INFLATION VIA HIGGS-DILATON POTENTIAL IN TWO TIME PHYSICS

VSOP 24
Mini-seminar

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Overview

1. Big Bang model
2. Inflation
3. Inflation via Higgs-Dilaton potential in Two-Time physics (2T)

I. Big Bang model

a. The Cosmological Principle: the universe is **homogeneity** and **isotropy** ($\gtrsim 100$ Mpc).

b. General Relativity

i. The FLRW metric

$$ds^2 = -c^2dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right). \quad (1)$$

ii. The Einstein equations

$$R_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right). \quad (2)$$

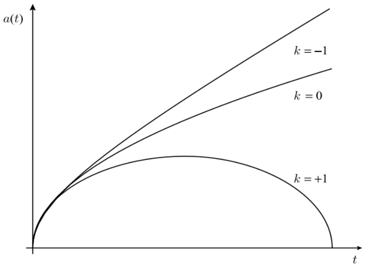

Figure: **Universe with the values of curvature constant.** Source: V. Mukhanov, *Physical Foundations of Cosmology*, Cambridge University Press, 2005.

c. Two problems of Big Bang model

The Horizon problem

The ratio between the size of homogeneous region l_i and Big Bang model l_c

$$\frac{l_i}{l_c} \sim 10^{28}. \tag{3}$$

The size of homogeneous universe is larger than the size of Big Bang expansion universe.

The Flatness problem

- Big Bang model: $a(t)$ grows with time $\Rightarrow$ the initial universe is curved.
- At the begining of the universe

$$|\Omega_i - 1| \leq 10^{-56}. \tag{4}$$

The initial universe is extremely flat.

II. INFLATION

An extension of Big Bang model ($10^{-34} \sim 10^{-36} s$ after Big Bang), accelerated expansion.

a. New inflation

The biquadratic potential

$$V(\varphi) = -\frac{\lambda}{4}\varphi^4 + \frac{m^2}{2}\varphi^2 + V(0). \tag{5}$$

Two Slow-roll conditions

$$\begin{cases} \dot{\varphi}^2 \ll V(\varphi), \\ \ddot{\varphi} \ll 3H\dot{\varphi}. \end{cases} \tag{6}$$

Slow-roll parameters

$$\begin{cases} \varepsilon_V = \left(\frac{V'}{V}\right)^2 \frac{M_P^2}{2}, & \eta_V = M_P^2 \frac{V''}{V}; \\ \varepsilon_H = -\frac{d\ln H}{d\ln a}, & \eta_H = -\frac{d\ln H'}{d\ln a}. \end{cases} \tag{7}$$

Inflation happens when $\varepsilon, \eta \ll 1$ and ends if $\varepsilon, \eta = 1$.

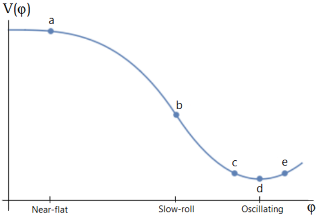

Figure: **The potential energy of Slow-roll inflation.** Source: P. A. Farago, *ΛCDM Cosmology + Chaotic Inflation*, Virgina Commonwealth University VCU Scholars Compass, 2015.

b. Chaotic inflation

The potential of Chaotic inflation

$$V(\varphi) = \frac{1}{2}m^2\varphi^2. \tag{8}$$

The equation of motion and Friedmann equation

$$\begin{cases} \ddot{\varphi} + 3H\dot{\varphi} & = -m^2\varphi, \\ H^2 + \frac{k}{a^2} & = \frac{1}{6}(\dot{\varphi}^2 + m^2\varphi^2). \end{cases} \tag{9}$$

Quasi-exponential expansion

$$a(t) = a_0 \exp\left[\int_0^t H(t)dt\right] \sim a_0 e^{Ht}. \tag{10}$$

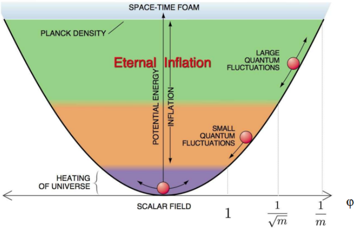

Figure: **A simple model of Chaotic Inflation.** Source: A. Linde, *Inflationary cosmology*, Lect.Notes Phys.738:1-54 (2008).

c. Hybrid inflation

The potential of two scalar fields

$$V(\varphi,\sigma) = \frac{1}{4\lambda}M^4 + \frac{1}{4}\lambda\sigma^4 + \frac{1}{2}(-M^2 + g^2\varphi^2)\sigma^2 + \frac{1}{2}m^2\varphi^2. \quad (11)$$

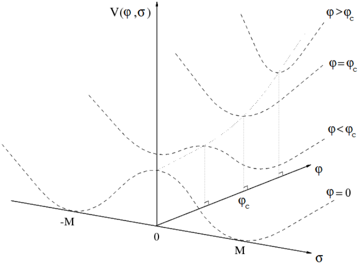

Figure: **A simple model of Hybrid Inflation.** Source: P. Jiun-Huei Wu, *Analytical and Numerical Approaches to Inflationary Cosmology*, M. Sc in Astronomy thesis at The University of Sussex (1996).

- $\varphi > \varphi_c$, (φ_c is the critical value of φ) $\sigma = 0$. The equation of motion of φ during inflation

$$3H\dot{\varphi} = -m^2\varphi. \quad (12)$$

- $\varphi = \varphi_c$, $\sigma = 0$. Assuming $m^2 \ll \frac{g^2M^2}{\lambda}$, we have $V(\varphi_c) \simeq \frac{M^4}{4\lambda}$.
- $\varphi < \varphi_c$, symmetry breaking occurs.

III. INFLATION VIA HIGGS-DILATON POTENTIAL IN TWO-TIME PHYSICS

a. Two-Time Physics (2T)

i. Tensor metric in 2T

1T: $(d-1)$ spacelike dimensions and 1 timelike dimension.

$$g_{\mu\nu} = \text{diag}(-1, 1, 1, 1), \qquad \mu, \nu = 0, 1, 2, 3. \tag{13}$$

2T: d spacelike dimensions and 2 timelike dimensions.

$$\eta_{MN} = \text{diag}(\mathbf{-1}, \mathbf{1}, -1, 1, 1, 1), \qquad M, N = \mathbf{0'}, \mathbf{1'}, 0, 1, 2, 3. \tag{14}$$

This model is based on symplectic group $Sp(2, \mathbf{R})$ which relates to the symmetry between position and momentum.

ii. Lagrangian of Standard Model in 2T

$$\begin{pmatrix} u^L \\ d^L \end{pmatrix}_{\frac{1}{3}}, \left(u^R\right)_{\frac{4}{3}}, \left(d^R\right)_{-\frac{2}{3}}, \begin{pmatrix} \nu_e^L \\ e^L \end{pmatrix}_{-1}, \left(\nu_e^R\right)_0, \left(e^R\right)_{-2} \qquad \text{the first generation.}$$

$$\begin{pmatrix} c^L \\ s^L \end{pmatrix}_{\frac{1}{3}}, \left(c^R\right)_{\frac{4}{3}}, \left(s^R\right)_{-\frac{2}{3}}, \begin{pmatrix} \nu_\mu^L \\ \mu^L \end{pmatrix}_{-1}, \left(\nu_\mu^R\right)_0, \left(\mu^R\right)_{-2} \qquad \text{the second generation.}$$

$$\begin{pmatrix} t^L \\ b^L \end{pmatrix}_{\frac{1}{3}}, \left(t^R\right)_{\frac{4}{3}}, \left(b^R\right)_{-\frac{2}{3}}, \begin{pmatrix} \nu_\tau^L \\ \tau^L \end{pmatrix}_{-1}, \left(\nu_\tau^R\right)_0, \left(\tau^R\right)_{-2} \qquad \text{the third generation.}$$

The Lagrangian of Standard Model in 4+2 dimensions

$$L(A, \Psi^{L,R}, H, \Phi) = L(A) + L(A, \Psi^{L,R}) + L(\Psi^{L,R}, H) + L(A, H, \Phi). \qquad (15)$$

$L(A)$: Lagrangian for gauge bosons A;
$L(A, \Psi^{L,R})$: Lagrangian interaction between fermions $\Psi^{L,R}$ and gauge bosons A;
$L(\Psi^{L,R}, H)$: Yukawa couplings lagrangian;
$L(A, H, \Phi)$: Higgs-Dilaton lagrangian (H: Higgs field, Φ: Dilaton field).

b. Higgs-Dilaton potential in 2T

Lagrangian of Higgs-Dilaton

$$L(A,H,\Phi) = \frac{1}{2}\Phi\partial^2\Phi + \frac{1}{2}\left[H^\dagger D^2 H + (D^2 H)^\dagger H\right] - \lambda(H^\dagger H - \alpha^2\Phi^2)^2 - V(\Phi), \quad (16)$$

The Higgs-Dilaton potential

$$V(H,\Phi) = \lambda(H^\dagger H - \alpha^2\Phi^2)^2 + V(\Phi). \quad (17)$$

The reduced formulations, using gauge fixing technology

$$\begin{cases} \Phi(X) & \longrightarrow \frac{1}{\kappa}\phi(x), \\ H(X) & \longrightarrow \frac{1}{\kappa}h(x). \end{cases} \quad (18)$$

The reduced Higgs-Dilaton potential from 2T to 1T

$$V(H,\Phi) \longrightarrow \frac{\lambda}{\kappa^4}(h^2 - \alpha^2\phi^2)^2 + V(\phi), \quad (19)$$

i. Higgs-Dilaton potential in New Inflation

Dilaton field ϕ plays the role of inflaton in New inflation.

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \phi_0^2)^2. \tag{20}$$

The scalar field and scale factor

$$\begin{cases} \phi = \phi_i \text{exp}\left(-2M_P\sqrt{\frac{\lambda}{3}}t\right), \\ a(t) = a_i \text{exp}\left[-\frac{\phi_i^2}{8M_P^2}\text{exp}\left(-4M_P\sqrt{\frac{\lambda}{3}}t\right) - \frac{\phi_0^2}{2M_P}\sqrt{\frac{\lambda}{3}}t\right]. \end{cases} \tag{21}$$

The two potential slow-roll parameters

$$\begin{cases} \varepsilon_V = 8M_P^2 \frac{\phi^2}{(\phi^2-\phi_0^2)^2}, \\ \eta_V = 4M_P^2\left[\frac{2\phi^2}{(\phi^2-\phi_0^2)^2} + \frac{1}{\phi^2-\phi_0^2}\right]. \end{cases} \tag{22}$$

The reduced Dilaton potential satisfies New inflation model.

ii. Higgs-Dilaton potential in Chaotic Inflation

The quartic potential of Dilaton is

$$V(\phi) = \lambda\phi^4. \tag{23}$$

λ: arbitrary parameter. The inflaton field and scale factor

$$\begin{cases} \phi(t) = \phi_i \text{exp}\left(-4M_P\sqrt{\frac{\lambda}{3}}t\right) \\ a(t) = a_i \text{exp}\left[-\frac{\phi_i^2}{8M_P^2}\text{exp}\left(-8M_P\sqrt{\frac{\lambda}{3}}t\right)\right]. \end{cases} \tag{24}$$

The initial values of two slow-roll parameters

$$\begin{cases} \varepsilon_i = \frac{1}{1-N}, \\ \eta_i = \frac{3}{2(1-N)}. \end{cases} \tag{25}$$

The reduced quadratic and quartic Dilaton potential satisfy the Chaotic inflation model.

iii. Higgs-Dilaton potential in Hybrid Inflation

Higgs plays the non-inflation role and Dilaton plays the inflation role.
Unitary gauge for Higgs-Dilaton potential in Eq.(17) with electroweak scale v

$$\begin{cases} H^0(x) = \frac{1}{\kappa}\left[v + h(x)\right], \\ \Phi(x) = \frac{1}{\alpha\kappa}\left[v + \alpha\phi(x)\right]. \end{cases} \tag{26}$$

The 1T Higgs-Dilaton potential

$$\begin{aligned} V(h,\phi) = &\frac{\lambda}{4}h^4 + \lambda v h^3 + \frac{\lambda}{2}(-\alpha^2\phi^2 - 2v\alpha\phi + 2v^2)h^2 - \lambda(v\alpha^2\phi^2 + 2v^2\alpha\phi)h \\ &+ \frac{\lambda}{4}\alpha^4\phi^4 + \lambda v\alpha^3\phi^3 + \lambda v^2\alpha^2\phi^2. \end{aligned} \tag{27}$$

$\Rightarrow$ The Higgs-Dilaton potential does not satisfy Hybrid inflationary scenario.

Summary

The reduced Higgs-Dilaton potential satisfies the New and Chaotic inflation and does not fit the Hybrid inflation.

Outlook

For further discuss

- EWPT.
- Put the reduced Higgs-Dilaton potential in other recent inflation models.
- Use other reduced potential in Hybrid inflation.
- Use other higher-dimensional models into Hybrid inflation.


THANKS
FOR
LISTENING

Tensor metric

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \dfrac{a^2}{1-kr^2} & 0 & 0 \\ 0 & 0 & r^2a^2 & 0 \\ 0 & 0 & 0 & r^2a^2\sin^2\theta \end{pmatrix}. \tag{28}$$

Energy-momentum tensor

$$T^{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}. \tag{29}$$

Ricci tensor

$$R_{\mu\nu} = \frac{\partial\Gamma^l_{l\nu}}{\partial x^l} - \frac{\partial\Gamma^l_{\nu l}}{\partial x^\nu} + \Gamma^l_{\mu\nu}\Gamma^m_{lm} - \Gamma^m_{\mu l}\Gamma^l_{\nu m}, \tag{30}$$

where Christoffel symbol Γ is the function of coordinates

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}). \tag{31}$$

The size of a homogenerous, isotropic region at the initial time

$$l_i \sim ct_0 \frac{a_i}{a_0}, \tag{32}$$

and the size of causal region in case if there were no inflation

$$l_c \sim ct_i. \tag{33}$$

The Friedman equation

$$\Omega(t) - 1 = \frac{k}{H^2 a^2}. \tag{34}$$

When $\Omega \sim 1$, $k \sim 0$: the universe is flat. By the definition of the Hubble parameter: $H = \frac{\dot{a}}{a}$, we get

$$\begin{aligned} |\Omega_i - 1| &= |\Omega_0 - 1| \left| \frac{\Omega_i - 1}{\Omega_0 - 1} \right| = |\Omega_0 - 1| \frac{k}{(Ha)_i^2} \frac{(Ha)_0^2}{k} \\ &= |\Omega_0 - 1| \frac{(Ha)_0^2}{(Ha)_i^2} = |\Omega_0 - 1| \left(\frac{\dot{a}_0}{\dot{a}_i} \right)^2 \\ &\leq 10^{-56}. \end{aligned} \tag{35}$$

Gauge fixing technology

The space-time metric in 2T

$$ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'} dX^{-'} + dX^\mu dX^\nu g_{\mu\nu}, \tag{36}$$

and the lightcone coordinates for extra dimensions are

$$X_i^{\pm'} = \frac{1}{\sqrt{2}}(X_i^{0'} + X_i^{1'}) \Leftrightarrow \begin{cases} X^2 & = 2X^{+'}X^{-'} + X^\mu X_\mu, \\ X^M P_M & = X^{+'}P^{-'} + X^{-'}P^{+'} + X^\mu P_\mu. \end{cases} \tag{37}$$

The components of X^M can be parameterized as

$$\begin{cases} X^{+'} & = \kappa \\ X^{-'} & = \kappa\lambda \\ X^\mu & = \kappa x^\mu \end{cases} \Leftrightarrow \begin{cases} \kappa & = X^{+'}, \\ \lambda & = \frac{X^{-'}}{X^{+'}}, \\ x^\mu & = \frac{X^\mu}{X^{+'}}. \end{cases} \tag{38}$$

Then, the field is

$$\Phi(X) = \Phi(\kappa, \lambda, x^\mu). \tag{39}$$

The scalar field follows

$$\left(X.\partial + \frac{d-2}{2}\right)\Phi(X) = (\kappa\partial_\kappa + 1)\Phi(\kappa, \lambda, x^\mu) = 0, \tag{40}$$

where $\Phi(X) = \kappa^{-1}\underline{\Phi}(\lambda, x^\mu)$. To make the field $\Phi(X)$ is independent on λ, we seperate Φ into

$$\underline{\Phi}(\lambda, x^\mu) = \phi(x) + (2\lambda + x^2)\tilde{\phi}(\lambda, x^\mu), \tag{41}$$

with $\phi(x) = \underline{\Phi}(0, x^\mu)$ and note that

$$X^2 = 2X^{+'}X^{-'} + X^\mu X_\mu = \kappa^2(2\lambda + x^2), \tag{42}$$

while $\tilde{\phi}(\lambda, x^\mu)$ is gauge freedom with respect to 2T gauge symmetry, we choose $\tilde{\phi}(\lambda, x^\mu) = 0$, the reduction of scalar field from 2T to 1T

$$\Phi(X) = \kappa^{-1}\phi(x). \tag{43}$$

S^T: transpose matrix of S; $\Omega = \begin{pmatrix} 0 & -I_n \\ I_n & 0 \end{pmatrix}$.