

Exercise 1

$$\begin{aligned}\partial_\mu q'(x) &= \partial_\mu (U(x) q(x)) = U(x) \partial_\mu q(x) + \partial_\mu U(x) q(x) \\ &= U \left(\partial_\mu q(x) + U^{-1} (\partial_\mu U) q(x) \right) \quad (1)\end{aligned}$$

$$A'_\mu(x) = U A_\mu U^{-1} + \frac{i}{\hbar} (\partial_\mu U) U^{-1} \quad (2)$$

$$\partial_\mu (\bar{A}) = \partial_\mu + i g_s \bar{A}_\mu \qquad \bar{A}_\mu = \sum_a t^a A_\mu^a$$

$$\Rightarrow \partial_\mu (\bar{A}') q'(x) = \underset{(1), (2)}{U} \left(\partial_\mu q(x) + U^{-1} (\partial_\mu U) q(x) \right)$$

$$+ i g_s \left(U \bar{A}_\mu U^{-1} + \frac{i}{\hbar} (\partial_\mu U) U^{-1} \right) U q(x)$$

$$= U \partial_\mu q(x) + i g_s U \bar{A}_\mu q(x)$$

$$= U (\partial_\mu (\bar{A}) q(x))$$

$$\tilde{q}' = \tilde{q} U^{-1}$$

$$\Rightarrow \mathcal{L}' = \tilde{q}(x) U^{-1} \left(i \delta^{\mu\nu} U (\partial_\mu A_q) - m U q \right)$$

$$= \mathcal{L}$$

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Exercise 2

$$\begin{array}{c} f^a \\ \rightarrow \\ t_{ij}^a \end{array}$$

$$F_{bc}^a = -if^{abc}$$

$$\sum_{a,j} t_{ij}^a t_{jk}^a = C_F \delta_{ik} \quad \sum_{a,cd} F_{bcd}^a F_{dcl}^a = G_d \delta_{lc}$$

$$\text{m } \bigcirc = \text{Trace}(t^a) = 0$$

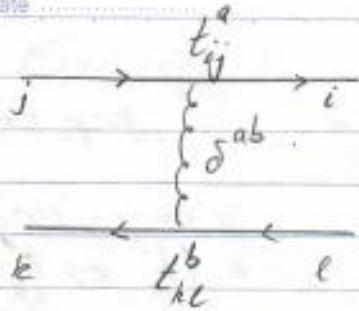
$$\text{m } \bigcirc \text{ m } \begin{array}{c} t^a \\ t^b \end{array} = \text{Trace}(t^a t^b) = \frac{1}{R} \delta^{ab} = \frac{1}{2} \delta^{ab}$$

$$\begin{array}{c} t^a \\ t^b \end{array} = C_F \delta_{ik}$$

$$\text{m } \begin{array}{c} d \\ d \\ m \\ m \\ \curvearrowleft \\ c \\ c \end{array} = f^{acd} f^{bdc} = F_{ac}^d F_{cb}^d = G_d \delta_{ab}$$

$V(p_1 g, +)$ of 1st vertex

$-V(p_2 g, r)$ of 2nd vertex



$$= T_R \left(\delta_{il} \delta_{kj} - \frac{1}{N_c} \delta_{ij} \delta_{kl} \right)$$

$$= T_R \left(\begin{array}{ccc} j & & i \\ \nearrow & \searrow & \\ k & & l \end{array} \right) - \frac{1}{N_c} \left(\begin{array}{ccc} j & & i \\ & \swarrow & \\ k & & l \end{array} \right)$$

contract with δ_{jk} δ_{ik} to check



$$= T_R \left(N_c^2 - \frac{1}{N_c} \cdot N_c \right) = T_R (N_c^2 - 1)$$

$$= C_F N_c \checkmark$$

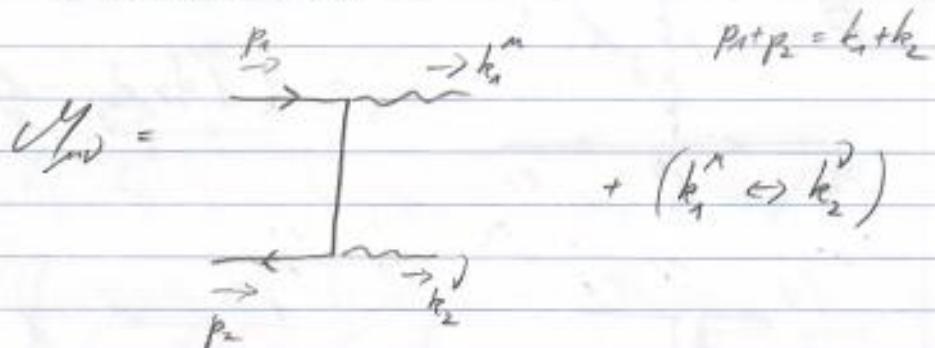
$$b) \quad \text{Diagram} \cdot \delta^{ab} = \text{Diagram} = T_R \delta_{ab} \delta^{ab}$$

$$= T_R (N_c^2 - 1)$$

$$= \text{Diagram} = t_{ij}^a t_{jk}^a \delta_{ik} = C_F \delta_{ii} = C_F N_c$$

$$\Rightarrow G = T_R \frac{N_c^2 - 1}{N_c}$$

Exercise 3a



$$p_1 + p_2 = k_1 + k_2$$

$$+ (k_1^\mu \leftrightarrow k_2^\mu)$$

$$= -i e^2 (\mathcal{M}_{\mu\nu}^{(1)} + \mathcal{M}_{\mu\nu}^{(2)})$$

$$\mathcal{M} = \epsilon^\mu(k_1) \epsilon^\nu(k_2) \mathcal{M}_{\mu\nu}$$

$$\mathcal{M}_{\mu\nu}^{(1)} = \bar{v}(p_2) \not{v}_\nu \frac{p_1 \cdot k_1}{(p_1 - k_1)^2} \not{v}_\mu u(p_1)$$

$$\mathcal{M}_{\mu\nu}^{(2)} = \bar{v}(p_2) \not{v}_\mu \frac{p_1 \cdot k_2}{(p_1 - k_2)^2} \not{v}_\nu u(p_1)$$

$$k_1^\mu \mathcal{M}_{\mu\nu}^{(1)} = \bar{v}(p_2) \not{v}_\nu \frac{(p_1 - k_1)(k_1 - p_1)}{(p_1 - k_1)^2} u(p_1)$$

use $p_1 u(p_1) = 0$

$$p_1 u(p_1) = 0$$

$$= -\bar{v}(p_2) \not{v}_\nu u(p_1)$$

$$k_1^\mu \mathcal{M}_{\mu\nu}^{(2)} = \bar{v}(p_2) \not{v}_\mu \frac{k_1 \cdot p_2}{(k_1 - p_2)^2} \not{v}_\nu u(p_1)$$

use $p_1 - k_2 = k_1 - p_2$

$$\text{use } \bar{v}(p_2) p_2 = 0 \quad \bar{v}(p_2) \frac{(k_1 - p_2)^2}{(k_1 - p_2)^2} \not{v}_\nu u(p_1)$$

$$= -k_1^\mu \mathcal{M}_{\mu\nu}^{(1)}$$

Exercise 3b

Lectures \Rightarrow

QED

$$M_{\mu\nu} = -ig^c \left\{ (t^a t^b)_{ij} (M_{\mu\nu}^{(1)} + M_{\mu\nu}^{(2)}) - i f^{abc} t_j^c M_{\mu\nu}^{(1)} \right. \\ \left. + M_{\mu\nu}^{(3)} \right\} \quad (*)$$

$$M_{\mu\nu}^{(3)} = if^{abc} t_j^c \bar{v}(p_2) \not{v}_S u(p_1) \frac{1}{p^2 + i\delta} V_{\mu\nu S} (k_1, k_2, -p)$$

$$V_{\mu\nu S} (k_1, k_2, -(k_1 + k_2)) = (k_1 - k_2)_S g_{\mu\nu} \quad p = k_1 + k_2 \\ + (2k_2 + k_1)_\mu g_{\nu S} \\ - (2k_1 + k_2)_\nu g_{\mu S}$$

$$k_1^\mu V_{\mu\nu S} = 2k_1 \cdot k_2 g_{\nu S} - k_{1D} (k_1 + k_2)_S - k_{1S} k_{2D} \\ \text{use } p^2 = (k_1 + k_2)^2 = 2k_1 \cdot k_2 \quad \begin{cases} = (p_1 + p_2)_S \\ \rightarrow 0 \text{ (Dirac eq.)} \end{cases}$$

$$\rightarrow k_1^\mu M_{\mu\nu}^{(3)} = if^{abc} t_j^c \bar{v}(p_2) \not{v}_S u(p_1) \left\{ g_{\nu S} - \frac{k_{1S} k_{2D}}{2k_1 \cdot k_2} \right\} \\ - if^{abc} t_j^c \left\{ \bar{v}(p_2) \not{k}_D u(p_1) - \bar{v}(p_2) \not{k}_1 u(p_1) \frac{k_{2D}}{2k_1 \cdot k_2} \right\} \\ \text{cancel with } M_{\mu\nu}^{(1)} \text{-term}$$

$$\rightarrow k_1^\mu M_{\mu\nu} = -g^c f^{abc} t_j^c \bar{v}(p_2) \frac{k_1}{2k_1 \cdot k_2} u(p_1) + \underline{\underline{k_{2D}}}$$

Exercise 4

$$\langle L_N^{M_1 M_2} \rangle = P(N) \int_0^{\infty} \pi d\zeta_i \delta(1 - \sum \zeta_i)$$

$$\underbrace{\int_{\frac{R^2}{\ell^2}}^{\ell^2} \frac{d\ell}{\ell^2} \ell^{m_1} \ell^{n_2} [\ell^2 - R^2 + i\delta]^{-N}}_{\ell} = k g^{m_1 n_2}$$

$$g_{m_1 n_2} \langle L_N^{M_1 M_2} \rangle = K \cdot D$$

$$= P(N) \int_0^{\infty} \pi d\zeta_i \delta(1 - \sum \zeta_i) \int d\ell \ell^2 [\ell^2 - R^2]^{-N}$$

$$\ell^2 = \ell^2 - R^2 + R^2$$

$$= P(N) \int_0^{\infty} \pi d\zeta_i \delta(1 - \sum \zeta_i) \int d\ell \left\{ [\ell^2 - R^2]^{-N+1} \right. \\ \left. + R^2 [\ell^2 - R^2]^{-N} \right\}$$

remember integral over loop momenta:

$$I_N^D = P(N) \int_0^{\infty} \pi d\zeta_i \delta(1 - \sum \zeta_i) \int d\ell [\ell^2 - R^2]^{-N} \\ = (-1)^N P(N - \frac{D}{2}) \int_0^{\infty} \pi d\zeta_i \delta(1 - \sum \zeta_i) [R^2]^{\frac{D-N}{2}}$$

$$\Rightarrow K \cdot D = (-1)^{N-1} \frac{P(N)}{P(N-1)} \frac{\Gamma(N-1-\frac{D}{2})}{\Gamma(\frac{D-N+1}{2})} \int_0^{\infty} \pi d\zeta_i \delta(1 - \sum \zeta_i) \\ + (-1)^N P(N - \frac{D}{2}) \int_0^{\infty} \pi d\zeta_i \delta(1 - \sum \zeta_i) [R^2]^{\frac{D-N+1}{2}}$$

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$$= (-1)^N \Gamma(N - \frac{D+2}{2}) \int \overline{\sigma} dz_i \cdot S(1 - \sum z_i) [R^2]^{\frac{D+2-N}{2}}$$

$$\left\{ = (N-1) + N-1 - \frac{D}{2} \right\}$$

$$= -\frac{D}{2} \int_N^{D+2}$$

$$\Rightarrow K = -\frac{1}{2} \int_N^{D+2}$$

Exercise 5

$$m \circ m = \text{Diagram } 1 + \text{Diagram } 2 + \text{Diagram } 3$$

$$\overline{T}_{\mu\nu}^{(F)}(q) \quad T_{\mu}^{(g)} \quad T_{\nu}^{(g)}$$

$$\overline{T}_{\mu\nu}^{(F)} = \text{Trace}(t^a t^b)(-1)(-ig)^2 c^{-2}$$

$$\text{from top } \int \frac{d^D k}{(2\pi)^D} \frac{\text{Tr}[t_\mu(k+q-m)t_\nu(k-m)]}{(k^2 - m^2 + i\delta)((k+q)^2 - m^2 + i\delta)}$$

$$\text{use } \text{Trace}[t_\mu \mp t_\nu k] = 4(k_{\mu \pm \nu} + k_{\nu \pm \mu} - g_{\mu\nu} k \cdot \tau)$$

$$\text{Trace}[t_\mu t_\nu] = 4g_{\mu\nu}$$

$$\overline{T}_{\mu\nu}^{(F)} = -4g^2 T_R \delta^{ab} \int \frac{d^D k}{(2\pi)^D} \frac{k^\mu (k+q)_\mu + k^\nu (k+q)_\nu - g^{ab} (k^2 + k \cdot q - m^2)}{(k^2 - m^2 + i\delta)((k+q)^2 - m^2 + i\delta)}$$

$$\text{use } k^2 + k \cdot q - m^2 = (k+q)^2 - m^2 - k \cdot q - q^2$$

$$\overline{T}_{\mu\nu}^{(F)} = -4g^2 T_R \delta^{ab} \frac{i\pi^2}{(2\pi)^D} \left\{ 2 \overline{I}_2^{(n)}(q, m, m) + 2g^2 \overline{I}_2^{(n)}(q, -m) \right. \\ \left. - g^{ab} (I_1(m) - g_s I_1^5(q, -m) - g_s^2 I_2^{(n)}) \right\}$$

we only need the pole part of the integrals

$$I_1(m) = \frac{1}{\epsilon} m^2 + O(\epsilon^0)$$

$$I_2(q^2, m, m) = \frac{1}{\epsilon} + \rho_m$$

$$T_2^{\mu}(g_{\mu\nu\mu}) = -\frac{1}{2} g^{\mu} \frac{1}{\varepsilon} + \text{fin}$$

$$T_2^{\mu\nu}(g_{\mu\nu\mu}) = \frac{1}{\varepsilon} \left\{ \frac{1}{3} g^{\mu\nu} + g^{\mu\nu} \left(-\frac{1}{12} g^2 + \frac{1}{2} m^2 \right) \right\}$$

$$\Rightarrow \{ \dots \} = \frac{1}{\varepsilon} \left(g^{\mu\nu} \left(-\frac{1}{6} g^2 + m^2 \right) + \frac{2}{3} g^{\mu\nu} - g^{\mu\nu} \right. \\ \left. - g^{\mu\nu} \left(m^2 + \frac{1}{2} g^2 - g^2 \right) \right) \\ = \frac{1}{\varepsilon} \frac{1}{3} \left(g^{\mu\nu} g^2 - g^{\mu\nu} \right)$$

$$\tilde{T}_{\mu\nu}^{(F)} = -\frac{4}{3} i \frac{ds}{4\pi} T_R \delta^{\mu\nu} \left(g^{\mu\nu} g^2 - g^{\mu\nu} \right) \frac{1}{\varepsilon} + \text{fin}$$

used $\frac{g^2 \pi^2}{16\pi^4} = \underbrace{\frac{g^2}{4\pi}}_{ds} \frac{1}{4\pi} = \frac{ds}{4\pi}$

$$= -i \delta^{\mu\nu} \left(g^{\mu\nu} - g^2 g^{\mu\nu} \right) \tilde{T}^{(F)}(g^2)$$

$$\tilde{T}(g^2) = \frac{4}{3} \frac{ds}{4\pi} T_R \cdot \frac{1}{\varepsilon} + \text{fin}, \quad \tilde{T}_{\text{ren}}^{(F)} = \tilde{T}(g^2) + \frac{2}{3} \tilde{Z}_3^{(F)} - 1$$

$$\Rightarrow \tilde{Z}_3^{(F)} = 1 + \delta \tilde{Z}_2^{(F)}, \quad \delta \tilde{Z}_2^{(F)} = \frac{ds}{4\pi} \left(-\frac{4}{3} \right) T_R \cdot \frac{1}{\varepsilon}$$

for up flavours running in the fermion loop

multiply by n_f

$$\text{mult.} : \tilde{Z}_2^{(F)} = (\tilde{Z}_2^{(F)})^{-\frac{1}{2}} = 1 - \frac{1}{2} \delta \tilde{Z}_2^{(F)}$$

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