

# Beam-Recoil Polarization in Virtual Compton Scattering from the Proton below Pion Threshold

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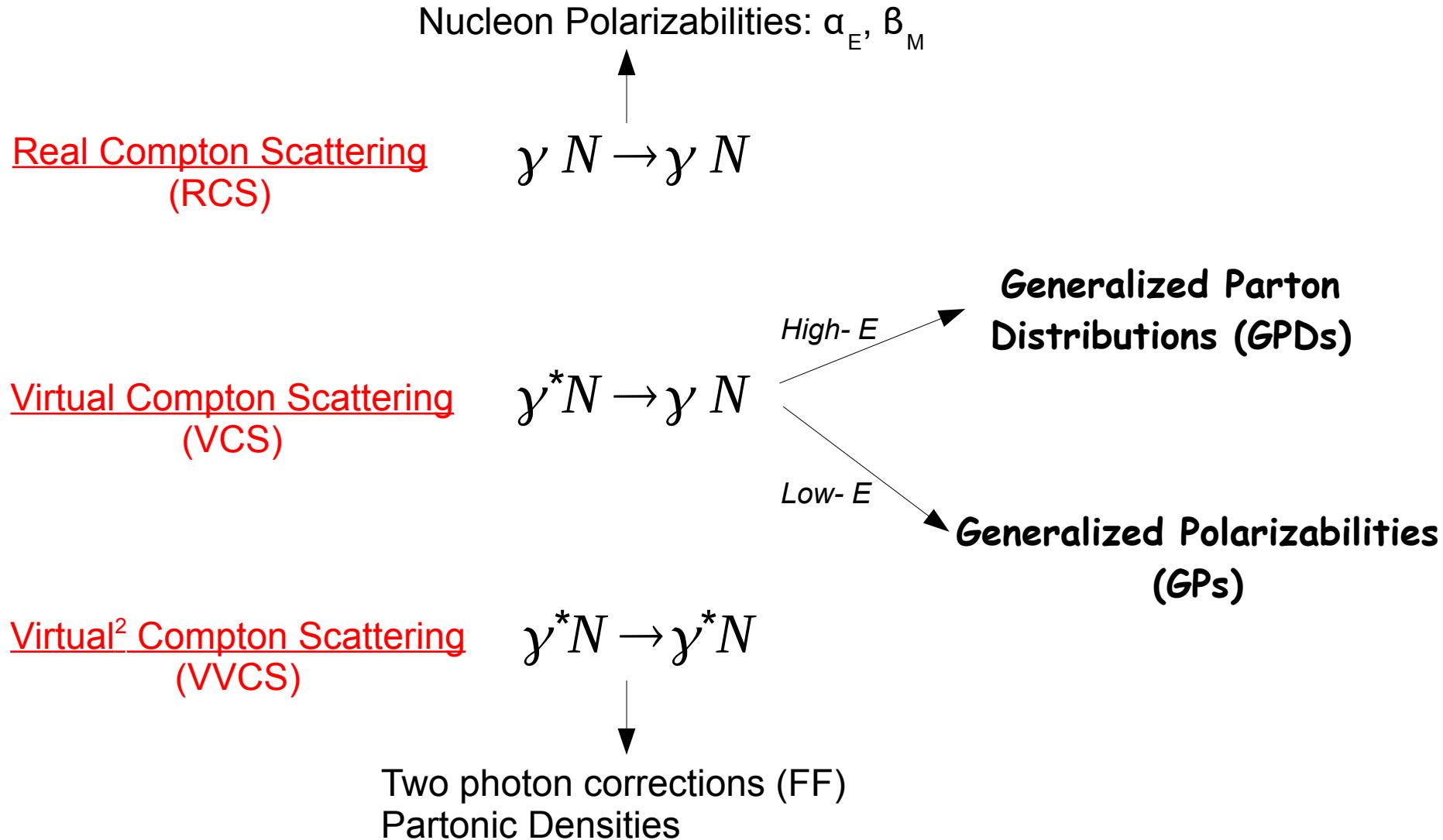
Propriété d'un consortium d'universités canadiennes, géré en co-entreprise à partir d'une contribution administrée par le Conseil national de recherches Canada

# Outline

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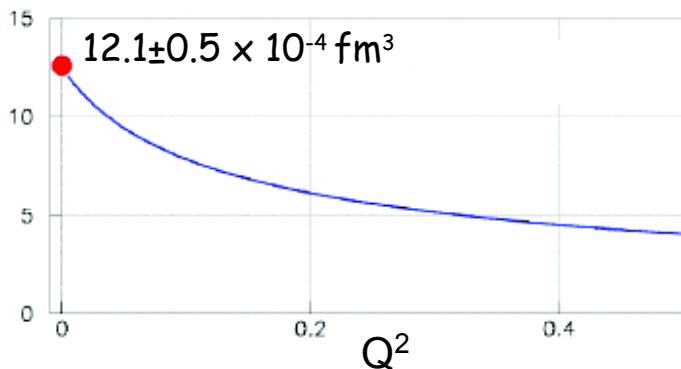
- Motivation
- Theoretical Aspects
- The Experiment
- Data Analysis and Results
- Conclusion and Outlook

# (Virtual) Photons and the Nucleon



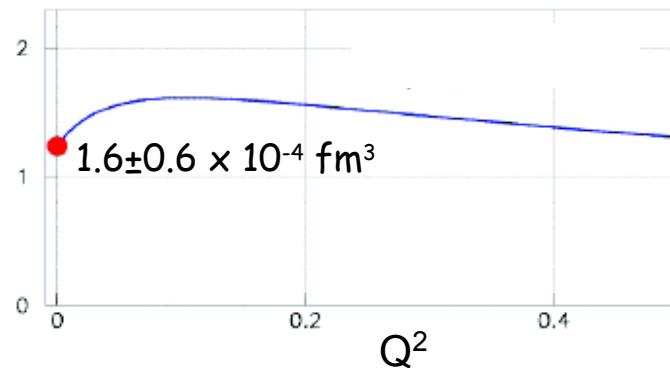
# Generalized Polarizabilities

## Electric Polarizability $\alpha$



- Displacement of charges
- Induced dipole moment  $p = \alpha E$
- Atomic Systems  $\alpha/V \sim 1$
- Nucleons:  $\alpha \sim 10^{-4} \text{ fm}^3$ ,  $V \sim 1 \text{ fm}^3$

## Magnetic Polarizability $\beta$



- Diamagnetism ( $\beta < 0$ )
- ChPT: Pions are the relevant degrees of freedom
- „Pion Cloud“ : Induced eddy currents of spinless charged particles

## Paramagnetism ( $\beta > 0$ )

- Resonant structure of the Nucleon
- Example:  $N \rightarrow \Delta$  Transition

+ 4 “spin” GPs

### History of the GPs:

H. Arenhoevel et al., NPA 233 (1974) 153

P. Guichon et al., NPA 591 (1995) 606

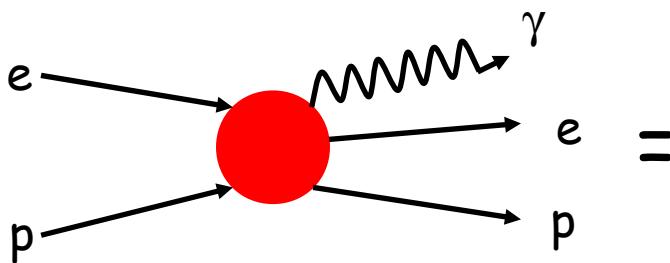
D. Drechsel et al., PRC 55 (1997) 424

D. Drechsel et al., PRC 57 (1998) 941

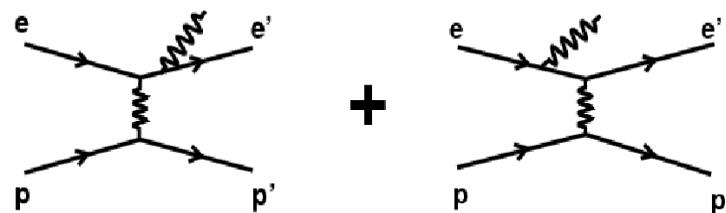
### RCS data:

V. Olmos de Leon EPJA 10 (2001) 207

# Virtual Compton Scattering



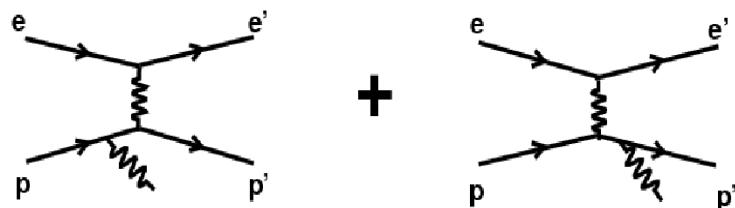
Photoelectronproduction



Bethe-Heitler

Contribution

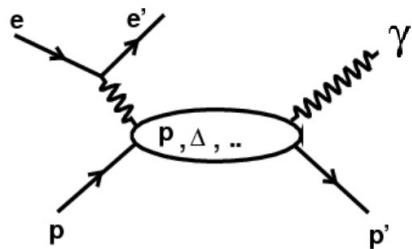
- Not separable from VCS
- Lorentz Boost:
  - in the direction of  $e$
- Cross Section  $\sim 1/k$
- Form Factors needed



Born Term

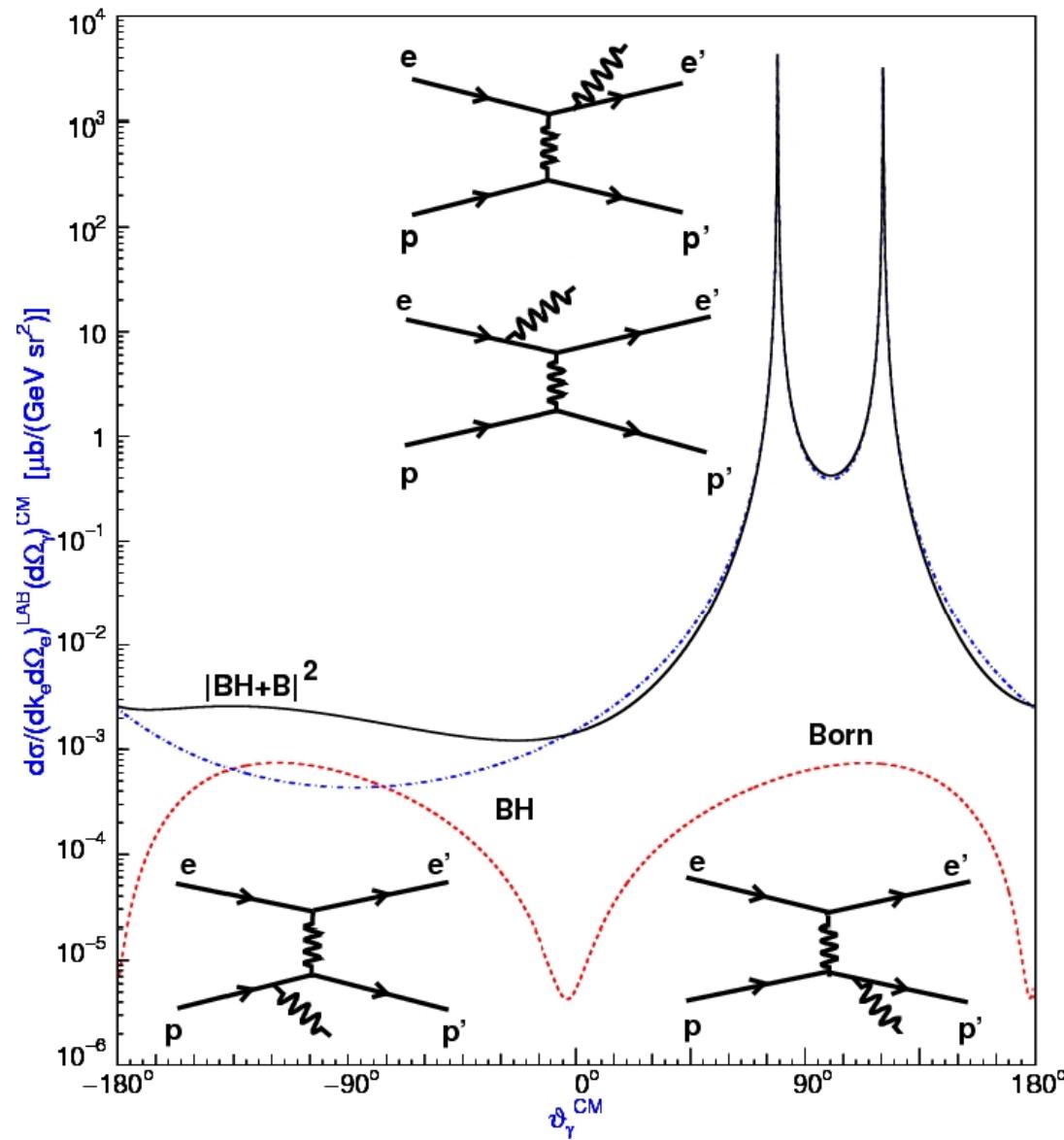
- First order term
- Suppressed by  $1/m_p$
- Form Factors needed

VCS Contribution



Parametrized by the  
Generalized Polarizabilities

# Virtual Compton Scattering

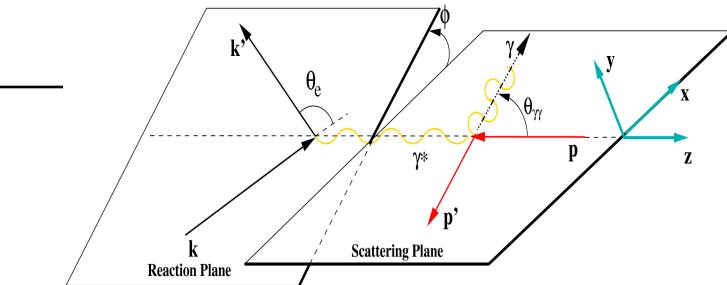


# Low Energy Expansion

## Cross Section

$$\frac{d^5\sigma}{dE'd\Omega'd\Omega_{\gamma}^{cm}} = d^5\sigma^{BH+Born} + \phi q' \Psi_0(q, \varepsilon, \theta, \phi) + O(q'^2)$$

$$\Psi_0 = v_1(\theta, \phi, \varepsilon)(P_{LL}(q^2) - P_{TT}(q^2)/\varepsilon) + v_2(\theta, \phi, \varepsilon)P_{LT}(q^2)$$



P.A.M. Guichon, et al., Nucl Phys A 591 (1995) 606-638

## Double Polarization Observables

$$P_{x,y,z} = \frac{d^5\sigma^{\uparrow\uparrow} + d^5\sigma^{\downarrow\downarrow} - d^5\sigma^{\uparrow\downarrow} - d^5\sigma^{\downarrow\uparrow}}{d^5\sigma^{\uparrow\uparrow} + d^5\sigma^{\downarrow\downarrow} + d^5\sigma^{\uparrow\downarrow} + d^5\sigma^{\downarrow\uparrow}} = \frac{d^5\sigma^{h\uparrow} - d^5\sigma^{h\downarrow}}{2 d^5\sigma}$$

## Structure Functions

$$\Psi_0 = v_1(P_{LL} - P_{TT}/\varepsilon) + v_2 P_{LT}$$

$$\Delta\Psi_0^z = 4 h [v_1^z P_{TT} + v_2^z P_{LT}^z + v_3^z P_{LT}'^z]$$

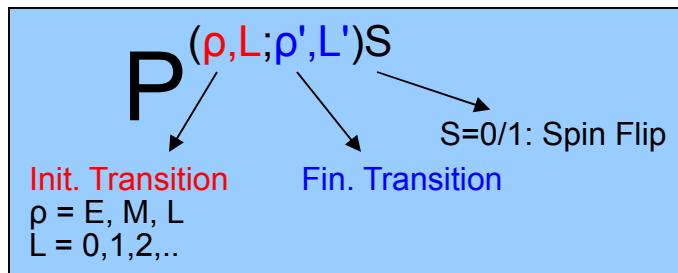
$$\Delta\Psi_0^x = 4 h [v_1^x P_{LT}^\perp + v_2^x P_{TT}^\perp + v_3^x P_{TT}'^\perp + v_4^x P_{LT}'^\perp]$$

$$\Delta\Psi_0^y = 4 h [v_1^y P_{LT}^\perp + v_2^y P_{TT}^\perp + v_3^y P_{TT}'^\perp + v_4^y P_{LT}'^\perp]$$

$$\Delta d^5\sigma_{x,y,z}^h = \Delta d^5\sigma_{x,y,z}^{BH+Born} + \phi q' \Delta\Psi_0^{x,y,z} + \phi O(q'^2)$$

M.Vanderhaeghen, PLB 402 (1997) 243

## Generalized Polarizabilities



$$\begin{aligned}
 P_{LL} &= a P^{C1 \rightarrow E1} \\
 P_{TT} &= c_1 P^{M1 \rightarrow M1(S)} + c_2 P^{M2 \rightarrow E1(S)} \\
 P_{LT} &= b P^{M1 \rightarrow M1} + c_3 \left[ P^{C0 \rightarrow M1(S)} + d_1 P^{C2 \rightarrow M1(S)} \right] \\
 P_{LT}^z &= c_4 P^{M1 \rightarrow M1(S)} + c_3 \left[ P^{C0 \rightarrow M1(S)} + d_1 P^{C2 \rightarrow M1(S)} \right] \\
 P_{LT}'^z &= c_5 P^{M1 \rightarrow M1(S)} + c_6 \left[ P^{C0 \rightarrow M1(S)} + d_1 P^{C2 \rightarrow M1(S)} \right] \\
 P_{LT}'^\perp &= \left[ d_2 P^{C0 \rightarrow M1(S)} + d_3 P^{C2 \rightarrow M1(S)} \right]
 \end{aligned}$$

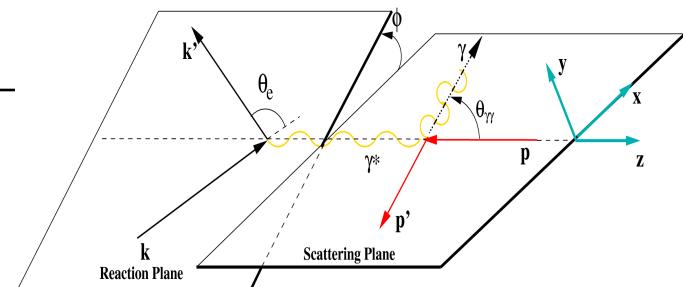
# Dispersion Relations

VCS defined by

$$Q^2 = -q^2$$

$$t = (q - q')^2$$

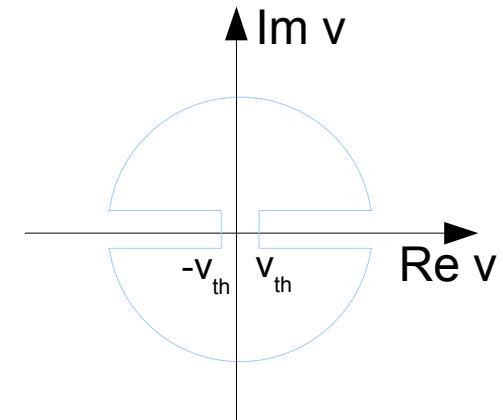
$$\nu = \frac{s-u}{4M} = E_\gamma^{LAB} + \frac{1}{4M}(t-Q^2)$$



Amplitudes analytical in  $\nu$ , Unitarity, Crossing Symmetry

Analytical continuation

$$\Re F_i^{nB}(Q^2, \nu, t) = \frac{2}{\pi} \int_{\nu_{th}}^{\infty} d\nu \frac{\nu' \Im F_i(Q^2, \nu, t)}{\nu'^2 - \nu^2}$$



MAID Parameterization

D.Drechsel, S.S.Kamalov, L.Tiator, Nucl.Phys. A645 (1999) 145-174

Parameterization of  $\alpha$  and  $\beta$

Fit to the experimental data

Prediction for the 4 Spin GPs

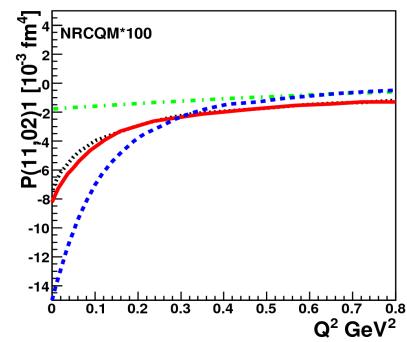
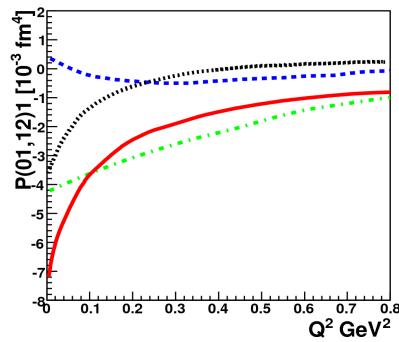
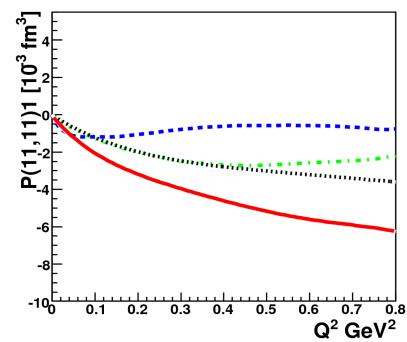
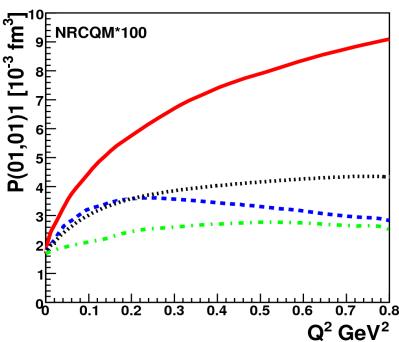
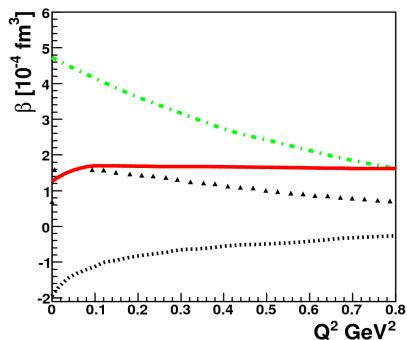
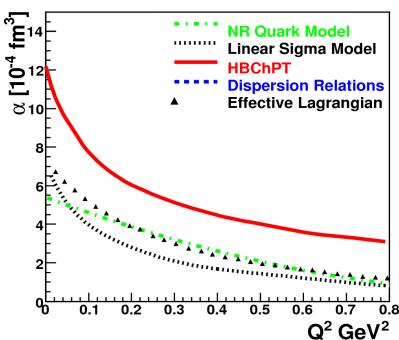
Dipole Form

B.Pasquini et al. , Eur. Phys. J. 11 (2001) 185-208

$$\alpha(Q^2) - \alpha^{\pi N} = \frac{\alpha - \alpha^{\pi N}}{(1+Q^2/\Lambda_\alpha^2)^2} \quad \rightarrow \quad P^{(C1 \rightarrow E1)0}(Q^2) = -\sqrt{\frac{2}{3}} \frac{4\pi}{e^2} \alpha(Q^2)$$

$$\beta(Q^2) - \beta^{\pi N} = \frac{\beta - \beta^{\pi N}}{(1+Q^2/\Lambda_\beta^2)^2} \quad \rightarrow \quad P^{(M1 \rightarrow M1)0}(Q^2) = -\sqrt{\frac{8}{3}} \frac{4\pi}{e^2} \beta(Q^2)$$

# Theoretical Predictions



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## NRCQM

P.A.M.Guichon et al. NPA 591 (1996) 606

Pasquini et al. nucl-th/0105074

## Linear Sigma Model

A.Metz et al. Z.Phys. A356 (1996) 351

.....

## HBChPT O(p3)

Hemmert et al. PRD 62 (2000) 014013

.....

## Dispersion Relations

B.Pasquini et al. PRC 62 (2000) 052201

.....

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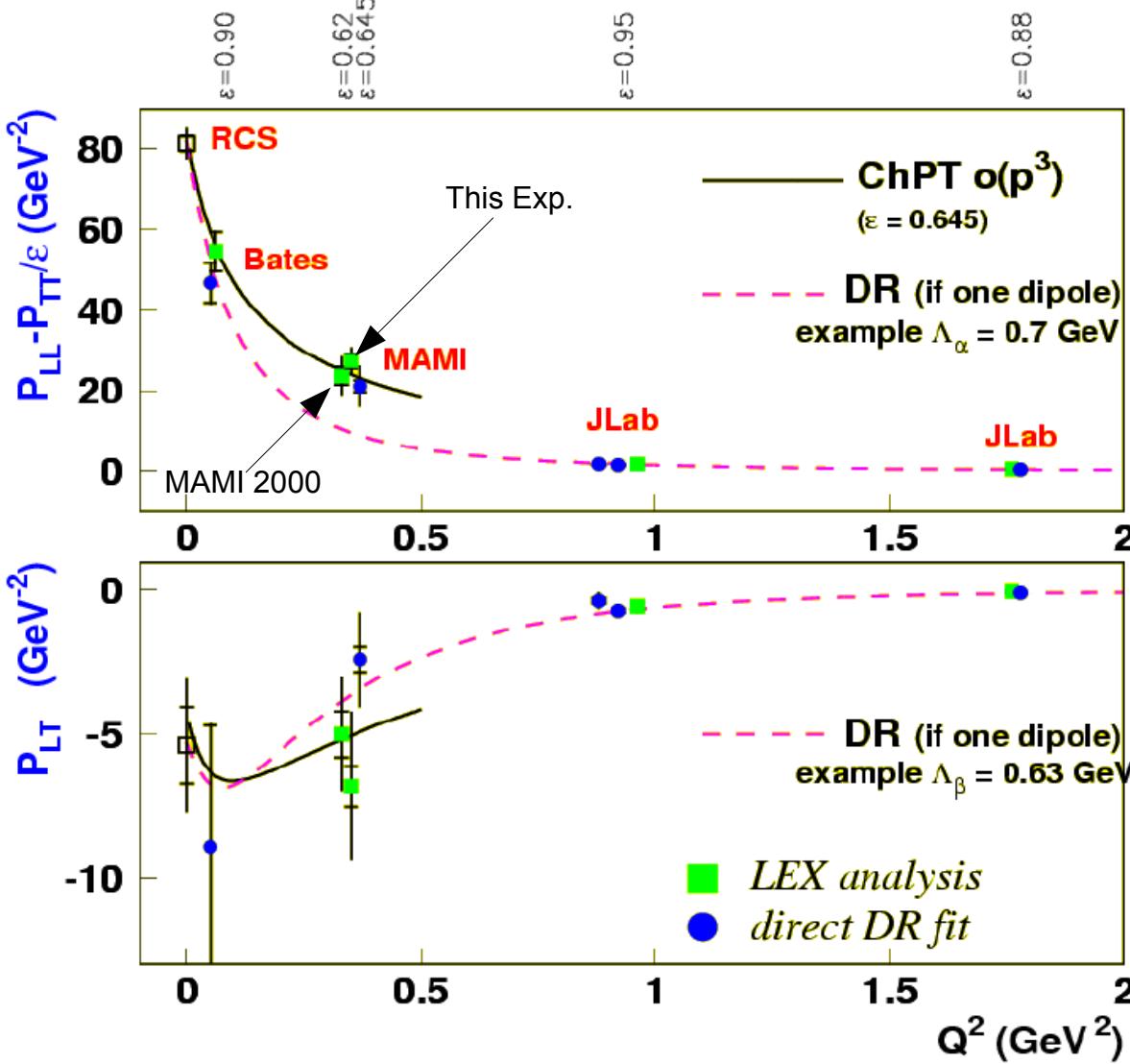
## Effective Lagrangian Model

A.Korchin et al. PRC 58 (1998) 1098

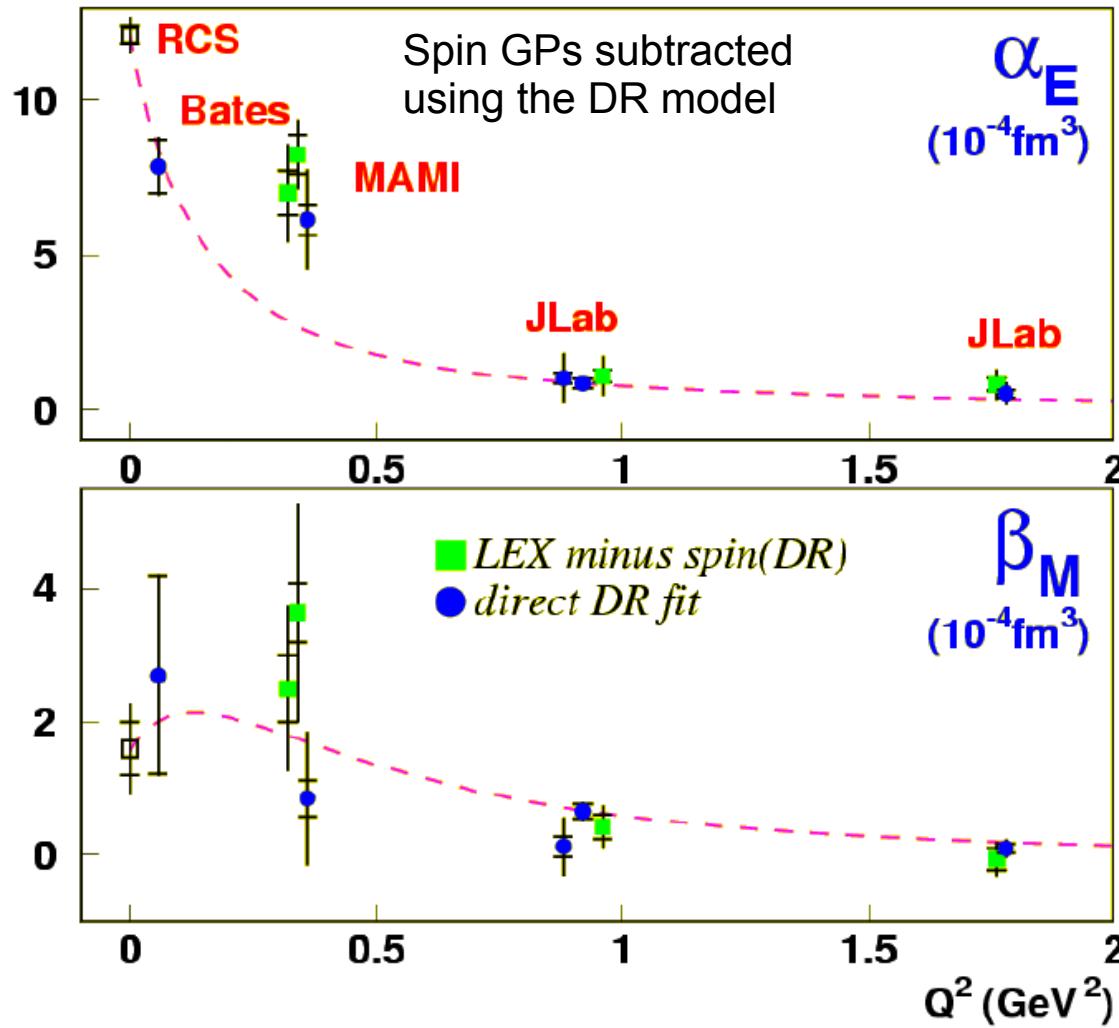
## Other ChPT calculations:

- **HBChPT O(4/5)**: Kao , Vanderhaeghen, PRD 70 (2004) 114004
- **Covariant ChPT O(4)** : D. Djukanovic (Mainz, PhD thesis)

# World Data: Structure Functions



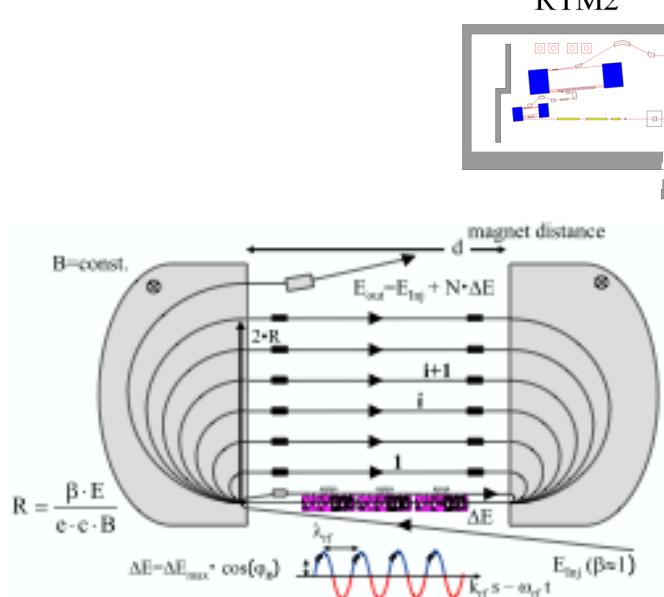
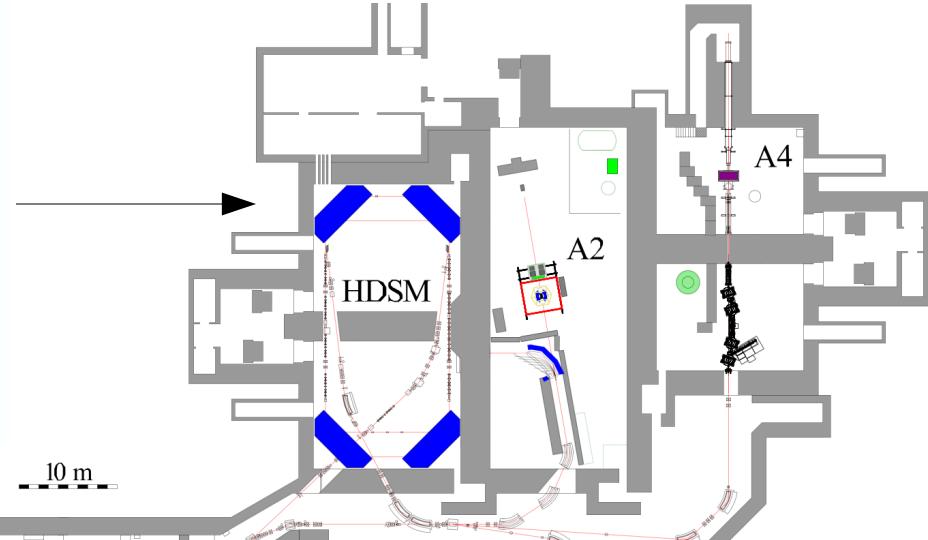
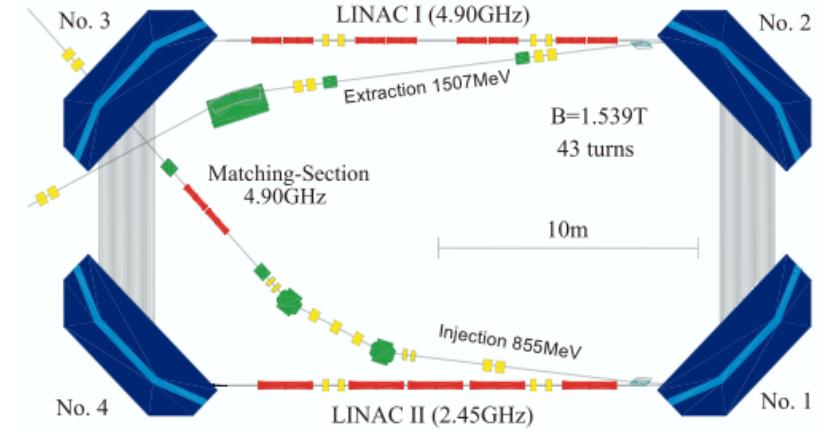
# World Data: Scalar GPs



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# Experimental Setup

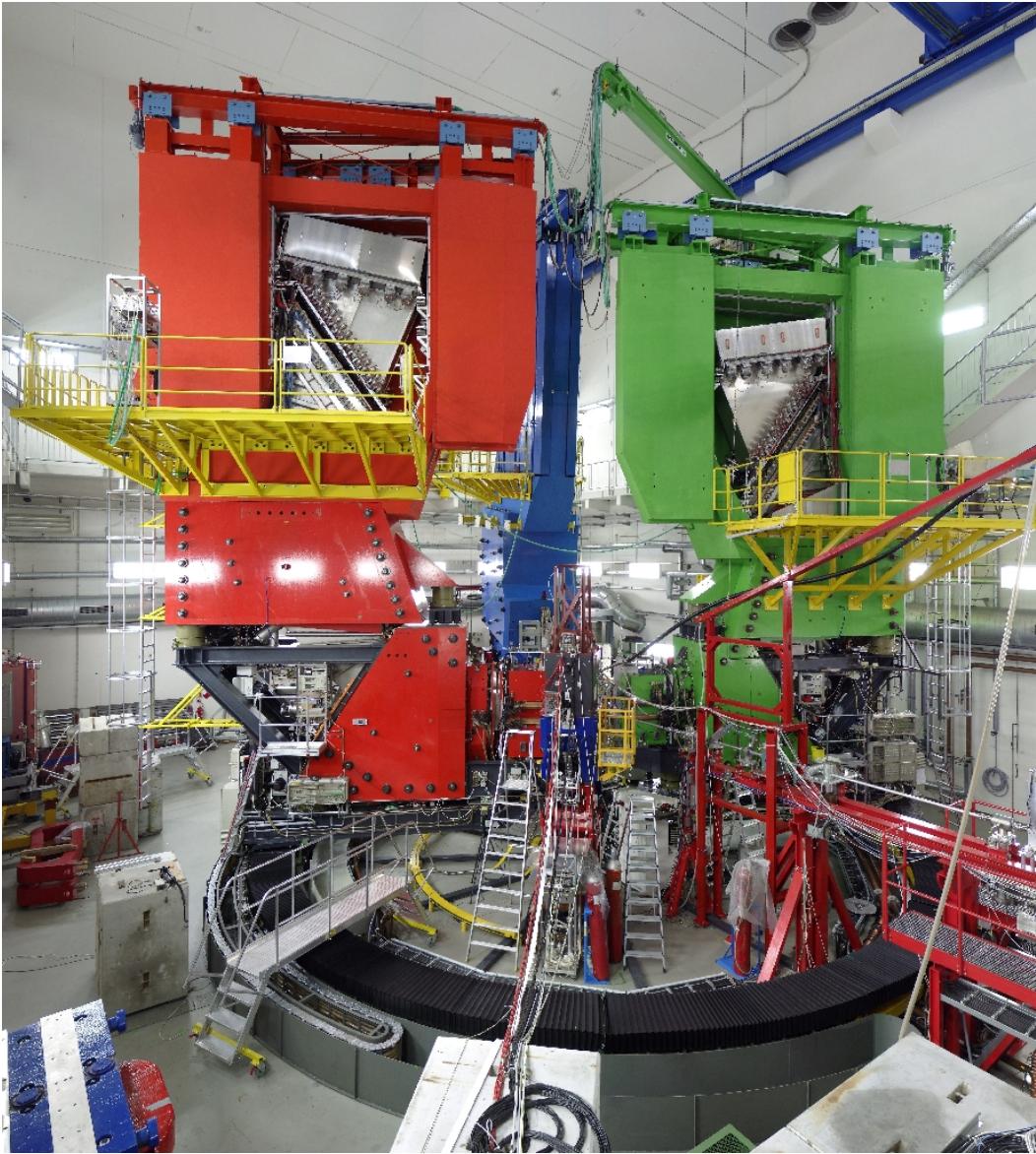
# The MAMI Accelerator Complex (KPH Mainz)



## MAMI Accelerator

- $E = 855 \text{ MeV}/c^2$
- Max. Current = 100 nA
- Duty Cycle: 100% (cw)
- Energy Spread 30 keV (FWHM)

# Experimental Setup



## 3 Spectrometers

- Momentum Resolution:  $10^{-4}$
- A,C: QSDD, Acceptance  $28\text{msr}$
- B: Clamshell, Acceptance  $5.6\text{msr}$

## Detectors

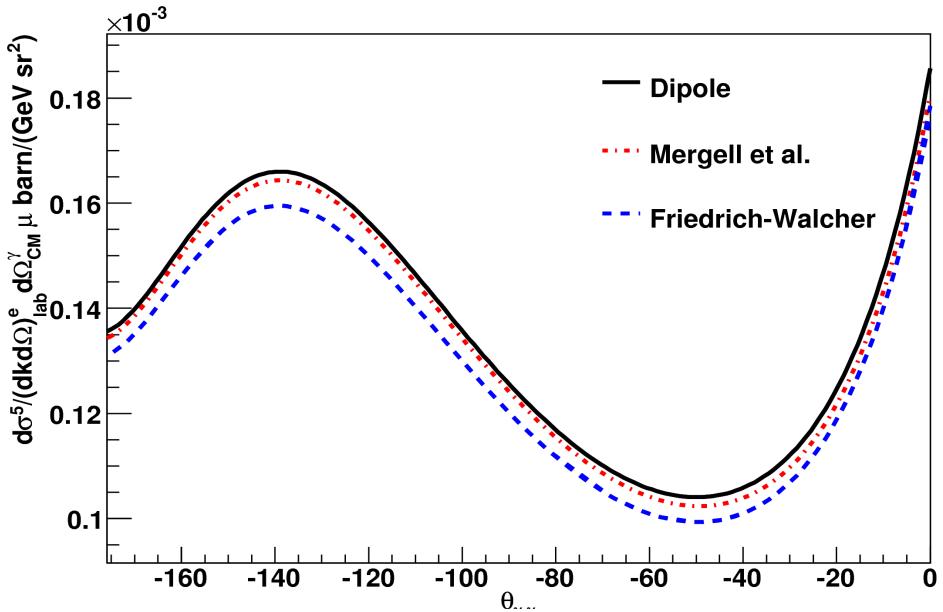
- Čerenkov Detector ( $e/\pi$  id.)
- Vertical Driftchambers (4 Planes)
- 2 Scintillator Planes

K. I. Blomqvist et al., NIM A403 (1998) 263-301

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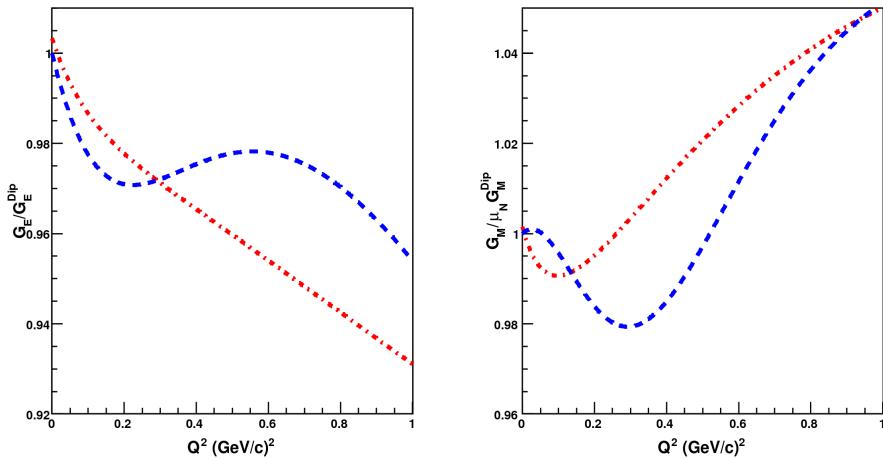
# Unpolarized CrossSection

# Form Factors



## Cross Section

- ~10% Difference
- 2 different analyses



## Parametrizations:

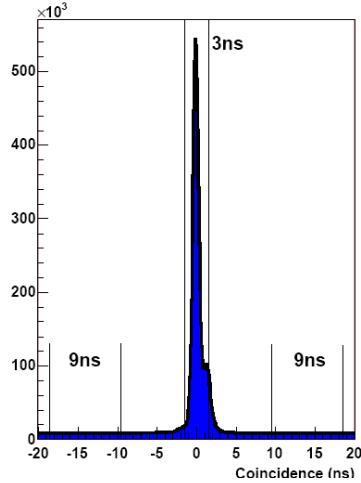
- Dipol: 
$$G_E(Q^2) = \frac{1}{(1+Q^2/\Lambda_d)^2}$$
- Mergell et al.
- Friedrich und Walcher

P.Mergell et al. , Nucl.Phys. A596 (1996) 367-391

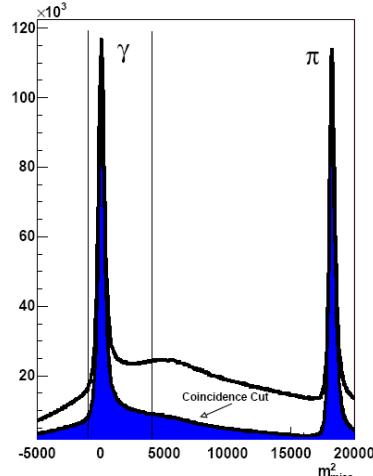
J. Friedrich and Th. Walcher, EPJ A 17 (2003) 607-623

# Cross Section

Coincidence



$$m^2 = (k+p-k'-p')^2$$



Mergell et al.

FW

HBChPT  $O(p^3)$

$$P_i - (1/\epsilon) P_{TT}$$

$$28.5 \pm 1.9 \pm 2.8$$

$$27.1 \pm 1.9 \pm 2.8$$

$$26.3/26.0 \text{ (GeV)}^2$$

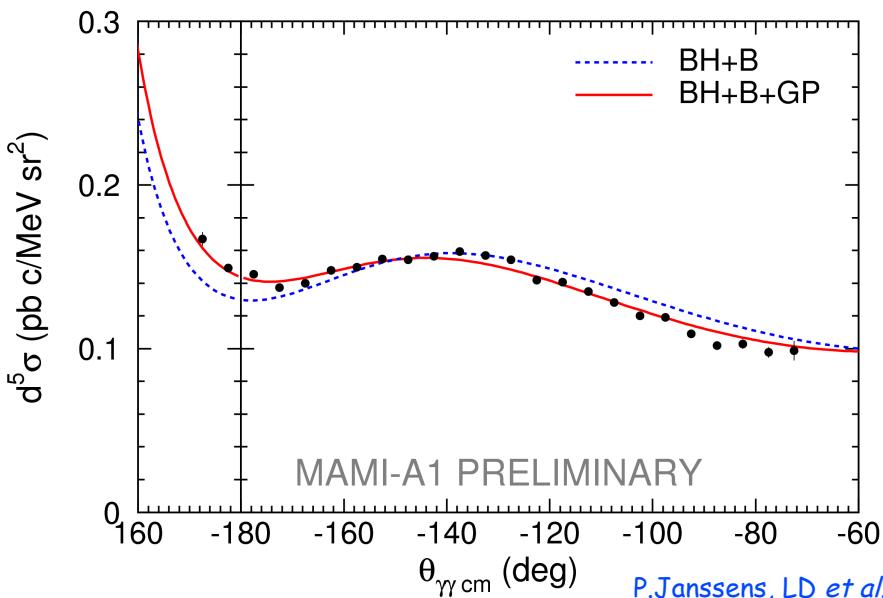
$$P_{LT}$$

$$-5.2 \pm 0.7 \pm 2.1$$

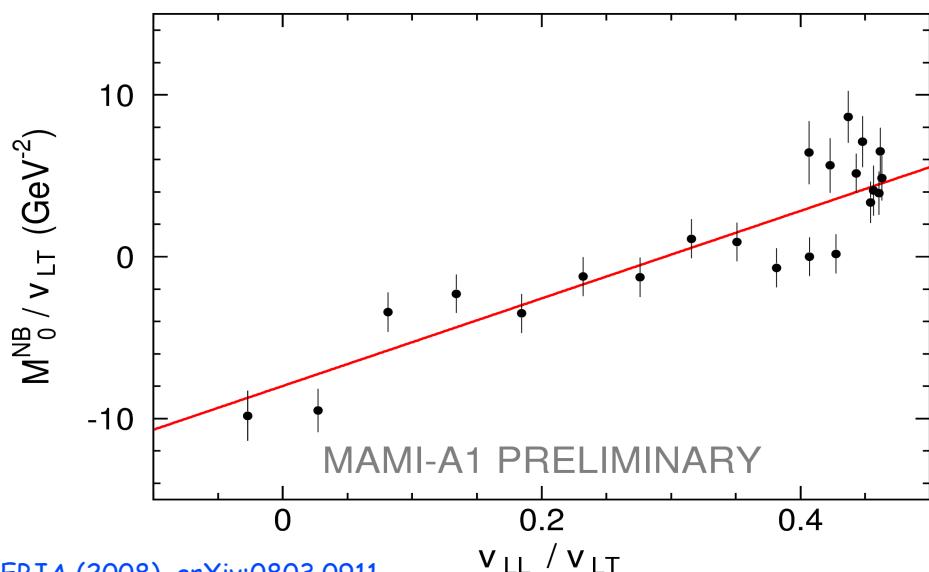
$$-8.0 \pm 0.7 \pm 2.1$$

$$-5.5/-5.4 \text{ (GeV)}^2$$

$$\chi^2/DoF = 2.42 \quad \chi^2/DoF = 2.63$$



P.Janssens, LD et al., EPJA (2008), arXiv:0803.0911

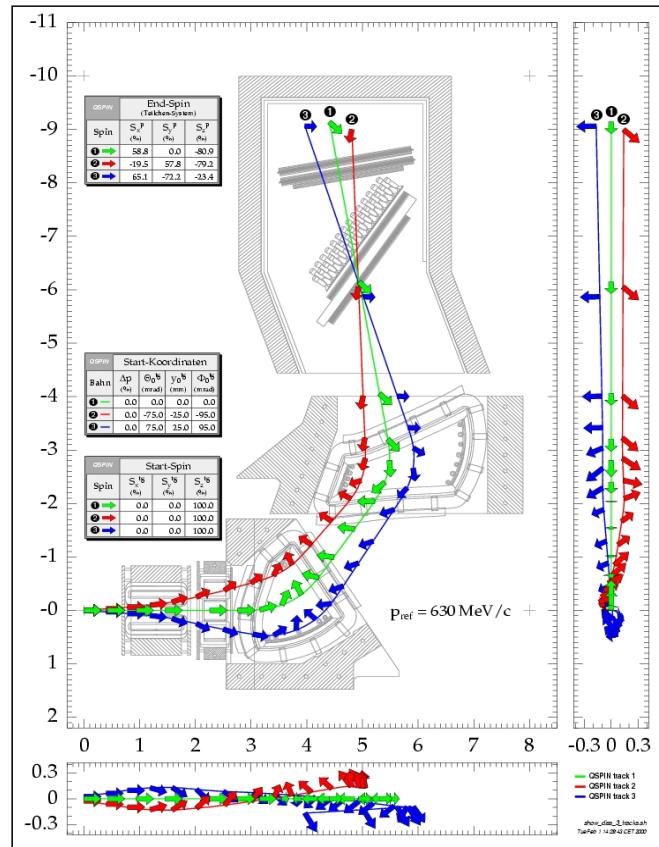
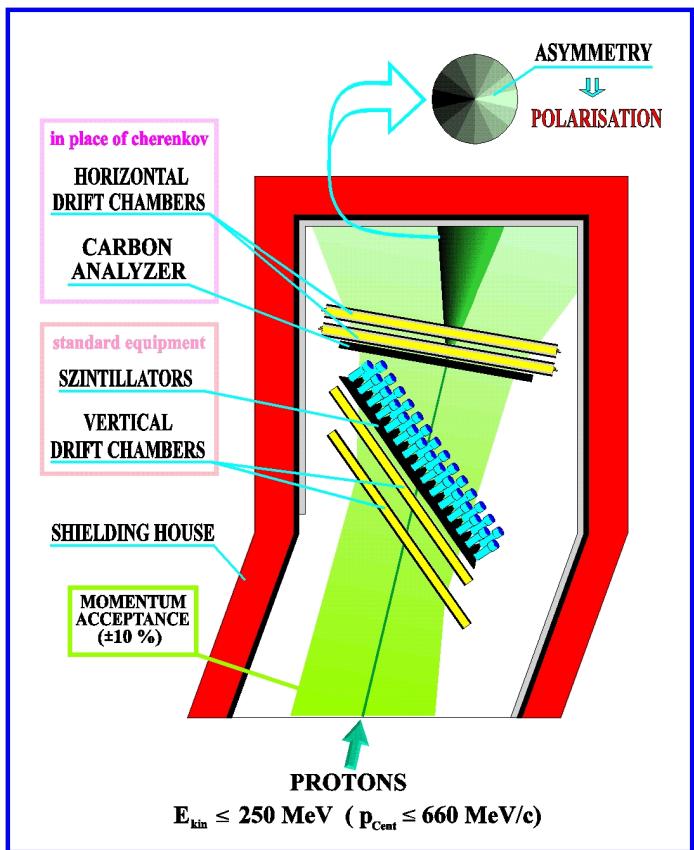


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# Double Polarization Observables

# Recoil Proton Polarimetry

Th. Pospischil, NIM A483 (2002) 746-733



- Cross Section  $pC^{12}$ : 
$$\sigma \propto 1 + P_b A_C(\Phi_i, E_i)(P_y^{fp} \sin \Phi_i - P_x^{fp} \cos \Phi_i)$$

Reconstruction of the Polarization

- 3 Components measurable
- Full spin precession calculated

Systematical Errors

- $P_x \sim \pm 0.8\%$
- $P_{y,z} \sim \pm 1.1\%$

# Maximum Likelihood Method

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Spin rotation, Lorentz Boost,..

- Parametrized in one single rotation
- 3 Parameters
- Computed for each event
- Rotation dependent on: Momentum, Target Coordinates, Spectrometer Angle

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix}^{fp} = \begin{pmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix}^{cm}$$

Likelihood:

- Three Polarization Components

$$\ln L = \sum_i \ln [1 + P_b h A_C(\Phi_i, E_i) (P_y^{fp} \sin \Phi_i - P_x^{fp} \cos \Phi_i)]$$

$$\ln L = \sum_i \ln [1 + P_b h A_C(\Phi_i, E_i) P_x^{cm} (a_{xx}^i \cos \Phi_i - a_{xy}^i \sin \Phi_i)]$$

$$+ P_y^{cm} (a_{yx}^i \cos \Phi_i - a_{yy}^i \sin \Phi_i)$$

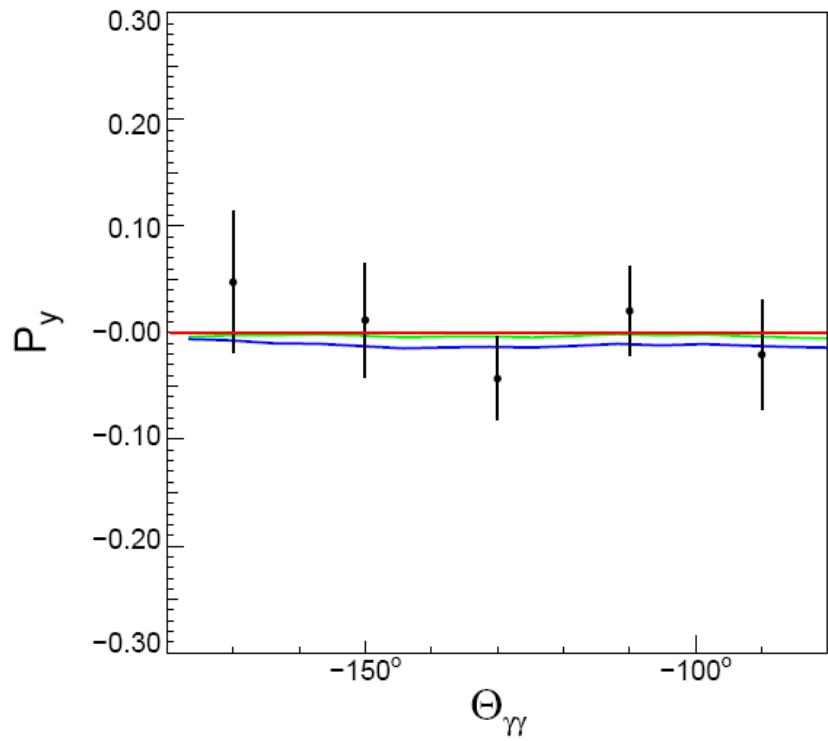
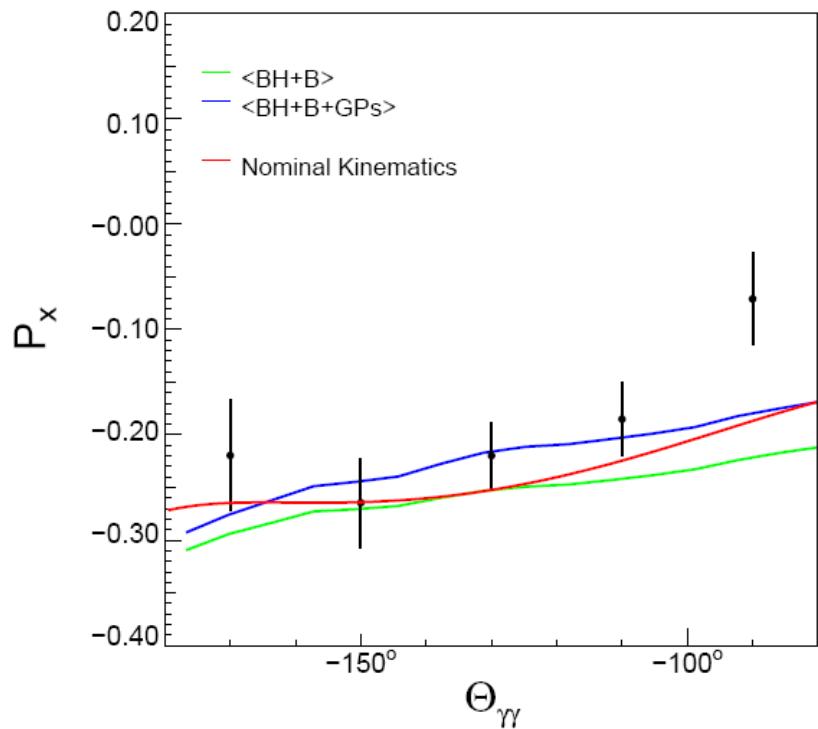
$$+ P_z^{cm} (a_{yz}^i \cos \Phi_i - a_{xz}^i \sin \Phi_i)]$$

# Maximum Likelihood Method

## General Procedure

- Measurement of the kinematics  
(FP angles, Target coordinates, ..)
  - Single Event Probability:  $p_i = 1 + P_b h A_C(\Phi_i, E_i) (P_y^{fp} \sin \Phi_i - P_x^{fp} \cos \Phi_i)$
  - Lorentz Boost, Spin Rotation  $P^{fp} \leftarrow \rightarrow P^{cm}$
  - Likelihood:  $L = \prod_i p_i \rightarrow \ln L = \sum_i \ln p_i$
  - Maximize:  $\frac{\partial \ln L}{\partial P_{x,y,z}^{cm}} = 0 \rightarrow \langle P_{x,y,z}^{cm} \rangle$
  - Statistical Error:  $\sigma_{ij} = \sqrt{\left( \frac{\partial^2 \ln L}{\partial P_i \partial P_j} \right)^{-1}}$
- Advantages
- No binning needed  
Statistically robust  
Efficient use of the information  
Fast algorithm on modern PCs

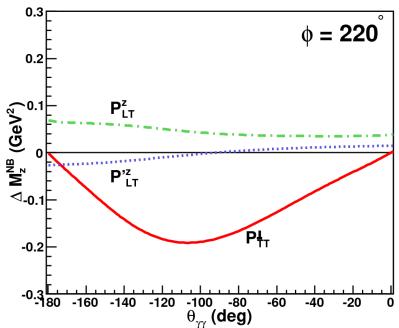
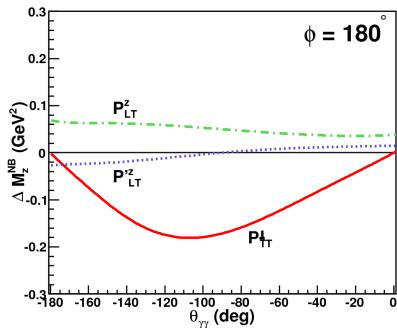
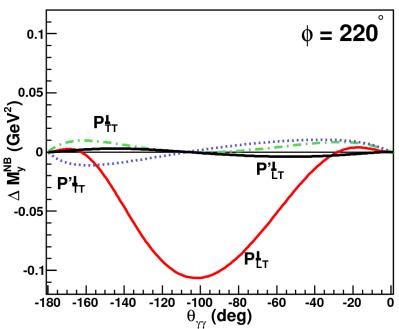
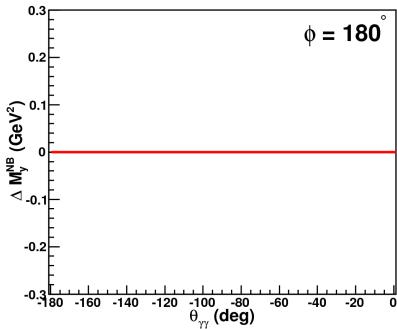
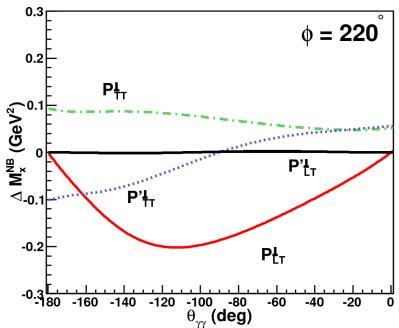
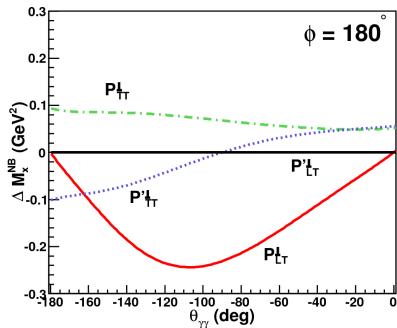
# Double Polarization Observables



- False and background asymmetries negligible

- $P_x$  bigger than  $BH+B$
- $P_y$  consistent with 0
- $P_z$  fixed on  $BH+B$ 
  - Small correlation with  $P_x$

# Sensibility to the Structure Functions



$$P_{x,y,z} = \frac{d^5\sigma^{\uparrow\uparrow} + d^5\sigma^{\downarrow\downarrow} - d^5\sigma^{\uparrow\downarrow} - d^5\sigma^{\downarrow\uparrow}}{d^5\sigma^{\uparrow\uparrow} + d^5\sigma^{\downarrow\downarrow} + d^5\sigma^{\uparrow\downarrow} + d^5\sigma^{\downarrow\uparrow}} = \frac{d^5\sigma^{h\uparrow} - d^5\sigma^{h\downarrow}}{2 d^5\sigma}$$

$$\Delta d^5\sigma_{x,y,z}^h = \Delta d^5\sigma_{x,y,z}^{BH+Born} + \phi q' \Delta \Psi_0^{x,y,z} + \phi O(q'^2)$$

$$\Psi_0 = v_1(\mathbf{P}_{\text{LL}} - \mathbf{P}_{\text{TT}}/\varepsilon) + v_2 \mathbf{P}_{\text{LT}}$$

$$\Delta \Psi_0^z = 4h [v_1^z \mathbf{P}_{\text{TT}} + v_2^z \mathbf{P}_{\text{LT}}^z + v_3^z \mathbf{P}'_{\text{LT}}^z]$$

$$\Delta \Psi_0^x = 4h [v_1^x \mathbf{P}_{\text{LT}}^\perp + v_2^x \mathbf{P}_{\text{TT}}^\perp + v_3^x \mathbf{P}'_{\text{TT}}^\perp + v_4^x \mathbf{P}'_{\text{LT}}^\perp]$$

$$\Delta \Psi_0^y = 4h [v_1^y \mathbf{P}_{\text{LT}}^\perp + v_2^y \mathbf{P}_{\text{TT}}^\perp + v_3^y \mathbf{P}'_{\text{TT}}^\perp + v_4^y \mathbf{P}'_{\text{LT}}^\perp]$$

- Larger effect from  $P_{\text{LT}}^\perp$

- Few Information from  $P_{\gamma}$

- $P_z$  (still) not precise enough

# Results (MAMI Double Pol. Experiment)

Form Factors	$P_{LT}^\perp \text{ (GeV}^{-2}\text{)}$
Arrington <i>et al.</i> [1]	$-17.6 \pm 3.3$
Hammer <i>et al.</i> [2]	$-17.7 \pm 3.3$
F-W [3]	$-17.8 \pm 3.3$

Overall Systematical Error: +2.12, -0.38

Sources:

- Beam Polarization
- Constraints
- CrossSection Input
- Effect of other Structure Functions

Theoretical Models

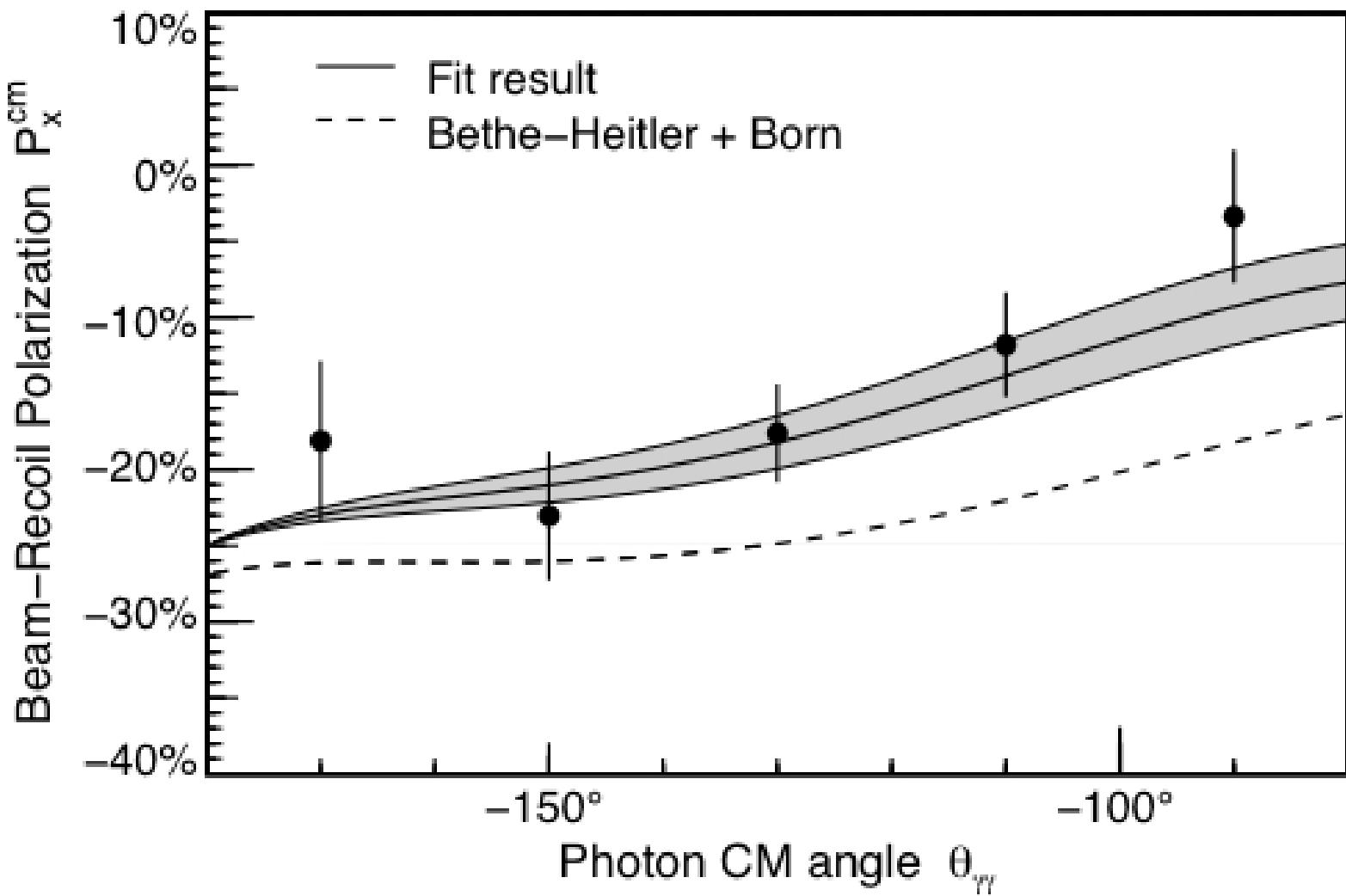
$$\text{HBChPT } O(p^3): P_{LT}^\perp = -10.4 \text{ GeV}^{-2}$$
$$\text{DR Model: } P_{LT}^\perp = -12.7 \text{ GeV}^{-2}, \Lambda_\alpha = \infty$$

[1] J. Arrington et al. PRC 76, 035205 (2007)

[2] H.-W. Hammer et al., PRC 75, 035202 (2007)

[3] J.Friedrich, Th.Walcher, EPJ A17, 607 (2003)

# Extraction of $P_{LT}^\perp$



LD, P.Janssens, et al., in preparation

# Conclusions

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## Virtual Compton Scattering

- Intuitive physical interpretation
- Fundamental as the Form Factors
- Contribution from different low- $q$  QCD degrees of freedom
- New test for the nucleon models

## Experimental Activity

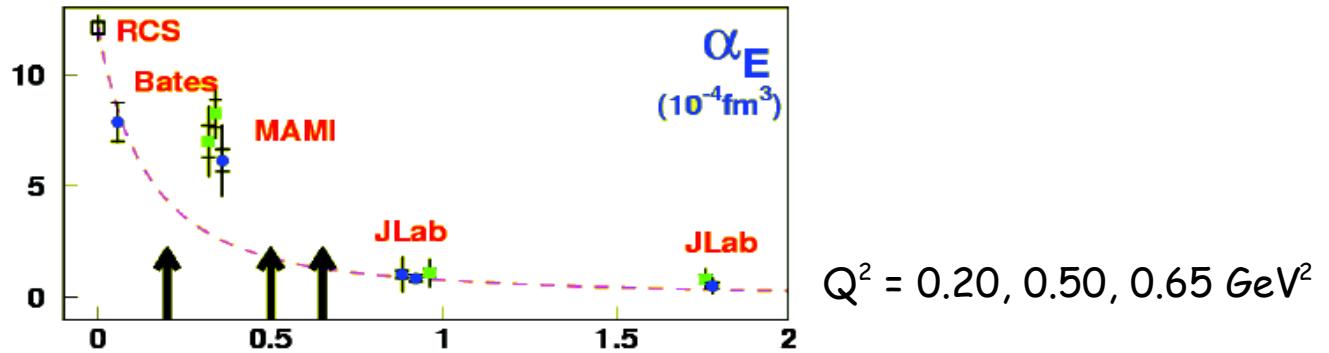
- Measurement through Photonelectroproduction
- First unpolarized experiment at MAMI ( $Q^2=0.33 \text{ GeV}^2/c^2$ )
- Experiments at MIT-Bates ( $Q^2=0.05 \text{ GeV}^2/c^2$ ) and JLab ( $Q^2= 0.92 ; 1.76 \text{ GeV}^2/c^2$ )
- Measurement of the Single Spin Asymmetry at MAMI [I. Bensafa et al., Eur. Phys. J. A 32, \(2007\) 69-75](#)
- NOW: First Double polarization observables

# Outlook

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## The Future

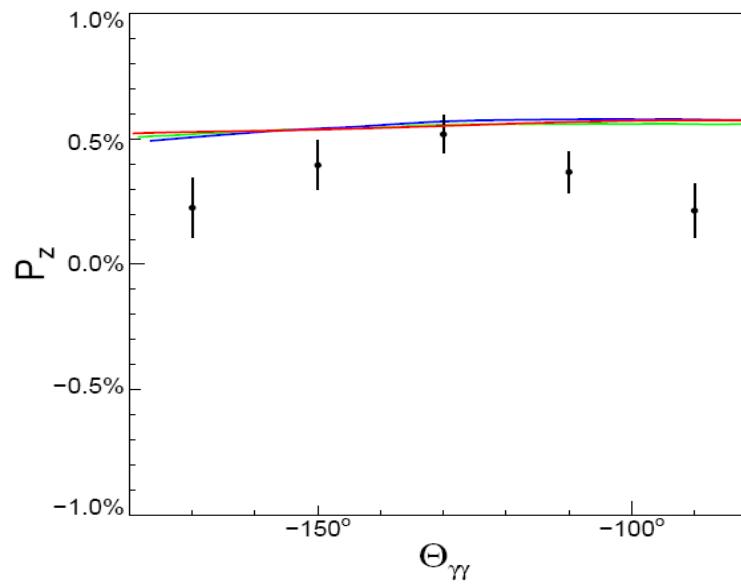
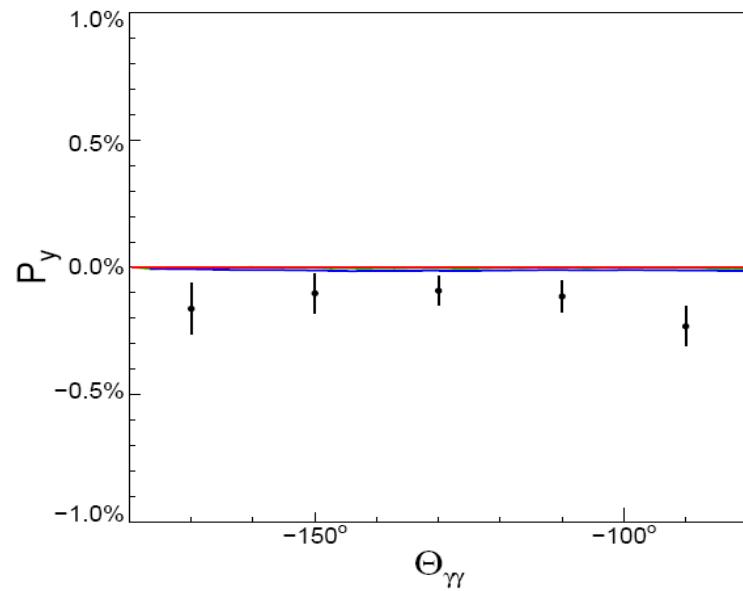
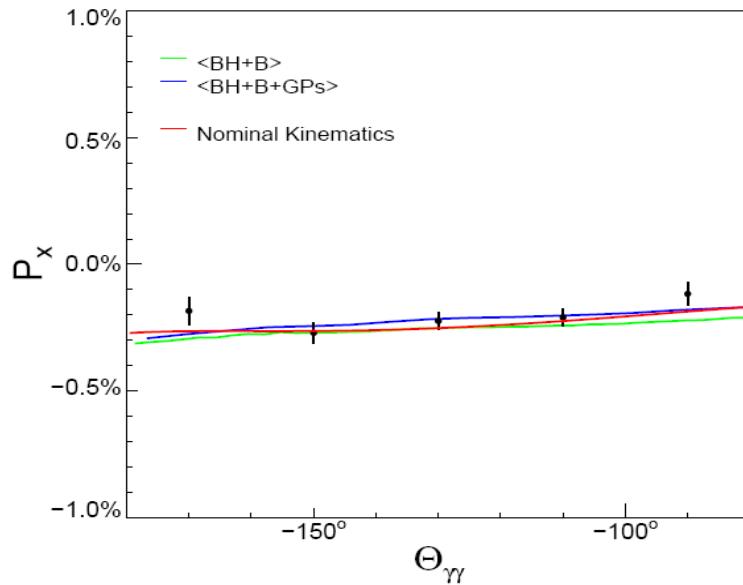
- More Statistics for the double polarization experiments: Separation of the GPs
- Measurement of new kinematics for clarifying the  $Q^2$  dependence of  $\alpha$  and  $\beta$



- New kinematical ranges with MAMI-C



# Double Polarization Observables



Reconstruction of 3 components in the CM system

$P_x$  well reconstructed

- Small spin precession

Correlation between  $P_y$  and  $P_z$

- Strong Spin Precession
- How to correct? → Constraint