Introduction to Quantum Chromodynamics and Loop Calculations

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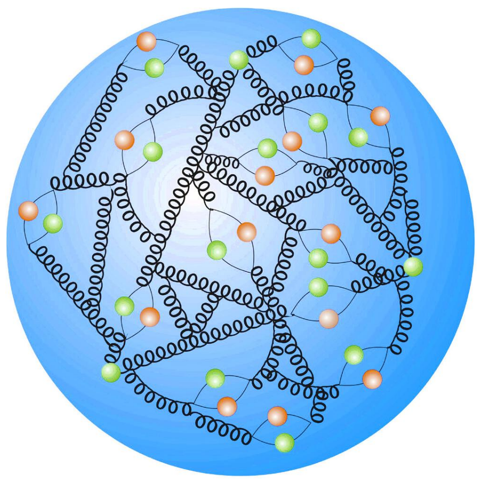
Lectures 1-3



Contents

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1 Quantum Chromodynamics as a non-Abelian gauge theory

Quantum Chromodynamics (QCD) is the theory of the strong interactions between quarks and gluons (also called *partons*).

The interactions are called "strong" since they are the strongest of the four fundamental forces at a length scale a bit larger than the proton radius. At a distance of 1 fm (1 fm $= 10^{-15}$m), its strength is about 10^{38} times larger than the gravitational force. However, we will see later that the strong coupling varies with energy. The higher the energy (i.e. the smaller the distance between the partons[1]), the weaker it will be. This phenomenon is called *asymptotic freedom*. At large distances, however, the interaction (coupling) becomes very strong. Therefore quarks and gluons cannot be observed as isolated particles. They are *confined* in hadrons, which are bound states of several partons. Quarks come in different *flavours*, u, d, c, s, t, b are known to exist (see Fig. 2). Baryons are hadrons regarded as bound states of 3 quarks, for example the proton *(uud)*, mesons are quark-antiquark bound states. The 9 mesons constructed from *up*, *down* and *strange* quarks are shown in Fig. 1.

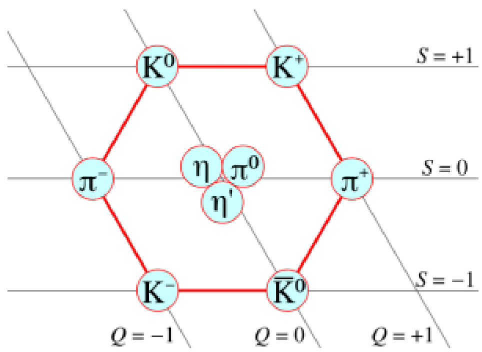


Figure 1: The Meson nonet. *Source: Wikimedia Commons.*

Why "Chromodynamics"? In addition to the well-known quantum numbers like electromagnetic charge, spin, parity, quarks carry an additional quantum number called *colour*. Bound states are colour singlets. Note that without the colour quantum number, a bound state consisting e.g. of 3 u-quarks (called Δ^{++}) would violate Pauli's exclusion principle if there was no additional quantum number (which implies that this state must be totally antisymmetric in the colour indices).

The emergence of QCD from the quark model [1–3] started more than 50 years ago, for a review see e.g. Ref. [4]. QCD as the theory of strong interactions is nowadays well established, however it still gives us many puzzles to solve and many tasks to accomplish in order to model particle interactions in collider physics.

[1]We will work in units where $\hbar = c = 1$.

There are various approaches to make predictions and simulations based on QCD. They can be put into two broad categories: (i) perturbative QCD, (ii) non-perturbative QCD (e.g. "Lattice QCD"). Our subject will be perturbative QCD.

1.1 Quarks and the QCD Lagrangian

The quark model and experimental evidence suggested that

- Hadrons are composed of quarks, which are spin 1/2 fermions.
- Quarks have electromagnetic charges $\pm 2/3$ (up-type) and $\mp 1/3$ (down-type) and come in 3 different colours ("colour charge").
- There is evidence that the colour charge results from an underlying local $SU(3)$ gauge symmetry.
- The mediators of the strong force are called gluons, which interact with both the quarks and themselves. The latter is a consequence of the non-Abelian structure of $SU(3)$.
- Quarks are in the fundamental representation of $SU(3)$, gluons in the adjoint representation.
- Quarks are believed to come in 6 flavours, forming 3 generations of up-type and down-type quarks: $\binom{u}{d}, \binom{c}{s}, \binom{t}{b}$. They transform as doublets under the electroweak interactions. The answer to the question why quarks and leptons come in 3 generations is still unknown.

QCD as a $SU(N_c)$ gauge theory

The strong interactions can be described as an $SU(3)$ local gauge theory, where the "charges" are denoted as *colour*. They are embedded in the Standard Model (SM) of elementary particle physics, with underlying gauge group $SU(3) \times SU(2)_L \times U(1)_Y$. The particle content of the SM as we know it right now is shown in Fig. 2.

The underlying structure of gauge theories can be described by *Lie groups*. QED is an Abelian gauge theory because the underlying group is the Abelian group $U(1)$. For QCD, the underlying group is the non-Abelian group $SU(N_c)$, where N_c is the number of colours (we believe that in Nature $N_c = 3$, but the concept is more general). The non-Abelian group structure implies that gluons interact with themselves (while photons do not), as we will see shortly. Non-Abelian gauge theories are also called *Yang-Mills* theories.

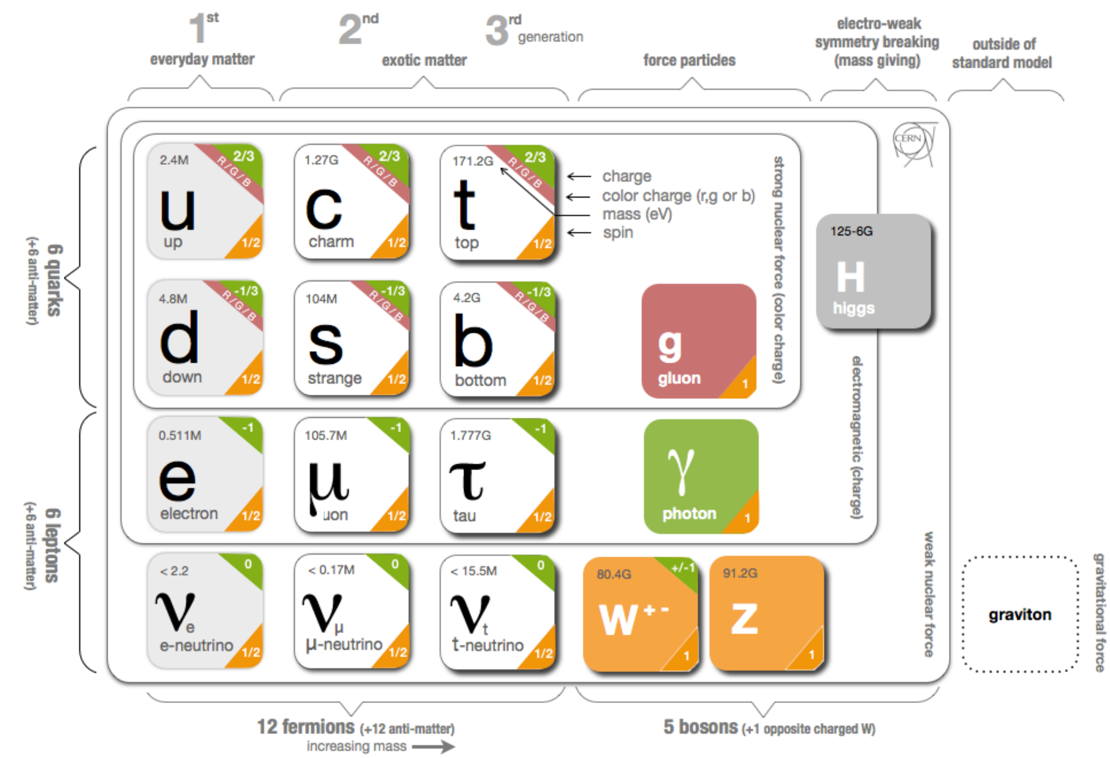


Figure 2: The particles of the Standard Model. *Source: CERN*

An important concept in QCD (and in the Standard Model in general) is the formulation as a *local* gauge theory. This means that the gauge transformation parameter depends itself on x, the position in space time.

Consider the quark fields $q_f^j(x)$ for just one quark flavour f. The index j labels the colour, $j = 1, \ldots, N_c$. Treating the quarks as free Dirac fields, we have

$$\mathcal{L}_q^{(0)}(q_f, m_f) = \sum_{j,k=1}^{N_c} \bar{q}_f^j(x) \; (i\,\gamma_\mu \partial^\mu - m_f)_{jk} \; q_f^k(x) \;, \tag{1.1}$$

where the γ_μ matrices in 4 space-time dimensions satisfy the Clifford algebra,

$$\{\gamma^\mu, \gamma^\nu\} \;=\; 2\, g^{\mu\nu}\,, \qquad \left\{\gamma^\mu, \gamma^5\right\} \;=\; 0\,. \tag{1.2}$$

Now let us apply a transformation $q_k \to q'_k = U_{kl}\, q^l$, $\bar{q}_k \to \bar{q}'_k = \bar{q}^{\,l} U^{-1}_{lk}$, with

$$U_{kl} = \exp\left\{ i \sum_{a=1}^{N_c^2-1} t^a\, \theta^a \right\}_{kl} \equiv \exp\left\{ i\, \boldsymbol{t} \cdot \boldsymbol{\theta} \right\}_{kl} \;, \tag{1.3}$$

where θ^a are the symmetry group transformation parameters. The Lagrangian of free Dirac fields remains invariant under this transformation as long as it is a *global* transformation, i.e. as long as the θ^a do not depend on x: $\mathcal{L}_q^{(0)}(q) = \mathcal{L}_q^{(0)}(q')$. The matrices $(t^a)_{kl}$ are $N_c \times N_c$ matrices representing the $N_c^2 - 1$ generators of $SU(N_c)$ in the so-called fundamental representation (see also Section 1.3).

However, we aim at *local* gauge transformations, i.e. the gauge transformation parameter θ in Eq. (1.3) depends on x. In QED, where the underlying gauge group is $U(1)$, a global transformation would just be a phase change. The requirement of a free electron field to be invariant under *local* transformations $\theta = \theta(x)$ inevitably leads to the introduction of a *gauge field* A_μ, the photon. The analogous is true for QCD: requiring local gauge invariance under $SU(N_c)$ leads to the introduction of gluon fields A^a_μ.
As the local gauge transformation

$$U(x) = \exp\left\{ i\, \boldsymbol{t} \cdot \boldsymbol{\theta(x)} \right\} \tag{1.4}$$

depends on x, the derivative of the transformed quark field $q'(x)$ reads

$$\partial_\mu\, q'(x) = \partial_\mu \left(U(x) q(x) \right) = U(x) \partial_\mu\, q(x) + \left(\partial_\mu U(x) \right) q(x) \;. \tag{1.5}$$

To keep $\mathcal{L}_q$ gauge invariant, we can remedy the situation caused by the second term above if we define a *covariant derivative* D^μ by

$$\left(D^\mu[A] \right)_{ij} = \delta_{ij} \partial^\mu + i\, g_s\, t^a_{ij} A^\mu_a \;, \tag{1.6}$$

or, without index notation

$$\boldsymbol{D}^\mu[\boldsymbol{A}] = \partial^\mu + i\, g_s\, \boldsymbol{A}_\mu \;, \tag{1.7}$$

where $\boldsymbol{A}^\mu = t^a\, A^\mu_a$ (sum over $a = 1 \ldots N_c^2 - 1$ understood). The fields A^μ_a are called *gluons*, they are coloured vector fields which transform under general $SU(N_c)$ transformations as follows:

$$\boldsymbol{A}_\mu \to \boldsymbol{A}'_\mu = U(x) \boldsymbol{A}_\mu U^{-1}(x) + \frac{i}{g_s} (\partial_\mu U(x)) U^{-1}(x) \;. \tag{1.8}$$

Therefore the Lagrangian for the quark fields which is invariant under local gauge transformations reads

$$\mathcal{L}_q(q_f, m_f) = \sum_{j,k=1}^{N_c} \bar{q}^{\,j}_f(x) \left(i\, \gamma_\mu \boldsymbol{D}^\mu[\boldsymbol{A}] - m_f \right)_{jk} q^k_f(x) \;. \tag{1.9}$$

What is the dynamics of the gauge fields? The purely gluonic part of the QCD Lagrangian can be described by the so-called Yang-Mills Lagrangian

$$\mathcal{L}_{YM} = -\frac{1}{4} F^a_{\mu\nu}\, F^{a,\mu\nu} \;, \tag{1.10}$$

where the non-Abelian field strength tensor $F^a_{\mu\nu}$ is given by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s\, f^{abc} A^b_\mu A^c_\nu \;. \tag{1.11}$$

The constants f^{abc} are the structure constants of the $SU(N_c)$ Lie algebra. They are completely antisymmetric and are related to the generators $(F^a)_{bc}$ of $SU(N_c)$ in the adjoint representation by $F^a_{bc} = -i\, f^{abc}$.

For $N_c = 3$, the matrices t^a are the 8 Hermitian and traceless generators of $SU(3)$ in the fundamental representation, $t^a = \lambda^a/2$, where the λ^a are called Gell-Mann matrices. They satisfy the commutator relation

$$[t^a,\, t^b] = i\, f^{abc}\, t^c \;, \tag{1.12}$$

with normalisation $\mathrm{Trace}(t^a t^b) = T_R \delta^{ab}$. Usually the convention is $T_R = 1/2$ for the fundamental representation. More details will be given in Section 1.3.

So finally we obtain for the "classical" QCD Lagrangian

$$\begin{aligned} \mathcal{L}_c &= \mathcal{L}_{YM} + \mathcal{L}_q \\ &= -\frac{1}{4} F^a_{\mu\nu}\, F^{a,\mu\nu} + \sum_{j,k=1}^{N_c} \bar{q}^j_f(x)\; (i\, \gamma_\mu \boldsymbol{D}^\mu[\boldsymbol{A}] - m_f)_{jk}\; q^k_f(x) \;. \end{aligned} \tag{1.13}$$

1.2 Feynman rules for QCD

We are not quite there yet with the complete QCD Lagrangian. The "classical" QCD Lagrangian $\mathcal{L}_c$ contains degenerate field configurations (i.e. they are equivalent up to gauge transformations). This leads to the fact that the bilinear operator in the gluon fields is not invertible, such that it is not possible to construct a propagator for the gluon fields. This will be outlined in the following.

The Feynman rules can be derived from the action,

$$S = i \int \mathrm{d}^4x\, \mathcal{L}_c \equiv S_0 + S_I \,, \quad \text{where} \quad S_0 = i \int \mathrm{d}^4x\, \mathcal{L}_0 \,, \quad \text{and} \quad S_I = i \int \mathrm{d}^4x\, \mathcal{L}_I .$$

In this decomposition $\mathcal{L}_0$ contains the free fields, i.e. terms bilinear in the fields (kinetic terms) and $\mathcal{L}_I$ contains all other terms (interactions). The gluon propagator $\Delta^{ab}_{\mu\nu}(p)$

is constructed from the inverse of the bilinear term in $A^a_\mu A^b_\nu$. In momentum space we have the condition (we suppress colour indices as these terms are diagonal in colour space, i.e. we leave out overall factors of the form δ^{ab})

$$i\,\Delta_{\mu\rho}(p)\,\left[p^2 g^{\rho\,\nu}\;-\;p^\rho p^\nu\right]=g^\nu_\mu\,. \tag{1.14}$$

However, we find

$$\left[p^2 g^{\rho\,\nu}-p^\rho p^\nu\right]p_\nu=0\,, \tag{1.15}$$

which means that the matrix $[p^2 g^{\rho\,\nu}\;-\;p^\rho p^\nu]$ is not invertible because it has at least one eigenvalue equal to zero. We have to remove the physically equivalent configurations from the classical Lagrangian. This is called *gauge fixing*. We can achieve this by imposing a constraint on the fields A^a_μ, adding a term to the Lagrangian with a Lagrange multiplier. For example, *covariant gauges* are defined by the requirement $\partial_\mu A^\mu\,(x)=0$ for any x. Adding

$$\mathcal{L}_{\rm GF}=-\frac{1}{2\lambda}\left(\partial_\mu A^\mu\right)^2,\qquad \lambda\in\mathbb{R},$$

to $\mathcal{L}$, the action S remains the same. The bilinear term then has the form

$$i\left(p^2 g^{\mu\nu}-\left(1-\frac{1}{\lambda}\right)p^\mu p^\nu\right),$$

with inverse

$$\Delta_{\mu\nu}(p)=\frac{-i}{p^2+i\,\varepsilon}\left[g_{\mu\nu}-(1-\lambda)\frac{p_\mu p_\nu}{p^2}\right]\,. \tag{1.16}$$

The so-called $i\,\varepsilon$ prescription ($\varepsilon>0$) shifts the poles of the propagator slightly off the real p^0-axis (where p^0 is the energy component) and will become important later when we consider loop integrals. It ensures the correct causal behaviour of the propagators. Of course, physical results must be independent of λ. Choosing $\lambda=1$ is called *Feynman gauge*, $\lambda=0$ is called *Landau gauge*.

In covariant gauges unphysical degrees of freedom (longitudinal and time-like polarisations) also propagate. The effect of these unwanted degrees of freedom is cancelled by the ghost fields, which are coloured complex scalars obeying Fermi statistics. Unphysical degrees of freedom and the ghost fields can be avoided by choosing *axial (physical) gauges*. The axial gauge is defined by introducing an arbitrary vector n^μ with $p\cdot n\neq 0$, to impose the constraint

$$\mathcal{L}_{\rm GF}=-\frac{1}{2\alpha}\left(n^\mu A_\mu\right)^2\qquad(\alpha\to 0)\,,$$

which leads to

$$\Delta_{\mu\nu}(p,n)=\frac{-i}{p^2+i\,\varepsilon}\left(g_{\mu\nu}-\frac{p_\mu n_\nu+n_\mu p_\nu}{p\cdot n}+\frac{n^2\,p_\mu p_\nu}{(p\cdot n)^2}\right).$$

A convenient choice is $n^2 = 0$, called *light-cone gauge*. Note that we have

$$\Delta_{\mu\nu}(p,n)\, p^\mu = 0\ ,\ \Delta_{\mu\nu}(p,n)\, n^\mu = 0\ .$$

Thus, only 2 degrees of freedom propagate (transverse ones in the $n^\mu + p^\mu$ rest frame). The price to pay by choosing this gauge instead of a covariant one is that the propagator looks more complicated and that it diverges when p^μ becomes parallel to n^μ. In the light-cone gauge we have

$$\begin{aligned} \Delta_{\mu\nu}(p,n) &= \frac{i}{p^2 + i\,\varepsilon}\, d_{\mu\nu}(p,n) \\ d_{\mu\nu}(p,n) &= -g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n} = \sum_{\lambda=1,2} \epsilon^\lambda_\mu(p)\left(\epsilon^\lambda_\nu(p)\right)^*\ , \end{aligned} \tag{1.17}$$

where $\epsilon^\lambda_\mu(p)$ is the polarisation vector of the gluon field with momentum p and polarisation λ. This means that only the two physical polarisations ($\lambda = 1, 2$) propagate. In Feynman gauge, we have

$$\sum_{pol} \epsilon^\lambda_\mu(p)\left(\epsilon^\lambda_\nu(p)\right)^* = -g_{\mu\nu}\ , \tag{1.18}$$

where the polarisation sum also runs over non-transverse gluon polarisations, which can occur in loops and will be cancelled by the corresponding loops involving ghost fields *(see later, exercises)*.

The part of the Lagrangian describing the Faddeev-Popov ghost fields can be derived using the path integral formalism, and we refer to the literature for the derivation. The result reads

$$\mathcal{L}_{FP} = \eta_a^\dagger\, M^{ab}\, \eta_b\ , \tag{1.19}$$

where the η_a are $N_c^2 - 1$ complex scalar fields which obey Fermi statistics and do not occur as external states. In Feynman gauge, the operator M^{ab} (also called Faddeev-Popov matrix) is given by

$$M^{ab}_{Feyn} = \delta^{ab}\, \partial_\mu\partial^\mu + g_s\, f^{abc} A^c_\mu \partial^\mu\ . \tag{1.20}$$

Note that in QED (or another Abelian gauge theory) the second term is absent, such that the Faddeev-Popov determinant $\det M$ does not depend on any field and therefore can be absorbed into the normalisation of the path integral, such that no ghost fields are needed in Abelian gauge theories.

In the light-cone gauge, the Faddeev-Popov matrix becomes

$$M^{ab}_{LC} = \delta^{ab}\, n_\mu \partial^\mu + g_s\, f^{abc}\, n_\mu A^\mu_c\ , \tag{1.21}$$

such that, due to the gauge fixing condition $n \cdot A = 0$, the matrix is again independent of the gauge field and therefore can be absorbed into the normalisation, such that no ghost fields propagate.

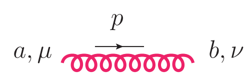
So finally we have derived the full QCD Lagrangian

$$\mathcal{L}_{QCD} = \mathcal{L}_{YM} + \mathcal{L}_q + \mathcal{L}_{GF} + \mathcal{L}_{FP} \; . \tag{1.22}$$

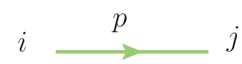
We will not derive the QCD Feynman rules from the action, but just state them below.

Propagators: ($i\varepsilon$ prescription understood)

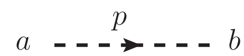
gluon propagator: $\Delta^{ab}_{\mu\nu}(p) = \delta^{ab}\,\Delta_{\mu\nu}(p)$



quark propagator: $\Delta^{ij}_q\,(p) = \delta^{ij}\, i\, \frac{\not{p}+m}{p^2-m^2}$

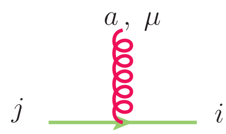


ghost propagator: $\Delta^{ab}\,(p) = \delta^{ab}\, \frac{i}{p^2}$

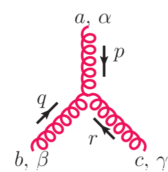


Vertices:

quark-gluon: $\Gamma^{\mu,\, a}_{gq\bar{q}} = -i\, g_s\, (t^a)_{ij} \gamma^\mu$

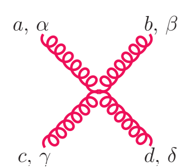


three-gluon: $\Gamma^{abc}_{\alpha\beta\gamma}(p,q,r) = -i\, g_s\, (F^a)_{bc}\, V_{\alpha\beta\gamma}(p,q,r)$

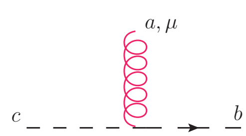


$$V_{\alpha\beta\gamma}(p,q,r) = (p-q)_\gamma g_{\alpha\beta} + (q-r)_\alpha g_{\beta\gamma} + (r-p)_\beta g_{\alpha\gamma}\,, \qquad p^\alpha + q^\alpha + r^\alpha = 0$$

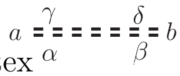
four-gluon: $\Gamma^{abcd}_{\alpha\beta\gamma\delta} = -i\, g_s^2 \begin{bmatrix} +f^{xac}\, f^{xbd}\, (g_{\alpha\beta} g_{\gamma\delta} - g_{\alpha\delta} g_{\beta\gamma}) \\ +f^{xad}\, f^{xcb}\, (g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\beta} g_{\gamma\delta}) \\ +f^{xab}\, f^{xdc}\, (g_{\alpha\delta} g_{\beta\gamma} - g_{\alpha\gamma} g_{\beta\delta}) \end{bmatrix}$



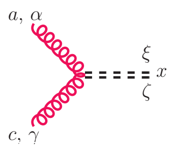
ghost-gluon: $\Gamma^{\mu,\, a}_{g\eta\bar{\eta}} = -i\, g_s\, (F^a)_{bc}\, p^\mu$



The four-gluon vertex differs from the rest of the Feynman rules in the sense that it is not in a factorised form of a colour factor and a Lorentz tensor. This is an inconvenient feature because it prevents the separate summation over colour and Lorentz indices and complicates automation. We can however circumvent this problem by introducing an auxiliary field with propagator



$= -\frac{i}{2}\delta^{ab}(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\delta}g^{\beta\gamma})$, that couples only to the gluon with vertex



$$= i\sqrt{2}g_s\, f^{xac}g^{\alpha\xi}g^{\gamma\zeta}.$$

We can check that a single four-gluon vertex can be written as a sum of three graphs. This way the summations over colour and Lorentz indices factorize completely.

Finally, we have to supply the following factors for incoming and outgoing particles:

- outgoing fermion: $\bar{u}(p)$
- outgoing antifermion: $v(p)$
- incoming fermion: $u(p)$
- incoming antifermion: $\bar{v}(p)$
- outgoing photon, or gluon: $\epsilon_\mu^\lambda(p)^*$
- incoming photon, or gluon: $\epsilon_\mu^\lambda(p)$.

1.3 Colour Algebra

For the generators of the group T^a, the commutation relation

$$[T^a,\, T^b] = i\, f^{abc}\, T^c \tag{1.23}$$

holds, independent of the representation.

The generators of $SU(3)$ in the fundamental representation are usually defined as $t^a_{ij} = \lambda^a_{ij}/2$, where the λ^a_{ij} are also called Gell-Mann matrices. They are traceless and hermitian and can be considered as the $SU(3)$ analogues of the Pauli-matrices for $SU(2)$.

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \; \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \; \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \; \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Quarks are in the fundamental representation of $SU(3)$. Therefore the Feynman rules for the quark-gluon vertex involve t^a_{ij} where $i, j = 1 \ldots N_c$ run over the colours of the quarks (the *degree* of the group), while $a = 1 \ldots N_c^2 - 1$ runs over the number of generators (the *dimension*) of the group. Gluons are in the adjoint representation of $SU(3)$, which we denote by the matrices $(F^a)_{bc}$, related to the structure constants by $(F^a)_{bc} = -i\, f^{abc}$. The adjoint representation is characterised by the fact that the dimension of the vector space on which it acts is equal to the dimension of the group itself, $a, b, c = 1 \ldots N_c^2 - 1$. The gluons can be regarded as a combination of two coloured lines, as depicted in Fig. 3. Contracting colour indices is graphically equivalent

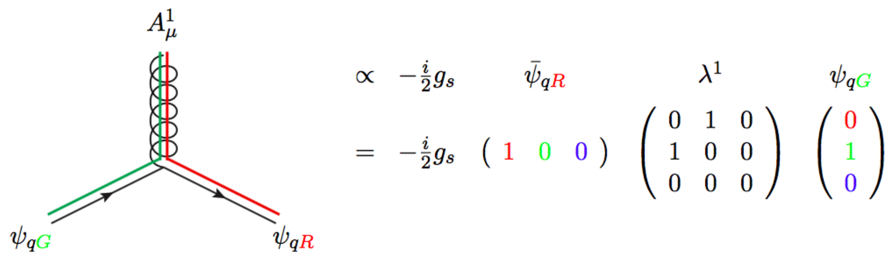


Figure 3: Representation of the gluon as a double colour line. *Picture: Peter Skands, arXiv:1207.2389.*

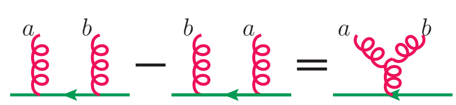
to connecting the respective colour (or anticolour) lines. The above representation of the quark-gluon vertex embodies the idea of colour conservation, whereby the colour-anticolour quantum numbers carried by the $q\bar{q}$ pair are transferred to the gluon.

The sums $\sum_{a,j} t^a_{ij} t^a_{jk}$ and $\sum_{a,d} F^a_{bd} F^a_{dc}$ have two free indices in the fundamental and adjoint representation, respectively. One can show that these sums are invariant under $SU(N)$ transformations, and therefore must be proportional to the unit matrix:

$$\sum_{j,a} t^a_{ij} t^a_{jk} = C_{\mathrm{F}}\, \delta_{ik}\,, \qquad \sum_{a,d} F^a_{bd} F^a_{dc} = C_{\mathrm{A}}\, \delta_{bc}\,. \tag{1.24}$$

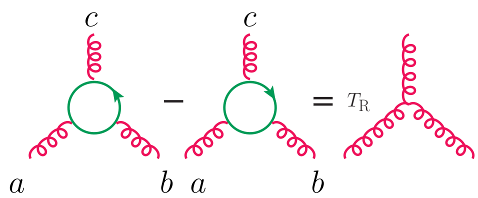
The constants C_{F} and C_{A} are the eigenvalues of the quadratic Casimir operator in the fundamental and adjoint representation, respectively.

The commutation relation (1.23) in the fundamental representation can be represented graphically by



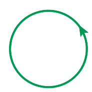
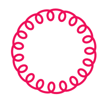
.

Multiplying this commutator first with another colour charge operator, summing over the fermion index and then taking the trace over the fermion line (i.e. multiplying with δ_{ik}) we obtain the representation of the three-gluon vertex as traces of products of colour charges:

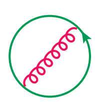
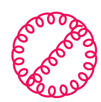


$$\mathrm{Trace}(t^a t^b t^c) - \mathrm{Trace}(t^c t^b t^a) = i\, T_R f^{abc}\,.$$

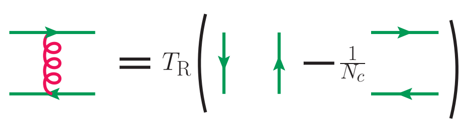
In the exercises we will see some examples of how to compute the colour algebra structure of a QCD diagram, independent of the kinematics. For example, taking the trace of the identity in the fundamental and in the adjoint representation we obtain

 $= N_c\,,$  $= N_c^2 - 1\;,$

respectively. Then, using the expressions for the fermion and gluon propagator insertions, we find

 $= C_{\mathrm{F}}\, N_c\;,$  $= C_{\mathrm{A}}\,(N_c^2 - 1)\,.$

There is also something like a Fierz identity, following from representing the gluon as a double quark line:



$$t^a_{ij} t^a_{kl} = T_{\mathrm{R}} \left(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl} \right)\;. \tag{1.25}$$

The Casimirs can be expressed in therms of the number of colours N_c as (*Exercise 2*)

$$C_F = T_R \frac{N_c^2 - 1}{N_c}\;,\; C_{\mathrm{A}} = 2\, T_R\, N_c\;. \tag{1.26}$$

The colour factors C_{F} and C_{A} can indirectly be measured at colliders. As they depend on N_c, these measurements again confirmed that the number of colours is three. One of the measurements will be discussed in the next section.

Acknowledgement: Some figures have been taken from Ref. [5].

1.4 Experimental evidence for colour

How can it be experimentally verified that the QCD colour quantum numbers exist? This is not straightforward, since colour is confined (hadrons are "white"), so that its existence can only be inferred. Here we describe one of the earliest and convincing measurements suggesting that there is a colour quantum number and that the number of colours is three.

Consider the total cross section for the production of a fermion-antifermion pair $f\bar{f}$ in an electron-positron collision, to lowest order in the electromagnetic coupling. The fermion has electromagnetic charge $e\,Q_f$ and mass m, and we approximate the electron to be massless. The leading order cross section is

$$\sigma_f(s) = \frac{4\pi\alpha^2 Q_f^2}{3s}\beta\left(1+\frac{2m_f^2}{s}\right)\theta(s-4m_f^2)\,, \tag{1.27}$$

where s is the center-of-mass energy squared. Note that we have attached a label f to the mass of the fermion of type f. The factor involving the electric charges also depends on the fermion type ("flavour" for quarks). Thus, for an electron, muon and tau $Q_f = -1$, for up, charm, and top quarks $Q_f = 2/3$, while for down, strange and bottom quarks $Q_f = -1/3$. The factor $\beta = \sqrt{1-4m_f^2/s}$ is a phase space volume factor, and the theta function is telling us that the available energy $\sqrt{s}$ must be larger than $2m_f$ in order to allow the production of a $f\bar{f}$ pair. When s is just a little bit larger than $4m^2$, β is close to zero, i.e. near the production threshold the cross section is small. Far above threshold $\beta \sim 1$.

We can use this result to infer the number of colour charges because quarks of each colour make their contribution to the total cross section. If the produced fermions are charged leptons (electrons, muons or taus), there are no additional quantum numbers to be taken into account. However, if the produced fermions are quarks, we have to sum over the flavours *and* colours in Eq. (1.27). The inclusive *hadronic* cross section (based in the production of quark-antiquark pairs) therefore reads

$$\sigma_{had}(s) = \sum_{f=u,d,s,c,\ldots} \frac{4\pi\alpha^2 Q_f^2}{3s}\beta\left(1+\frac{2m_f^2}{s}\right)\theta(s-4m_f^2)\,N_c \tag{1.28}$$

The extra factor N_c at the end accounts for the fact that quarks come in $N_c = 3$ colours. We may interpret this as a prediction for the inclusive hadronic cross section because the quarks in the final state must, before they reach any detector, make a transition to a hadronic final state, see the illustration in Fig. 4. In Fig. 5 we see the confrontation of this result with data, and that the agreement is very good, except that we did not anticipate the huge peak near $\sqrt{s} \simeq 90\,\text{GeV}$. That is because we did

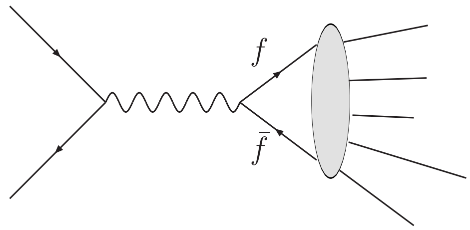


Figure 4: $e^+e^- \to$ hadrons; the blob represents the "hadronization" process, where the quarks get confined into hadrons.

not include in our calculation of $\sigma_f(s)$ in eq. (1.27) a second diagram in which not a photon (as in Fig. 4) but a Z-boson of mass $M_Z \simeq 91$GeV is exchanged between the e^+e^- and the $f\bar{f}$ pair. Had we done so, we would have more terms in the final answer for $\sigma(s)$ in Eq. (1.27), with the factor $1/s$ replaced by $1/(s - M_Z^2 + \Gamma_Z^2)$, where Γ_Z is the Z-boson decay width (about 2.5 GeV). The good agreement also implies that the effect of higher order corrections to $\sigma(s)$ should be small, and indeed they turn out to be so, after calculation. We can now define an observable traditionally called the R-ratio:

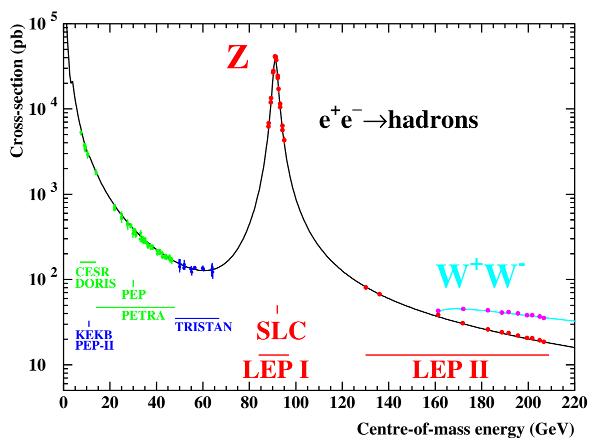


Figure 5: Total cross section for e^+e^- to hadrons.

$$R(s) = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \,. \tag{1.29}$$

The benefit of defining such a ratio is that a many common factors cancel in the theoretical prediction, and that many experimental uncertainties cancel in the measurement. We have

$$R(s) = \frac{\sum_{f=u,d,s,c,\ldots} \sigma(e^+e^- \to f\bar{f})}{\sigma(e^+e^- \to \mu^+\mu^-)} \,. \tag{1.30}$$

For large center-of-mass energy $\sqrt{s}$ we can derive from (1.28) that

$$R(s) \overset{s\to\infty}{\longrightarrow} N_c \sum_{f=u,d,s,c,\ldots} Q_f^2\, \theta(s-4m_f^2) \tag{1.31}$$

In Fig. 6 we confront this result with experiment. As the jumps cannot be seen so

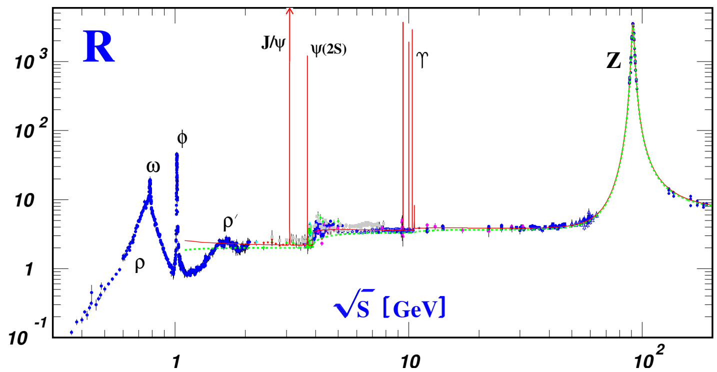


Figure 6: R-ratio vs. center of mass energy

well if all the resonances are included, in Fig. 7 shows the results from the program `rhad` [6] where only the regions which are accessible perturbatively are shown. It shows the jumps beyond the charm-meson production threshold, $\sqrt{s} > 2m_c$, and beyond the B-meson production threshold, $\sqrt{s} > 2m_b$, much better.

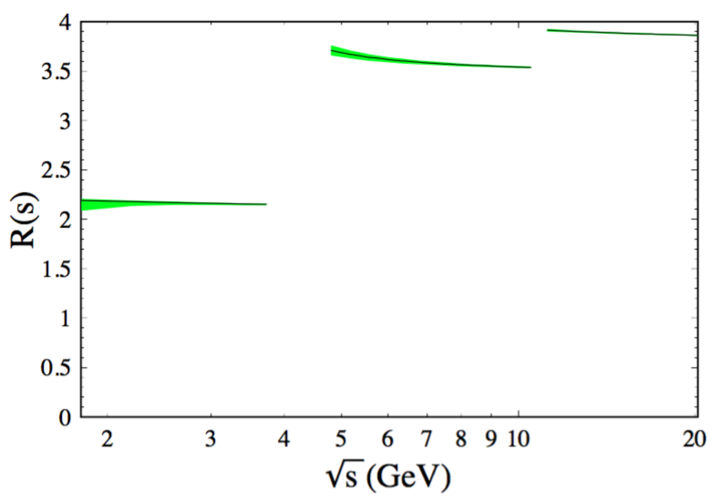


Figure 7: Mass thresholds in the hadronic R-ratio, calculated in Ref. [6], resonance regions excluded.

We can draw the conclusions that (i) there is again fairly good agreement between prediction and measurement; (ii) we see the effects of new quark flavour f being "turned on" as the energy increases beyond the production threshold $2m_f$ ($m_c \simeq 1.5$ GeV, $m_b \simeq 4.5$ GeV); (iii) the larger step at charm than at bottom (proportional to $Q_c^2 = 4/9$ and $Q_b^2 = 1/9$, respectively) is well-predicted; (iv) the value of $R(s)$ beyond the bottom quark threshold

$$R(s) = N_c \sum_{f=u,d,s,c,b} Q_f^2\, \theta(s - 4m_f^2) = 3\left(\tfrac{4}{9} + \tfrac{1}{9} + \tfrac{1}{9} + \tfrac{4}{9} + \tfrac{1}{9}\right) = \tfrac{11}{3} \tag{1.32}$$

agrees with experiment, and indicates that quarks come indeed in 3 colours.

Acknowledgement: Subsection 1.4 was inspired by Ref. [7].

2 Tree level amplitudes

In this section we will not only see how to calculate tree level matrix elements (squared), but also how unphysical polarisations in the QCD case arise and cancel when including ghost fields.

Let us first consider a simple process in QED, $e^+e^- \to \gamma\gamma$, where the contributing diagrams are shown in Fig. 8.

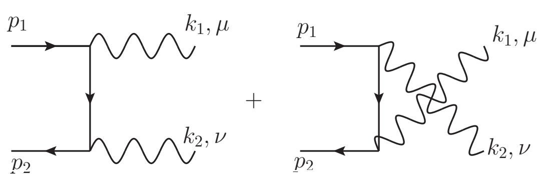


Figure 8: The process $e^+e^- \to \gamma\gamma$ at leading order.

If p_1 and p_2 are the two incoming momenta and k_1 and k_2 the two photon momenta, where $p_1 + p_2 = k_1 + k_2$, and neglecting the electron mass, we can write the amplitude as

$$\begin{aligned} \mathcal{M} &= -i\, e^2\, \epsilon_1^\mu(k_1)\epsilon_2^\nu(k_2)\, M_{\mu\nu}\ ,\ M_{\mu\nu} = M^{(1)}_{\mu\nu} + M^{(2)}_{\mu\nu}\ , \\ M^{(1)}_{\mu\nu} &= \bar{v}(p_2)\,\gamma_\nu \frac{\not{p}_1 - \not{k}_1}{(p_1 - k_1)^2}\gamma_\mu\, u(p_1)\ , \\ M^{(2)}_{\mu\nu} &= \bar{v}(p_2)\,\gamma_\mu \frac{\not{p}_1 - \not{k}_2}{(p_1 - k_2)^2}\gamma_\nu\, u(p_1)\ . \end{aligned} \tag{2.1}$$

Gauge invariance requires that $\epsilon_2^\nu \partial^\mu M_{\mu\nu} = 0, \epsilon_1^\mu \partial^\nu M_{\mu\nu} = 0$. In fact, $J_\mu \equiv \epsilon_2^\nu M_{\mu\nu}$ is a conserved current (charge conservation) coupling to the photon k_1. In momentum space, this means $k_1^\mu J_\mu = 0$.

Exercise: Verify explicitly that $M_{\mu\nu}$ is gauge invariant, and that this is the case independently of $p_i \cdot \epsilon(p_i) = 0$ being fulfilled or not.

Now let us look at the QCD analogue, the process $q\bar{q} \to gg$.
Due to the non-Abelian structure of QCD, we have a third diagram containing gluon

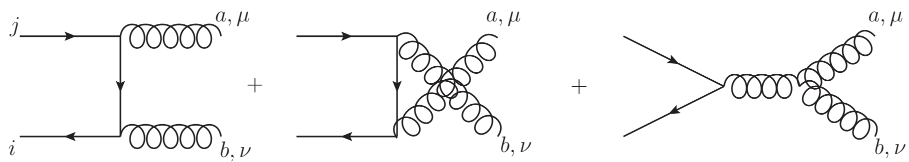


Figure 9: The process $q\bar{q} \to gg$ at leading order.

self-interactions. The leading order amplitude (in an expansion in α_s) is given by

$$\begin{aligned} \mathcal{M} &= -i\, g_s^2\, \epsilon_1^\mu(k_1)\epsilon_2^\nu(k_2)\, M_{\mu\nu}^{\mathrm{QCD}} \\ M_{\mu\nu}^{\mathrm{QCD}} &= (t^b t^a)_{ij}\, M_{\mu\nu}^{(1)} + (t^a t^b)_{ij}\, M_{\mu\nu}^{(2)} + M_{\mu\nu}^{(3)}\ , \end{aligned} \tag{2.2}$$

where $M_{\mu\nu}^{(1)}$ and $M_{\mu\nu}^{(2)}$ are exactly the same as in the QED case. Now we can use $(t^b t^a)_{ij} = (t^a t^b)_{ij} - i\, f^{abc} t^c_{ij}$ to write Eq. (2.2) as

$$M_{\mu\nu}^{\mathrm{QCD}} = (t^a t^b)_{ij}\, \left[M_{\mu\nu}^{(1)} + M_{\mu\nu}^{(2)}\right] - i\, f^{abc} t^c_{ij}\, M_{\mu\nu}^{(1)} + M_{\mu\nu}^{(3)}\ , \tag{2.3}$$

The term in square brackets in Eq. (2.3) is the QED amplitude, for which we know that $k_i^\mu\, M_{\mu\nu} = 0$. Therefore, the full $M_{\mu\nu}^{\mathrm{QCD}}$ in Eq. (2.3) can only be gauge invariant if $M_{\mu\nu}^{(3)}$ cancels the extra term $\sim f^{abc} t^c_{ij}\, M_{\mu\nu}^{(1)}$ when contracted with k_i^μ.

In fact, we find that

$$k_1^\mu M_{\mu\nu}^{(1)} = -\bar{v}(p_2)\, \gamma_\nu\, u(p_1) \tag{2.4}$$

$$k_1^\mu M_{\mu\nu}^{(3)} = i\, f^{abc} t^c_{ij} \bar{v}(p_2)\, \gamma_\nu\, u(p_1) - i\, f^{abc} t^c_{ij}\, \bar{v}(p_2)\, \not{k}_1\, u(p_1)\, \frac{k_{2,\nu}}{2k_1 \cdot k_2}\ . \tag{2.5}$$

The first term in Eq. (2.5) cancels the one proportional to $M_{\mu\nu}^{(1)}$ in Eq. (2.3). The second term in Eq. (2.5) is left over! However, it is zero upon contraction with the polarisation vector $\epsilon^\nu(k_2)$ if k_2 is the momentum of a *physical* gluon, i.e. if $\epsilon^\nu(k_2) \cdot k_2 = 0$.

2.1 Polarisation sums

In order to obtain cross sections, we need to calculate the modulus of the scattering amplitude, $|\mathcal{M}|^2$. For unpolarised cross sections, we sum over the polarisations/spins of the final states, so we need to evaluate

$$\sum_{\text{phys. pol}} \epsilon_{\mu_1}(k_1)\epsilon_{\nu_1}(k_2)\mathcal{M}^{\mu_1\nu_1}\epsilon^\star_{\mu_2}(k_1)\epsilon^\star_{\nu_2}(k_2)\left(\mathcal{M}^{\mu_2\nu_2}\right)^\star \ . \tag{2.6}$$

In QED, we can replace the polarisation sum by $-g_{\mu\nu}$, i.e.

$$\sum_{\text{phys. pol}} \epsilon_{\mu_1}(k_1)\epsilon^\star_{\mu_2}(k_1) \to -g_{\mu_1\mu_2} \ . \tag{2.7}$$

Note that Eq. (2.7) is *not* an equality, but holds because the amplitude $\mathcal{M}$ must fulfill the QED Ward Identity. This can be seen as follows:
Let us pick a reference frame where the momentum of the photon 1 (simply denoted by k instead of k_1) is $k = (k^0, 0, 0, k^0)$ and the polarisation vectors are given by $\epsilon_{L,R} = (0, 1, \pm i, 0)/\sqrt{2}$, satisfying the usual normalisation properties $\epsilon_L\epsilon^\star_L = \epsilon_R\epsilon^\star_R = -1, \epsilon_L\epsilon^\star_R = 0$. Introducing a light-like vector n which is dual to k, $n = (k^0, 0, 0, -k^0)$, we can write the physical polarisation sum as

$$\sum_{i=L,R} \epsilon^\mu_i(k)\epsilon^{\nu,\star}_i(k) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = -g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k\cdot n} \ . \tag{2.8}$$

However, in QED the second term can be dropped. This is because $k^\mu\epsilon^\nu(k_2)M_{\mu\nu} = 0$, $\epsilon^\mu(k)k_2^\nu M_{\mu\nu} = 0\,$. If we define $\mathcal{M}_\mu = \epsilon^\nu(k_2)M_{\mu\nu}$ we have $k^\mu\mathcal{M}_\mu = 0 \Rightarrow \mathcal{M}_0 = \mathcal{M}_3$ and therefore

$$\sum_{i=L,R} \epsilon^\mu_i(k)\epsilon^{\nu,\star}_i(k)\mathcal{M}_\mu\mathcal{M}^\star_\nu = |\mathcal{M}_1|^2+|\mathcal{M}_2|^2 = |\mathcal{M}_1|^2+|\mathcal{M}_2|^2+|\mathcal{M}_3|^2-|\mathcal{M}_0|^2 = -g^{\mu\nu}\mathcal{M}_\mu\mathcal{M}^\star_\nu \ . \tag{2.9}$$

The longitudinal ($\mathcal{M}_3$) and time-like ($\mathcal{M}_0$) components cancel each other. Therefore we have always (for n photons) $k_1^{\mu_1}\ldots k_n^{\mu_n}\mathcal{M}_{\mu_1\ldots\mu_n} = 0$, regardless whether $\epsilon(k_j)\cdot k_j = 0$ or not.
This is not the case in QCD. In QCD we just showed that $k_1^\mu\mathcal{M}_\mu \sim \epsilon(k_2)\cdot k_2$, which vanishes only for physical polarisations. If $\epsilon(k_2)\cdot k_2 \neq 0$, then $\mathcal{M}_0 \neq \mathcal{M}_3$ in Eq. (2.9) and therefore we can *not* just use $-g_{\mu\nu}$ for the polarisation sum. However, it can be shown that

$$S_{\text{unphys.}} \equiv \sum_{\text{unphysical pol.}} |\epsilon_\mu(k_1)\epsilon_\nu(k_2)\mathcal{M}^{\mu\nu}|^2 = \left| i\, g_s^2 f^{abc}t^c\bar{v}(p_2)\,\frac{\not{k}_1}{(k_1+k_2)^2}\,u(p_1)\right|^2 \ . \tag{2.10}$$

Calculating the ghost contribution shown in Fig. 10 however leads to the expression in Eq. (2.10) with opposite sign. This shows that the ghost fields cancel the unphysical polarisations of the gluon fields. Therefore it is also possible to use $-g_{\mu\nu}$ for the polarisation sum if the ghost fields are taken into account when calculating the squared amplitude. Note that closed ghost loops get an additional factor of -1 from the Feynman rules because they obey Fermi statstics.

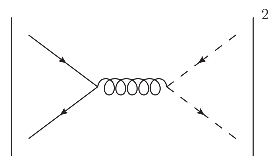


Figure 10: Ghost fields in the polarisation sum.

2.2 From amplitudes to cross sections

Let us first look at the scattering in a very general way and scatter particles of type a with number density n_a on a fixed target with particle density n_b and depth d, see Fig. 11. If F is the area of the beam and v_a the velocity of the beam particles, the flux is given by

$$\text{flux} = n_a\, v_a = \frac{\dot{N}_a}{F} \ , \tag{2.11}$$

where $\dot{N}_a$ is the number of particles per time unit [s]. The number of target particles situated within the beam area is $N_b = n_b\, F\, d$, and $L = \text{flux} \cdot N_b$ is called the *luminosity*. The *reaction rate* is defined as

$$R = L \cdot \sigma_r \ ,$$

where σ_r is the *cross section* for reaction r.

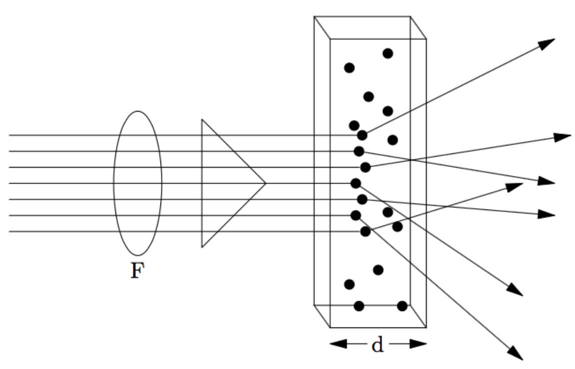


Figure 11: Scattering on a fixed target with depth d.

Differential cross sections can be defined for example in terms of the angular distributions of the scattered particles. The reaction rate per volume element $d\Omega$ is given by

$$R(\theta,\phi) = L\,\frac{d\sigma(\theta,\phi)}{d\Omega}\,d\Omega \;\text{ such that }\; \sigma = \int_0^{2\pi} d\phi \int_{-1}^{1} d\cos\theta\,\frac{d\sigma(\theta,\phi)}{d\Omega}\;. \tag{2.12}$$

For a circular collider with two bunches crossing each other rather than hitting a fixed target, we have

$$L = f\cdot n\cdot\frac{N_a N_b}{F}\;, \tag{2.13}$$

where f is the bunch frequency, n is the number of bunches, N_a and N_b are the number of particles per bunch, and F is the bunch crossing area. Cross sections are usually given in units of 'barn', where 1 barn $\simeq 10^{-24}\mathrm{cm}^2$. The LHC Run II luminosity is $L = 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$. So for cross sections of $\mathcal{O}(100)$ pb (1 picobarn $= 10^{-12}$ barn) we have a rate of 1 event per second. Taking $t\bar{t}$ production at 14 TeV: $\sigma \simeq 1000$ pb such that we have ~ 10 events per second.

For a reaction $q_a + q_b \to p_1 + \ldots + p_N$, the reaction rate is calculated according to Fermi's golden rule based on the *transition matrix element* $|\mathcal{M}|^2$. We have

$$\mathrm{d}\sigma = \frac{J}{\text{flux}}\cdot|\mathcal{M}|^2\cdot \mathrm{d}\Phi_N\;, \tag{2.14}$$

where

$$\text{flux} = 4\sqrt{(q_a\cdot q_b)^2 - m_a^2 m_b^2}$$

and $J = 1/j\,!$ is a statistical factor to be included for each group of j identical particles in the final state. The phase space volume spanned by the final state particles is denoted by $\mathrm{d}\Phi_N$, see Section 3.6.
For a decay process $Q \to p_1 + \ldots + p_N$ we have

$$\mathrm{d}\Gamma = \frac{J}{2\sqrt{Q^2}}\cdot|\mathcal{M}|^2\cdot \mathrm{d}\Phi_N\;. \tag{2.15}$$

For unpolarized incoming particles and if the spins of the final state particles are not measured, the matrix element is given by

$$|\mathcal{M}|^2 \to \overline{\sum}|\mathcal{M}|^2 = \prod_{\text{initial}}\frac{1}{N_{\text{pol}}N_{\text{col}}}\sum_{\text{final pol,col}}|\mathcal{M}|^2 \tag{2.16}$$

$$\begin{aligned}
&\text{quarks:} && N_{\text{col}} = N_c,\; N_{\text{pol}} = 2 \text{ (massless)}\\
&\text{gluons:} && N_{\text{col}} = N_c^2 - 1,\\
& && N_{\text{pol}} = \begin{cases} D-2 & \text{in conventional dim reg (CDR)}\\ 2 & \text{other schemes (HV, DRED)}\end{cases}\;.
\end{aligned}$$

For polarised amplitudes we only average over colours in the initial state and sum over colours in the final state.

A Appendix

A.1 The strong CP problem

In addition to the Yang-Mills Lagrangian, one can construct an additional gauge invariant dimension-four operator, the Θ-term:

$$\mathcal{L}_\Theta = \frac{\Theta g_s}{32\pi^2} \sum_a F^a_{\mu\nu} \tilde{F}^{a,\mu\nu} , \quad \text{with} \quad \tilde{F}^{a,\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F^a_{\alpha\beta} .$$

As it would give a (CP-violating) contribution to the electric dipole moment of the neutron, we know that Θ must be very small, $\Theta < 10^{-10}$. Why this term is so small (or probably zero) is not known. The is called the "strong CP problem". A possible solution has been suggested by Peccei and Quinn (1977), where the Θ-term belongs to an additional $U(1)$ symmetry, associated with a complex scalar field, called the *axion*. This symmetry is spontaneously broken by the vacuum expectation value obtained by this scalar field; the axion is the (almost) massless Goldstone boson of this broken symmetry. Searches for the axion are ongoing in several experiments.

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