Conformal Neutrinos

with M. Quirós, 0901.0006

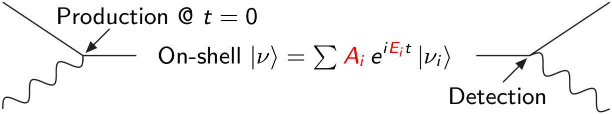
Gero von Gersdorff

March 15, 2009

Outline

Neutrino Masses - Experimental Results

- Neutrino masses measured via oscillations



- Amplitude gives Mixing angles . . . Frequency gives Δm^2
  - $m_2^2 - m_1^2 = 7.6 \; 10^{-5}$ eV^2 [KAMLAND]
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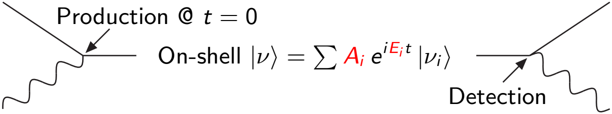

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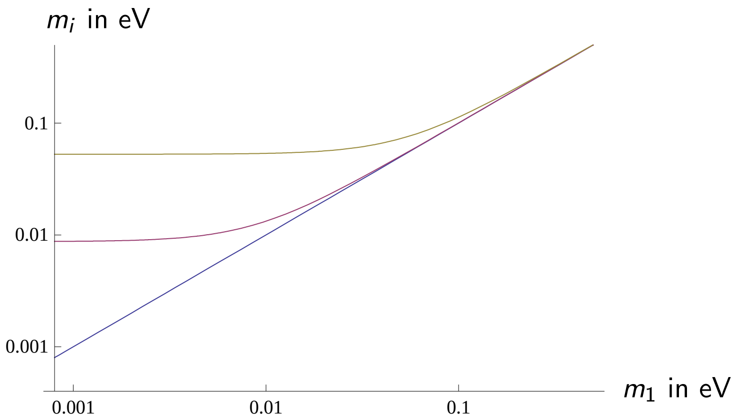
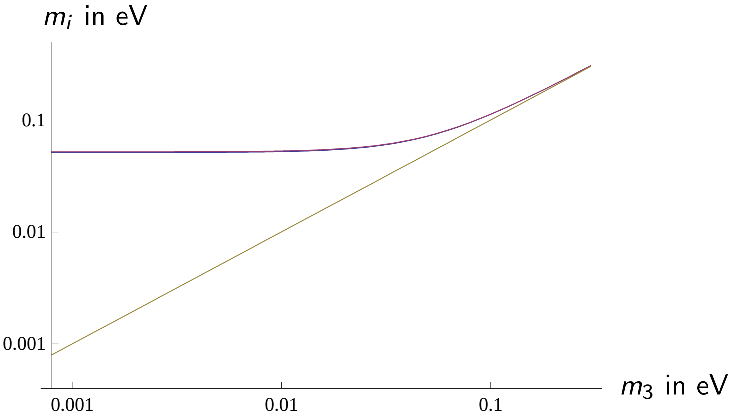
- Still leaves one free continues parameter and one sign
  - $m_3 > m_2 > m_1 \Rightarrow$ regular "hierarchy"
  - $m_2 > m_1 > m_3 \Rightarrow$ inverted "hierarchy"

Regular "Hierarchy": $m_3 > m_2 > m_1$



Inverted "Hierarchy": $m_2 > m_1 > m_3$



Neutrino Masses - Theory

- Fermion masses are multiplicatively renormalized
- Still, scale $m_\nu \lesssim 1$ eV clearly requires some explanation
- Creating m_ν with Yukawa couplings

$$\mathcal{L}^Y = h_\nu \, H \, \ell_L \bar{n}_R \qquad \Longrightarrow \qquad h_\nu = \frac{m_\nu}{v} = 10^{-12}$$

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- OR add large mass term for RH neutrino

$$m_R^M \, (n_R)^2 \xrightarrow{\text{integr. out } n_R} \frac{h_\nu^2}{m_R^M} (H\ell_L)^2 \xrightarrow{EWSB} \frac{v^2 h_\nu^2}{m_R^M} (n_L)^2$$

- See Saw mechanism: $h_\nu = \mathcal{O}(1)$ for $m_R^M \approx 10^{14}$ GeV.

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- Allow fo large anomalous dimension $1/2 \lesssim \gamma < 1$
- Impose lepton number conservation (can be relaxed)
- Lagrangian

$$\mathcal{L} = \mathcal{L}_{CFT}(n_R, \dots) + \mathcal{L}_{SM} + \Lambda^{-\gamma} H \bar{n}_R \ell_L$$

Breaking of Scale Invariance

- EWSB induces a Dirac mass term

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Conclusion

The scale of conformal breaking defines the neutrino mass scale. Parametrically suppressed wrt electroweak scale.

The RG Flow

- Use rescaled fields to display running Yukawa/mass

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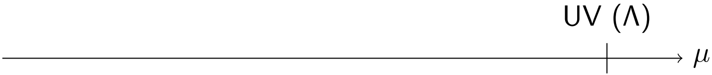

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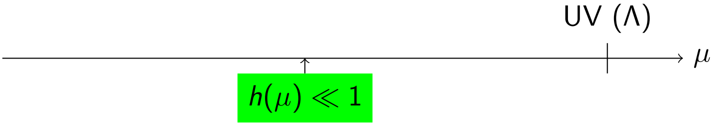

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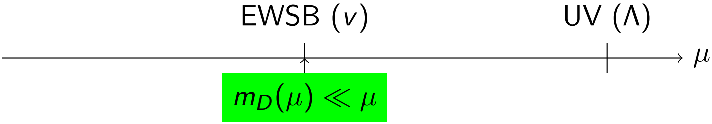

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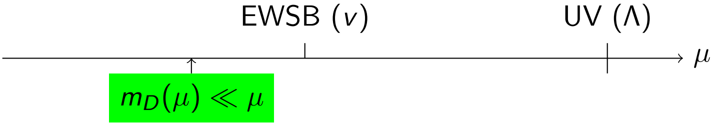

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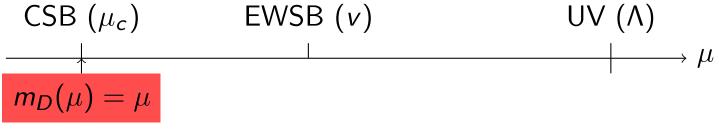

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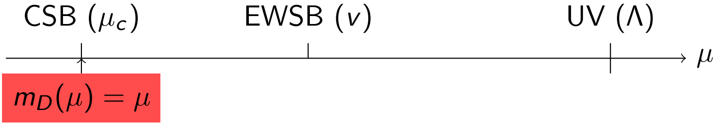


- Yields previous result

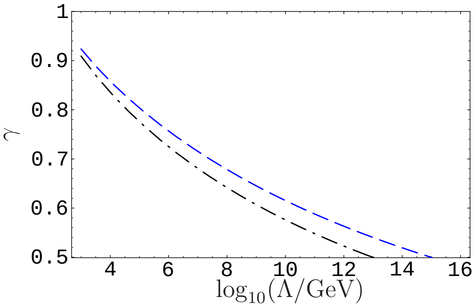
$$\mu_c = m^D = v\left(\frac{v}{\Lambda}\right)^{\frac{\gamma}{1-\gamma}}$$

UV-scale vs anomalous dimension

- For fixed neutrino mass scale, Λ and γ are related

$$m^D = v\left(\frac{v}{\Lambda}\right)^{\frac{\gamma}{1-\gamma}}$$

$$\begin{cases} 1 \text{ for } \gamma = 0.5 \\ 9 \text{ for } \gamma = 0.9 \end{cases}$$



Here: $m^D = 0.05 \ldots 1$ eV

Three Generations

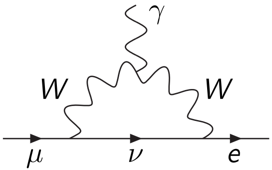
- Degenerate masses natural if anomalous dimensions equal
- Hierarchies can be achieved with different anomalous dimensions
- assume $\gamma_3 = \gamma_2 < \gamma_1$. CSB occurs at $\mu_c = m_3 = m_2$ but

$$m_1(\mu_c) = \mu_c \left(\frac{\mu_c}{\Lambda}\right)^{\gamma_1 - \gamma}$$

- Can be suppressed w.r.t. $m_{2,3}$
- Just two RH neutrinos with same γ give $m_3 = m_2$, $m_1 = 0$.
- Also differences in Yukawas at high scales, SM corrections, etc....

FCNC mediated by SM neutrinos

- Consider $\mu \to e\gamma$ in SM with Dirac Neutrinos
- Vanishes at tree level (GIM)
- Contribution at 1 loop



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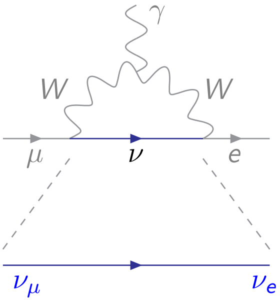

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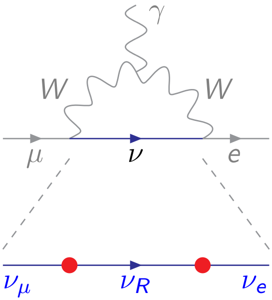

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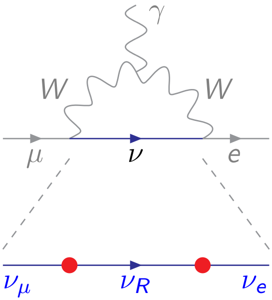
- Conformal Neutrinos: Dirac masses grow with energy!



FCNC mediated by conformal neutrinos

- Result for conformal neutrinos:

$$B(\mu \to e\gamma) = \frac{3}{32}\frac{\alpha}{\pi}\left|\frac{\pi\gamma}{\sin(\pi\gamma)}\sum_i U_{ei}U^*_{\mu i}\left(\frac{m_i}{M_W}\right)^{2-2\gamma}\right|^2$$

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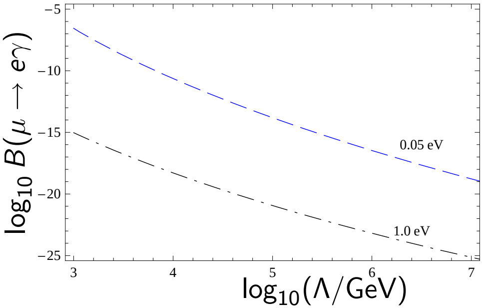
- Parametric enhancement for $\gamma > 0$ w.r.t SM.
- Can be attributed to running of Dirac masses.

Comparing to Experiment



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- (using best fits for U)
- Regular hierarchy

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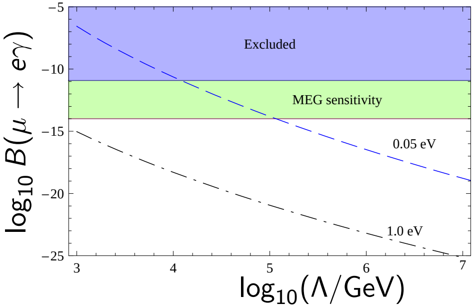

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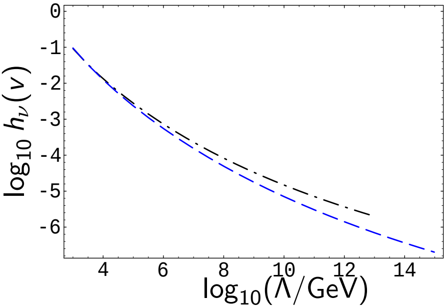
- (using best fits for U)
- Regular hierarchy
- Bound (for 0.05 eV ν) $\Rightarrow \Lambda \gtrsim 10$ TeV ($\gamma \lesssim 0.86$)
- Future sensitivity $\Rightarrow \Lambda \sim 100$ TeV ($\gamma \sim 0.81$)

Coupling to Higgs

- Yukawa becomes stronger at larger scales
- Can we detect the running Yukawa?
- For $\Lambda = 10$ TeV Neutrino Yukawas are comparable to c, τ!



Yukawa coupling at the weak scale

The width of the Higgs

- Partial width of Higgs to $\nu\bar{\nu}$

$$\Gamma(H \to \nu\bar{\nu}) = h^2(m_H)\,\frac{m_H}{16\pi}\,\frac{2}{\Gamma(1-\gamma)\Gamma(3+\gamma)}$$

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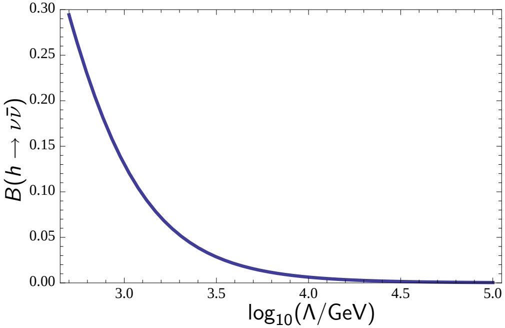
- Also some mild suppression for $\gamma = 1$

Higgs Branching in Neutrinos



- $m^D = 0.1$ eV.
- $m_H = 130$ GeV.

Lepton Number Violation

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- EWSB again creates LH Majorana mass, does not run, dominates at low energies...
- Like see saw, $m_L^M \sim c\, v^2/\Lambda$, but no heavy states!

An Inverse See-Saw

- Parametric dependence of mass matrix (1 flavour)

$$\begin{pmatrix} m_L^M & m^D \\ m^D & m_R^M \end{pmatrix} = v \begin{pmatrix} \frac{v}{\Lambda} & \left(\frac{v}{\Lambda}\right)^{\frac{\gamma}{1-\gamma}} \\ \left(\frac{v}{\Lambda}\right)^{\frac{\gamma}{1-\gamma}} & \left(\frac{v}{\Lambda}\right)^{\frac{3\gamma-1}{1-\gamma}} \end{pmatrix}$$

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- Hierarchy $m_L^M \gg m^D \gg m_R^M$
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- Leads to inverse See-saw mechanism (very light sterile states...)

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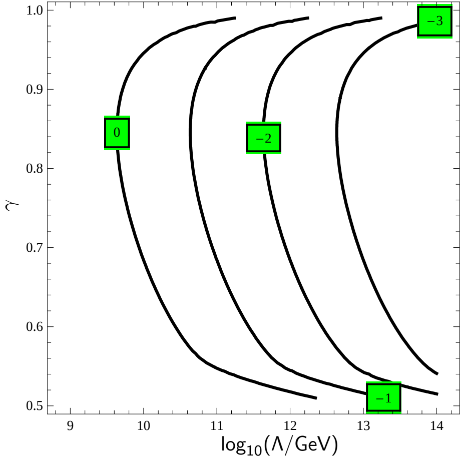

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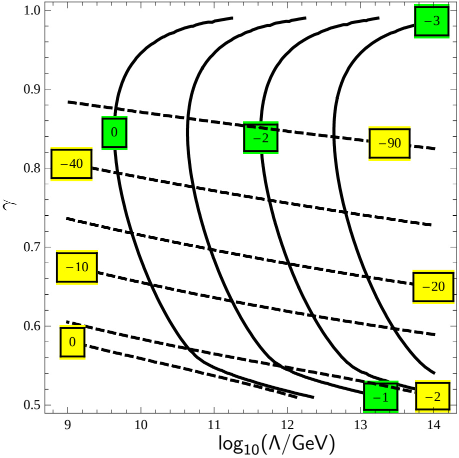

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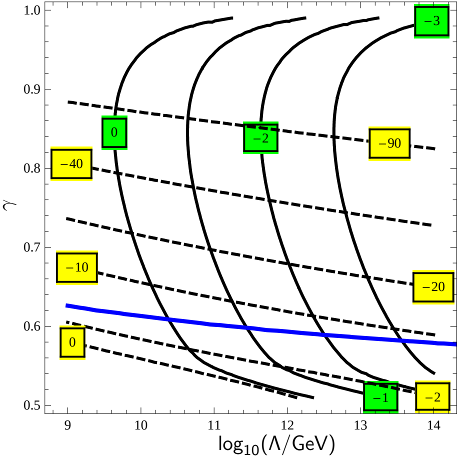

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Mixing = 1%

- Conformal Neutrinos are an interesting method to obtain small neutrino masses
- RH neutrinos are conformal operators with large anomalous dimension
- Conformal Symmetry breaking naturally triggered by EWSB
- $\mu \to e\gamma$ and $H \to \nu\bar{\nu}$ strongly enhanced w.r.t. SM
- Conformal Majorana neutrinos also possible, similar to Seesaw, but no heavy states
- Lots of things to be done: cosmology, supersymmetry...