Calculation of the dark matter relic density in supersymmetric models including SUSY-QCD corrections

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Outline

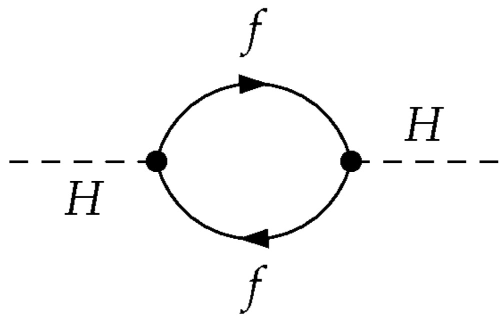
1. Introduction and Motivation
2. First example: “A-Funnel” region
3. Neutralino Annihilation into “Heavy” Quarks
4. Conclusion and Outlook

Motivations for Physics beyond the Standard Model

Standard Model rather "low-energy" limit of a more fundamental theory

☑ "Hierarchy problem"

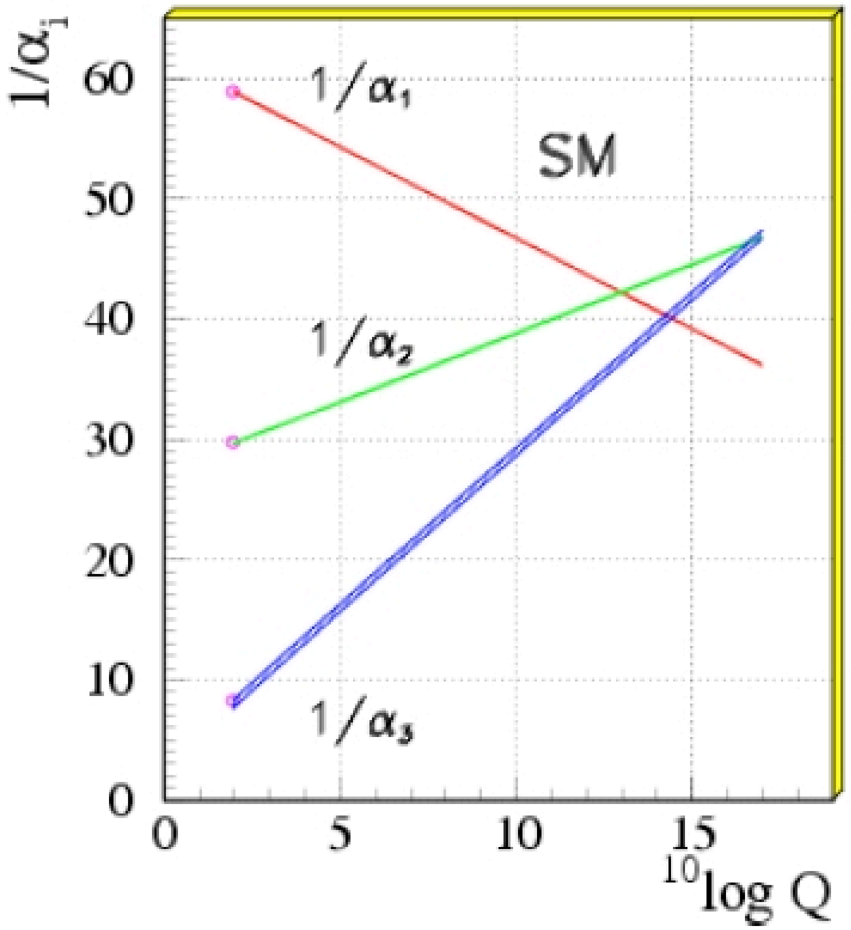
Higgs mass receives quadratically divergent fermionic loop corrections [Witten 1981, Kaul 1982]



$$\Delta m_H^2 = \frac{|\lambda_f|^2}{16\pi^2}\left[-2\ \Lambda_{\rm UV}^2 + 6m_f^2 \log\frac{\Lambda_{\rm UV}}{m_f} + \ldots\right]$$

☑ Gauge coupling unification

Running coupling constants should meet in single point if corresponding interactions unify



☑ Gravity

Cannot be ignored at energies near the Planck scale

☑ "Aesthetic" reasons

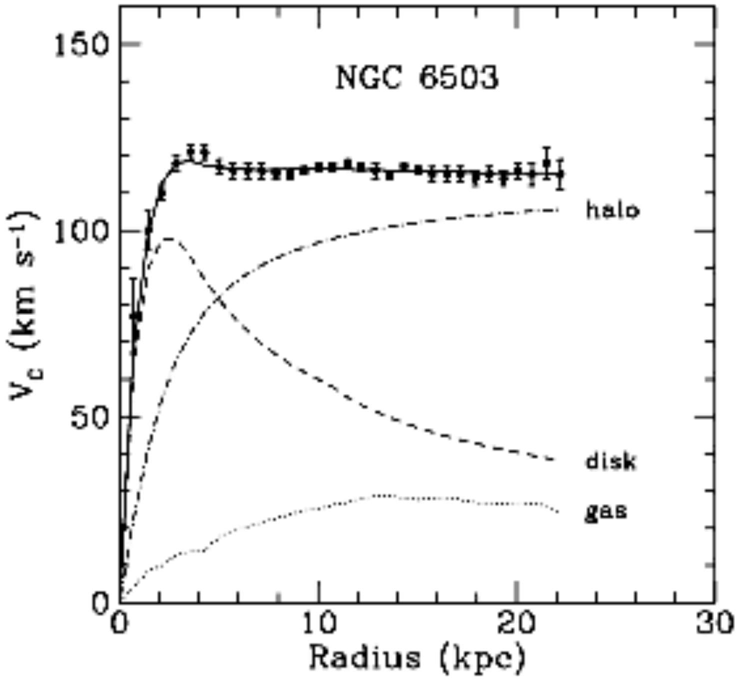
Number of parameters in Standard Model, ...

Observational Evidence for Dark Matter

First observational hints

Velocity dispersion and rotation curves

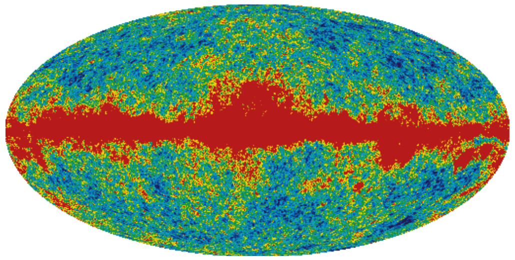
[Zwicky 1933, Rubin *et al.* 1970]



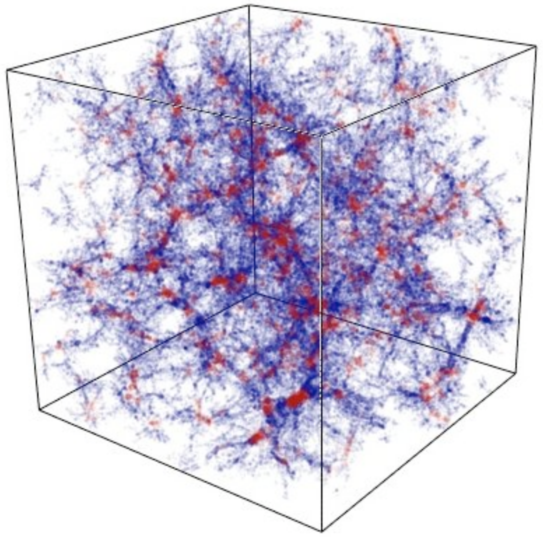
CMB anisotropies

Cosmological parameters from WMAP mission

[Hinshaw *et al.* (WMAP) 2008]



$$\Omega_{\rm tot} = 1.005 \pm 0.034$$
$$\Omega_{\rm CDM} = 0.223 \pm 0.013$$



Structure formation

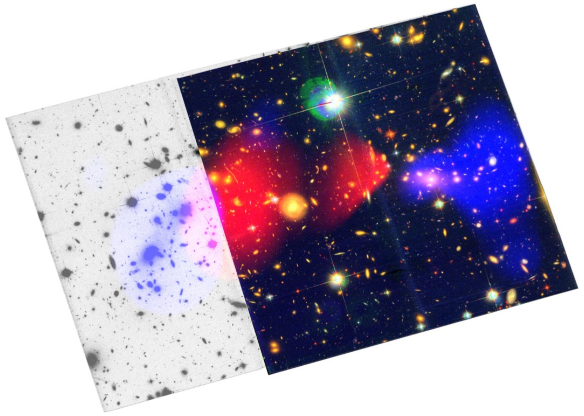
Cold dark matter needed to explain large structures

[Blumenthal *et al.* 1984]

First direct observation

Observation of "Bullet Cluster" proofs existence of dark matter

[Clowe *et al.* 2006]



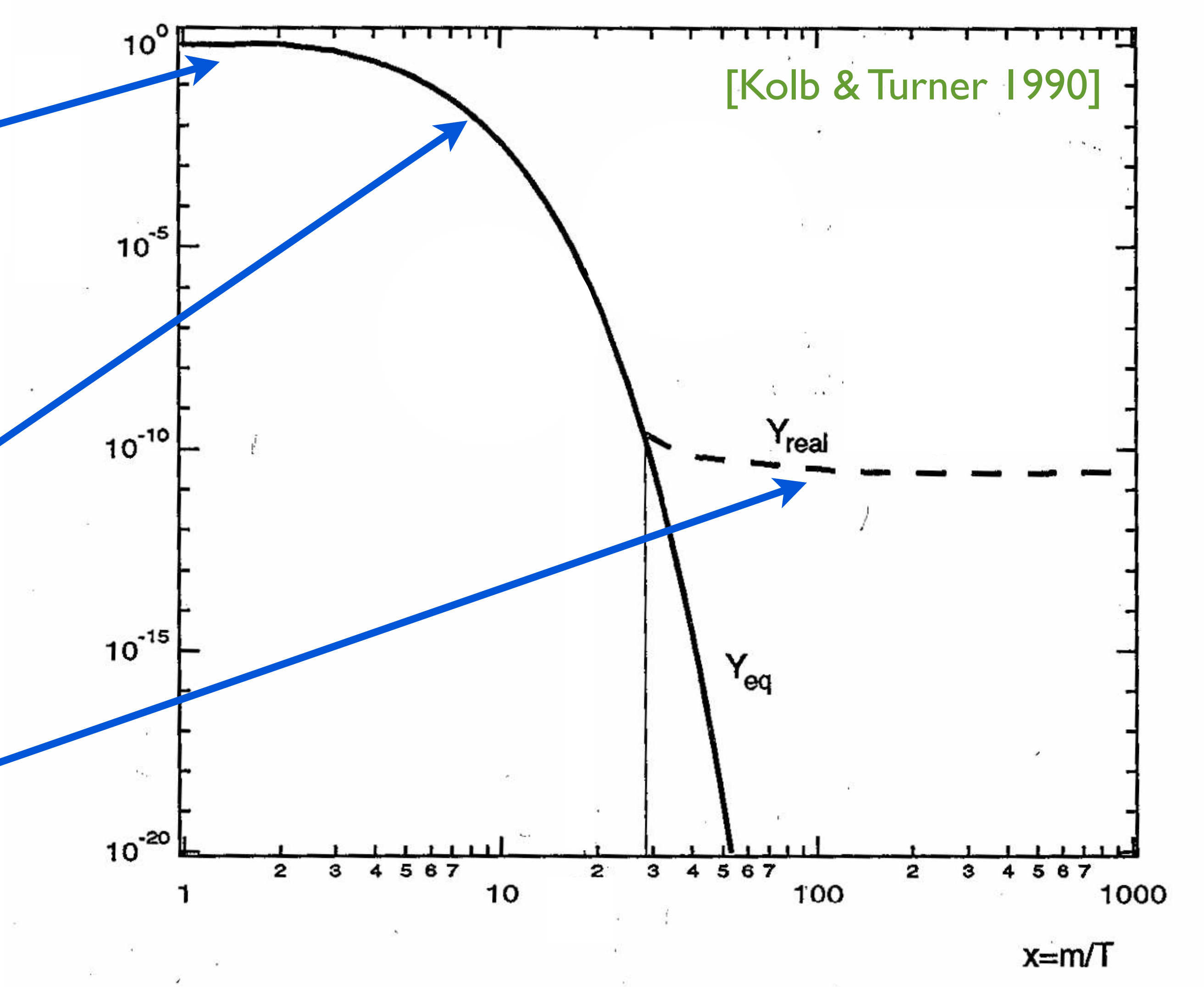
Today's Picture

Dark matter consists of stable non-relativistic "WIMPs" (weakly interacting massive particles)

In early Universe, equilibrium is maintained through production and annihilation

The equilibrium density drops exponentially with decreasing temperature

When the annihilation rate falls under the expansion rate, the particle "freezes out"



Direct / indirect detection experiments put limits on mass and scattering cross sections

→ Edelweiss, DAMA, CDMS, CRESST, MIMAC, ...

→ EGRET, PAMELA, ATIC, BESS, ANTARES, IceCube, GLAST, ...

New physics, e.g. Supersymmetry, provides promising candidates for "WIMPs"...

The Minimal Supersymmetric Standard Model (MSSM)

Supersymmetry relates bosons and fermions

$$\mathcal{Q}\,|b\rangle = |f\rangle \qquad \mathcal{Q}\,|f\rangle = |b\rangle$$

Minimal supersymmetric theory including the Standard Model particles

SM Particles		Spin		Spin		Superpartners
Quarks	$(u_L \;\; d_L)$	$1/2$	Q	0	$(\tilde{u}_L \;\; \tilde{d}_L)$	Squarks
	$u_R^\dagger$	$1/2$	$\bar{u}$	0	$\tilde{u}_R^*$	
	$d_R^\dagger$	$1/2$	$\bar{d}$	0	$\tilde{d}_R^*$	
Leptons	$(\nu \;\; e_L)$	$1/2$	L	0	$(\tilde{\nu} \;\; \tilde{e}_L)$	Sleptons
	$e_R^\dagger$	$1/2$	$\bar{e}$	0	$\tilde{e}_R^*$	
Higgs	$(H_u^+ \;\; H_u^0)$	0	H_u	$1/2$	$\tilde{\chi}^0_{1,2,3,4}$	Neutralinos
	$(H_d^0 \;\; H_d^-)$	0	H_d	$1/2$	$\tilde{\chi}^\pm_{1,2}$	Charginos
W bosons	$W^0, W^\pm$	1				
B boson	B^0	1				
Gluon	g	1		$1/2$	$\tilde{g}$	Gluino
Graviton	G	2		$3/2$	$\tilde{G}$	Gravitino

Particle mixing after electroweak symmetry breaking gives rise to neutralinos and charginos

Supersymmetric Dark Matter

Introduce R-parity in the MSSM in order to avoid possible proton decay

$$P_R = (-1)^{3(B-L)+2s} = \begin{cases} +1 & \text{for Standard Model particles} \\ -1 & \text{for their superpartners} \end{cases}$$

R-parity conservation only allows for vertices involving an even number of superpartners

→ supersymmetric particles always produced in pairs

→ lightest supersymmetric particle (LSP) cannot decay and must be stable

Dark matter candidate ("WIMP") must be massive, stable, neutral in electric charge and colour

→ Two candidates within the MSSM: Lightest neutralino (mSUGRA)
Gravitino (GMSB)

Here: Focus on minimal supergravity (mSUGRA)

→ SUSY breaking mediated between hidden and observable sectors by gravity

→ five universal parameters at the grand unification scale

$$m_0, \ m_{1/2}, \ A_0, \ \tan\beta, \ \mathrm{sgn}(\mu)$$

Constraining the Parameter Space

☑ **Direct mass limits from LEP and Tevatron** [Amsler *et al.* (PDG) 2008]

$$m_{\tilde{\chi}_1^0} \gtrsim 36 \text{ GeV} \qquad m_{\tilde{\chi}_1^\pm} \gtrsim 45 \text{ GeV}$$

☑ **Precision and low-energy observables**

Inclusive branching ratio $b \to s\gamma$

$$\text{BR}(b \to s\gamma) = (3.52 \pm 0.25) \cdot 10^{-4}$$

[Barbiero *et al.* (HFAG) 2007]

New physics contributions to the electroweak ρ-parameter and the anomalous magnetic moment of the muon [Amsler *et al.* (PDG) 2008]

$$\Delta\rho = (1.02 \pm 0.86) \cdot 10^{-3} \qquad \Delta a_\mu = (29.2 \pm 8.6) \cdot 10^{-10}$$

☑ **Cosmology**

Scenario should include a candidate for "WIMP" (i.e. no charged LSP)

Relic density of cold dark matter required to agree with WMAP+SN+BAO data

$$0.1097 < \Omega_{\text{CDM}} h^2 < 0.1165$$

[Hinshaw *et al.* (WMAP) 2008]

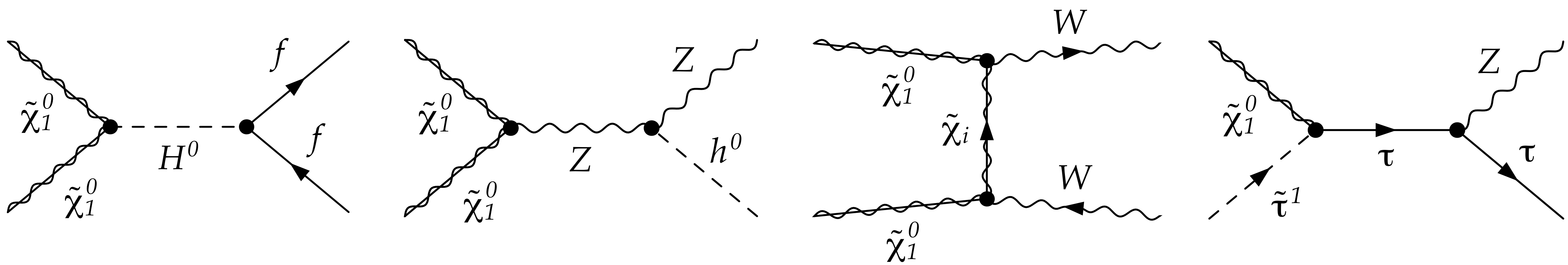
Relic density calculation gives additional information w.r.t. collider and precision data

Relic Density Calculation

Number density of relic particle governed by the Boltzmann equation

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3Hn - \langle \sigma_{\mathrm{ann}} v \rangle \left(n^2 - n_{\mathrm{eq}}^2 \right) \longrightarrow \Omega_{\mathrm{CDM}} h^2 = \frac{m_{\tilde{\chi}}\, n_0}{\rho_c} \propto \frac{1}{\langle \sigma_{\mathrm{ann}} v \rangle}$$

Cross section σ_{ann} includes all annihilation and coannihilation processes



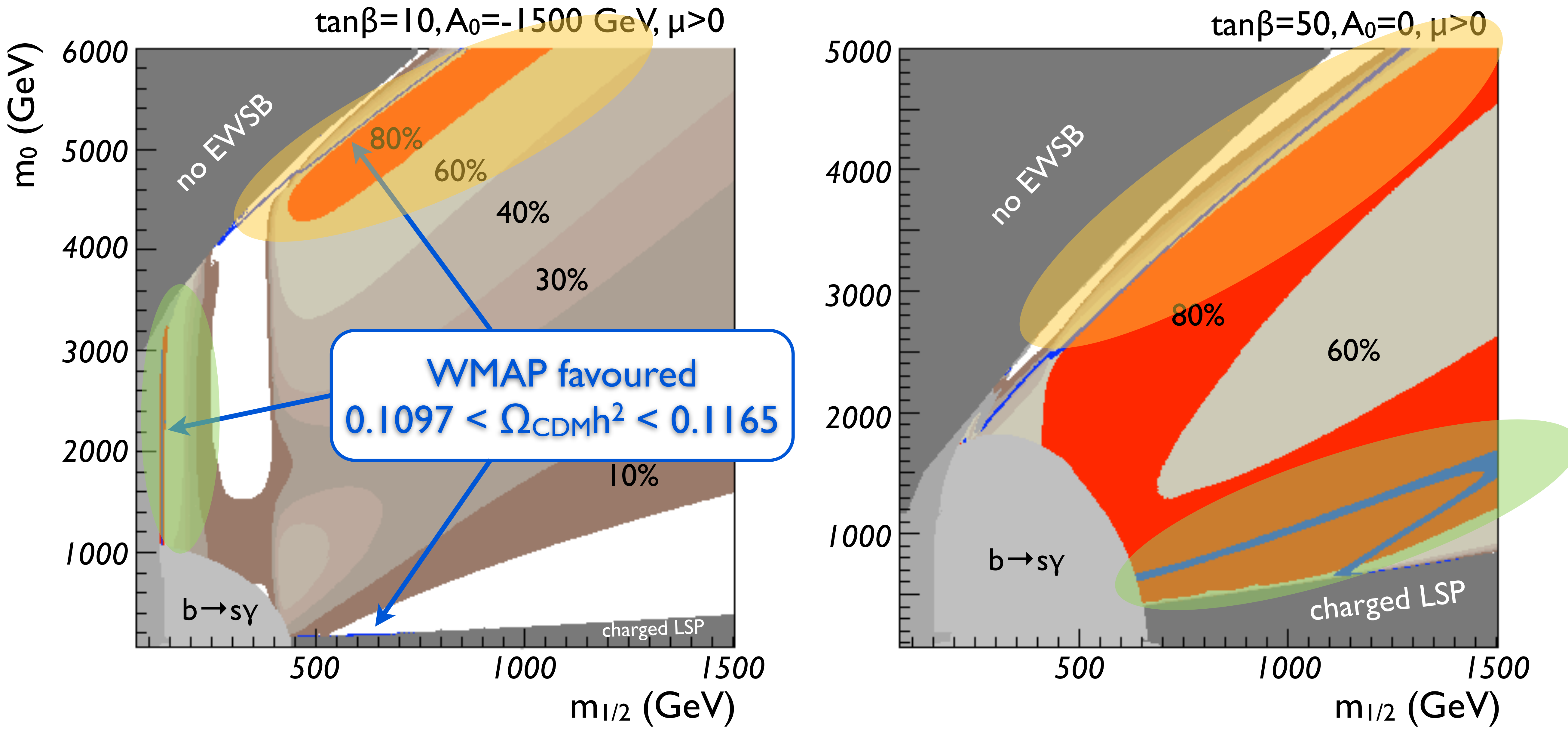
Thermal average involves velocity distribution of the non-relativistic relic particle

$$\langle \sigma_{\mathrm{ann}} v \rangle = \int \mathrm{d}v \tilde{f}(v) \sigma_{\mathrm{ann}} v = \int \mathrm{d}s f(s) \sigma_{\mathrm{ann}}(s)$$

Public codes perform a calculation of the relic density for given scenario

- ➞ DarkSUSY (only neutralino) [Gondolo *et al.* 2004]
- ➞ micrOMEGAs (all kinds of LSP) [Bélanger *et al.* 2003]

Neutralino Annihilation into Quarks



Interesting regions:

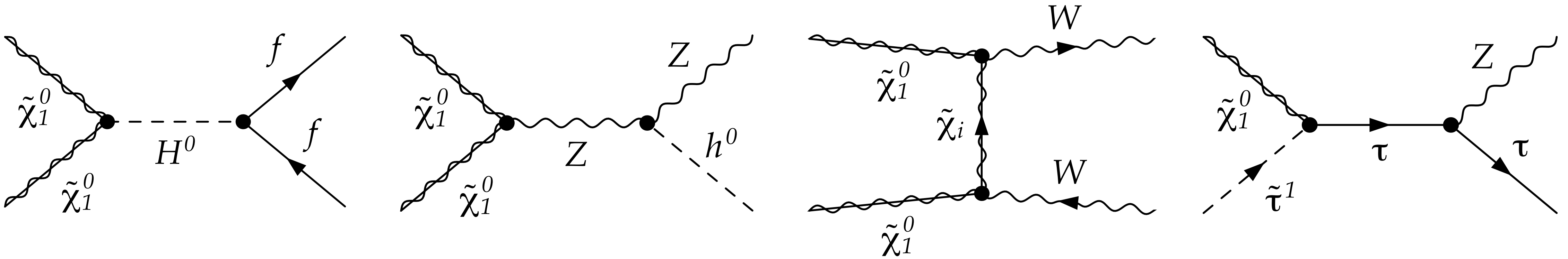
- Focus point region ($t\bar{t}$ dominated)
- Higgs resonances ($b\bar{b}$ dominated)

Light quark final states do not play a significant role in mSUGRA scenarios

Motivation for SUSY-QCD Corrections

(Co)annihilation processes only implemented in public codes at tree-level

→ corrections included only for some very sensitive quantities



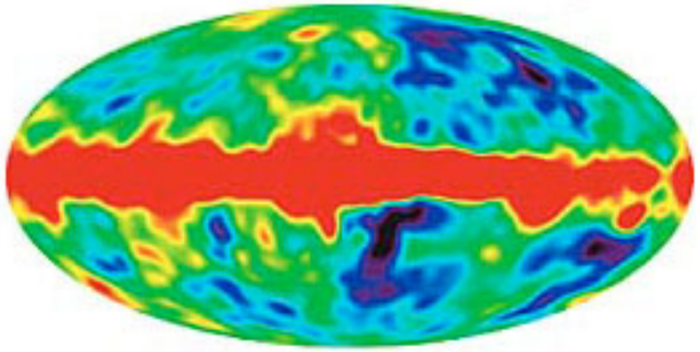
Higher order corrections can have important impact on cross sections

→ QCD corrections significant due to strong coupling constant

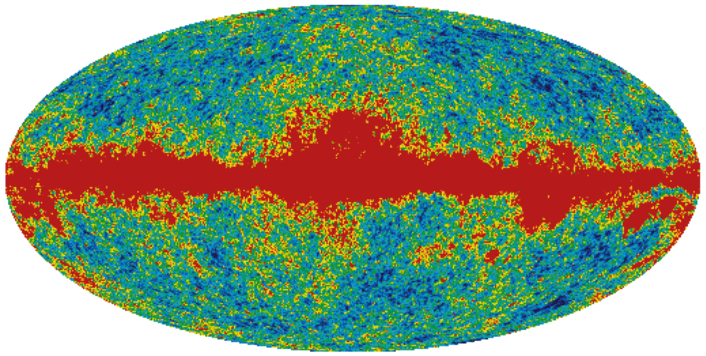
→ quark final states have sizeable contributions in relic density calculation

Planck satellite will deliver new cosmological data in near future

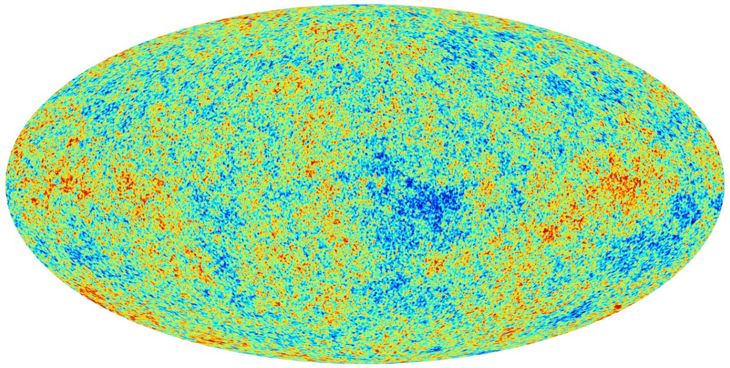
→ higher precision in theoretical predictions needed to match experimental improvements



COBE
1989

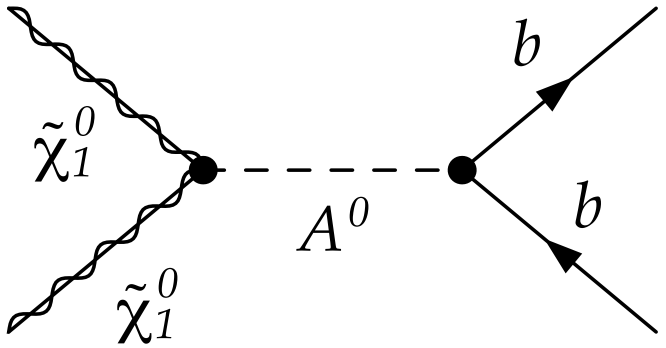


WMAP
2002



Planck
2009 ?

First Example: “A-Funnel” Region

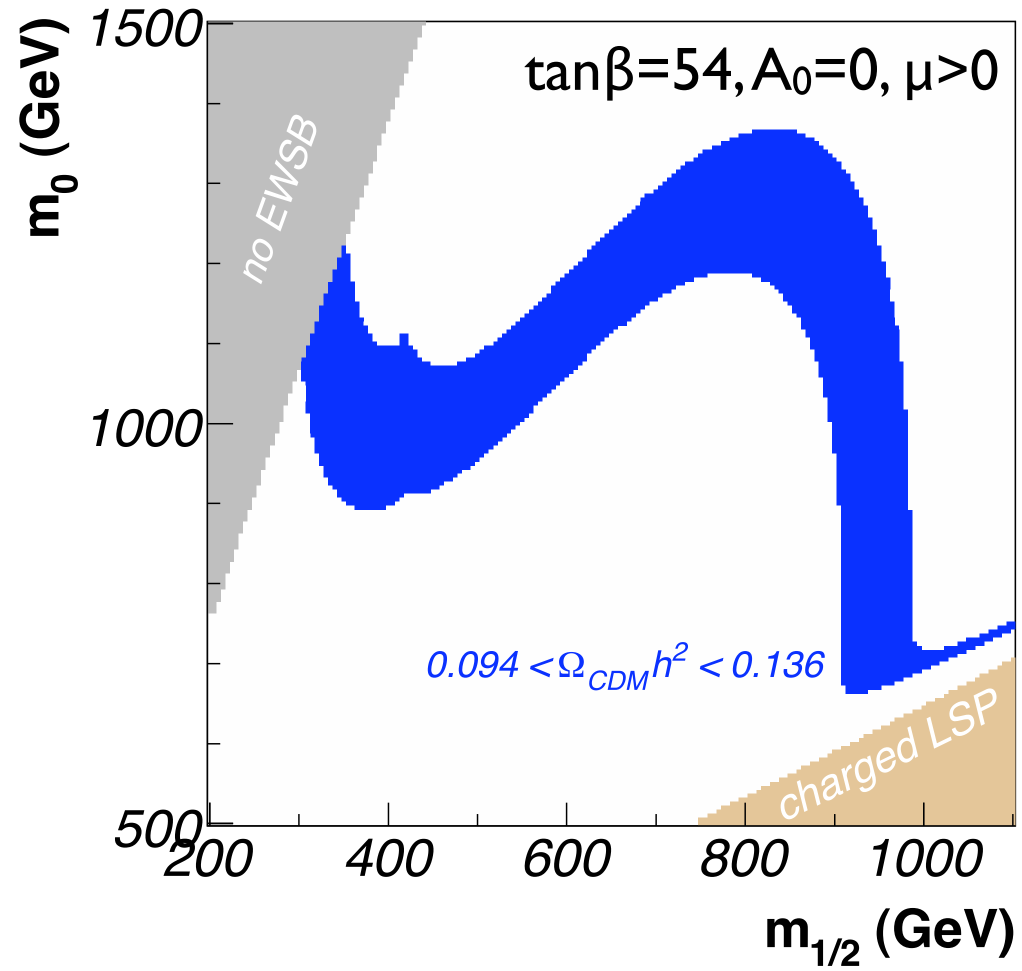


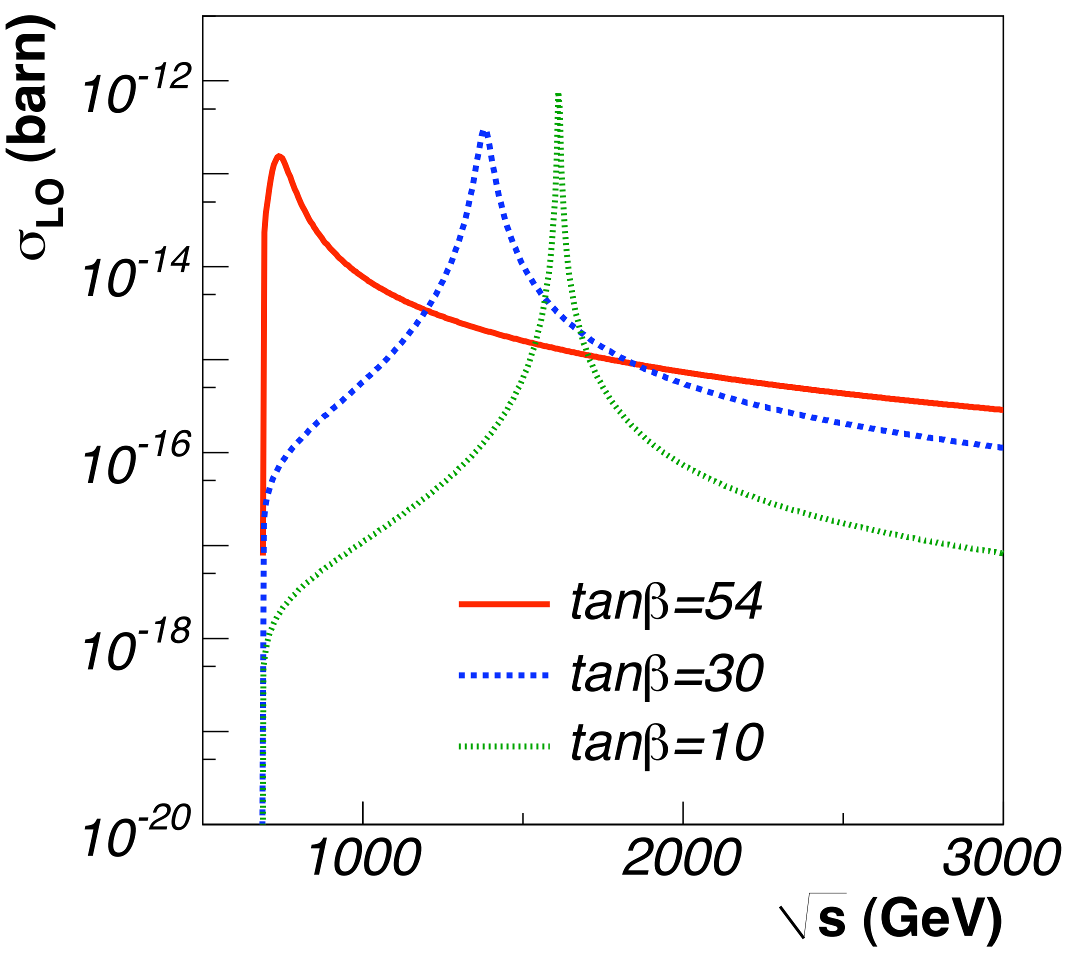
Annihilation into bottom quarks dominates at high tanβ

→ important Yukawa coupling of bottom quarks

→ neutralino pair behaves like pseudoscalar particle

→ large tanβ favoured by unifcation of Yukawa couplings in GUT theories [Carena *et al.* 1994]



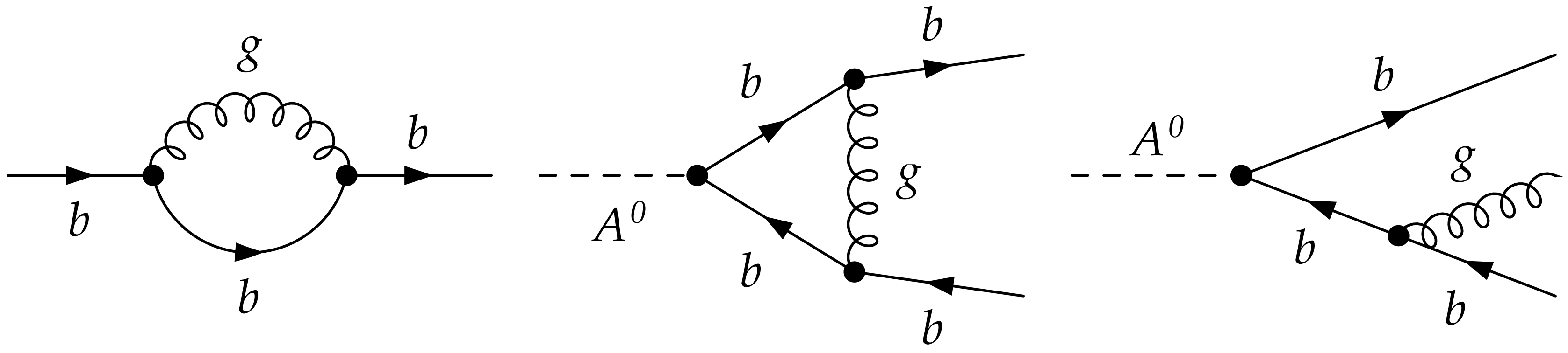


Process claimed to be compatible with the gamma ray excess observed by EGRET

→ problem with observed antiproton flux...? [de Boer *et al.* 2004, Bergström *et al.* 2006]

QCD Corrections at O(α_s)

Virtual loop corrections and real gluon emission



On-shell renormalization to cancel UV-singularities in virtual corrections

Dipole subtraction method to combine virtual and real emission contributions [Catani *et al.* 2000]

$$\sigma_{\rm NLO} \;=\; \Big[\sigma_{\rm V} + \int {\rm d}\sigma_{\rm aux}\Big]_{\epsilon=0} + \int \Big[{\rm d}\sigma_{\rm R} - {\rm d}\sigma_{\rm aux}\Big]_{\epsilon=0}$$

UV- and IR-finite annihilation cross section

$$\sigma_{\rm ann} \;=\; \sigma_{\rm LO}\Big[1 + \Delta_{\rm QCD}\Big]$$

$$\Delta_{\rm QCD} \;=\; \frac{\alpha_s(s)}{\pi} C_F \Bigg[\frac{1+\beta_b^2}{\beta_b} \bigg(4{\rm Li}_2 \frac{1-\beta_b}{1+\beta_b} + 2{\rm Li}_2 \frac{\beta_b - 1}{1+\beta_b} - 3\log\frac{2}{1+\beta_b}\log\frac{1+\beta_b}{1-\beta_b} - 2\log\beta_b \log\frac{1+\beta_b}{1-\beta_b} \bigg)$$
$$-3\log\frac{4}{1-\beta_b^2} - 4\log\beta_b + \frac{3}{8}\big(7-\beta_b^2\big) + \frac{1}{16\beta_b}\big(19 + 2\beta_b^2 + 3\beta_b^4\big)\log\frac{1+\beta_b}{1-\beta_b} \Bigg]$$

QCD Corrections: Improvements

Corrected cross section contains logarithmic mass singularity at high energies

$$\Delta_{\mathrm{QCD}}^{(\mathrm{HE})} \simeq \frac{\alpha_s C_F}{\pi}\left[-\frac{3}{2}\log\frac{s}{m_b^2}+\frac{9}{4}\right]$$

Resummation to all orders using the renormalization group

→ use running bottom quark mass in Yukawa coupling [Braaten & Leveille 1980]

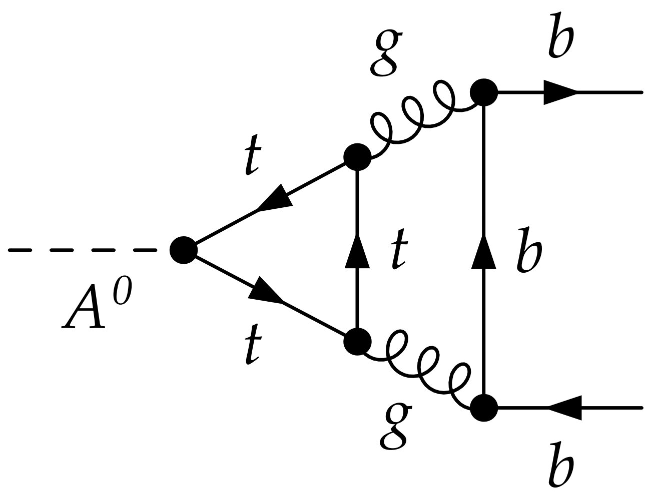
$$h_{Abb} = -\frac{g m_b \tan\beta}{2 m_W} \longrightarrow -\frac{g \overline{m}_b(s)\tan\beta}{2 m_W}$$

Higher order corrections are known up to O(α_s^4) [Chetyrkin *et al.* 1996, Chetyrkin 1997, Chetyrkin *et al.* 2005]

$$\Delta_{\mathrm{QCD}}^{(n_f=5)} = 29.15\,\frac{\alpha_s^2(s)}{\pi^2} + 41.77\,\frac{\alpha_s^3(s)}{\pi^3} - 825.6\,\frac{\alpha_s^4(s)}{\pi^4}$$

Top quark loop correction negligible at large tanβ [Chetyrkin *et al.* 1996]

$$\Delta_{\mathrm{top}} = \frac{1}{\tan^2\beta}\frac{\alpha_s^2(s)}{\pi^2}\left[\frac{23}{6}-\log\frac{s}{m_t^2}+\frac{1}{6}\log^2\frac{\overline{m}_b^2(s)}{s}\right]$$

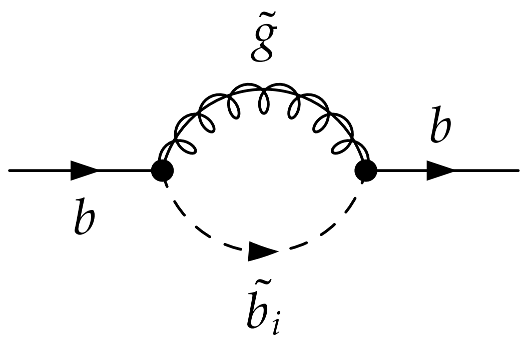


SUSY-QCD Corrections

Virtual loop corrections include self-energy and vertex correction diagrams

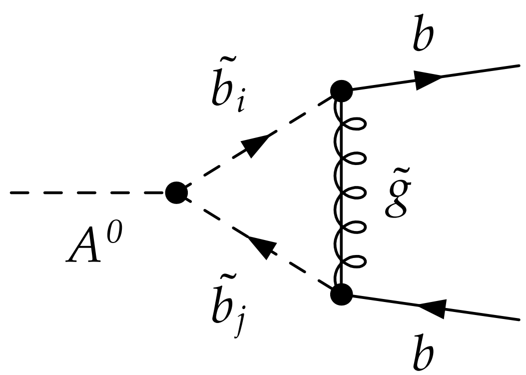
On-shell mass renormalization in "low-energy" limit of large SUSY masses

$$\left(\frac{\Delta m_b}{m_b}\right)_{\tilde{g}\tilde{b}} = \frac{\alpha_s(s)}{\pi} C_F \frac{m_{\tilde{g}}}{2}\left(A_b - \mu \tan\beta\right) I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}}),$$



Vertex correction equals mass renormalization up to a factor 1/tan²β

$$\Delta^{(\mathrm{LE})}_{\mathrm{SUSY}} = \frac{\alpha_s(s)}{\pi} C_F \left[1 + \frac{1}{\tan^2\beta}\right] m_{\tilde{g}} \mu \tan\beta \; I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}}).$$



Mass resummation in Yukawa coupling significant for large tanβ

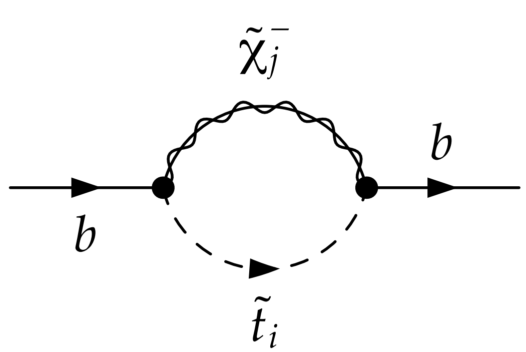
$$\overline{m}_b(s) \rightarrow \frac{\overline{m}_b(s)}{1 + \Delta m_b / m_b}$$

[Carena *et al.* 2000, Guasch *et al.* 2003]

Stop-chargino loop contributes at same order of magnitude

[Carena *et al.* 2000]

$$\left(\frac{\Delta m_b}{m_b}\right)_{\tilde{t}\tilde{\chi}^{\pm}} = -\frac{\lambda_t^2}{16\pi^2}\left[A_t \mu \tan\beta - \mu^2\right] I(\mu^2, m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2})$$



QCD and SUSY-QCD Corrections

Final annihilation cross section including corrections

$$\sigma_{\rm ann} \;=\; \overline{\sigma}_{\rm LO}(s)\Big[1+\Delta_{\rm QCD}+\Delta_{\rm top}+\Delta_{\rm SUSY}\Big]$$

"Leading order" part contains QCD running bottom mass and SUSY bottom mass resummation

Finite remainders:

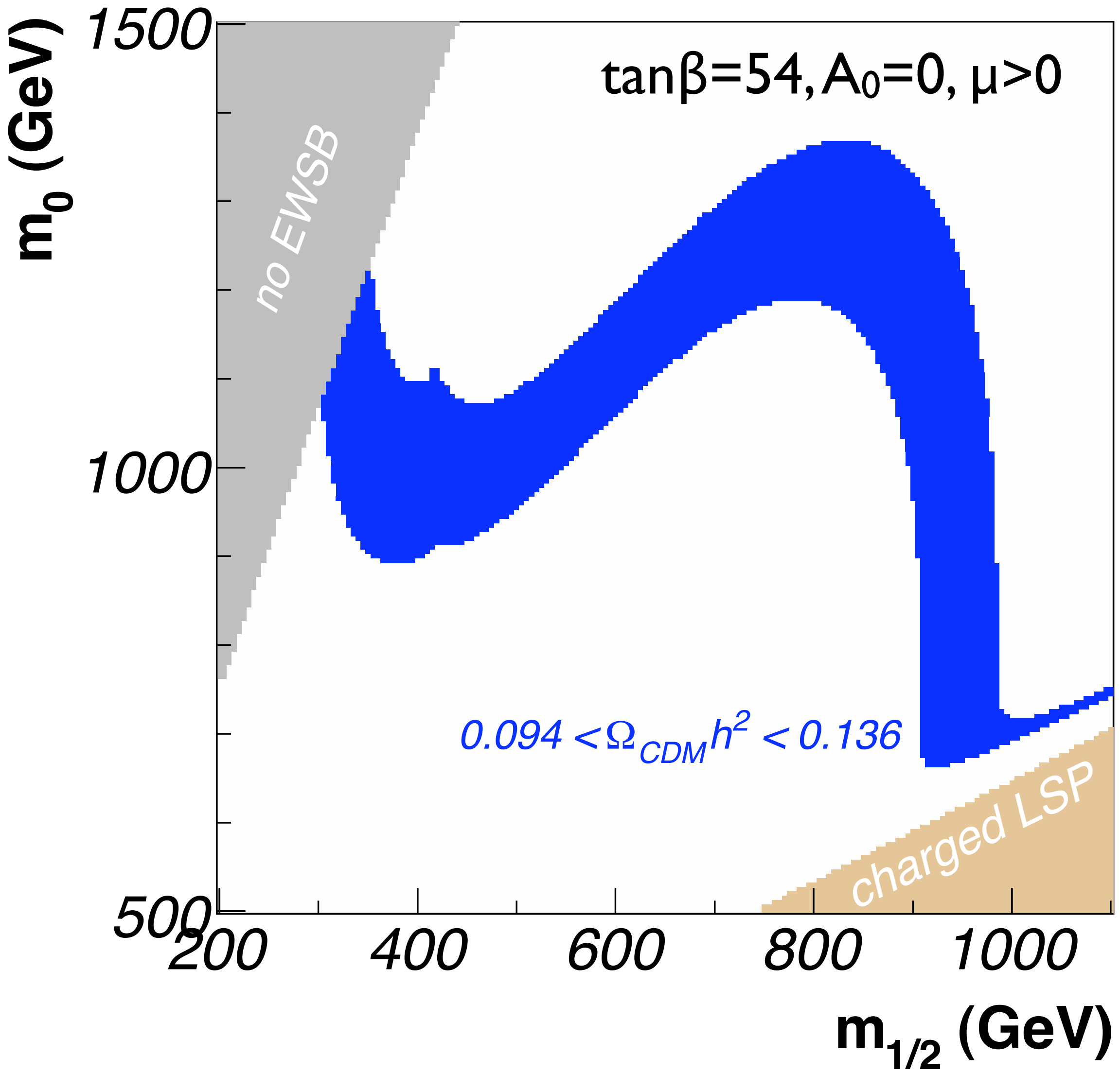
$$\Delta_{\rm QCD} \;=\; C_F f_{\rm QCD}(s)\frac{\alpha_s(s)}{\pi}+29.15\frac{\alpha_s^2(s)}{\pi^2}+41.77\frac{\alpha_s^3(s)}{\pi^3}-825.6\frac{\alpha_s^4(s)}{\pi^4}$$

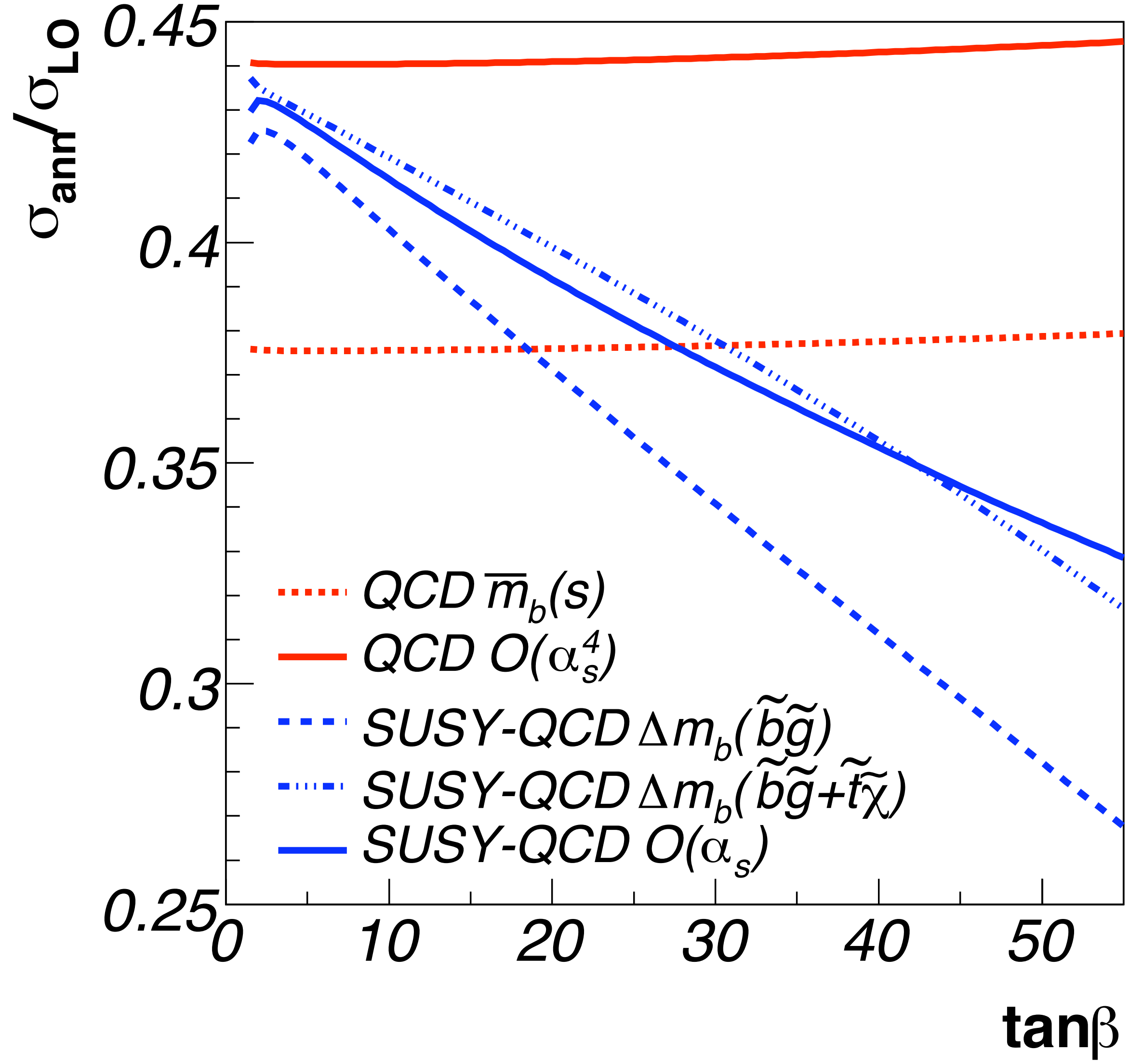
$$\Delta_{\rm top} \;=\; \frac{1}{\tan^2\beta}\frac{\alpha_s^2(s)}{\pi^2}\left[\frac{23}{6}-\log\frac{s}{m_t^2}+\frac{1}{6}\log^2\frac{\overline{m}_b^2(s)}{s}\right]$$

$$\Delta_{\rm SUSY} \;=\; \frac{\alpha_s(s)}{\pi}C_F\frac{1+\tan^2\beta}{\tan\beta}m_{\tilde g}\mu \times\Big[C_0(m_b^2,s,m_b^2;m_{\tilde b_1}^2,m_{\tilde b_2}^2,m_{\tilde g}^2)-C_0(0,0,0;m_{\tilde b_1}^2,m_{\tilde b_2}^2,m_{\tilde g}^2)\Big]$$

Impact on the Annihilation Cross Section

Study numerically the impact of the different corrections





Main impact due to QCD running mass: Cross section decreased by about 60%

SUSY-QCD mass resummation decreases by up to further 20% at large tanβ

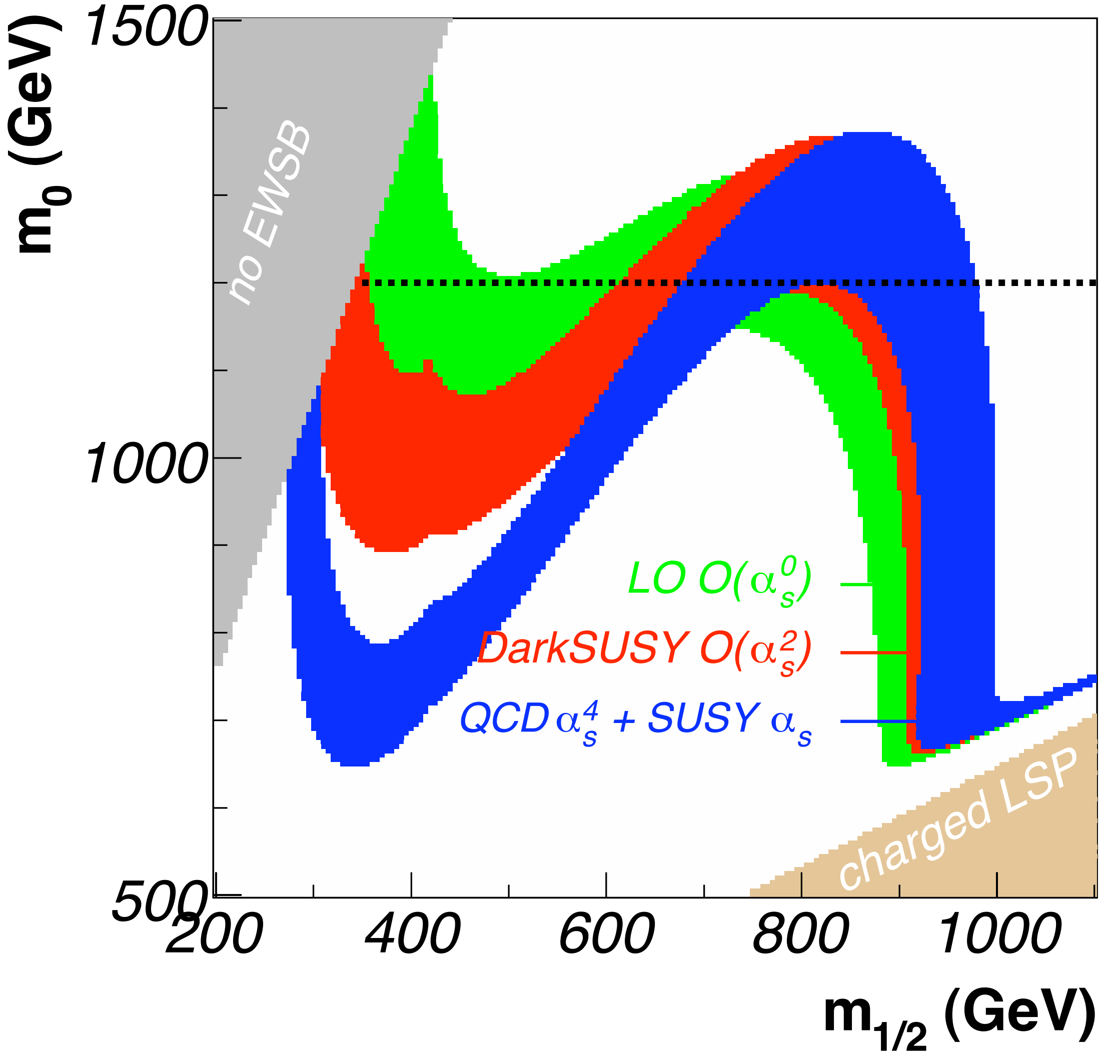
Impact on the Relic Density

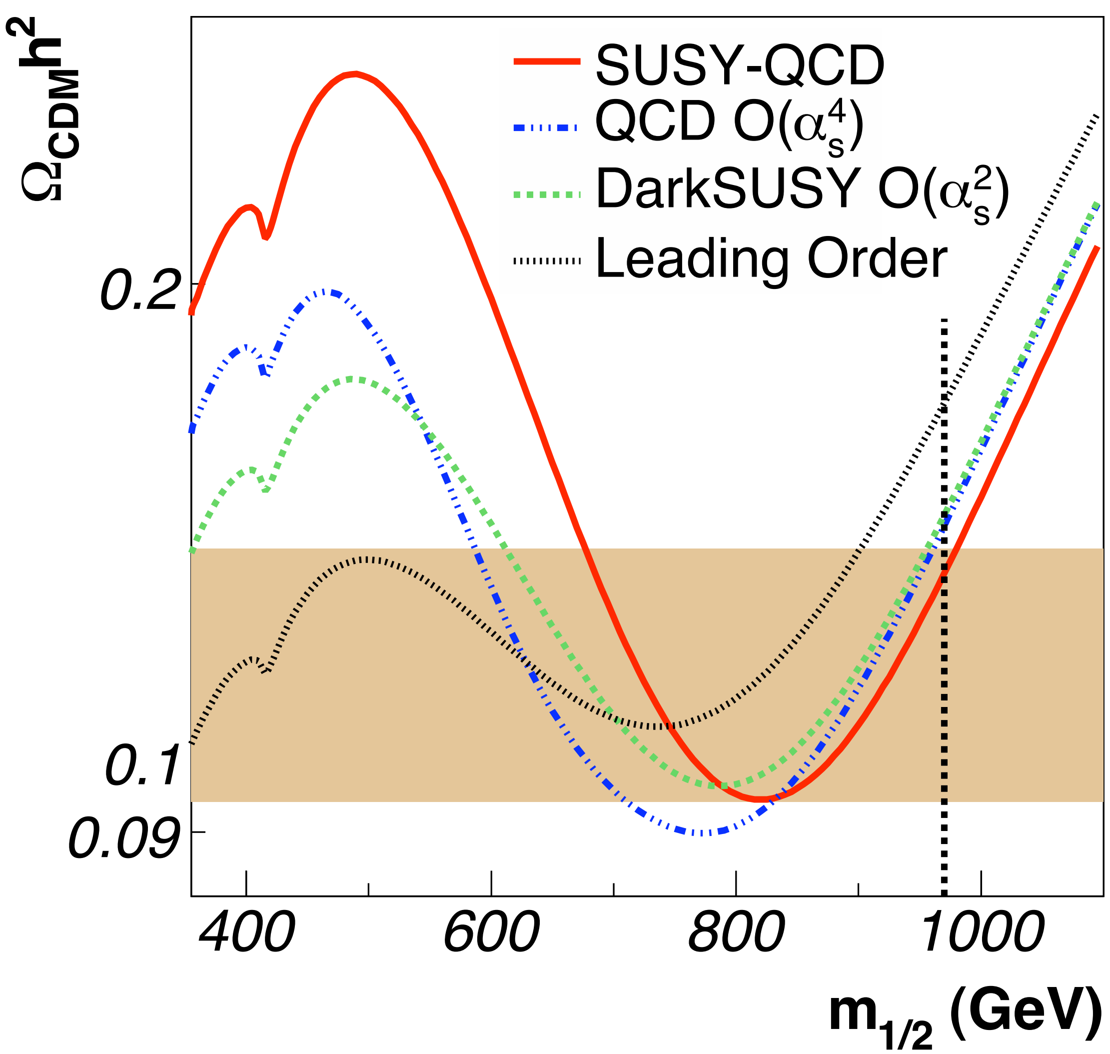
Scan m_0-$m_{1/2}$ plane with and without including corrections for tanβ=54 and μ>0

Renormalization group running: SPheno 2.2.3 [Porod 2004]

Physical mass spectrum: FeynHiggs 2.5.1 [Heinemeyer *et al.* 2000]

Neutralino relic density: DarkSUSY 4.1 (corrections included)



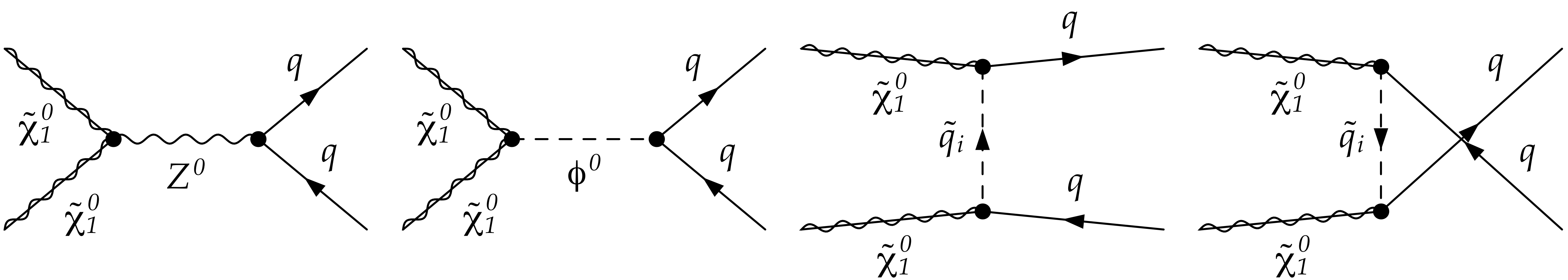


WMAP favoured contour shifted to smaller masses to compensate smaller cross section

Higgs decay width receives same QCD and SUSY-QCD corrections as cross section

General Neutralino Annihilation into Quarks

Now consider all diagrams leading to "heavy" quark final states



Full one-loop SUSY-QCD calculation

including resummation of logarithmically enhanced terms as discussed for the "A-Funnel"

Analytical calculation and verification:

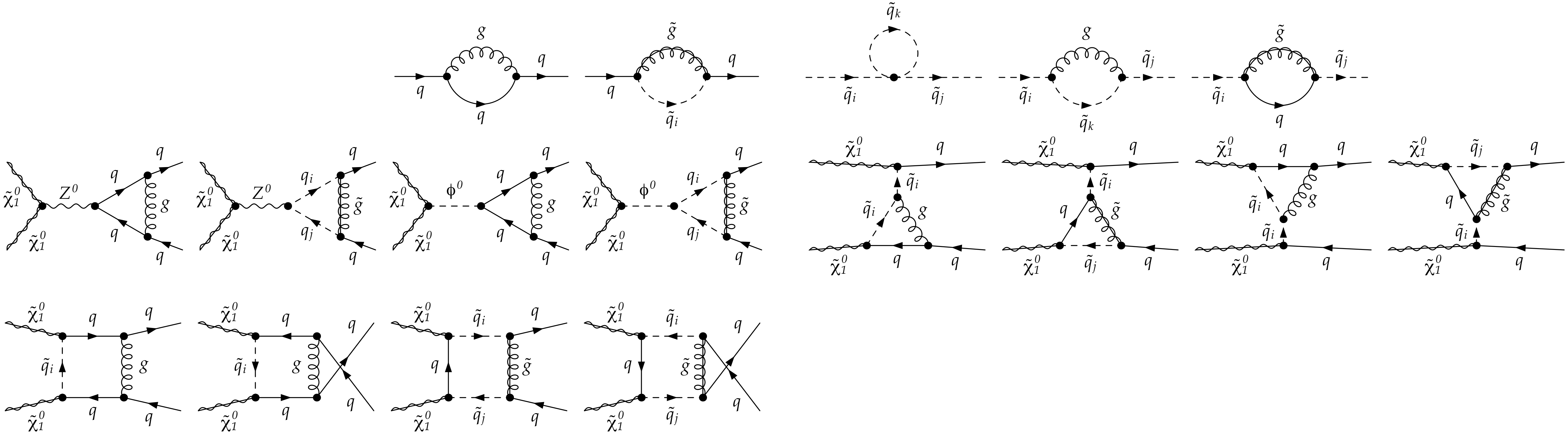
- FORM [Vermaseren 1991-2008]
- FeynArts / FormCalc / LoopTools [Hahn 1998-2008]
- FeynCalc [Mertig 1991-2008]
- calculation by hand...

Numerical evaluation:

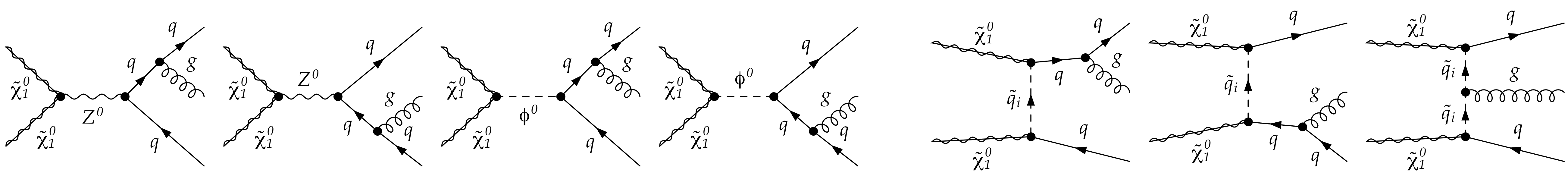
- SPheno 2.2.3 (mass spectrum) [Porod 2003]
- micrOMEGAs 2.1 (constraints and relic density) [Bélanger *et al.* 2003]
- own Fortran code (annihilation cross section)

QCD and SUSY-QCD Corrections

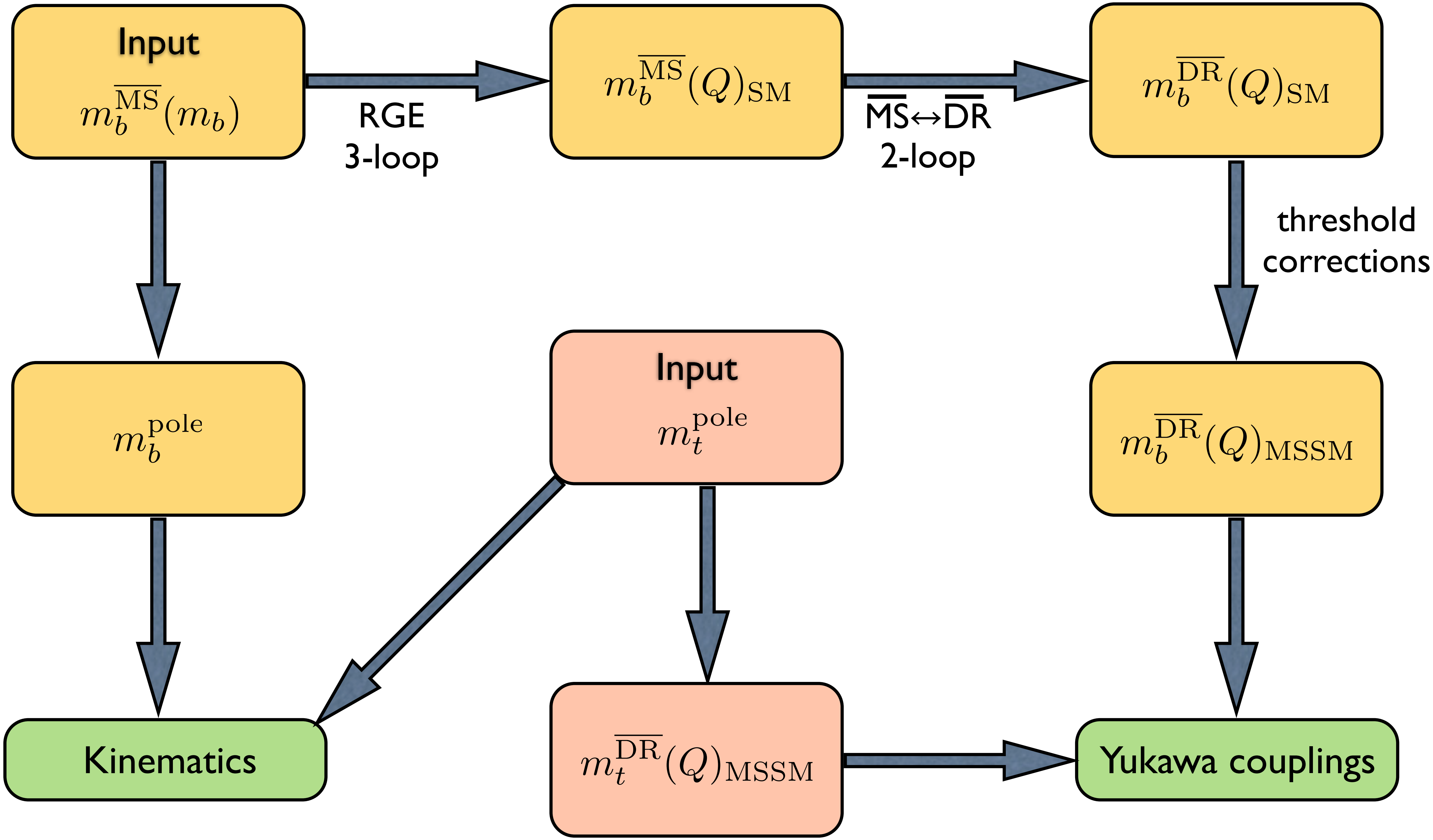
Virtual loop corrections: On-shell renormalization



Real emission corrections: Dipole subtraction method

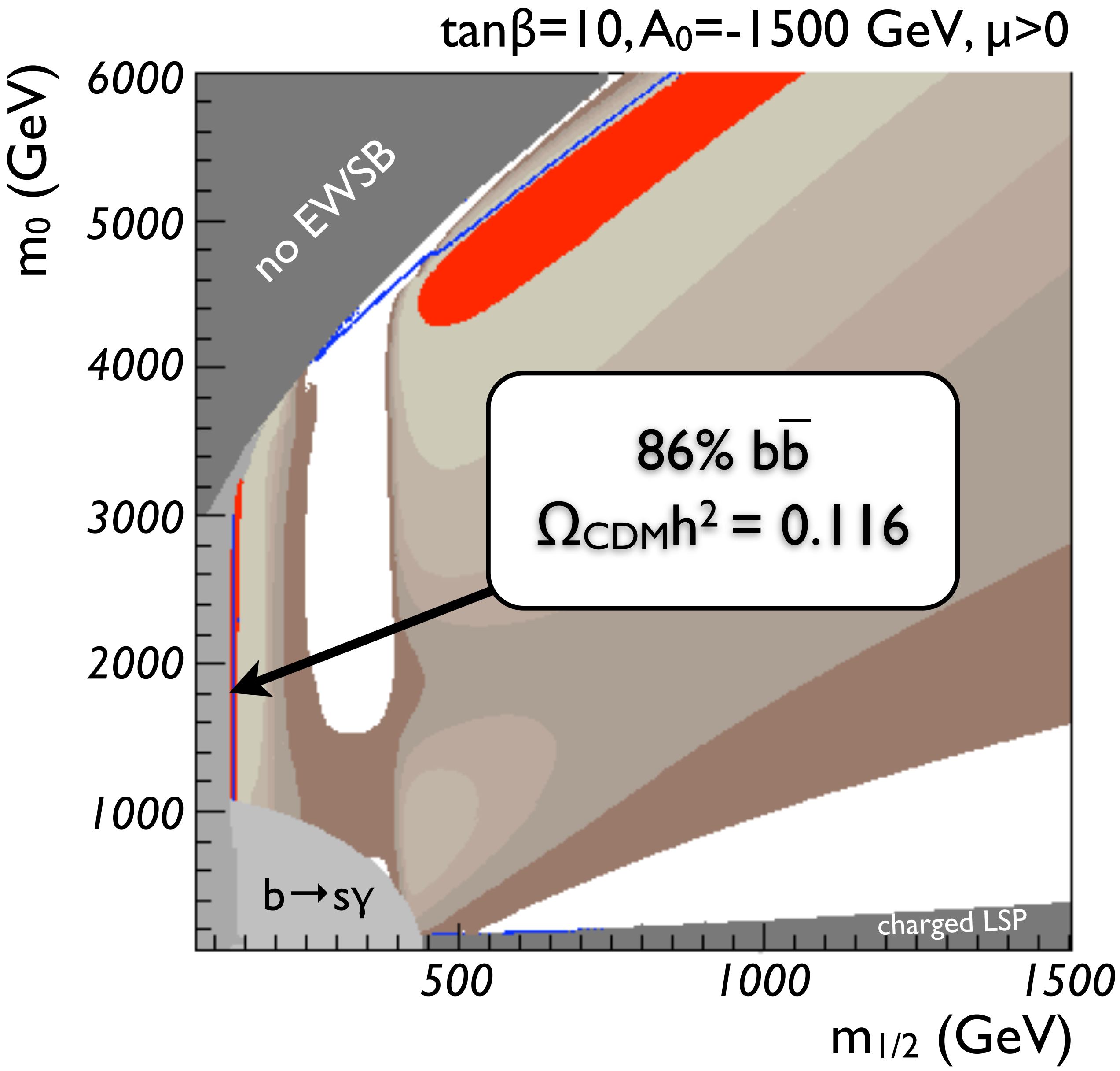


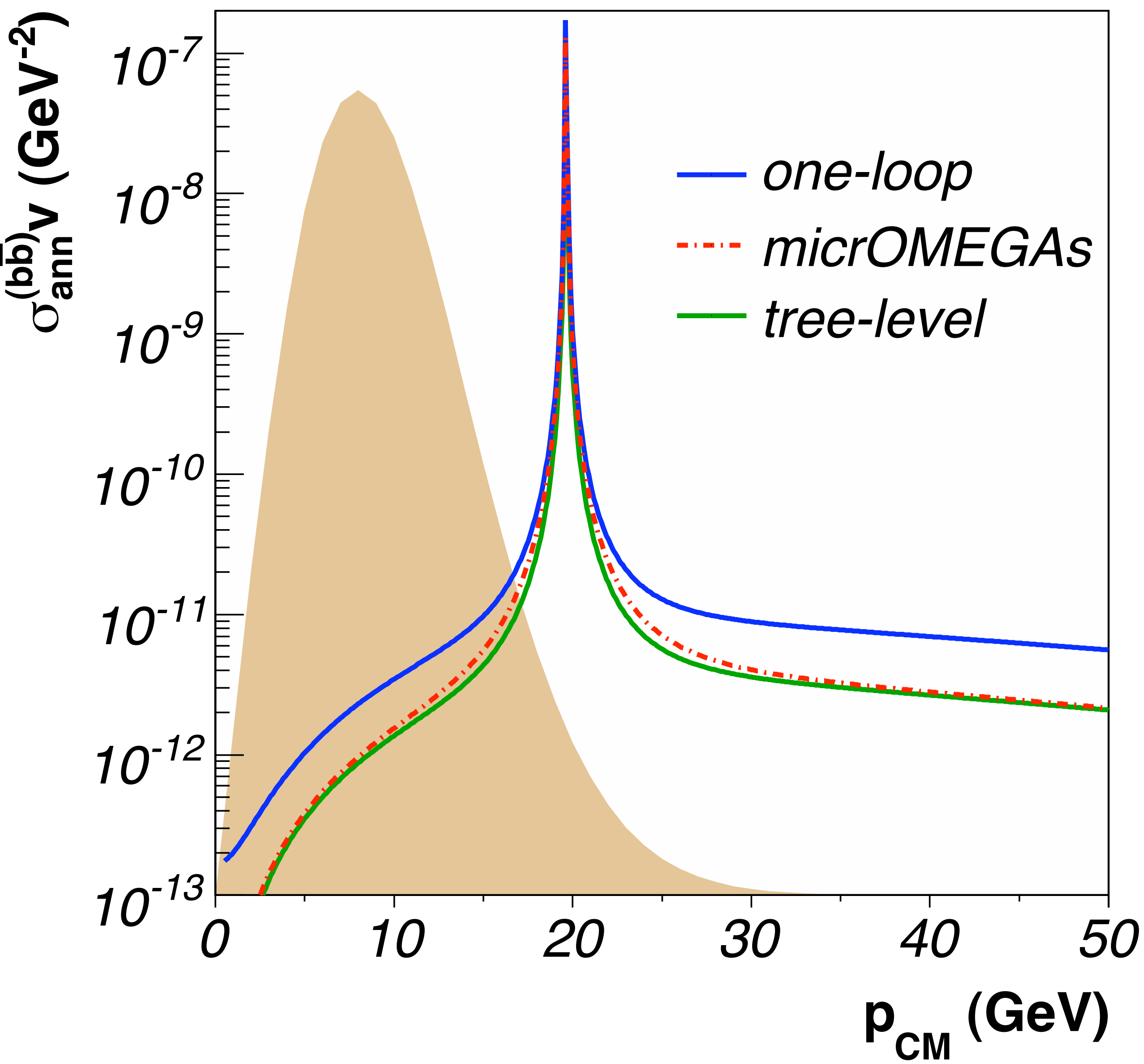
Treatment of Quark Masses



Numerical values: $m_b^{\overline{\rm MS}}(m_b) \;=\; 4.2\ {\rm GeV}$ [Amsler *et al.* (PDG) 2008] $\qquad m_t^{\rm pole} \;=\; 172.4\ {\rm GeV}$ [D0 + CDF coll. 2008]

Impact around the Higgs Resonance



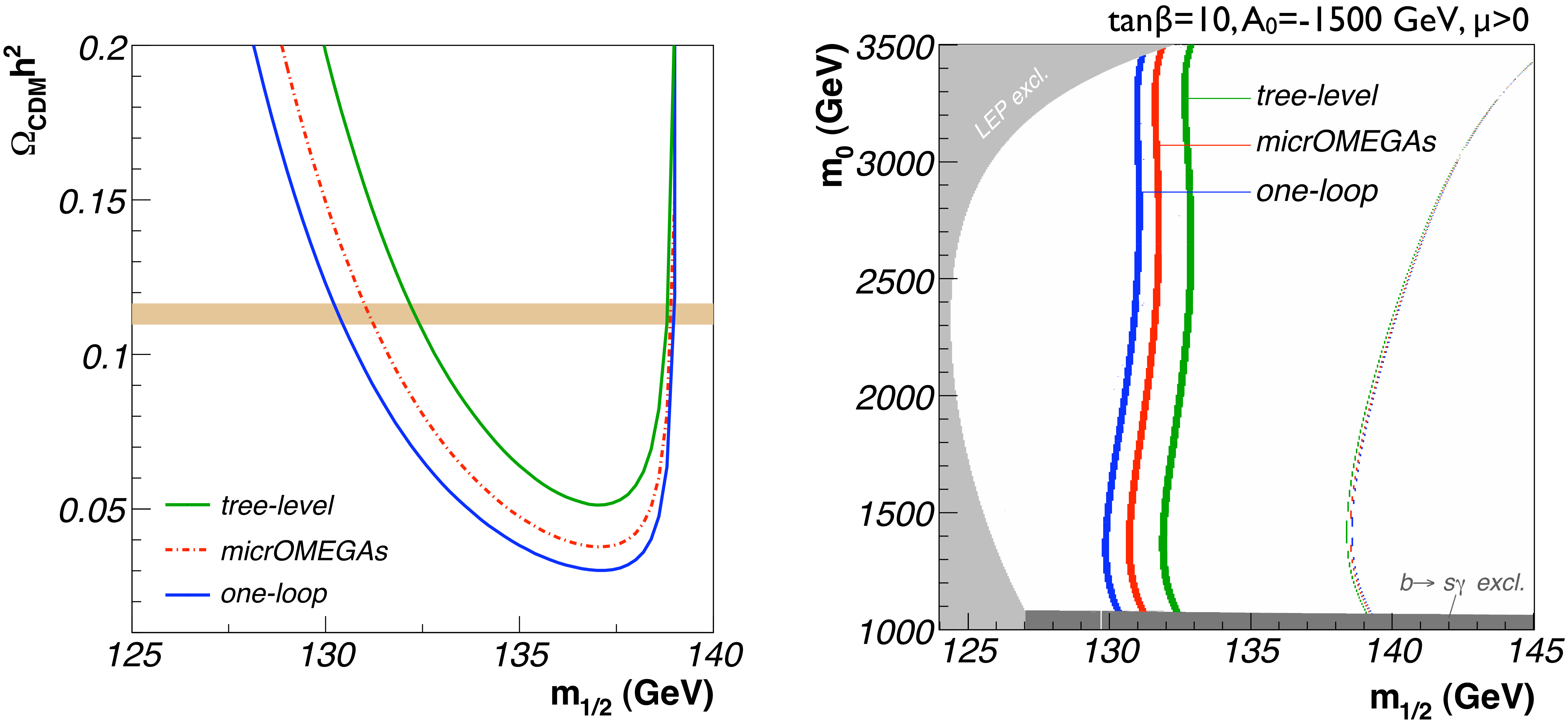


"Effective" Yukawa coupling very good approximation around the Higgs resonance

Position of resonance does not coincide with peak of velocity distribution for thermal average

Cross section enhanced significantly by one-loop QCD and SUSY-QCD corrections

Impact around the Higgs Resonance

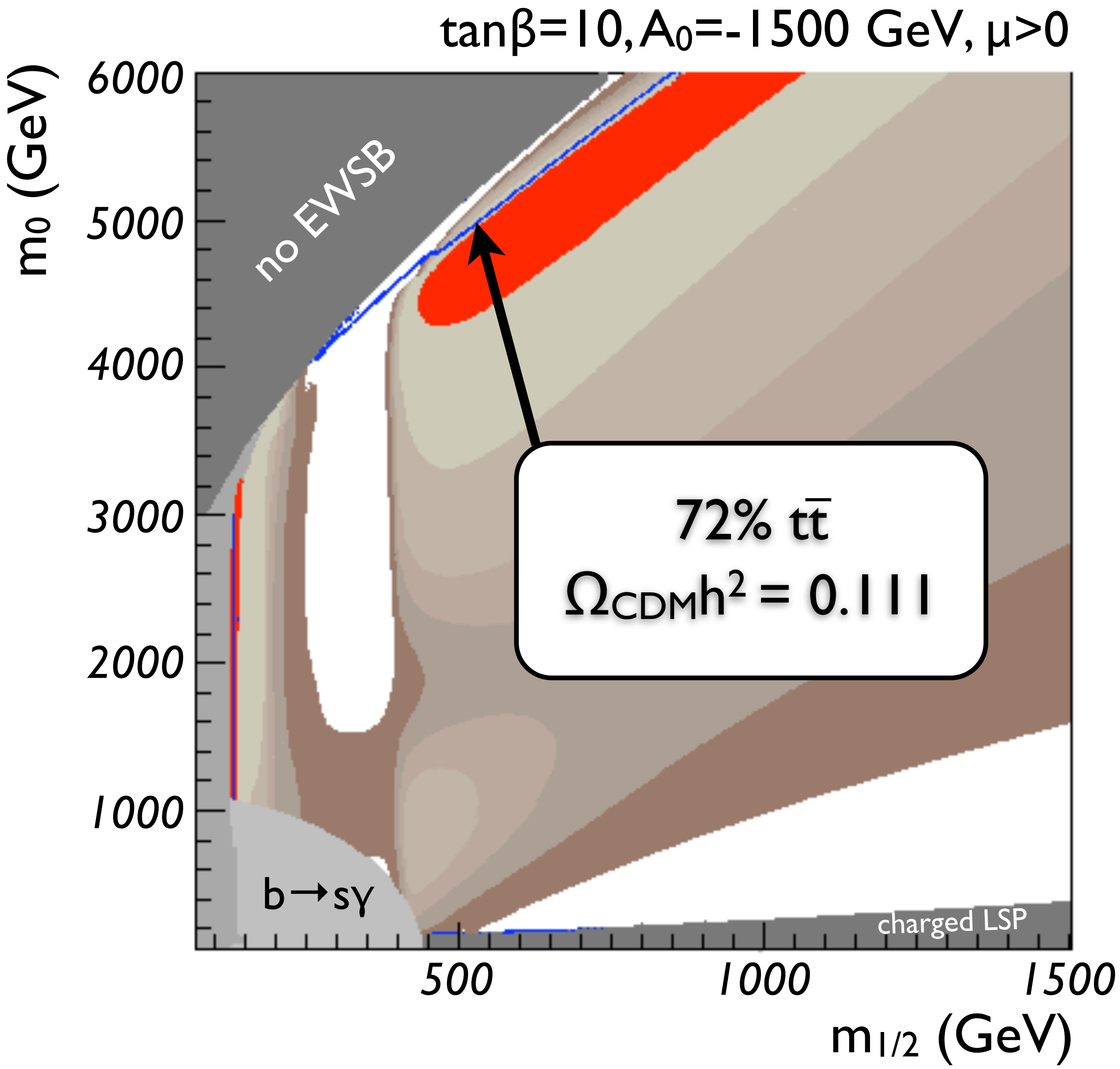


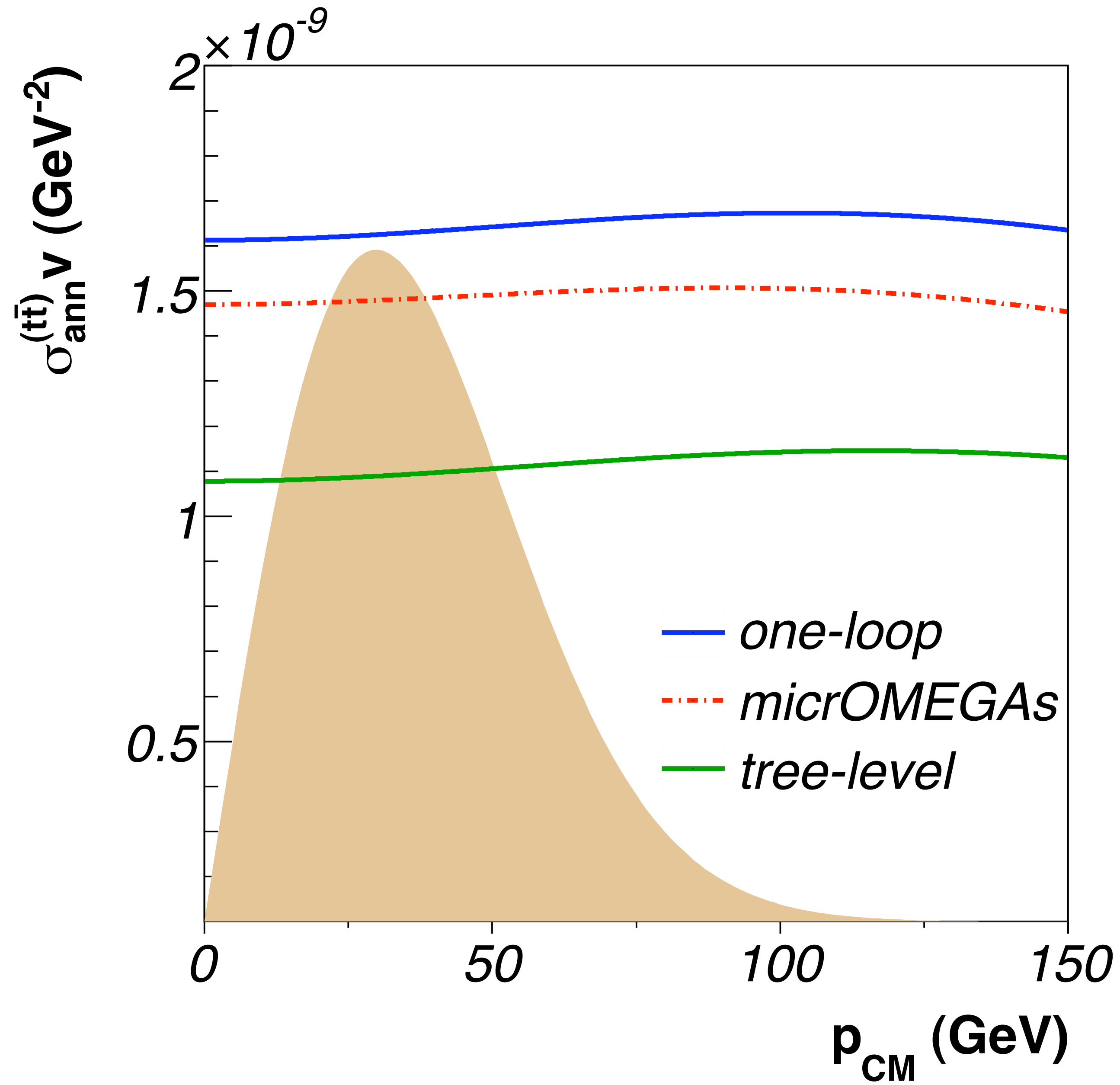
Effect on cross section influences directly the prediction of the relic density

Impact of corrections larger than experimental uncertainty

Favoured interval shifted towards chargino mass limit $m_{1/2} \geq 126$ GeV

Impact on Focus Point Region at low tanβ



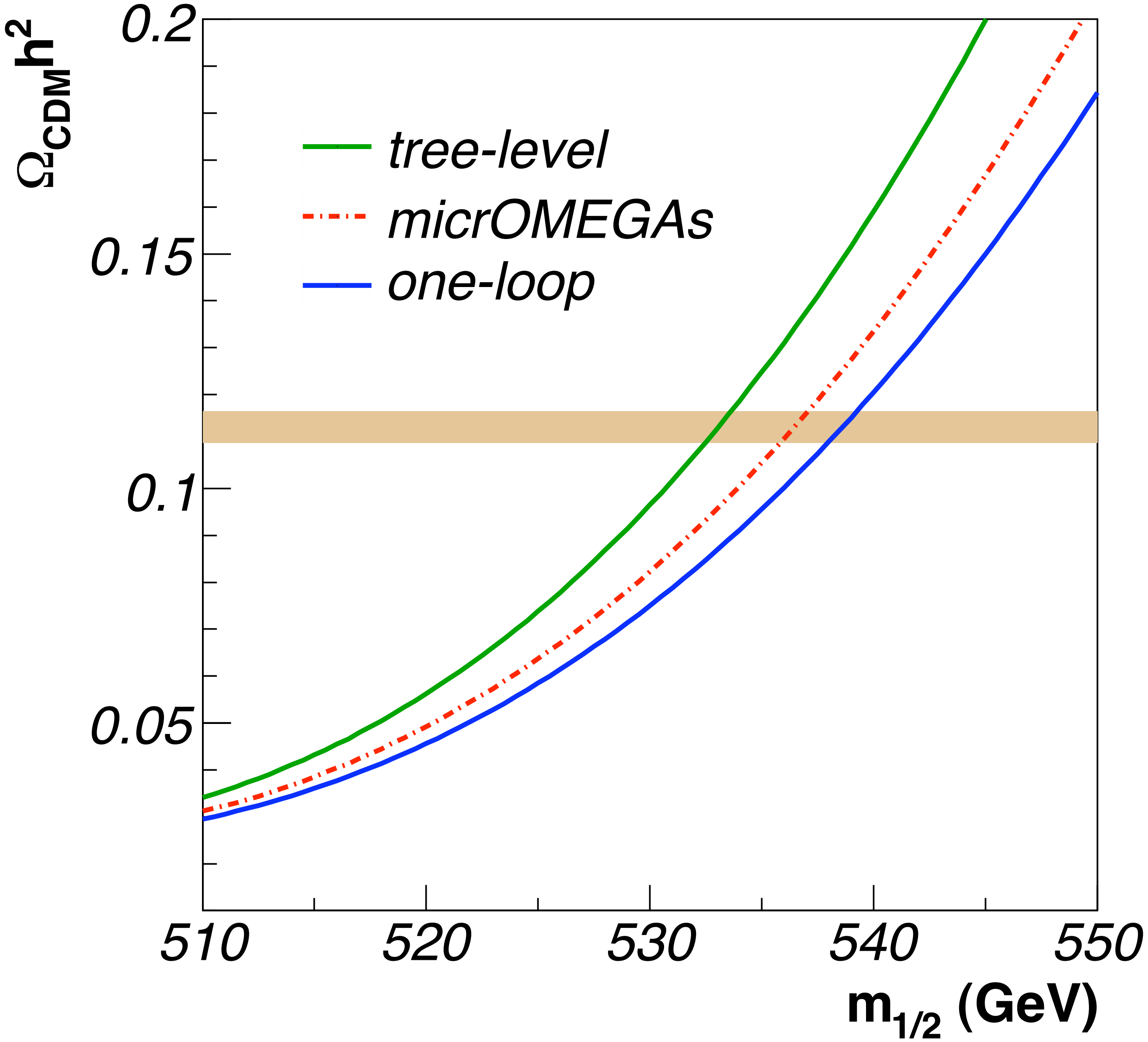


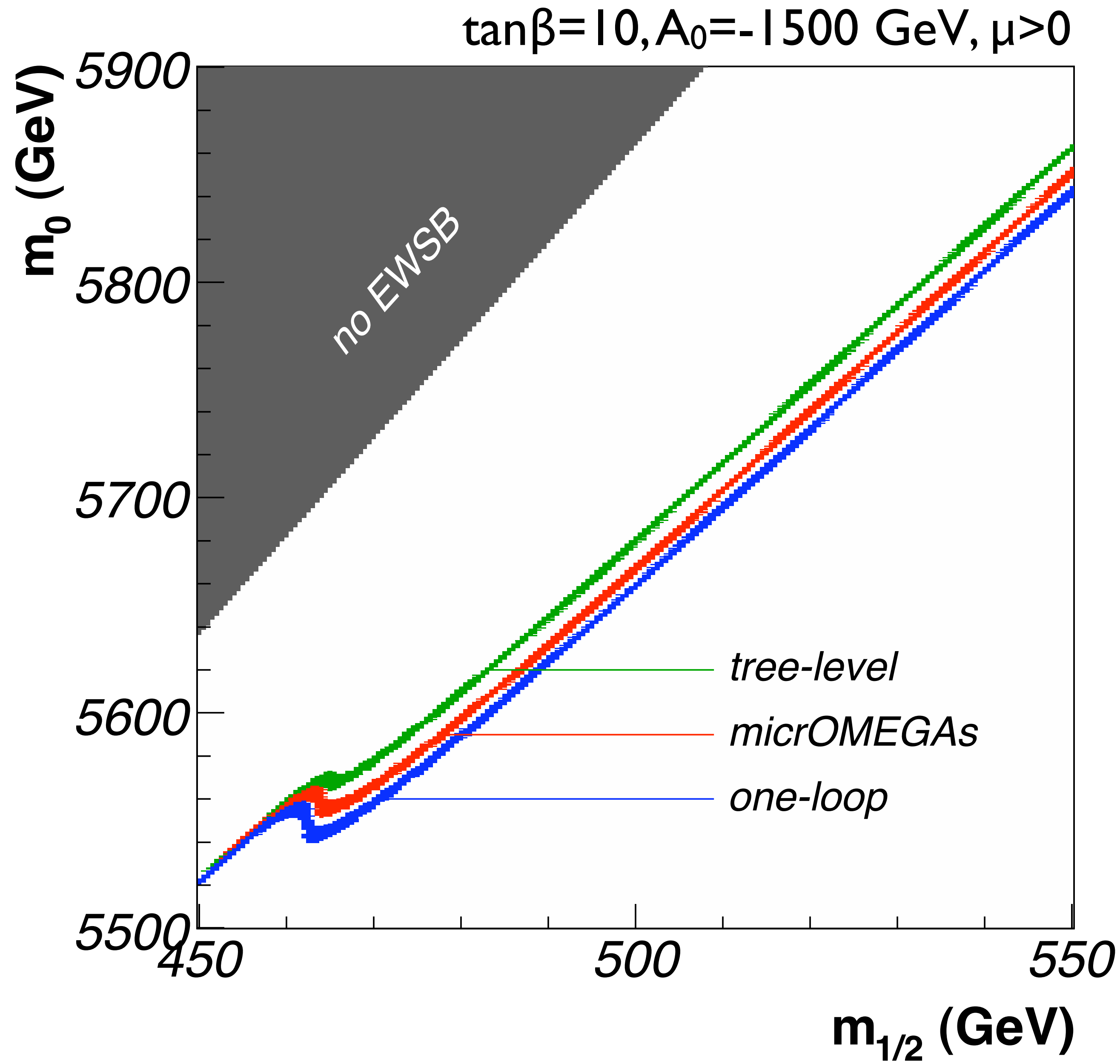
Higgs resonance situated at rather high energies around $p_{cm} \approx 2610$ GeV

Annihilation cross section sizeable due to large top quark mass

Cross section enhanced significantly by one-loop QCD and SUSY-QCD corrections

Impact on Focus Point Region at low tanβ

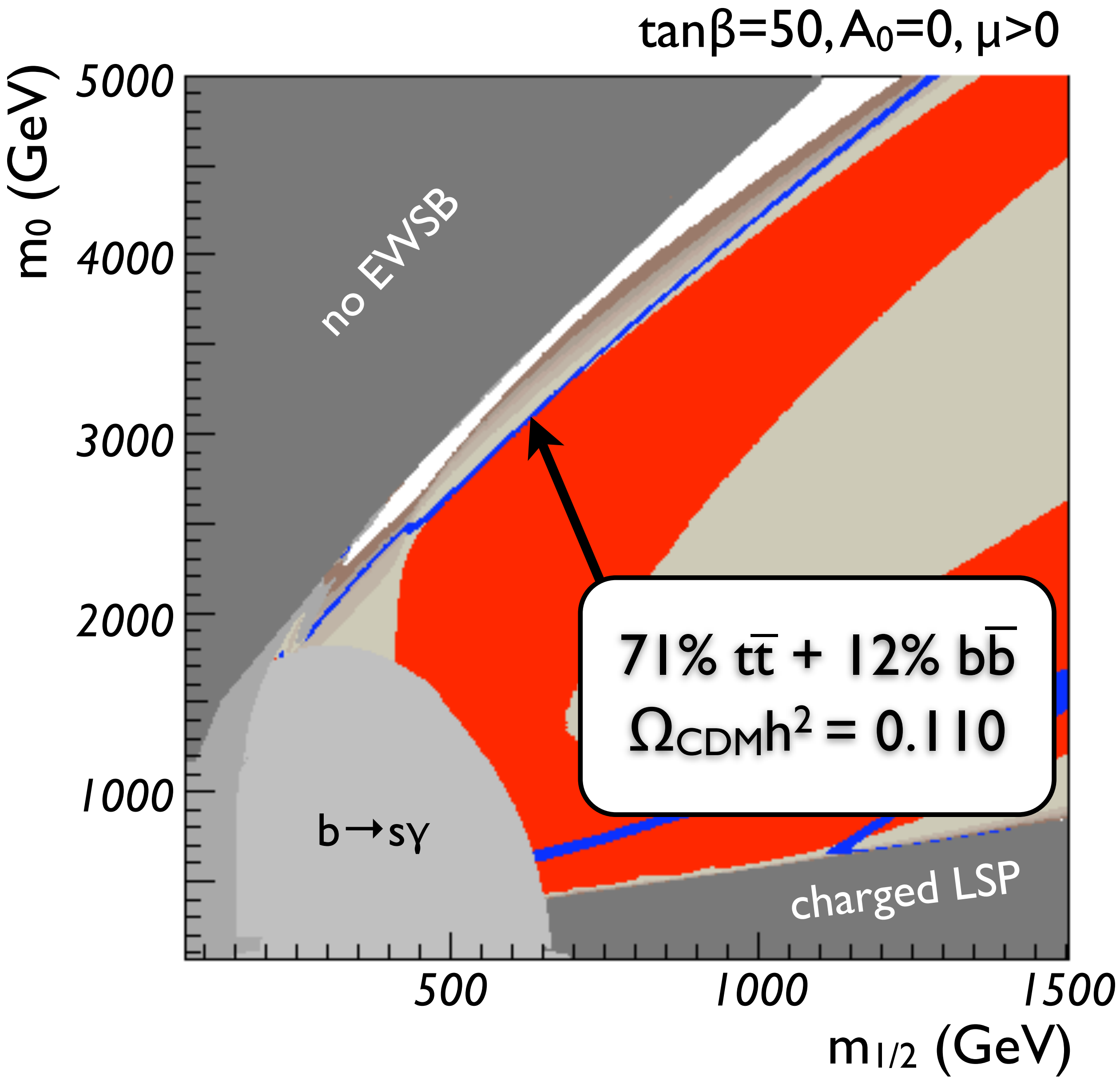


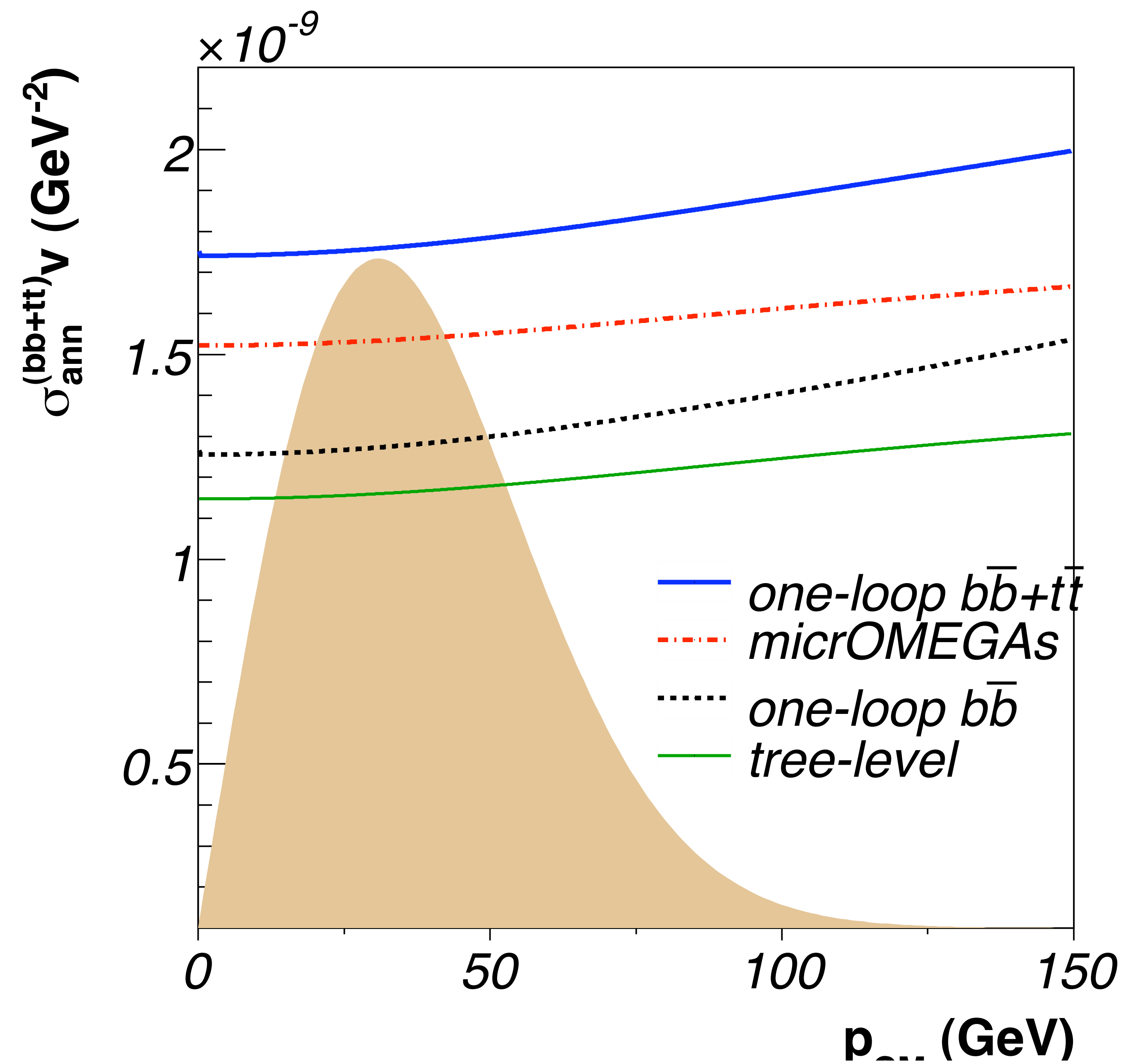


Impact of corrections on prediction of relic density larger than experimental uncertainty

Favoured region shifted to lower scalar masses m_0 and higher gaugino masses $m_{1/2}$

Impact on Focus Point Region at high tanβ

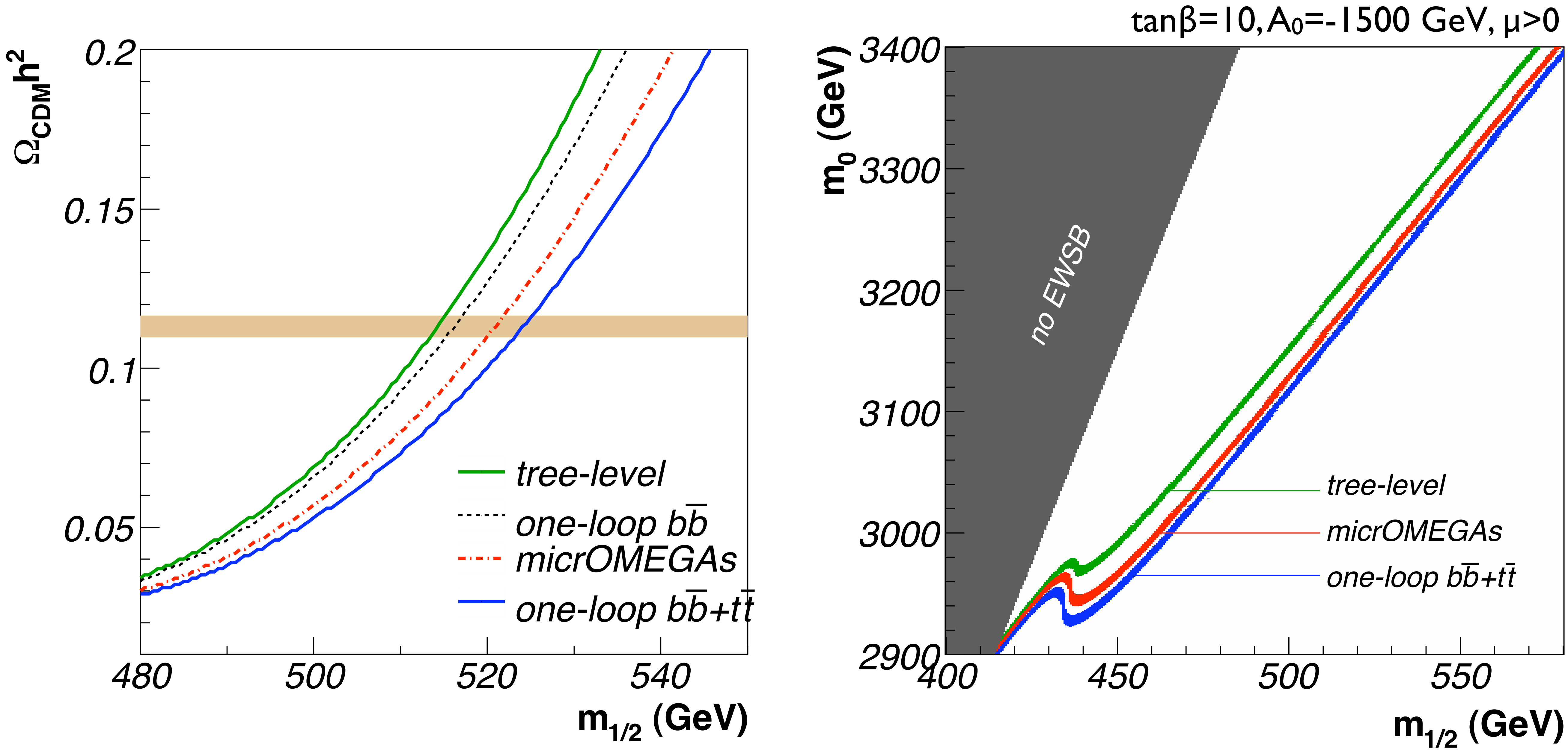




Important top and bottom quark contributions due to large top mass and large tanβ

Each cross section enhanced significantly by one-loop QCD and SUSY-QCD corrections

Impact on Focus Point Region at high tanβ



Bottom quark correction sizeable due to large tanβ

Top and bottom corrections shift relic density in the same direction

Favoured region shifted to lower scalar masses m_0 and higher gaugino masses $m_{1/2}$

Conclusion and Outlook

Dark matter annihilation is an interesting tool to constrain the SUSY parameter space

- → Complementary information with respect to collider and low-energy data
- → Neutralino annihilation into "heavy" quark-antiquark pairs dominant in mSUGRA
- → Radiative corrections important in the light of new cosmological precision measurements

SUSY-QCD corrections have significant impact on extraction of SUSY mass parameters

- → Calculation of full one-loop QCD and SUSY-QCD corrections
- → Significant impact on annihilation cross section and neutralino relic density
- → Include corrections in common analysis tools

Perspectives

- → Study scenarios with important contributions from light quark final states...
- → Include corrected cross section also in DarkSUSY and compare...
- → Include SUSY-QCD corrections to co-annihilation processes [Freitas 2007] and electroweak corrections [Baro & Boudjema, *in progress*]
- → Provide complete code including all one-loop corrections
- → Application to direct / indirect dark matter detection...