Black holes in scalar tensor theories

LPT Orsay, CNRS

Dark Energy workshop

Scalar-tensor theories

- are the simplest modification of gravity with one additional degree of freedom
- Admit a uniqueness theorem due to Horndeski 1973... and beyond Horndeski
- contain or are limits of other modified gravity theories. $f(R)$, $f(\mathcal{G})$, massive gravity etc.
- Have insightful screening mechanisms (Vainshtein) providing a "classical" limit to GR
- Can have late time de Sitter behavior. Include theories that can screen classically a big cosmological constant.
- There are de Sitter or flat solutions for which the scalar field is not trivial.
- For hairy black holes this turns out to be crucial

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Galileons/Horndeski [Horndeski 1973]

$$S_H = \int d^4x\sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$L_2 = K(X),$$
$$L_3 = -G_3(X)\Box\phi,$$
$$L_4 = G_4(X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$
$$L_5 = G_5(X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- Horndeski theory includes Shift symmetric theories where G_i's depend only on X and $\phi \to \phi + c$.
  Associated with the symmetry there is a Noether current, J^μ which is conserved $\nabla_\mu J^\mu = 0$.
  Presence of this symmetry permits a very general no hair argument

Black holes have no hair [Wheeler]

During gravitational collapse...
Black holes eat or expel surrounding matter
their stationary phase is characterized by a limited number of charges
and no details
black holes are bald...

No hair arguments/theorems dictate under some reasonable hypotheses that adding degrees of freedom lead to singular solutions...

For example in vanilla scalar-tensor theories black hole solutions are GR black holes with constant scalar.

Warning : beyond GR Birkhoff's theorem is not valid.
Spherical symmetry thus does not guarantee staticity.
Scalar tensor black holes radiate monopole gravity waves.
There is no reason for metric and scalar not to radiate for spherical symmetry
Let us now see a classical example of a hairy solution...

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Example: BBMB solution

- Consider a conformally coupled scalar field ϕ:

$$S[g_{\mu\nu},\phi,\psi]=\int_{\mathcal{M}}\sqrt{-g}\left(\frac{R}{16\pi G}-\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi-\frac{1}{12}R\phi^2\right)\mathrm{d}^4x+S_m[g_{\mu\nu},\psi]$$

- **Invariance of the EOM of ϕ under the conformal transformation**

$$\begin{cases} g_{\alpha\beta}\mapsto\tilde{g}_{\alpha\beta}=\Omega^2 g_{\alpha\beta} \\ \phi\mapsto\tilde{\phi}=\Omega^{-1}\phi \end{cases}$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair.
The BBMB solution [N. Bocharova et al.-70 , J. Bekenstein-74]

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- Static and spherically symmetric solution

$$ds^2 = -\left(1 - \frac{m}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1 - \frac{m}{r}\right)^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

with **secondary** scalar hair

$$\boxed{\phi = \sqrt{\frac{3}{4\pi G}}\frac{m}{r-m}}$$

- Geometry is that of an extremal RN.
  Problem:The scalar field is **unbounded** at $(r = m)$.
- A cosmological constant can cure this; [MTZ] family of solutions
- Secondary hair black hole

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Summary so far

- Vacua in Horndeski can be non trivial. Non trivial vacua lead to time dependent scalars even for flat spacetime.
- Time independence for spherical symmetry is not guaranteed. We dont have Birkhoff's theorem in scalar tensor theories
- No hair theorems are not valid for time dependent spacetimes.

Let us now look at a specific no hair theorem for static and spherically symmetric spacetimes...

...and shift symmetric theories

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...and shift symmetric theories

No hair [Hui, Nicolis] [Sotiriou, Zhou] [Babichev, CC, Lehébel]

Static no hair theorem

Consider shift symmetric Horndeski theory with G_2, G_3, G_4, G_5 arbitrary functions of X. We have a Noether current J^μ which is conserved, $\nabla_\mu J^\mu = 0$.

We now suppose that:

1. spacetime and scalar are spherically symmetric and static,
$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dK^2, \ \phi = \phi(r)$$
2. spacetime is asymptotically flat, $\phi' \to 0$ as $r \to \infty$ and the norm of the current J^2 is finite on the horizon,
3. there is a canonical kinetic term X in the action,
4. and the G_i functions are such that their X-derivatives contain only positive or zero powers of X.

Under these hypotheses, ϕ is constant and thus the only black hole solution is locally isometric to Schwarzschild.

Most interesting part of no go theorem: Breaking any of these hypotheses leads to black hole solutions!

Theorem can be extended for star solutions. [Lehébel et al.]

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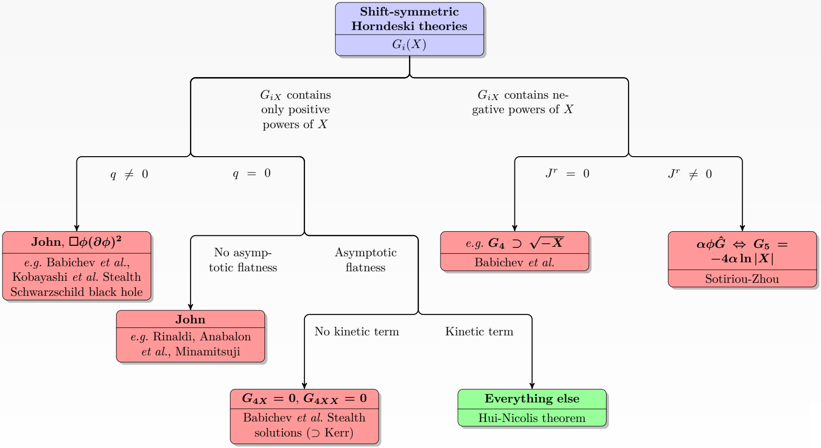
Hair versus no hair [figure: Lehébel]



A stealth solution

- Consider $S = \int d^4x\sqrt{-g}\,[\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]$
- take spherical symmetry and $\phi = \phi(t,r)$
- We have $\phi = qt + \psi(r)$ and at the end...
- All the field equations are solved if wa solve a single algebraic equation: $q^2\beta^3 - 2\zeta\beta k + C_0 k^{3/2} = 0 \to k = constant$!
- $f(r) = h(r) = 1 - \mu/r$
- $\phi_\pm = qt \pm q\mu\left[2\sqrt{\frac{r}{\mu}} + \log\frac{\sqrt{r}-\sqrt{\mu}}{\sqrt{r}+\sqrt{\mu}}\right] + \phi_0$...
- Consider $v = t + \int (fh)^{-1/2}dr$ then $ds^2 = -hdv^2 + 2\sqrt{h/f}\,dvdr + r^2d\Omega^2$ Regular chart for horizon, EF coordinates
- $\phi_+ = q\left[v - r + 2\sqrt{\mu r} - 2\mu\log\left(\sqrt{\frac{r}{\mu}} + 1\right)\right]$ + const
- Scalar regular at future black hole horizon.

Schwarzschild geometry with a non-trivial regular scalar field.

Exterior geometry for star

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Star solutions [Cisterna, Delsate, Rinaldi], [Maselli, Silva, Minamitsuji, Berti]

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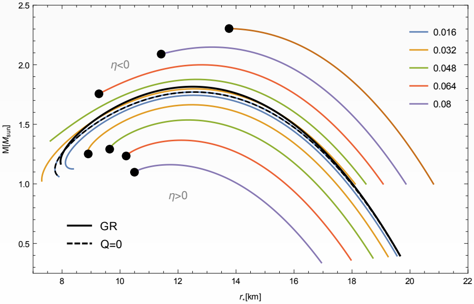
- Take stealth solution for exterior and consider PF matter for interior with ρ and P that does not couple to scalar.
- Presence of scalar field effects star interior which is different from GR.
- For fixed star radius $\beta > 0$ ($\beta < 0$) gives heavier (lighter) stars than GR.
- No GR limit for $q \to 0$
- Note no Vainshtein screening here



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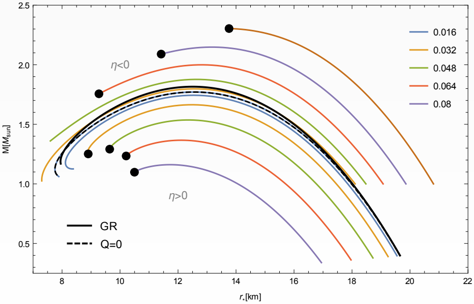

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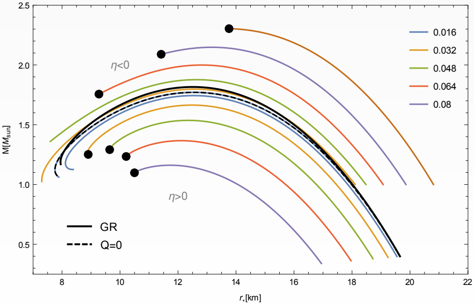

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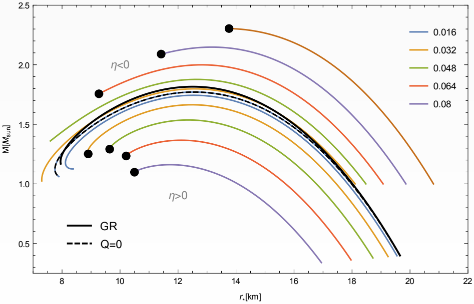
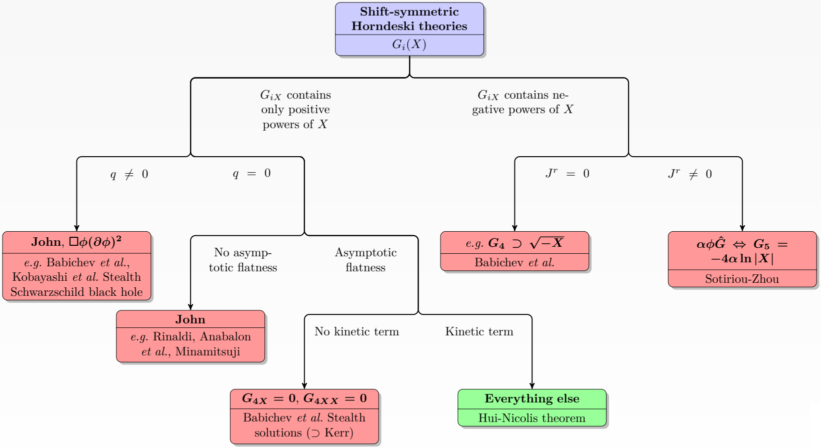


Hair versus no hair [Lehébel]



The special case of the Gauss-Bonnet invariant

[Sotiriou, Zhou] [Duncan et.al] [Mavromatos et.al]

The Gauss-Bonnet term, $\hat{G} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2$, is a topological invariant in 4 dimensions.
Variation with respect to the metric gives the 4 dim Lovelock identity, $H_{\mu\nu} = -2P_{\mu cde}R_{\nu}{}^{cde} + \frac{g_{\mu\nu}}{2}\hat{G} = 0$. If we couple to scalar then $\phi\hat{G}$ ceases to be trivial. The theory

$$\mathcal{L}^{\rm GB} = \frac{R}{2} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \alpha\phi\hat{G}$$

is non trivial and shift symmetric. Here, $\hat{G}$ (is independent of ϕ) and acts as a source to the scalar which cannot be set to zero.

- $\Box\phi + \alpha\hat{G} = 0$

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- Numerical solution can be found where the scalar and mass integration constants are fixed so that the solution is regular at the horizon.

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- $\Box\phi + \alpha\hat{G} = 0$
- The solution has infinite current norm at the horizon because $J^r \neq 0$

So far...

- For $q \neq 0$ we can find solutions analytically for G_2, G_4 and otherwise numerically
- For $q = 0$ we need to source the scalar field equation breaking one of the hypotheses of the theorem [Babichev, CC, Lehébel]
- For generic Horndeski we can use KK of known Lovelock solutions to construct black holes [CC, Gouteraux, Kiritsis]
- Slow rotation gives identical correction to GR. Stationary solutions not known except for stealth Kerr...

Purely quartic model, G_4 [Babichev, CC, Lehébel]

Suppose now that we keep G_4 analytic but do away with the kinetic term.
Consider again spherical symmetry ($q = 0$)

- Field equations dictate that $X = X_0$
- Regularity, $G_{4X}(X_0) = 0$ and $G_{4XX}(X_0) = 0$.

Any theory of the type, $G_4(X) = \zeta + \sum_{n\geq 2} \beta_n (X - X_0)^n$ will admit a stealth Schw. solution.
Further examination shows that, the Kerr metric is also an exact solution of the theory with

$$\phi(r,\theta) = \sqrt{-2X_0}\left[a\sin\theta - \sqrt{a^2 - 2mr + r^2} - m\ln\left(\sqrt{a^2 - 2mr + r^2} - m + r\right)\right]$$

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Conclusions

- Starting from a no hair theorem we have seen how to construct hairy black holes.
- Similar theorem exists for neutron stars.
- Using Lovelock solutions we can construct black holes in Horndeski theory.
- Many questions about stability of solutions;
- Higher order terms essential for novel branches of black holes
- One can construct solutions with EM fields and black hole solutions with primary hair
- Techniques for shift symmetric Horndeski can be extended to Maxwell-Proca theories.

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