# The Thermal Abundance of Semi-Relativistic Relics

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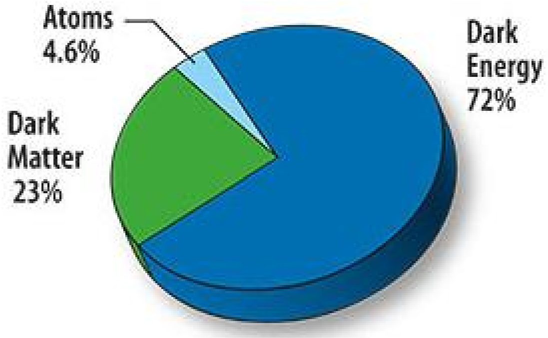
Refs:

- work in progress

# 1. Motivation



[http://wmap.gsfc.nasa.gov]

- The abundance of thermal relics (e.g. DM) is determined by the Boltzmann equation:

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma_{\mathrm{eff}} v \rangle (n_\chi^2 - n_{\chi,\mathrm{eq}}^2)$$

- Numerical calculation needed in evaluating the relic density in many cases

⇒ Analytic methods should be developed in various scenarios

- Approximate analytical solutions established for particles that are either relativistic (hot) or non-relativistic (cold) at decoupling

- No analytical formula for the relic density of particles that are semi-relativistic at decoupling

# Outline

This work

- Analytic treatment that connects the hot and cold relic solutions
- Late entropy production by semi-relativistic relics

1. Motivation
2. Thermal abundances of hot and cold relics (review)
3. Thermal abundance of semi-relativistic relics
4. Application of semi-relativistic relics
5. Summary

# 2. Thermal abundances of hot and cold relics

[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- The abundance of thermal relics (e.g. DM) is determined by the Boltzmann eq.:

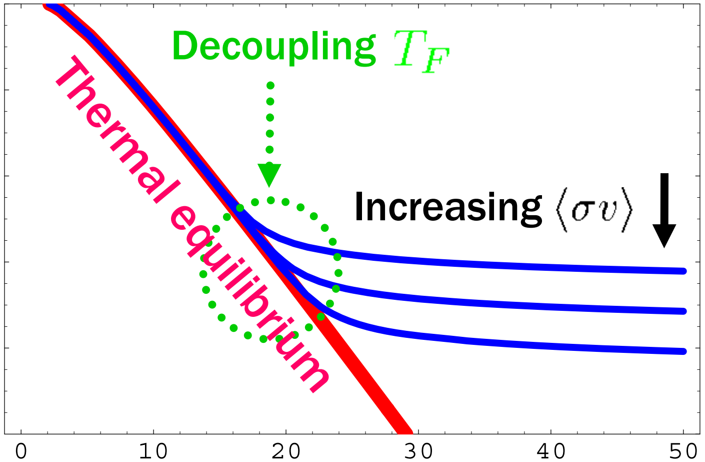
$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma_{\rm eff} v\rangle (n_\chi^2 - n_{\chi,{\rm eq}}^2)$$

$n_{\chi,({\rm eq})}$ : (Equilibrium) number density   $H$ : Hubble parameter

$\langle \sigma v\rangle$ :Thermally averaged annihilation cross section times velocity

$Y_\chi = n_\chi / s$

Co-moving number density




- At high temperatures:

$$\Gamma = n_\chi \langle \sigma v\rangle > H = R/\dot{R}$$

Thermal equilibrium was maintained

- When $\Gamma < H$ , the number density is fixed:

Decoupling, freeze-out

- $\chi$ decoupled in the RD epoch: $H = \dfrac{\pi T^2}{M_{\rm Pl}}\sqrt{\dfrac{90}{g_*}}$

# Thermal abundances of hot and cold relics

- Hot relics (decouple for $x_F < 3$ ):

$$Y_{\chi,\mathrm{eq}} \equiv \frac{n_{\chi,\mathrm{eq}}}{s}$$ almost constant

Final abundance is insensitive to the freeze out temperature:

$$Y_{\chi,\infty} = Y_{\chi,\mathrm{eq}}(x_F) = \frac{45}{2\pi^4} \frac{g_\chi}{g_{*s}(x_F)}$$

- Cold relics (decouple for $x_F > 3$ ):

$$\langle \sigma v \rangle = a + 6b/x + \mathcal{O}(1/x^2),$$

$$n_{\chi,\mathrm{eq}} = g_\chi \, (m_\chi T/2\pi)^{3/2} \mathrm{e}^{-m_\chi/T}$$

$$\Omega_\chi h^2 = 2.7 \times 10^8 \, Y_\chi \left( \frac{m_\chi}{1 \ \mathrm{GeV}} \right)$$

$$\boxed{\Omega_{\chi,\mathrm{standard}} h^2 \simeq 0.1 \times \left( \frac{a + 3b/x_F}{10^{-9} \ \mathrm{GeV}^{-2}} \right)^{-1} \left( \frac{x_F}{22} \right) \left( \frac{g_*}{90} \right)^{-1/2} \sim \Omega_{\mathrm{DM}} h^2}$$

# 3. Thermal abundance of semi-relativistic relics

- Precise evaluation of the abundance of particles that freeze out when they are semi-relativistic $(x_F \sim 3)$ is complicated

Goal: simple analytic treatment that describes the transition from non-relativistic to relativistic relics

- Assume the Maxwell-Boltzmann distribution:

$$Y_{\chi,\mathrm{eq}} \equiv \frac{n_{\chi,\mathrm{eq}}}{s} = 0.115 \frac{g_\chi}{g_{*s}} x^2 K_2(x)$$

($K_n(x)$: modified Bessel function)

Thermal average of cross section $\sigma$ :

$$\langle \sigma v \rangle = \frac{1}{8 m_\chi^4 T K_2^2(m_\chi/T)} \int_{4m_\chi^2}^{\infty} \mathrm{d}s \; \sigma (s - 4m_\chi^2) \sqrt{s} \; K_1(\sqrt{s}/T)$$

# Ansatz for approximate cross sections

- Consider neutrinos as stable relic:
- Annihilation cross section:

$$\sigma v^{\text{Dirac }\nu} = \frac{G^2 s}{16\pi}$$

$$\sigma v^{\text{Majorana }\nu} = \frac{G^2 s v^2}{16\pi}$$

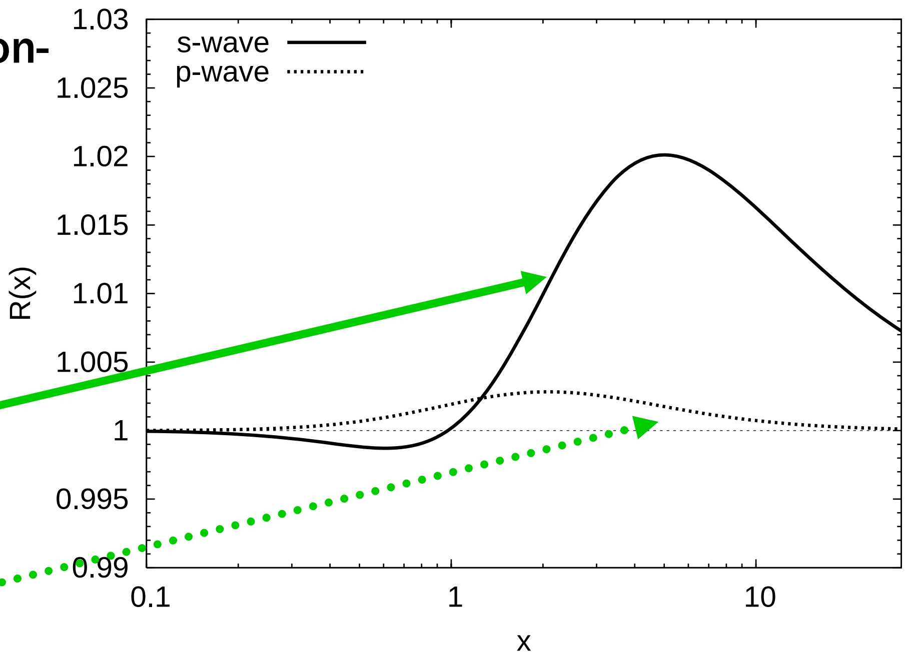
$G$ : Effective dimension-six coupling

- Ansatz for the thermally-averaged annihilation cross section:

$$\langle \sigma v \rangle_{\text{app}}^{\text{Dirac}} = \frac{G^2 m_\chi^2}{16\pi}\left(\frac{12}{x^2} + \frac{5+4x}{1+x}\right)$$

$$\langle \sigma v \rangle_{\text{app}}^{\text{Majorana}} = \frac{G^2 m_\chi^2}{16\pi}\left(\frac{12}{x^2} + \frac{3+6x}{(1+x)^2}\right)$$

- $\langle \sigma v \rangle_{\text{app}} / \langle \sigma v \rangle_{\text{exact MB}}$ :




**The approx. cross sections reproduce the exact results with accuracy of a few %**

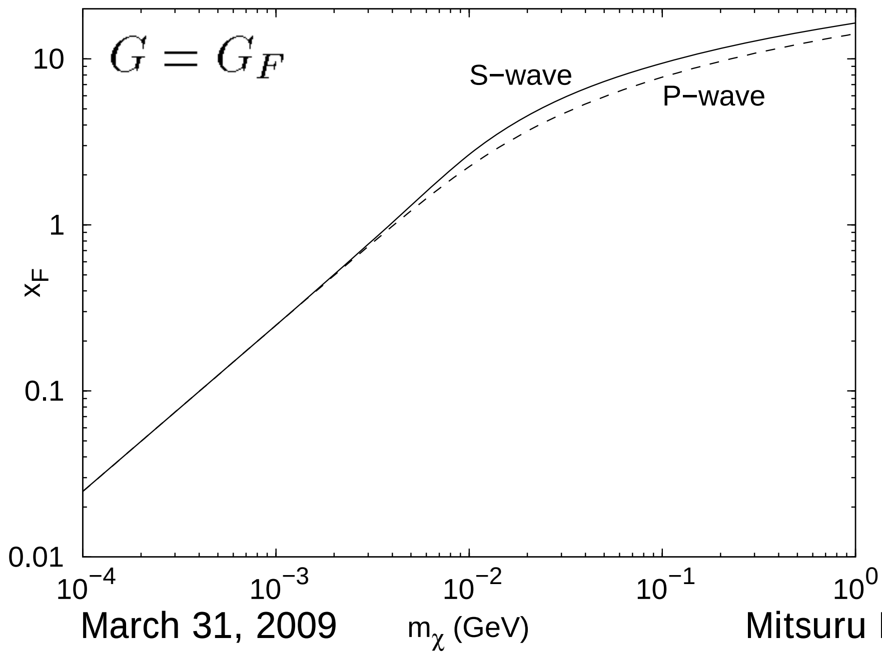
# Approximate abundance of semi-relativistic relics

- Define the freeze–out temperature by

$$\Gamma(x_F) = H(x_F)$$

$$\Gamma(x_F) = n_{\chi,\mathrm{eq}}(x_F)\langle\sigma v\rangle(x_F)$$

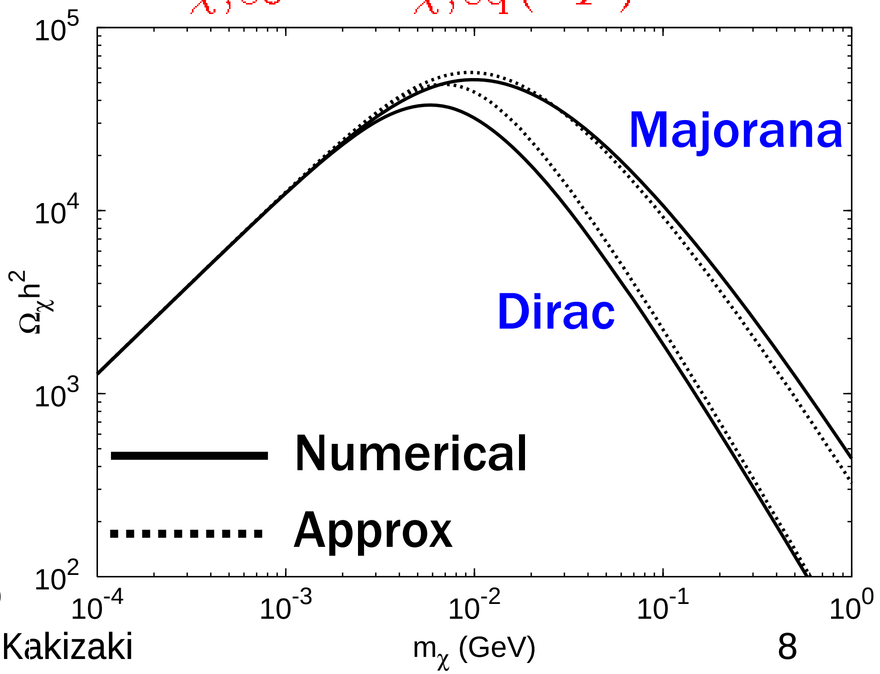
(different from the standard $x_F$ )




- Assume the relic abundance does not change after decoupling

Final abundance:

$$Y_{\chi,\infty} = Y_{\chi,\mathrm{eq}}(x_F)$$



# 4.1. Semi-relativistic dark matter?

- Observed dark matter abundance: $\Omega_{\rm DM} h^2 \simeq 0.1$

  Final density of a semi-relativistic particle: $Y_{\chi,\rm eq}(x \simeq 3) \sim 10^{-2}$

  ⇒ $m_\chi \sim 100\ \mathrm{eV}, \quad T_F \sim \mathcal{O}(10)\ \mathrm{eV}$

  Semi-relativistic dark matter affects BBN

- Such light particle decouples when semi-relativistic

  ⇒ Effective dimension-six coupling is too large:

  $$G \simeq 10^3\ \mathrm{GeV}^{-2}$$

# 4.2. Entropy production by decaying particles

- Suppose a decaying particle dominates the energy of Universe
  - Out-of-eq. decay ⇨ Large entropy production ⇨ Unwanted relics diluted
  - Ratio of the final to initial entropy: $$\frac{S_f}{S_i} = g_*^{1/4} \frac{m_\chi Y_{\chi,i} \tau_\chi^{1/2}}{M_{\rm Pl}^{1/2}} \propto \Omega_\chi h^2$$
    [Steinhardt, Turner (1983)]
- In the case of the late decay of non-relativistic particles
  - $\chi$ energy density can dominate at low temperate: $T \ll {\rm e}^{-x_F} T_F$
  - Very large mass, very long lifetime needed for large entropy production
- In the case of the late decay of semi-relativistic particles
  - The abundance at decoupling is large

⇨ **Significant entropy can be produced even if the mass is small**

# Example: sterile neutrino

- Consider a sterile neutrino mixed with an active neutrino (mixing angle: $\theta$)
- Decay rate of the sterile neutrino:

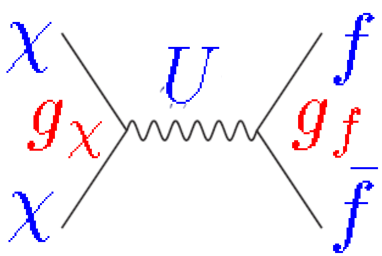
$$\Gamma_\chi = \frac{G_F^2 m_\chi^5}{192\pi^3}\sin^2\theta, \qquad \frac{G_F m_\chi^3}{16\pi}\sin^2\theta$$

(for small $m_\chi$ ) (for large $m_\chi$ )

should be large enough not to spoil BBN

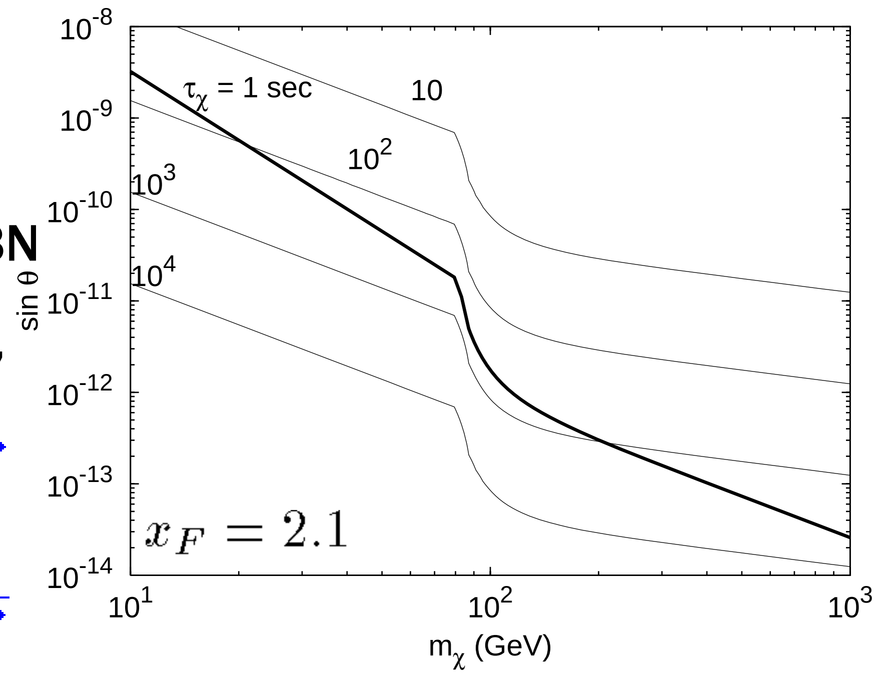
- By introducing a new heavy particle, U, large pair annihilation can be induced:

$$\sigma v = \frac{sv^2}{12\pi}\frac{g_\chi^2 g_f^2}{M_U^4}$$




$\Rightarrow$ $x_F \sim 3$ possible

- Entropy production $S_f/S_i$ by the decay of semi-relativistic sterile neutrinos



# 5. Summary

- We find an useful approximate analytic formula for the abundance of semi-relativistic relics

- Semi-relativistic relics are useful for producing a large amount of entropy

# Backup slides

- Observations of
  - cosmic microwave background
  - structure of the universe
  - etc.

⇒ **Non-baryonic dark matter: $\Omega_{\rm DM}h^2 = 0.1143 \pm 0.0034$**

Physics beyond the standard model (SM) of particle physics necessary

- Weakly interacting massive particles (WIMPs) $\chi$ are good candidates for dark matter (DM)

**The predicted thermal relic abundance naturally explains the observed dark matter abundance: $\Omega_{\chi,\rm standard}h^2 \sim 0.1$**

- Neutralino (LSP); 1st KK mode of the B boson (LKP); etc.

# 2. Standard calculation of the WIMP relic abundance

[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- Conventional assumptions for WIMPs as DM particle:
  - $\chi = \bar{\chi}$, single production of $\chi$ is forbidden
- WIMP abundance $n_\chi$ is determined by the Boltzmann eq.:

$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\mathrm{eq}}^2)$$

$H = \dot{R}/R$: Hubble expansion parameter

$\langle \sigma v \rangle$: thermal average of the annihilation cross section $\sigma(\chi\chi \to \mathrm{SM\ particles})$ times relative velocity $v$

$n_{\chi,\mathrm{eq}}$: equilibrium number density

- Introduce $Y_{\chi(,\mathrm{eq})} = \dfrac{n_{\chi(,\mathrm{eq})}}{s}, x = \dfrac{m_\chi}{T}$

$$\Rightarrow \frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\langle \sigma v \rangle s}{Hx}(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2)$$