Dark Energy & Modified Gravity: a theoretical perspective

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Astroparticules
et Cosmologie

Why trying to modify gravity ?

- So far, **General Relativity** appears compatible with all observations:
  - Laboratory experiments
  - Solar system measurements
  - Binary pulsars
  - Direct observation of GWs (NEW !)

"Why do you spend so much time and energy testing GR? We know that the theory is right." (Chandrasekhar to C. Will)

Why trying to modify gravity ?

- **Cosmic acceleration**
  - Cosmological constant seems “unnaturally” tiny
  - Models of **dark energy** & **modified gravity:** Scalar-tensor theories, f(R) gravity, massive gravity…

- **High energy modifications**
  - Beyond classical GR ?
  - Resolution of the singularities predicted by GR ?

- Modified gravity also provides benchmark models to **test General Relativity**
  - Parametrized classes of models
  - Guide for analysing data

How to modify gravity ?

- In **four** dimensions, the only **diffeomorphism** invariant action for the **metric** leading to at most **second order** equations of motion is the **Einstein-Hilbert** term plus a **cosmological constant** (Weyl, Cartan, Lovelock).

- **Additional fields** (scalar, vector, tensors)

  Scalar-tensor theories, …

- **Higher dimensions**

  Braneworlds, e.g. Randall-Sundrum, Dvali-Gabadadze-Porrati

- **Breaking of diffeomorphism invariance**
  - Lorentz-breaking theories: Einstein-aether, Horava-Lifshitz
  - Massive gravity

Modifying gravity

In practice, it is rather difficult to modify gravity:

1. The theory must be **internally consistent** (e.g. no problematic instabilities)

2. The theory **must look like GR** in all regimes where GR has been tested

   Lab tests, Solar system, Binary pulsars, GW from binary BH

3. Hopefully (but not necessarily), the theory should **account for the observed acceleration** and exhibit some **distinctive signatures**.

Scalar-tensor theories

- Simplest extension of GR: add a **scalar field**

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[F(\phi)R - Z(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi)\right] + S_m \left[\psi_m, g_{\mu\nu}\right]$$

- Simplest example: Brans-Dicke theories

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega_{\rm BD}}{\phi} (\partial\phi)^2\right] + S_m \left[\psi_m, g_{\mu\nu}\right]$$

where $\omega_{\rm BD}$ is a constant parameter.

Gravitation in Brans-Dicke theory

- Perturbed fields: $\phi = 1 + \delta\phi$

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Psi)\delta_{ij}dx^i dx^j$$

- Linearized eqs of motion: $(3 + 2\omega_{\rm BD})\nabla^2\delta\phi = -8\pi G\rho$

$$\nabla^2\Psi = 4\pi G\rho + \frac{1}{2}\nabla^2\delta\phi\,, \qquad \Psi - \Phi = \delta\phi$$

- **Gravitation:** $\nabla^2\Phi = 4\pi G\,\mu\,\rho\,, \qquad \Psi = \eta\,\Phi$

$$\mu = \frac{4 + 2\omega_{\rm BD}}{3 + 2\omega_{\rm BD}}\,, \qquad \eta = \frac{1 + \omega_{\rm BD}}{2 + \omega_{\rm BD}}$$

GR limit: $\omega_{\rm BD} \to \infty$

Constraint: $\omega_{\rm BD} > 4 \times 10^4$

Jordan vs Einstein "frame"

- **Jordan frame:** matter minimally coupled to the metric

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[F(\phi) R - Z(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right] + S_m \left[\psi_m, g_{\mu\nu} \right]$$

- **Conformal transformation:** $g^*_{ab} = \Omega^2 g_{ab}$

$$\int d^4x \sqrt{-g_*}\, R_* = \int d^4x \sqrt{-g} \left[\Omega^2 R + 6 \nabla^a \Omega \nabla_a \Omega \right]$$

- **Einstein frame:**
  - Dynamical term for the metric like in GR
  - Matter **non minimally coupled** to matter

$$S = \int d^4x \sqrt{-g_*} \left[\frac{M_P^2}{2} R_* - \frac{1}{2} g_*^{\mu\nu} \partial_\mu \phi_* \, \partial_\nu \phi_* - V(\phi_*) \right] + S_m \left[\psi_m, A^2(\phi_*) g^*_{\mu\nu} \right]$$

$$\xi = M_P^{-1} \frac{A_{\phi_*}}{A} \qquad \left[BD : \xi^2 = 1/(4\omega_{\rm BD} + 6) \right]$$

Chameleon mechanism

[Khoury, Weltman 2003]

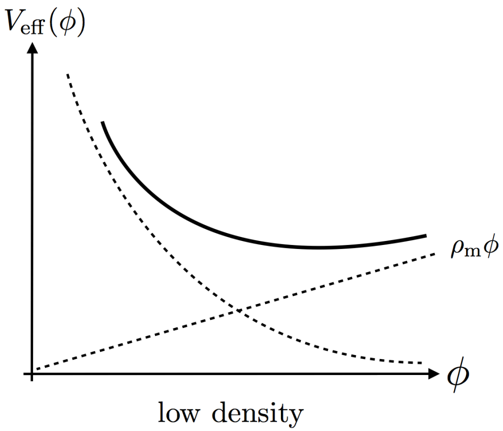
- **Einstein frame**

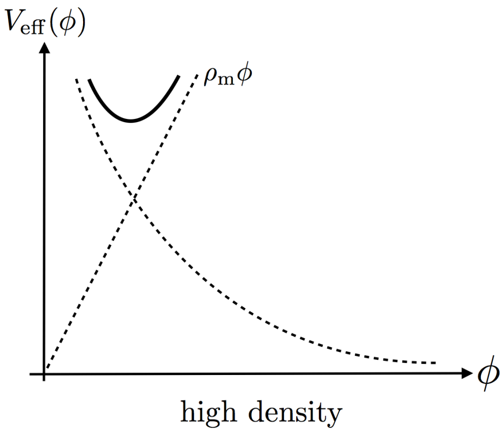
$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right] + S_m \left[\psi_m, \tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu} \right]$$

- **Effective potential**
  - Scalar field equation: $\nabla^\mu \nabla_\mu \phi = \frac{dV}{d\phi} + \frac{A_\phi}{A} \rho$
  - Hence

$$V_{\rm eff} = V(\phi) + [A(\phi) - 1]\hat{\rho} \simeq V(\phi) + \xi \frac{\phi}{M_p} \hat{\rho}$$

Chameleon mechanism





$$V_{\rm eff} = V(\phi) + \xi \frac{\phi}{M_p} \hat{\rho}$$

with

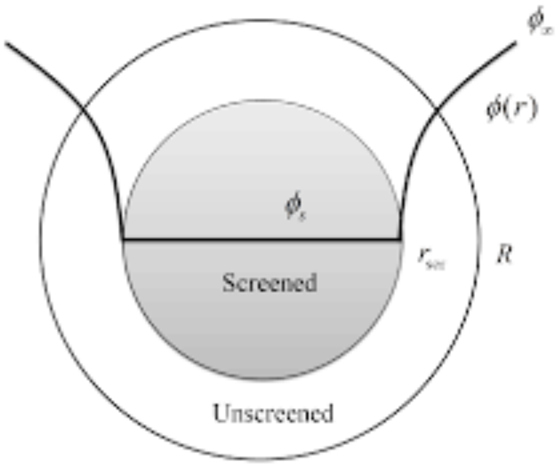
$$V(\phi) = V_0 + \frac{\mu^{4+n}}{\phi^n}$$

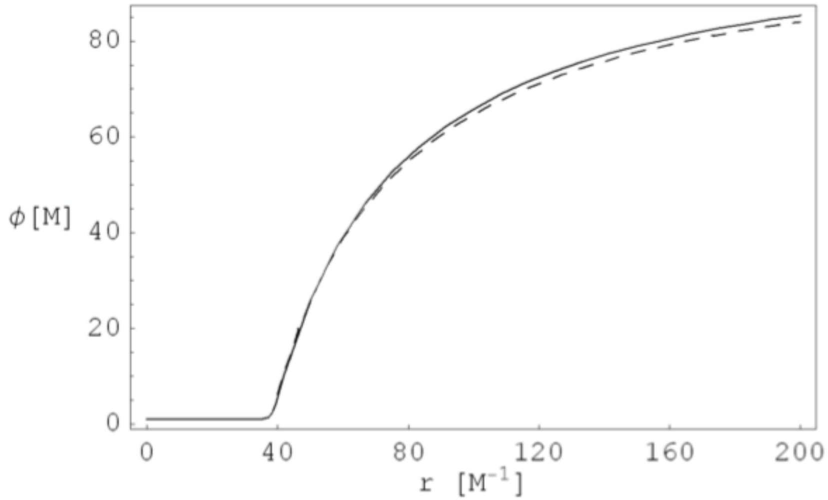
- Effective mass $m_{\rm eff}^2 \propto \rho^{\frac{n+2}{n+1}}$

- Outside the object, the scalar field profile is:

$$\phi = -\frac{Q}{M_P} \frac{e^{-m_\infty r}}{4\pi r} + \phi_\infty \qquad\qquad Q = \xi_{\rm eff}\, M$$

Chameleon mechanism





- The effective scalar charge of the object can be suppressed by the thin shell effect

$$Q = \xi_{\rm eff} M \quad \text{with} \quad \xi_{\rm eff} = \xi \quad \text{for unscreened objects}$$

$$\xi_{\rm eff} = 3\frac{\Delta R}{R}\xi \ll \xi \quad \text{for screened objects}$$

F(R) theories

[see e.g.review Sotiriou & Faraoni '10]

- Modify directly the gravitational action

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g}\, F(R) + S_{\rm matter}[\psi_m, g_{\mu\nu}]$$

- This is equivalent to a scalar-tensor theory

$$S = \frac{M_p^2}{2} \int d^4x \sqrt{-g}\, [F(\sigma) + (R-\sigma)F'(\sigma)]$$

- In the Einstein frame, coupling to matter

$$A(\phi) = \exp\left(-\frac{1}{\sqrt{6}} \frac{\phi}{M_P}\right) \qquad \xi = -\frac{1}{\sqrt{6}}$$

- **Chameleon-type screening**

Screening mechanisms

- Deviations from GR on cosmological scales should be compatible with small-scale observations (solar system, binary systems)

- **Screening mechanism**

$$Z(\phi_0)\,\nabla^2\delta\phi - m^2(\phi_0)\delta\phi = -\beta(\phi_0)\,\frac{\delta T}{M_P}$$

  – Chameleon: $m(\phi_0)$ is large

  – Dilaton & symmetron: $\beta(\phi_0) \ll 1$

  – Vainshtein: $Z(\phi_0) \gg \beta^2(\phi_0)$

Higher order scalar-tensor theories

- Usual scalar-tensor theories : $\mathcal{L}(\nabla_\lambda\phi, \phi)$

- Generalized theories with second order derivatives

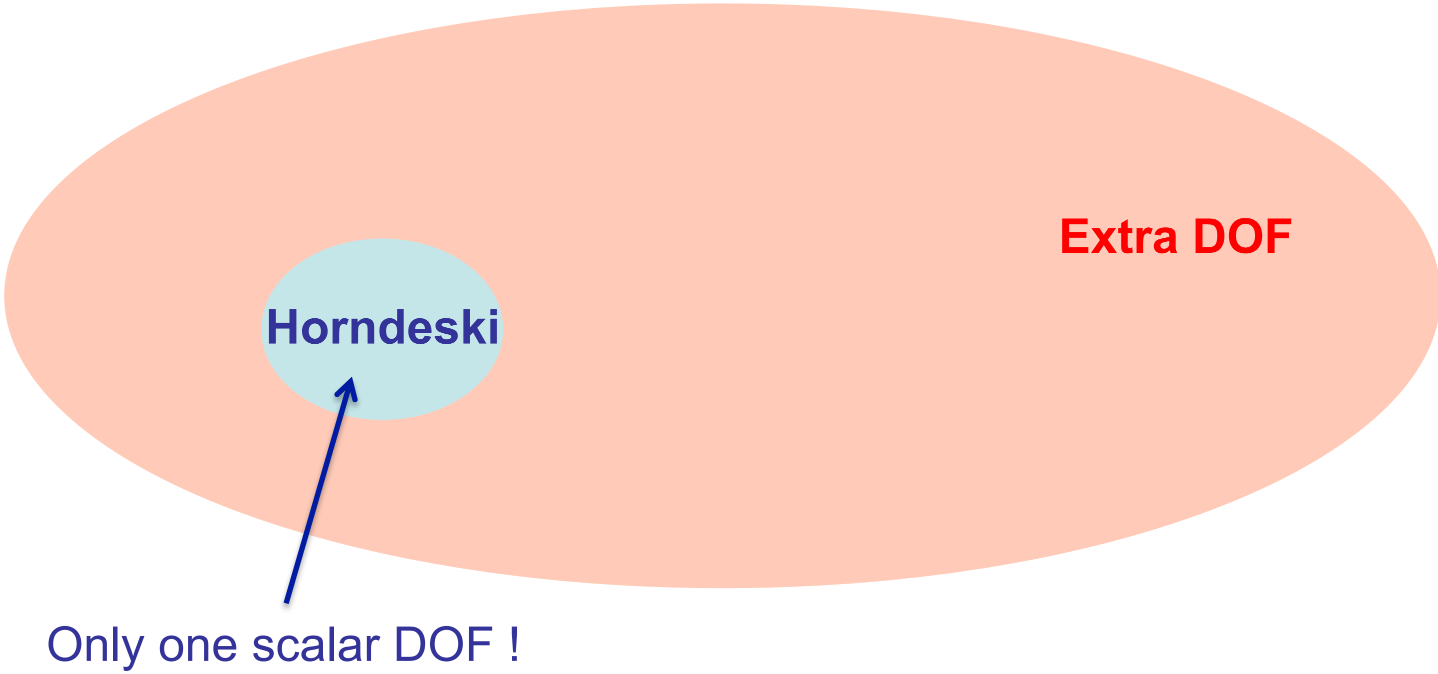
$$\mathcal{L}(\nabla_\mu\nabla_\nu\phi,\ \nabla_\lambda\phi,\ \phi)$$

- In general, they contain an **extra degree of freedom**, expected to lead to **Ostrogradsky instabilities**

$$L(\ddot{q}, \dot{q}, q)$$

Higher order scalar-tensor theories

- Usual theories (Brans-Dicke theories) $\mathcal{L}(\nabla_\lambda \phi, \phi)$
- Generalized theories: $\mathcal{L}(\nabla_\mu \nabla_\nu \phi,\ \nabla_\lambda \phi,\ \phi)$



Horndeski theories

Horndeski 74

- Combination of the following four Lagrangians

$$L_2^H = G_2(\phi, X)$$
$$L_3^H = G_3(\phi, X)\,\Box\phi$$
$$L_4^H = G_4(\phi, X)\,{}^{(4)}R - 2G_{4X}(\phi, X)(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu})$$
$$L_5^H = G_5(\phi, X)\,{}^{(4)}G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5X}(\phi, X)(\Box\phi^3 - 3\,\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\mu\sigma}\phi^{\nu}{}_{\sigma})$$

with
$$X \equiv \nabla_\mu\phi\nabla^\mu\phi$$
$$\phi_{\mu\nu} \equiv \nabla_\nu\nabla_\mu\phi$$

- **Second order equations of motion** for the scalar field and the metric

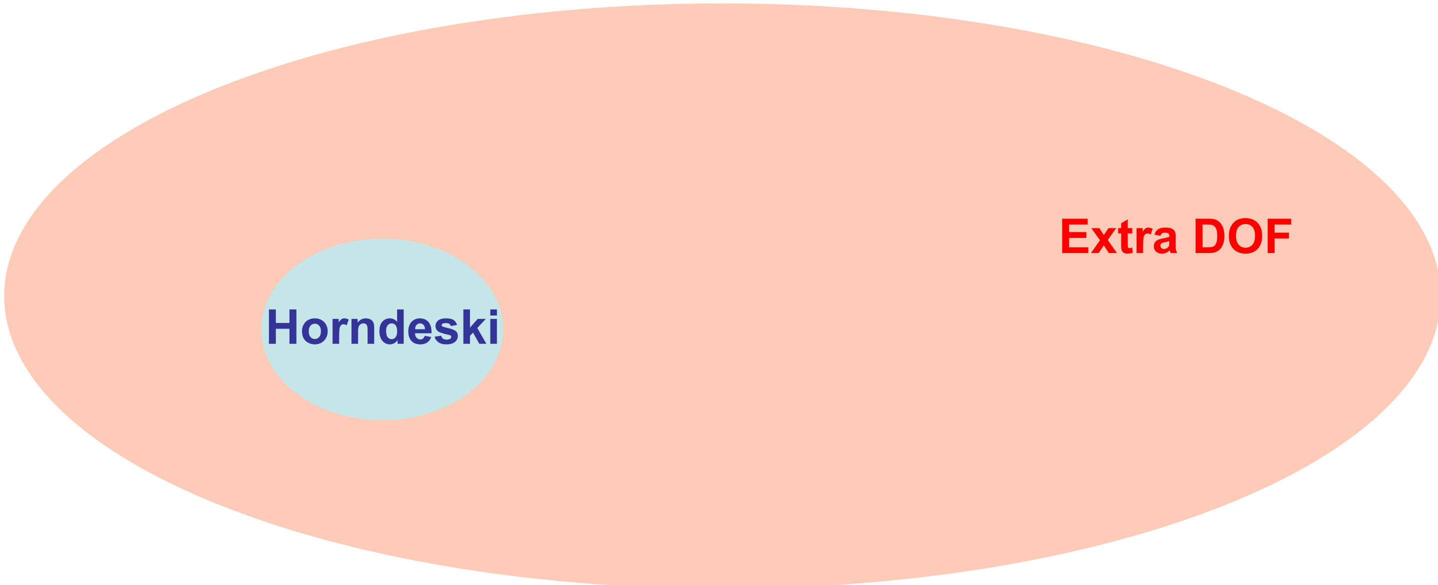
- They contain 1 scalar DOF and 2 tensor DOF.

  No dangerous extra DOF !

Higher order scalar-tensor theories

- Usual theories: $\mathcal{L}(\nabla_\lambda\phi, \phi)$

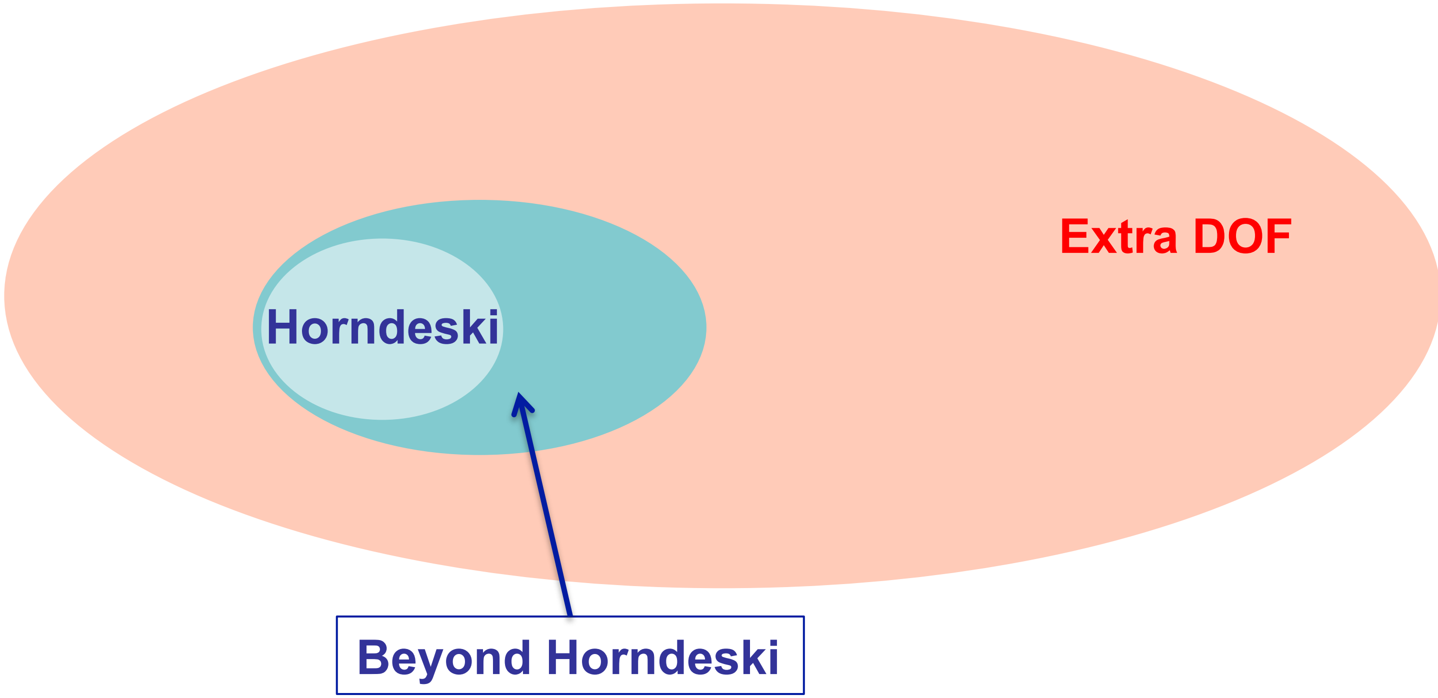
- Generalized theories: $\mathcal{L}(\nabla_\mu\nabla_\nu\phi,\ \nabla_\lambda\phi,\ \phi)$



Higher order scalar-tensor theories

- Usual theories: $\mathcal{L}(\nabla_\lambda \phi, \phi)$

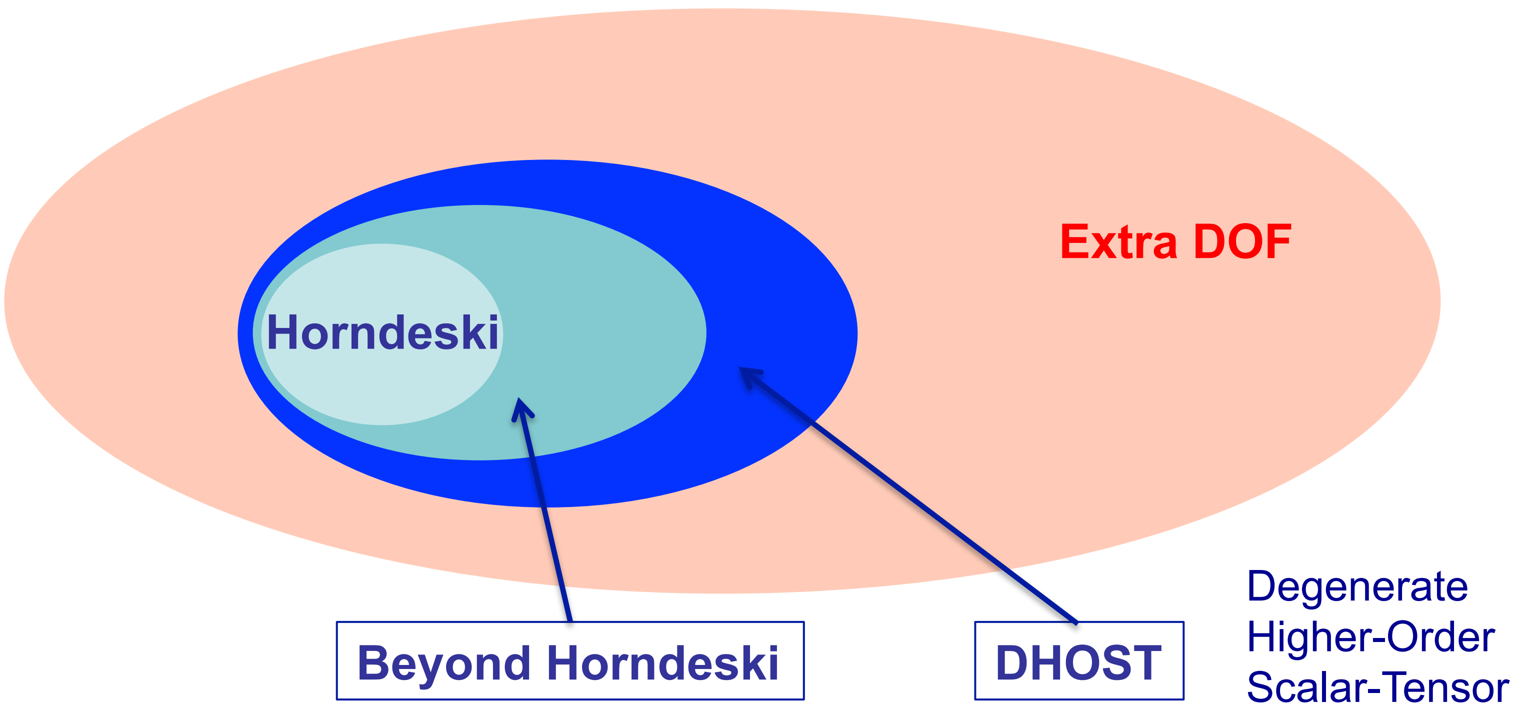
- Generalized theories: $\mathcal{L}(\nabla_\mu \nabla_\nu \phi,\ \nabla_\lambda \phi,\ \phi)$



Higher order scalar-tensor theories

- Usual theories: $\mathcal{L}(\nabla_\lambda\phi, \phi)$

- Generalized theories: $\mathcal{L}(\nabla_\mu\nabla_\nu\phi,\ \nabla_\lambda\phi,\ \phi)$



Beyond Horndeski

- First hint: disformal transformation of Einstein-Hilbert

Zumalacarregui & Garcia-Bellido '13

- Two extensions "beyond Horndeski"

[Gleyzes, DL, Piazza & Vernizzi '14]

$$L_4^{\rm bH} \equiv F_4(\phi, X)\, \epsilon^{\mu\nu\rho}{}_{\sigma}\, \epsilon^{\mu'\nu'\rho'\sigma} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'}$$

$$L_5^{\rm bH} \equiv F_5(\phi, X) \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma'} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'} \phi_{\sigma\sigma'}$$

leading to **third order** equations of motion.

- In contrast with earlier belief, no extra DOF if the total Lagrangian is "**degenerate**".

Presence of constraints in the system

Degenerate Lagrangians

DL & K. Noui ‘1510

- **Scalar-tensor theories**: scalar field + metric

- Simple toy model: $\phi(x^\lambda) \to \phi(t) \,, \quad g_{\mu\nu}(x^\lambda) \to q(t)$

- Lagrangian

$$L = \frac{1}{2} a \, \ddot{\phi}^2 + b \, \ddot{\phi} \, \dot{q} + \frac{1}{2} c \, \dot{q}^2 + \frac{1}{2} \dot{\phi}^2 - V(\phi, q)$$

- Equations of motion are higher order
  (4th order if a nonzero, 3rd order if a=0)

Degrees of freedom

- Introduce the auxiliary variable $Q \equiv \dot{\phi}$

$$L = \frac{1}{2} a\, \dot{Q}^2 + b\, \dot{Q}\dot{q} + \frac{1}{2} c\, \dot{q}^2 + \frac{1}{2} Q^2 - V(\phi, q) - \lambda(Q - \dot{\phi})$$

- Equations of motion

$$a\, \ddot{Q} + b\, \ddot{q} = Q - \lambda \qquad\qquad \dot{\phi} = Q\,, \quad \dot{\lambda} = -V_\phi$$

$$b\, \ddot{Q} + c\, \ddot{q} = -V_q$$

- If the Hessian matrix is **invertible**, one finds **3 DOF.**

$$M \equiv \left(\frac{\partial^2 L}{\partial v^a \partial v^b} \right) = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

[6 initial conditions]

Degrees of freedom

- If the Hessian matrix is degenerate, i.e. $ac - b^2 = 0$ then only **2 DOF** (at most).

$$M \equiv \left(\frac{\partial^2 L}{\partial v^a \partial v^b} \right) = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

[$\ddot{\phi}$ can be absorbed in $\dot{x} \equiv \dot{q} + \frac{b}{c}\ddot{\phi}$]

- Hamiltonian analysis: primary constraint and secondary constraint

[$p_a = \frac{\partial L}{\partial v^a}(v)$ cannot be inverted]

Higher Order Scalar-Tensor theories

- Consider all Lagrangians of the form [DL & Noui '1510]

$$S[g,\phi] = \int d^4x \sqrt{-g}\left[f_2{}^{(4)}R + C_{(2)}^{\mu\nu\rho\sigma}\,\nabla_\mu\nabla_\nu\phi\,\nabla_\rho\nabla_\sigma\phi\right]$$

where $f_2 = f_2(X,\phi)$ and $C_{(2)}^{\mu\nu\rho\sigma}$ depends only on ϕ and $\nabla_\mu\phi$.

- Equivalently: $C_{(2)}^{\mu\nu\rho\sigma}\,\phi_{\mu\nu}\,\phi_{\rho\sigma} = \sum_{i=1}^{5} a_i(X,\phi)\,L_i^{(2)}$

- Includes Horndeski $L_4^{\rm H}$ and Beyond Horndeski $L_4^{\rm bH}$

- **Full classification:** 7 degenerate subclasses

  (4 with $f_2 \neq 0$, 3 with $f_2 = 0$)

[See also Crisostomi et al '1602; Ben Achour, DL & Noui '1602; de Rham & Matas '1604]

Disformal transformations

- Transformations of the metric [Bekenstein '93]

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X,\phi)\, g_{\mu\nu} + D(X,\phi)\, \partial_\mu\phi\, \partial_\nu\phi$$

- Starting from an action $\tilde{S}\,[\phi, \tilde{g}_{\mu\nu}]$, one can define the new action

$$S[\phi, g_{\mu\nu}] \equiv \tilde{S}\,[\phi, \tilde{g}_{\mu\nu} = C\, g_{\mu\nu} + D\, \phi_\mu \phi_\nu]$$

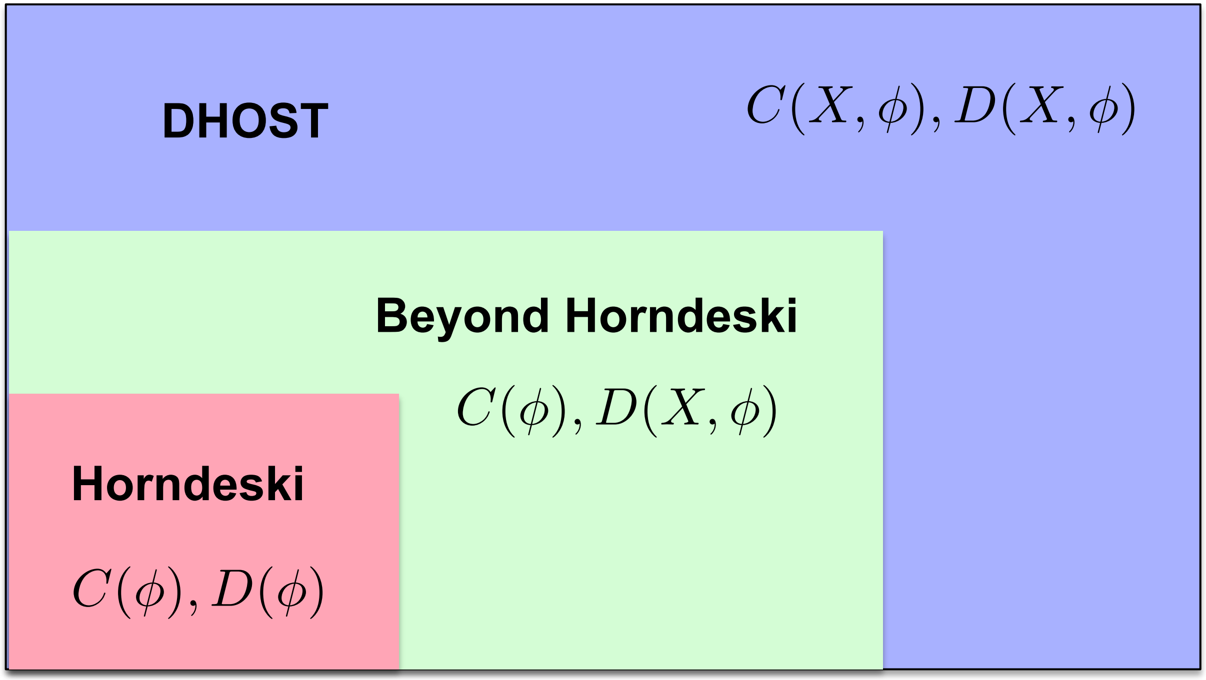
- The structure of quadratic DHOST theories is preserved under disformal transformations and all 7 subclasses are stable.

[Ben Achour, DL & Noui '1602]

Disformal transformations

- Stability under the transformations:

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C\, g_{\mu\nu} + D\, \partial_\mu\phi\, \partial_\nu\phi$$



Cubic DHOST theories

[Ben Achour, Crisostomi, Koyama, DL, Noui & Tasinato '1608]

- Action of the form

$$S[g,\phi] = \int d^4x\,\sqrt{-g}\left[f_3\,G^{\mu\nu}\phi_{\mu\nu} + C_{(3)}^{\mu\nu\rho\sigma\alpha\beta}\,\phi_{\mu\nu}\,\phi_{\rho\sigma}\,\phi_{\alpha\beta}\right]$$

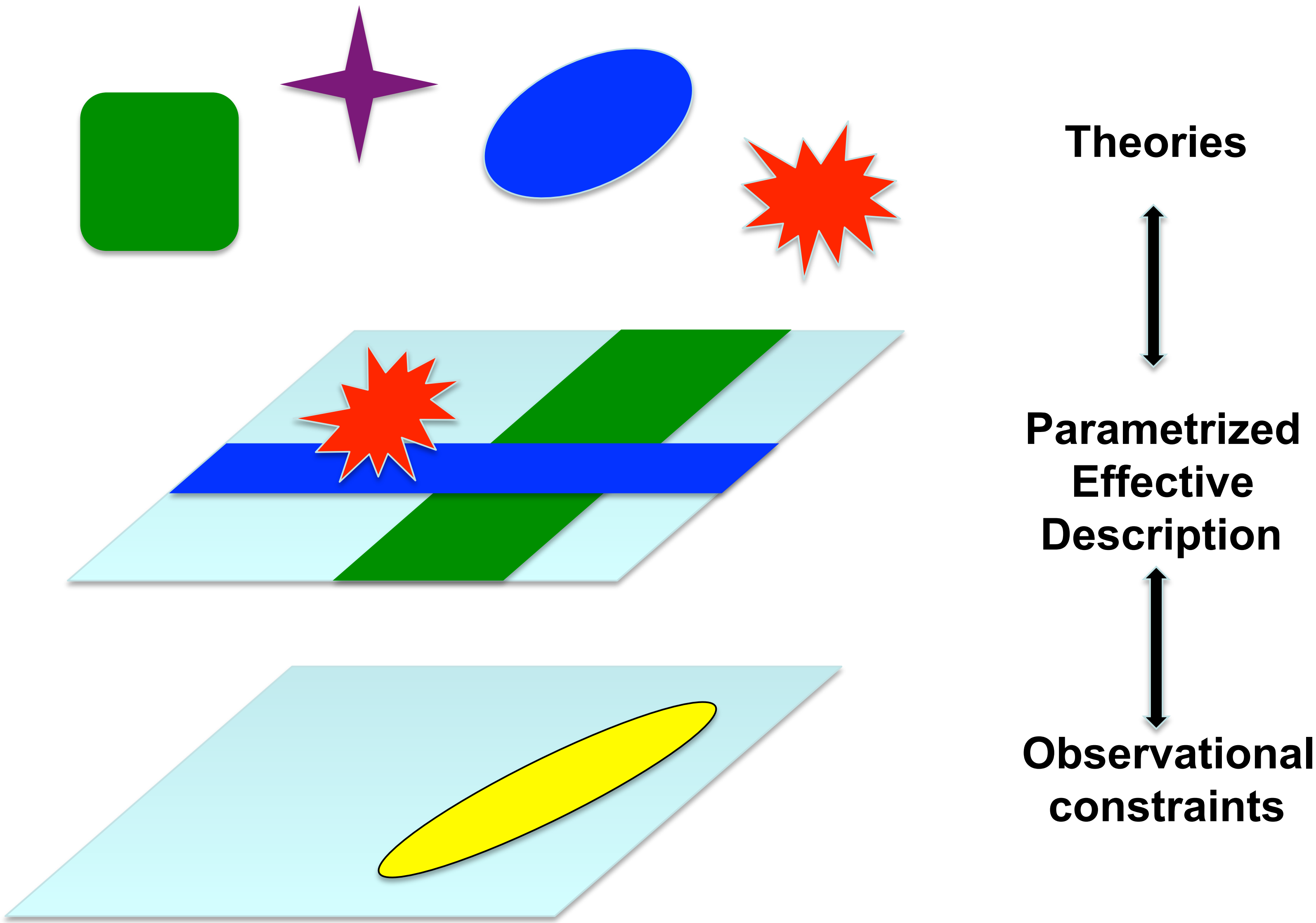
depends on eleven functions: $C_{(3)}^{\mu\nu\rho\sigma\alpha\beta}\,\phi_{\mu\nu}\,\phi_{\rho\sigma}\,\phi_{\alpha\beta} = \sum_{i=1}^{10} b_i(X,\phi)\,L_i^{(3)}$

- This includes the Lagrangians L_5^{H} and L_5^{bH} .

- 9 degenerate subclasses: 2 with $f_3 \neq 0$, 7 with $f_3 = 0$

- 25 **combinations of quadratic and cubic theories** (out of 7x9) are degenerate.

Cosmology:
Effective description of Dark Energy & Modified Gravity


Theories
Parametrized
Effective
Description
Observational
constraints

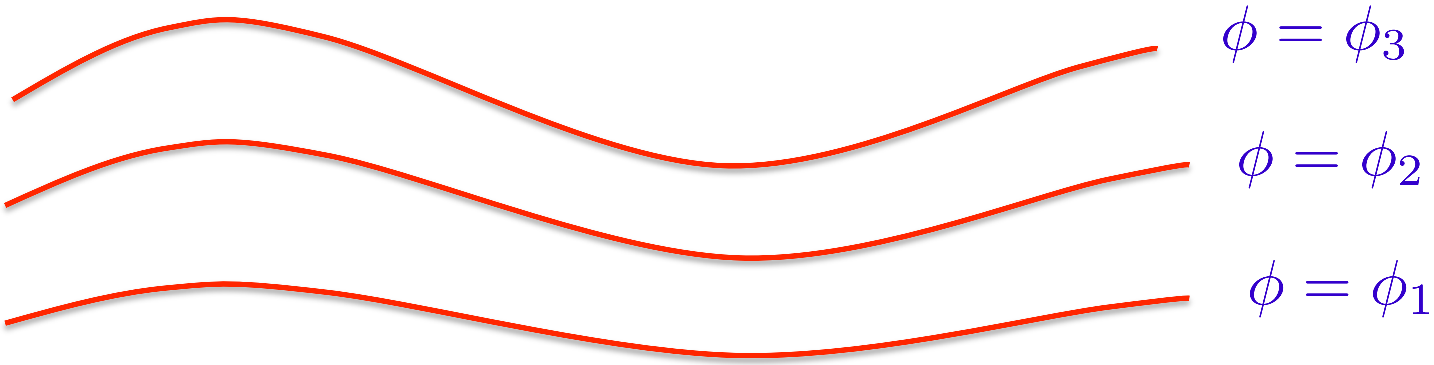
Effective description of Dark Energy

[See e.g review: Gleyzes, DL & Vernizzi 1411.3712]

- Restriction: **single scalar field** models

- The scalar field defines a **preferred slicing**

  Constant time hypersurfaces = uniform field hypersurfaces



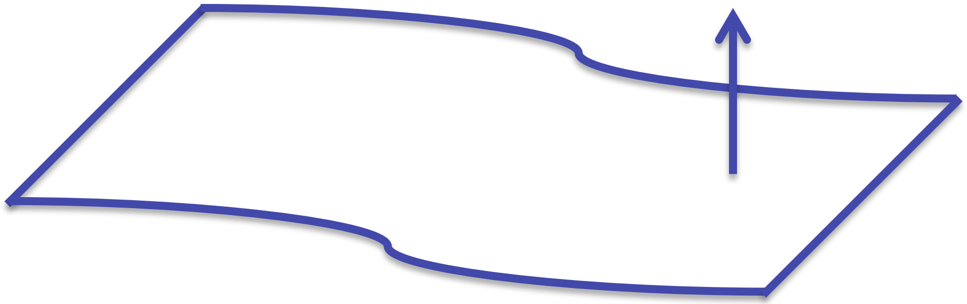
- All perturbations embodied by the metric only

Uniform scalar field slicing

- **3+1 decomposition** based on this preferred slicing

- Basic ingredients
  - **Unit vector normal** to the hypersurfaces

$$n^\mu = -\frac{\nabla^\mu \phi}{\sqrt{-(\nabla\phi)^2}}$$

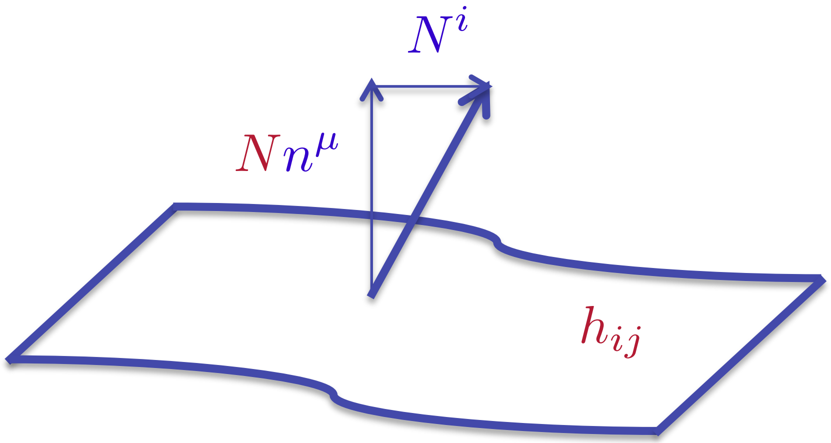


  - **Projection** on the hypersurfaces: $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$

ADM formulation

- **ADM decomposition of spacetime**

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$



Extrinsic curvature:

$$K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - D_i N_j - D_j N_i\right)$$

Intrinsic curvature: R_{ij}

$$X \equiv g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = -\frac{\dot{\phi}^2(t)}{N^2}$$

- **Generic Lagrangians of the form**

$$S_g = \int d^4x\, N\sqrt{h}\, L(N, K_{ij}, R_{ij}; t)$$

Homogeneous background & linear perturbations

- **Background** $ds^2 = -\bar{N}^2(t)\,dt^2 + a^2(t)\,\delta_{ij}dx^i dx^j$

$$\bar{L}(a,\dot{a},\bar{N}) \equiv L\left[K^i_j = \frac{\dot{a}}{\bar{N}a}\,\delta^i_j, R^i_j = 0, N = \bar{N}(t)\right]$$

- **Perturbations:** $\delta N \equiv N - \bar{N}\,,\ \delta K^i_j \equiv K^i_j - H\delta^i_j\,,\ \delta R^i_j \equiv R^j_i$

- Expanding the Lagrangian $L(q_A)$ with $q_A \equiv \{N, K^i_j, R^i_j\}$

yields $$L(q_A) = \bar{L} + \frac{\partial L}{\partial q_A}\delta q^A + \frac{1}{2}\frac{\partial^2 L}{\partial q_A \partial q_B}\delta q_A \delta q_B + \dots$$

- The **quadratic** action describes the **dynamics of linear perturbations**

Linear perturbations

- Quadratic action

Gleyzes, DL, Piazza & Vernizzi '13,
[notation: Bellini & Sawicki '14]

$$S^{(2)} = \int dx^3 dt\, a^3 \frac{M^2}{2} \Big[\delta K^i_j \delta K^j_i - \delta K^2 + \alpha_K H^2 \delta N^2 + 4\, \alpha_B H\, \delta K\, \delta N$$

$$+ (1+\alpha_T)\delta_2\left(\frac{\sqrt{h}}{a^3} R\right) + (1+\alpha_H) R\, \delta N \Big]$$

$$\alpha_M \equiv \frac{d \ln M^2}{H\, dt}$$

	α_K	α_B	α_M	α_T	α_H
Quintessence, K-essence	✓				
Kinetic braiding, DGP	✓	✓			
Brans-Dicke, f(R)	✓	✓	✓		
Horndeski	✓	✓	✓	✓	
Beyond Horndeski	✓	✓	✓	✓	✓

Scalar degree of freedom

- Scalar perturbations: $\delta N\,,\quad N_i \equiv \partial_i \psi\,,\quad h_{ij} = a^2(t)e^{2\zeta}\delta_{ij}$

- Quadratic action for the **physical degree of freedom**:

$$S^{(2)} = \frac{1}{2}\int dx^3 dt\, a^3 \left[\mathcal{K}_t\, \dot{\zeta}^2 + \mathcal{K}_s \frac{(\partial_i \zeta)^2}{a^2}\right]$$

$$\mathcal{K}_t \equiv \frac{\alpha_K + 6\alpha_B^2}{(1+\alpha_B)^2}\,, \quad \mathcal{K}_s \equiv 2M^2\left\{1+\alpha_T - \frac{1+\alpha_H}{1+\alpha_B}\left(1+\alpha_M - \frac{\dot{H}}{H^2}\right) - \frac{1}{H}\frac{d}{dt}\left(\frac{1+\alpha_H}{1+\alpha_B}\right)\right\}$$

- Stability
  - No ghost: $\mathcal{K}_t > 0$
  - No gradient instability: $c_s^2 \equiv -\dfrac{\mathcal{K}_s}{\mathcal{K}_t} > 0$

Tensor degrees of freedom

- Quadratic action for the **tensor modes**:

$$S_\gamma^{(2)} = \frac{1}{2}\int dt\, d^3x\, a^3 \left[\frac{M^2}{4}\dot\gamma_{ij}^2 - \frac{M^2}{4}(1+\alpha_T)\frac{(\partial_k \gamma_{ij})^2}{a^2}\right]$$

- Stability
  - No ghost: $M^2 > 0$
  - No gradient instability: $c_T^2 \equiv 1 + \alpha_T > 0$

Extension to DHOST theories

DL, Mancarella, Noui & Vernizzi '1703

- Quadratic action in terms of **9 functions of time**

$$S_{\rm quad} = \int d^3x\, dt\, a^3\, \frac{M^2}{2}\left\{\delta K_{ij}\delta K^{ij} - \left(1+\frac{2}{3}\alpha_{\rm L}\right)\delta K^2 + (1+\alpha_T)\left(R\frac{\delta\sqrt{h}}{a^3} + \delta_2 R\right)\right.$$

$$\left. + H^2\alpha_K \delta N^2 + 4H\alpha_B \delta K \delta N + (1+\alpha_H) R\delta N + 4\beta_1 \delta K \delta \dot N + \beta_2 \delta \dot N^2 + \frac{\beta_3}{a^2}(\partial_i \delta N)^2\right\}$$

- **Degeneracy conditions:** 2 possible sets

$$\mathcal{C}_{\rm I}:\ \alpha_{\rm L} = 0\,,\ \beta_2 = -6\beta_1^2\,,\ \beta_3 = -2\beta_1\left[2(1+\alpha_{\rm H}) + \beta_1(1+\alpha_{\rm T})\right]$$

$$\mathcal{C}_{\rm II}:\ \beta_1 = -(1+\alpha_{\rm L})\frac{1+\alpha_{\rm H}}{1+\alpha_{\rm T}}\,,\ \beta_2 = -6(1+\alpha_{\rm L})\frac{(1+\alpha_{\rm H})^2}{(1+\alpha_{\rm T})^2}\,,\ \beta_3 = 2\frac{(1+\alpha_{\rm H})^2}{1+\alpha_{\rm T}}$$

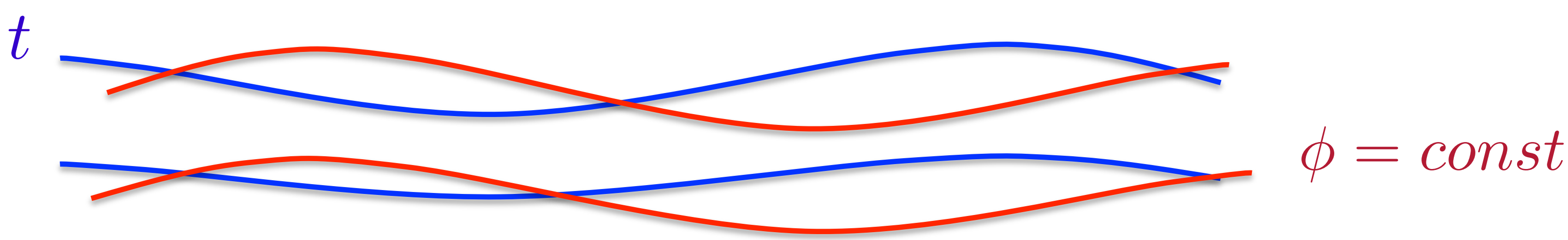
$\mathcal{C}_{\rm II}$: gradient instability either in the scalar or the tensor sector

Confrontation with observations

- Use a traditional gauge, e.g. Newtonian gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)\,(1-2\Psi)\,\delta_{ij}\,dx^i dx^j$$

- Description in an arbitrary slicing ?



- Coordinate change $\quad t \to t + \pi(t, \vec{x})$

- Perturbations: $\quad \Phi\,,\ \ \Psi\,,\ \ \pi\,,\delta_m\,,\ \ \vec{v}_m$

Cosmological perturbations

- **Standard equations** (in GR)

$$\begin{tikzcd} \Psi \arrow[r, leftrightarrow, "\Psi = \Phi"] \arrow[d, leftrightarrow] & \Phi \arrow[d, leftrightarrow] \\ \delta_m \arrow[r, leftrightarrow] & \vec{v}_m \end{tikzcd}$$

$$\nabla^2 \Psi = 4\pi G a^2 \rho_m \delta_m$$

(relativistic Poisson)

$$\dot{\vec{v}}_m + H\,\vec{v}_m = -\vec{\nabla}\Phi$$

(Euler)

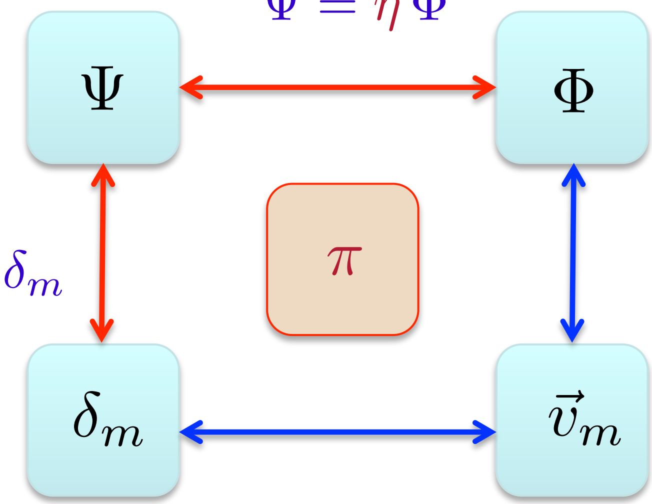
$$\dot{\delta}_m + \vec{\nabla} \cdot \vec{v}_m = 0$$

(continuity)

Cosmological perturbations

- **Modified equations**

$$\Psi = \eta\,\Phi$$



$$\nabla^2\Psi = 4\pi G(1+\Upsilon_G)a^2\rho_m\delta_m$$

Quasi-static approximation
(valid on scales
$kc_s \gg aH$
[Sawicki & Bellini ’15])

$$G_{\rm eff} = G_{\rm eff}(\alpha_i)\,, \qquad \eta = \eta(\alpha_i)$$

which can be confronted to observations (galaxy clustering, weak lensing…)

[Bellini et al ’15, Peronon et al. 15, Gleyzes et al 15, D’Amico et al 16, Alonso et al 16, …]

Neutron stars
in Modified Gravity

Stars in beyond Horndeski theories

Saito, Yamauchi, Mizuno, Gleyzes & DL ‘15
(see also Koyama & Sakstein ‘15)

- Partial breaking of Vainshtein mechanism inside matter

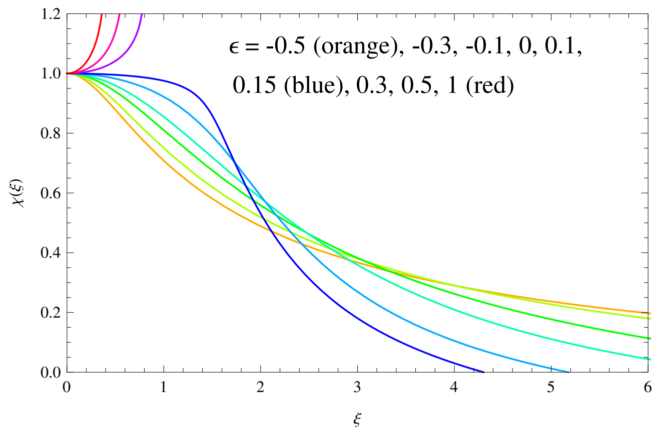
Kobayashi, Watanabe & Yamauchi ‘14

- Spherical symmetry & nonrelativistic limit:

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = G_{\mathrm{N}} \left(\frac{\mathcal{M}}{r^2} - \epsilon \frac{\mathrm{d}^2 \mathcal{M}}{\mathrm{d}r^2} \right) , \qquad \mathcal{M}(r) = 4\pi \int_0^r r'^2 \rho(r') \mathrm{d}r'$$

- Modified **Lane-Emden equation**
  (for $P = K\rho^{1+\frac{1}{n}}$)
  - Universal bound $\epsilon < 1/6$
  - Astrophysical constraints on $\Upsilon \equiv -4\epsilon$

$-0.18 < \Upsilon < 0.027$ [Sakstein 15, Jain et al 15]



Neutron stars in beyond Horndeski

Babichev, Koyama, DL, Saito & Sakstein '16

- Model

$$S = \int d^4x \sqrt{-g} \left[M_P^2 \left(\frac{R}{2} - \Lambda \right) - k_2 X + f_4 L_4^{\mathrm{bH}} \right]$$

with

$$L_4^{\mathrm{bH}} = -X \left[(\Box\phi)^2 - (\phi_{\mu\nu})^2 \right] + 2\phi^\mu \phi^\nu \left[\phi_{\mu\nu} \Box\phi - \phi_{\mu\sigma} \phi^\sigma{}_\nu \right]$$

- Cosmological solution: **de Sitter** with $\dot{\phi} = v_0 \neq 0 \,, \quad H \neq 0$

$$ds^2 = -(1 - H^2 r^2)\, dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega_2^2$$

$$\phi(r,t) = v_0 t + \frac{v_0}{2H} \ln\left(1 - H^2 r^2\right)$$

Neutron stars in beyond Horndeski

Babichev, Koyama, DL, Saito & Sakstein '16

- Spherical symmetric solutions

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2 d\Omega_2^2$$

with $\nu(r) = \nu_{\rm cosmo} + \delta\nu(r)\,, \quad \lambda(r) = \lambda_{\rm cosmo} + \delta\lambda(r)$

$$\phi(t,r) = \phi_{\rm cosmo}(t,r) + \delta\phi(r)$$

- **External solution**: Schwarzschild-de Sitter

$$ds^2 = -f dt^2 + f^{-1}dr^2 + r^2 d\Omega_2^2\,, \quad f \equiv 1 - \frac{2G_N M}{r} - H^2 r^2$$

$$\phi(t,r) = v_0\left[t - \int dr \frac{\sqrt{1-f}}{f}\right] \qquad G_N \equiv \frac{3G}{5\sigma^2 - 2}$$

Neutron stars in beyond Horndeski

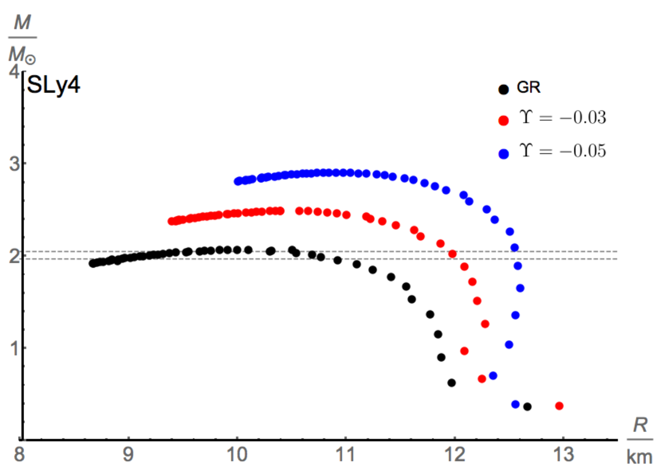
Babichev, Koyama, DL, Saito & Sakstein '16

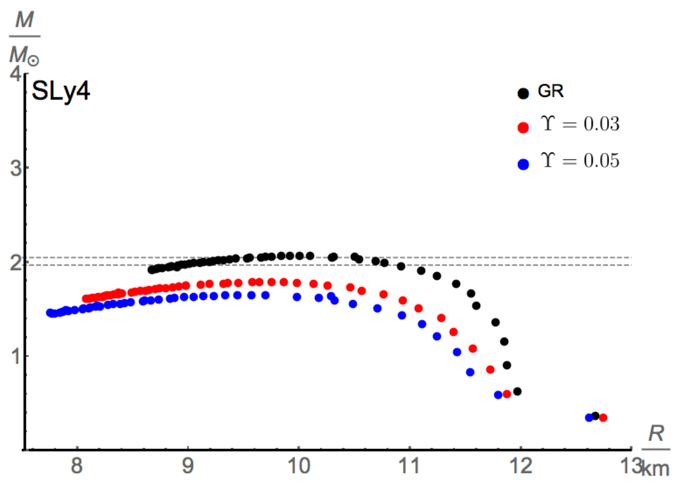
- Internal solution

  System analog to TOV equations

- Mass-radius relations

  For $\Upsilon < 0$ the maximum mass is larger than in GR.





- See also Sakstein, Babichev, Koyama, DL & Saito '16

Conclusions

- Modified gravity is a very active field of research, mainly motivated by
  - Cosmic acceleration
  - Testing GR
- Recent developments in the construction of scalar-tensor theories, in particular with the classification of DHOST theories, which contain a single scalar dof.
- Most existing scalar-tensor theories of modified gravity can be tested in cosmology, by using the effective description of dark energy and modified gravity.
- Modified gravity theories must also be tested in compact objects.