{20})} - 24m_Q^4(v_{20}) \ln \frac{b_F T^2}{m_Q^2(v_{20})}\Bigg]$$

$$\theta = \frac{1}{4\pi v_{20}^3}\left[6m_{W'^\pm}^3(v_{20}) + 3m_{Z'}^3(v_{20}) + m_H^3(v_{20}) + m_{H_s}^3(v_{20})\right]$$

$$\gamma = \frac{1}{12v_{20}^2}\left[6m_{W'^\pm}^2(v_{20}) + 3m_{Z'}^2(v_2) + m_H^2(v_{20}) + m_{H_s}^2(v_{20}) + 12m_Q^2(v_{20})\right]$$

$$T_0^2 = \frac{1}{2\gamma}\Bigg\{m_H^2(v_{20}) + m_{H_s}^2(v_{20})$$
$$- \frac{1}{8\pi^2 v_{20}^2}\left[6m_{W'^\pm}^4(v_{20}) + 3m_{Z'}^4(v_{20}) + m_H^4(v_{20}) + m_{H_s}^4(v_{20}) - 24m_Q^4(v_{20})\right]\Bigg\}.$$

Range of masses of new particles

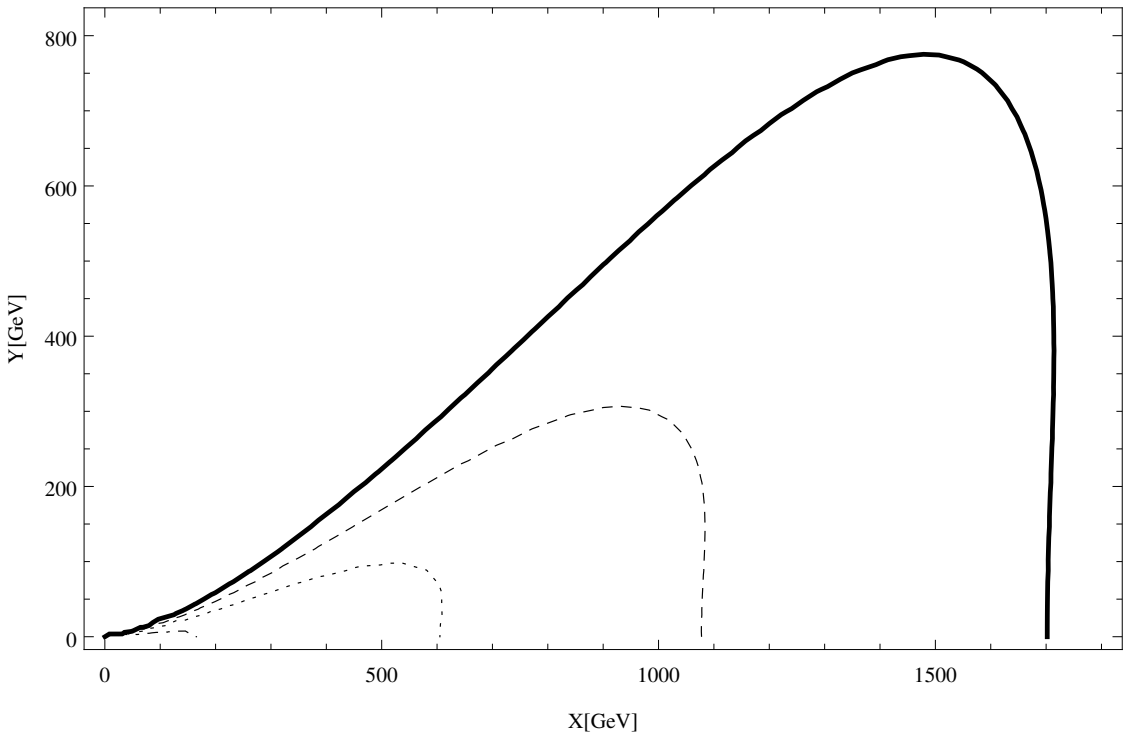

Figure: The symmetry breaking scale $v_{20} = 1110$ GeV. Thick contour $S = 1$, dashed contour $S = 1.5$, dotted contour $S = 2.5$, dashed-dotted contour $S_{max} = 8$.

$$0 \text{ GeV} < Y = m_H(v_{20}) = m_{H_S}(v_{20}) < 776 \text{ GeV}$$
$$0 \text{ GeV} < X = m_{Z'}(v_{20}) = m_{W'}(v_{20}) = m_Q(v_{20}) < 1700 \text{ GeV}$$

Correlation length of the SM-like EWPT

For $SU(2)_L \otimes U(1)_Y \to U(1)_Q$ phase transition, we have the effective potential:

$$V_{eff}(v) = \frac{\lambda_T}{4} v^4 - \frac{\theta}{3} T v^3 + \frac{\gamma(T^2 - T_0'^2)}{2} v^2. \tag{16}$$

We define the correlation length ξ as below :

$$\frac{\partial^2 V_{eff}(v)}{\partial v^2}(v_{eq}, T)\Big|_{v_{eq}} = \xi^{-2}, \tag{17}$$

where

$$v_{eq} = \begin{cases} 0 & , T > T_C \\ v_m = \dfrac{\theta T - \sqrt{(\theta T)^2 - 4\lambda_T \gamma (T^2 - T_0^2)}}{2\lambda_T} & , T < T_C \end{cases} \tag{18}$$

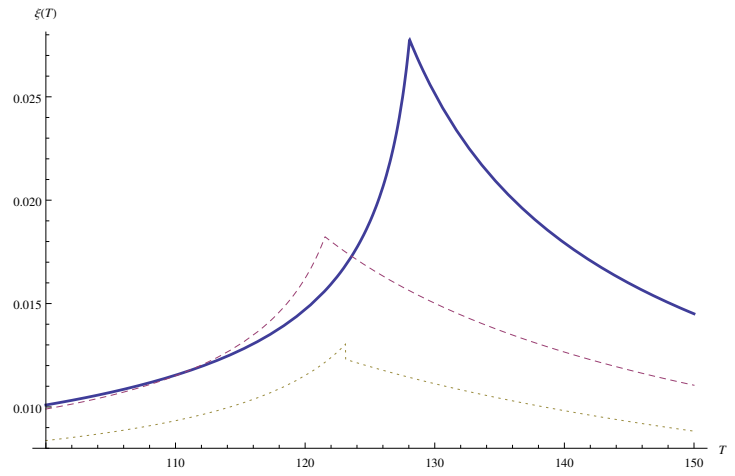

Figure: The dotted line: $m_{Z'}(v_0) = 252.2$ GeV, $m_{H_s}(v_0) = 404.7$ GeV for the transition strength $S = 2$, the critical temperature $T_C = 123.122$K. The dashed line: $m_{Z'}(v_0) = 163.4$ GeV, $m_{H_s}(v_0) = 381.5$ GeV for the transition strength $S = 1.5$, the critical temperature $T_C = 121.538$K. The thick line: $m_{Z'}(v_0) = 136.2$ GeV, $m_{H_s}(v_0) = 307.4$ GeV for the transition strength $S = 1$, the critical temperature $T_C = 128.054$K.

Lambda parameter

With the above phase transition structure, we determine a domain for the quaternary coupling of the Higgs, which is the parameter that the LHC is interested in. The effective potential at 0K of the SM-like electroweak phase transition:

$$V_{eff}(v,0) = \frac{\lambda_R}{4}v^4 + \frac{m_R^2}{2}v^2 + \Lambda_R, \tag{20}$$

where

$$\begin{cases} m_R^2 = -\dfrac{m_h^2}{2} + \dfrac{1}{16\pi^2} \displaystyle\sum_{i=h,W,Z,Z',H_S,t} n_i \frac{m_i^4}{v_0^2} \\ \lambda_R = \dfrac{m_h^2}{2v_0^2} - \dfrac{1}{32\pi^2} \displaystyle\sum_{i=h,W,Z,Z',H_S,t} n_i \frac{m_i^4}{v_0^4}\left(2\ln\frac{m_i^2}{v_0^2} + 3\right) \\ \Lambda_R = \dfrac{m_h^2}{8}v_0^2 + \dfrac{1}{128\pi^2} \displaystyle\sum_{i=h,W,Z,Z',H_S,t} n_i m_i^4 \end{cases} \tag{21}$$

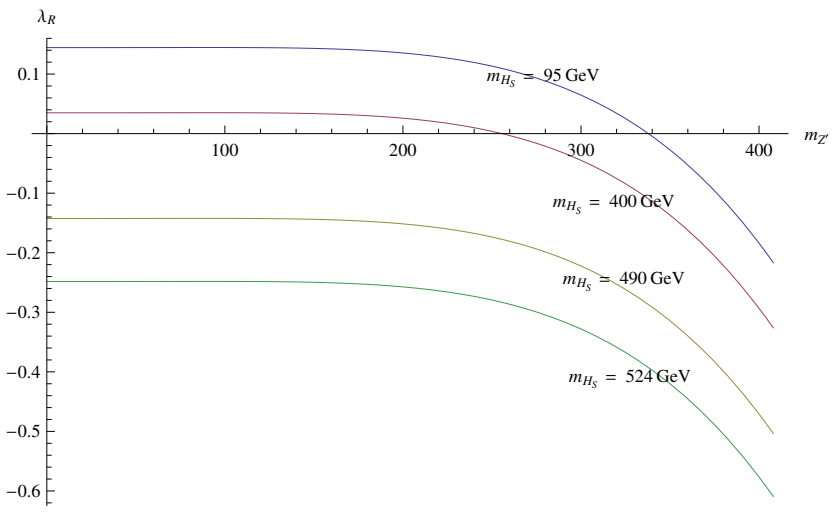

Figure:
$\lambda_R(m_{Z'}(v_0), m_{H_S}(v_0), T = 0)$

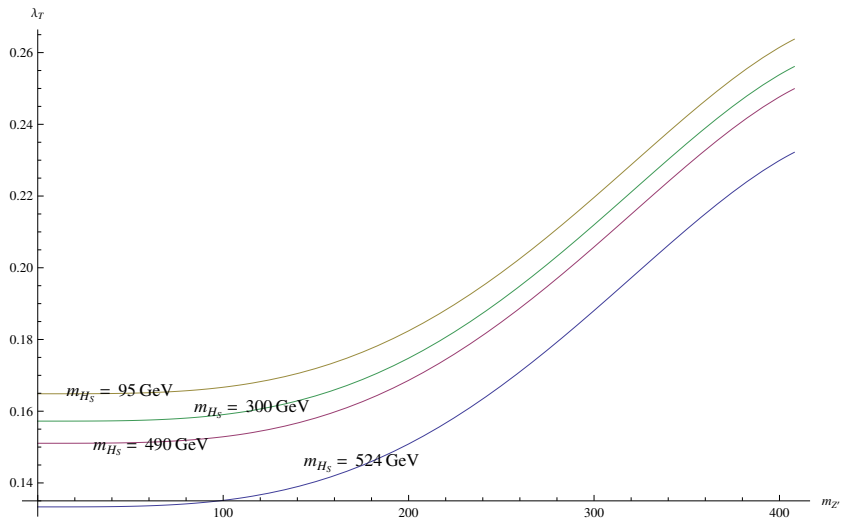

Figure:
$\lambda_T(m_{Z'}(v_0), m_{H_S}(v_0), T = 100\text{K})$

Conclusion

- Each transition in 2 scenarios is first-order.
- The SM-like EWPT is first-order with the strength $1 < S < 2.7$.
- In the three-stage picture, the second vev has to be larger than 535 GeV or 1.11 TeV in the two-stage one.
- The masses of new gauge bosons are bigger than 1.7 TeV and the new gauge coupling of $SU(2)_2$, $g_2 \geq 2$

Outlooks

- Make a correction to mass of H_s.
- Analyze EWPT again using neutrino data.
- Test the sphaleron solution with CosmoTransitions package.

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- Make a correction to mass of H_s.
- Analyze EWPT again using neutrino data.
- Test the sphaleron solution with CosmoTransitions package.