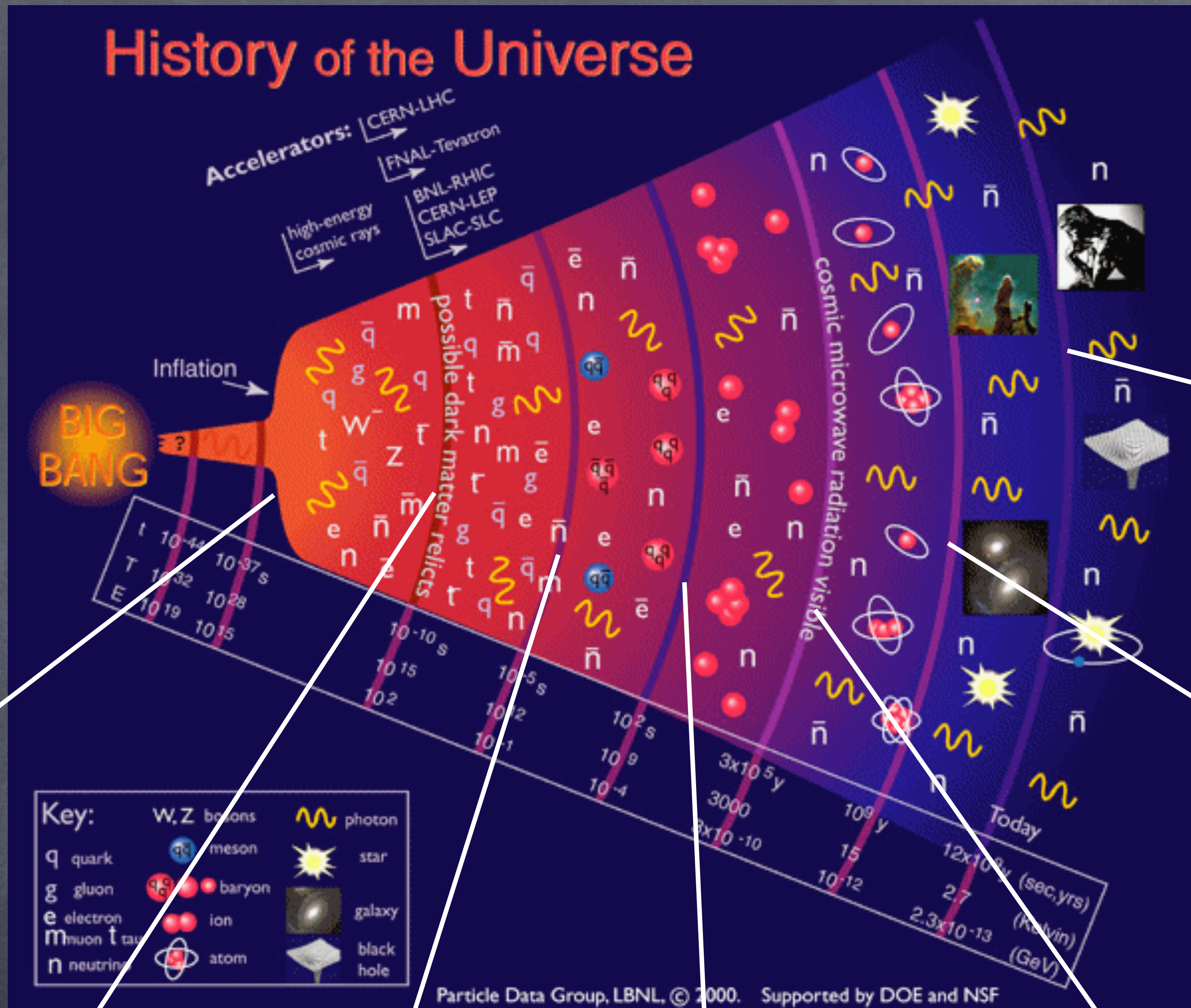
Cosmological Magnetic Fields



today

inflation

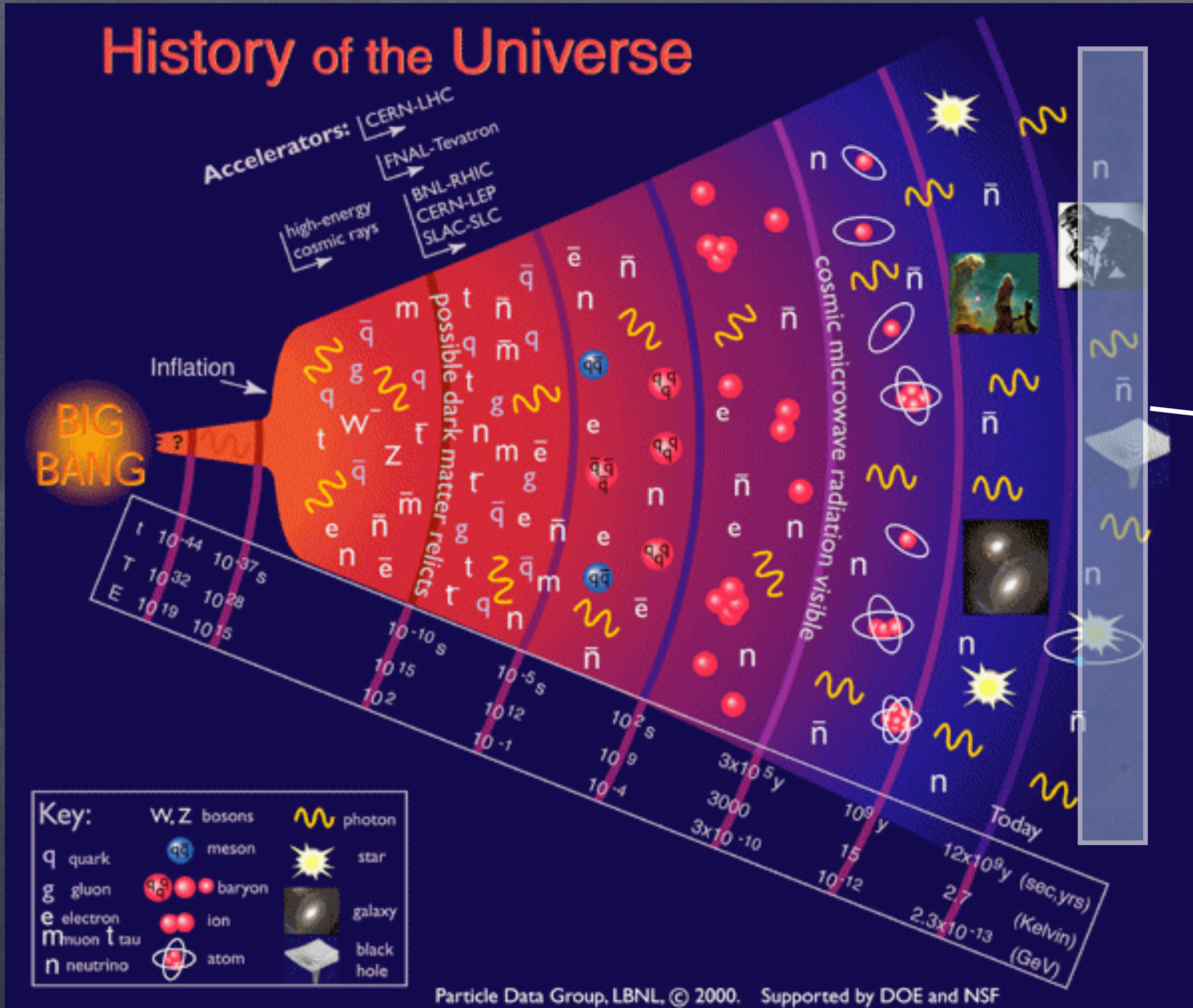
structure
formation
reionisation

EW phase
transition

QCD phase
transition

nucleosynthesis
neutrino decoupling
electron non relativistic

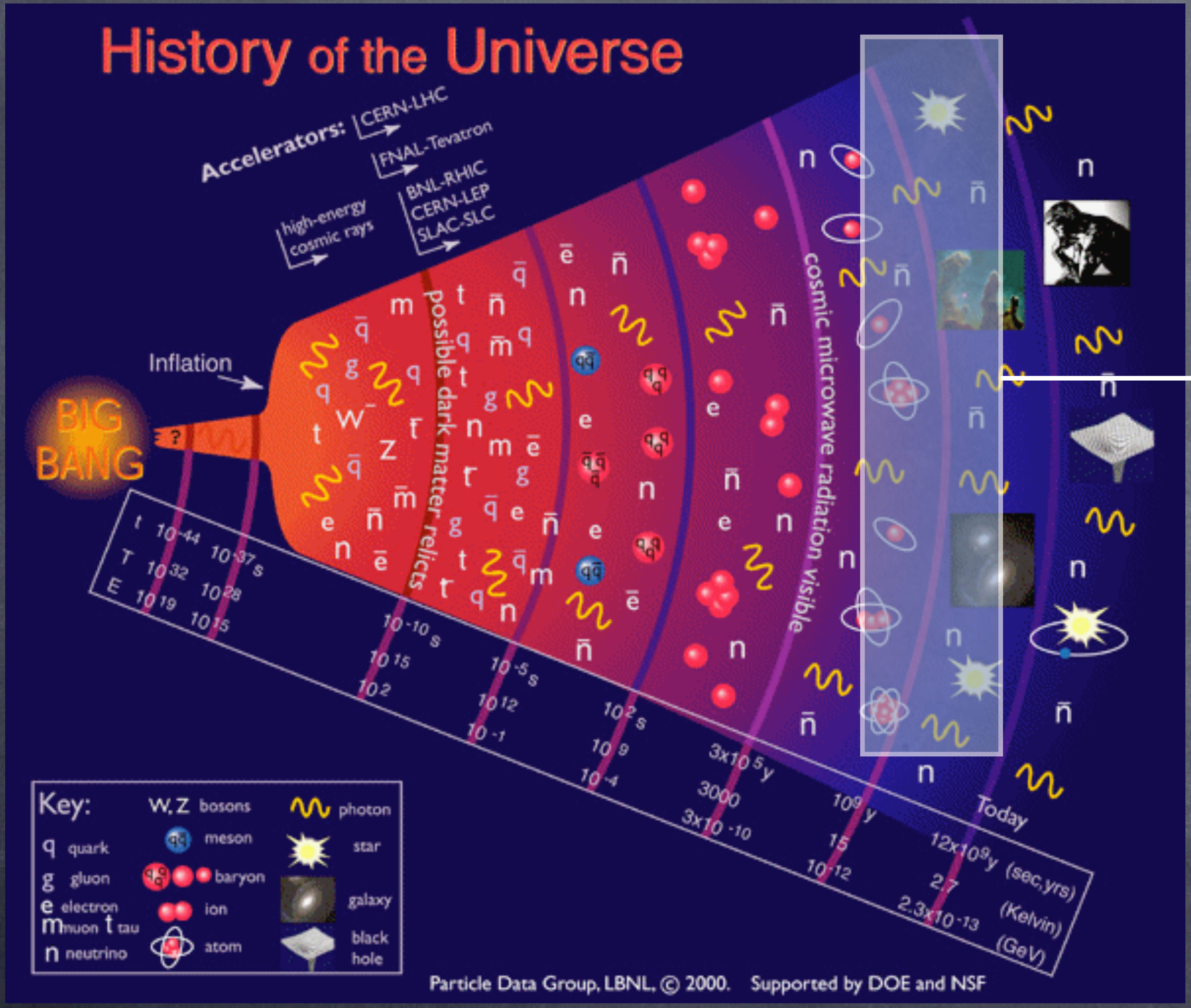
recombination
formation of the CMB
equality



Today: large scale magnetic fields are observed in galaxies and clusters

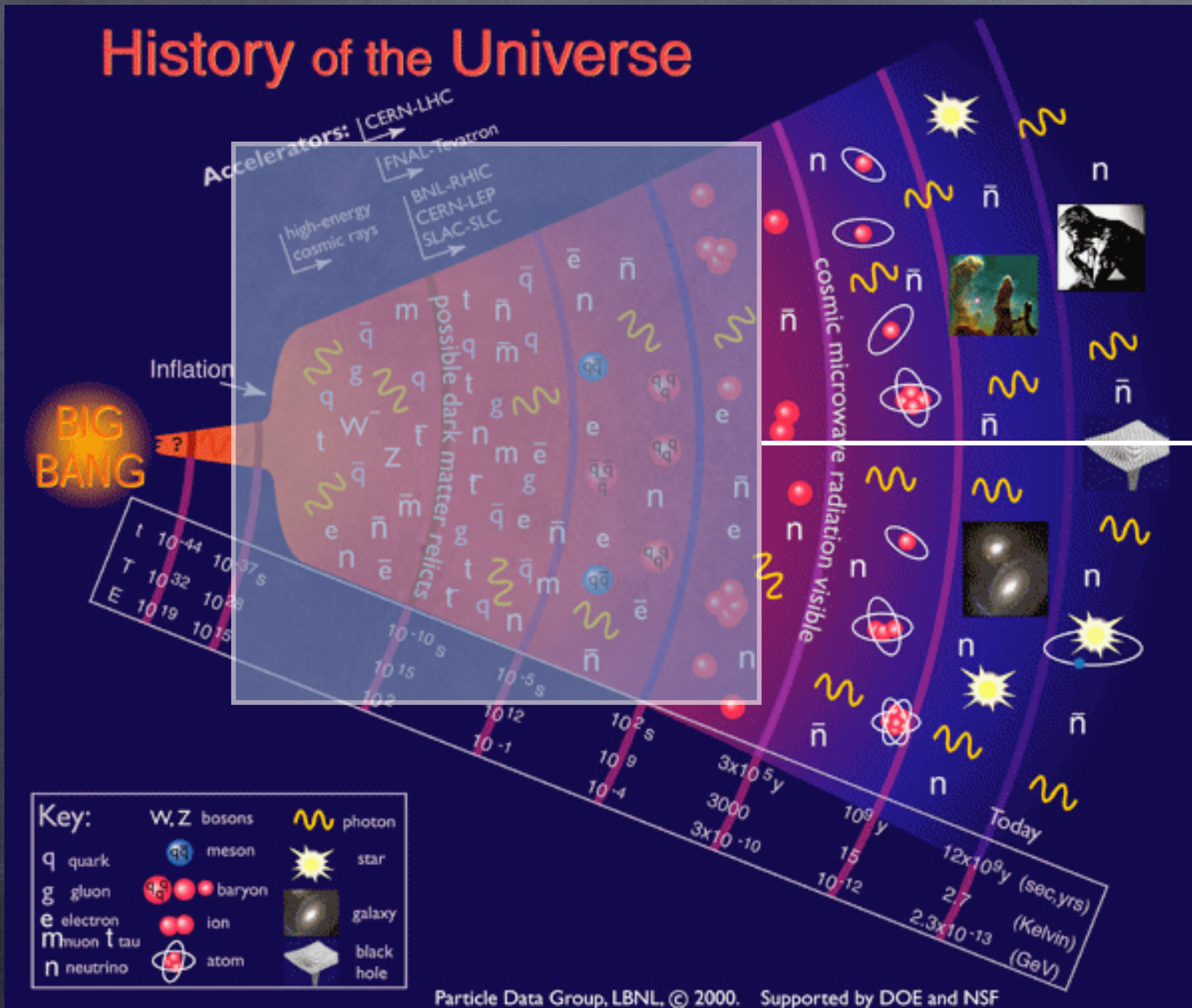
PROBLEM: WHAT IS THE ORIGIN OF THESE MAGNETIC FIELDS?

TWO POSSIBILITIES FOR THE ORIGIN:



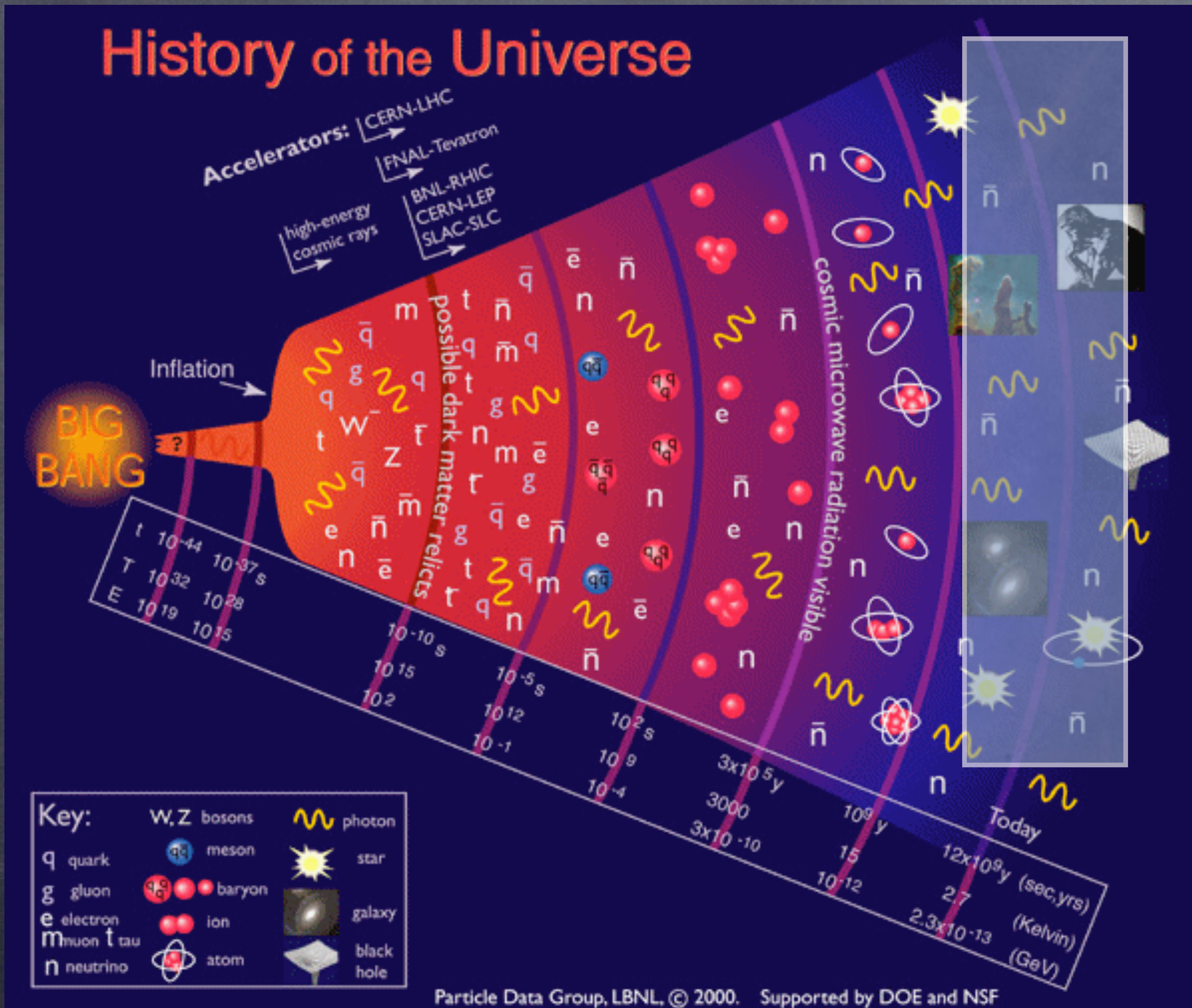
GENERATED DURING THE FORMATION OF STRUCTURES

TWO POSSIBILITIES FOR THE ORIGIN:



GENERATED IN THE PRIMORDIAL UNIVERSE

OUTLINE OF THE SEMINAR:

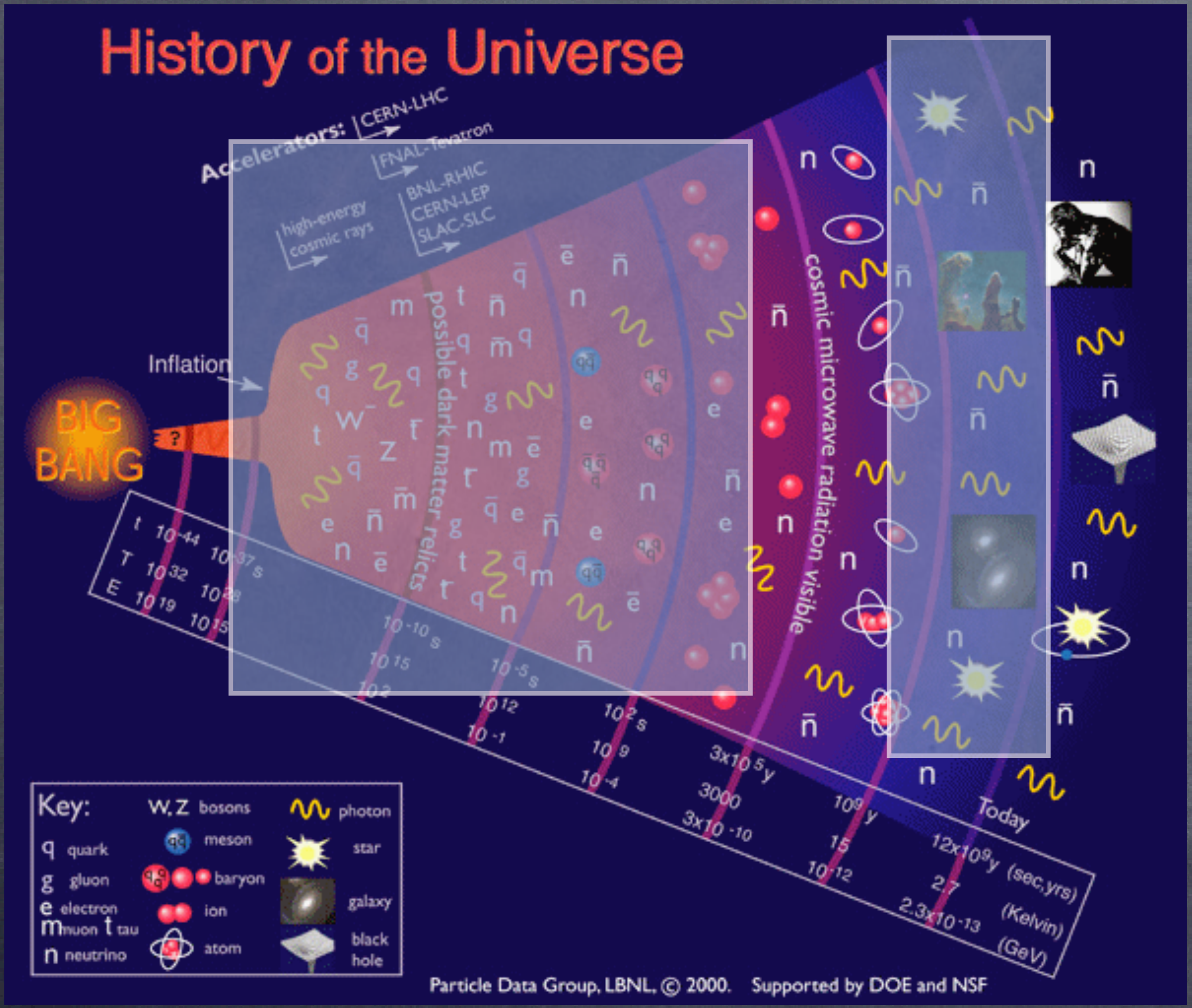


OBSERVATIONAL TECHNIQUES

CHARACTERISTICS OF THE FIELDS

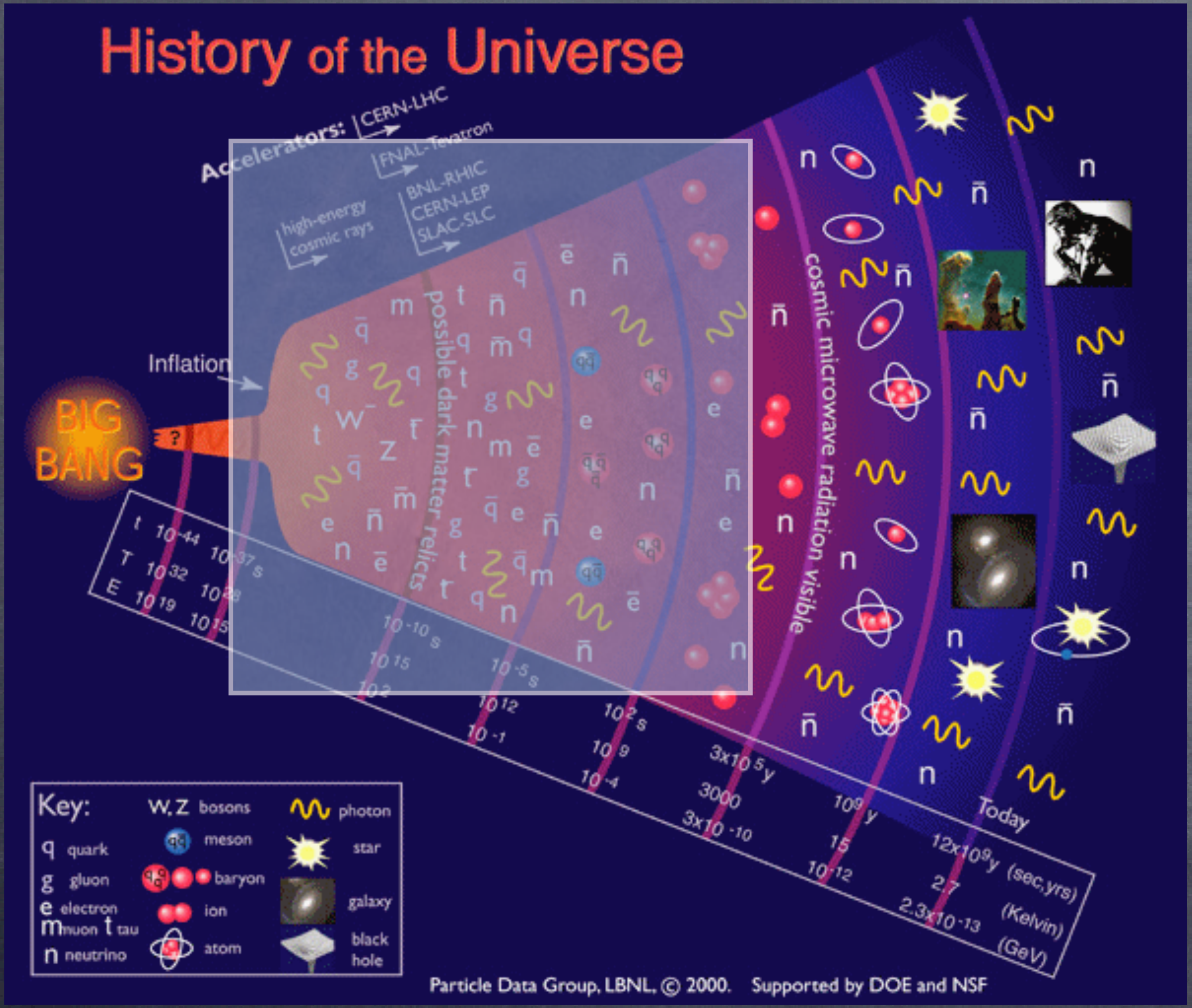
AMPLIFICATION DURING STRUCTURE FORMATION

OUTLINE OF THE SEMINAR:



POSSIBLE GENERATION MECHANISMS

OUTLINE OF THE SEMINAR:



IF THEY ARE PRIMORDIAL

EVOLUTION

CONSTRAINTS

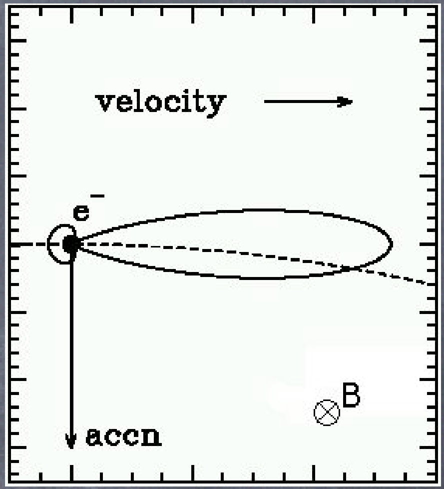
OBSERVATIONAL EFFECTS

OBSERVATIONS – Techniques

- Zeeman splitting: direct measure from hydrogen emission lines
  used only for the Milky Way

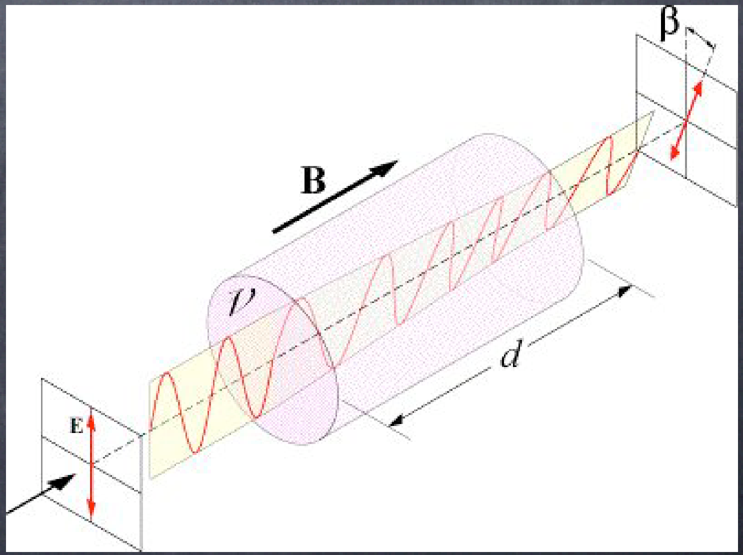
- Synchrotron radiation:

  plane of the sky component
  independent measure of electron density



- Faraday rotation:

  line of sight component
  independent measure of electron density
  several wavelengths (proportional to λ^2)



(Grasso and Rubinstein 2001, Giovannini 2003)

OBSERVATIONS – Galactic magnetic fields

General characteristics:

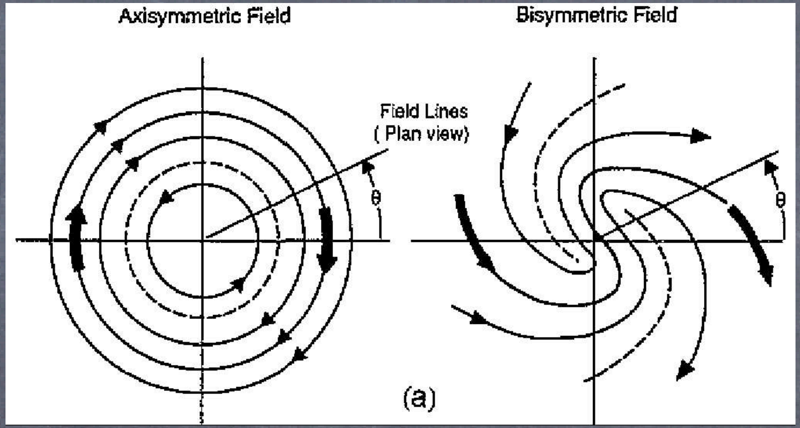
$$B_{\rm tot}^2 = B_{\rm u}^2 + B_{\rm r}^2$$

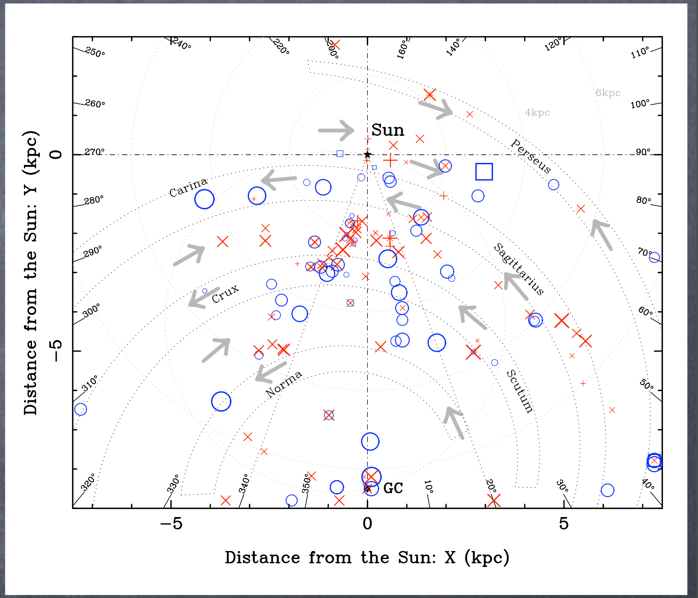
$$B_{\rm u} \simeq \mu{\rm G}$$

$$B_{\rm r} \simeq \text{several } \mu{\rm G}$$

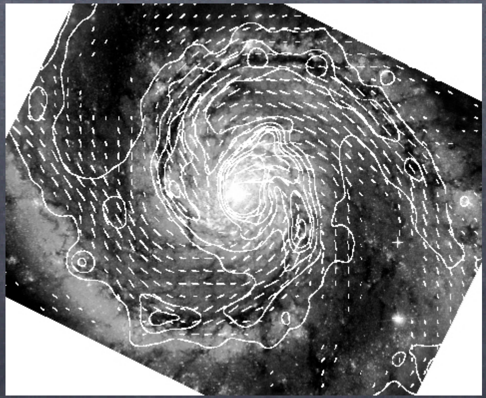
$$\ell_{\rm corr} \simeq {\rm kpc}$$

Structure:





- Axisymmetric or bisymmetric?
- Equipartition holds?
- Follows optical spiral arms?



M51

(Zweibel and Heiles 1997, Han 2007...)

- Dynamically important and not yet completely understood

OBSERVATIONS - Cluster magnetic fields

General characteristics: $B \simeq \mu\mathrm{G}$ $\ell_{\mathrm{corr}} \simeq 1 - 100\ \mathrm{kpc}$

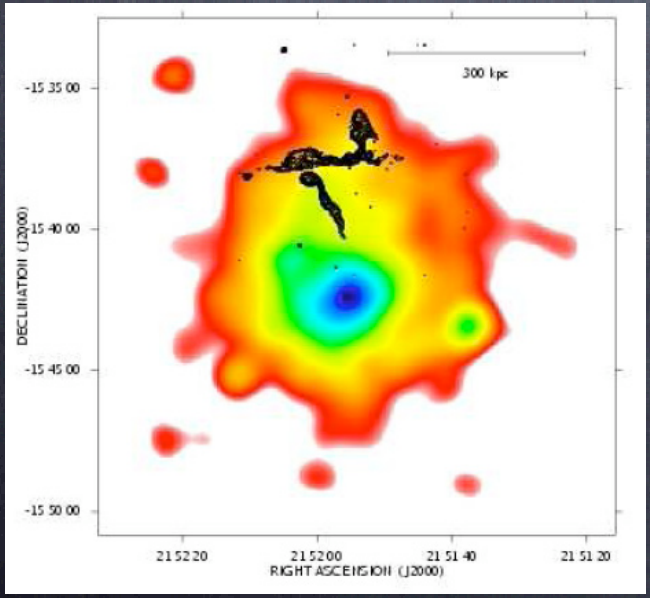
Examples:

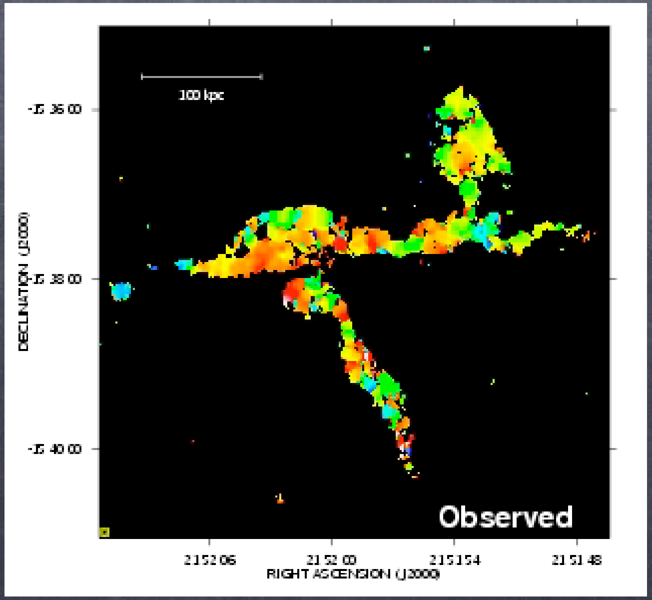
- Synchrotron emission of cluster wide diffuse sources: COMA

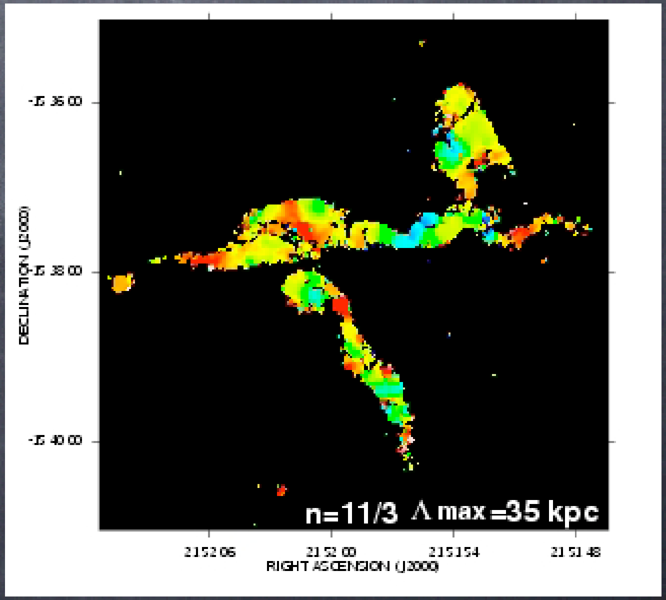
  Average value: $B \simeq 0.1 - 1\mu\mathrm{G}$ (Carilli and Taylor 2002)

- Faraday rotation of radio sources inside or behind the cluster + X-ray observation of the hot gas + simulated images

Abell 2382 (Guidetti et al 2007)



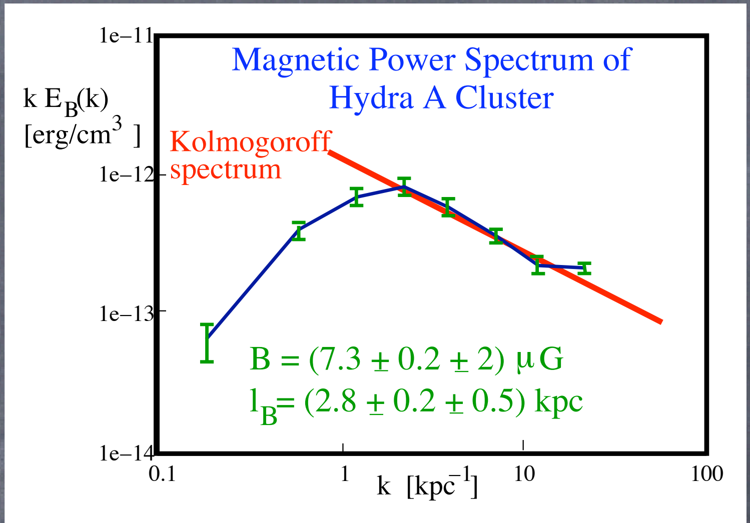




OBSERVATIONS – Cluster magnetic fields

Abell 2382 $B \simeq 3.3\mu G$ Kolmogorov spectrum $\Lambda_{max} \simeq 35 kpc$

Hydra A $B \simeq 7\mu G$ $\Lambda_{max} \simeq 3 kpc$

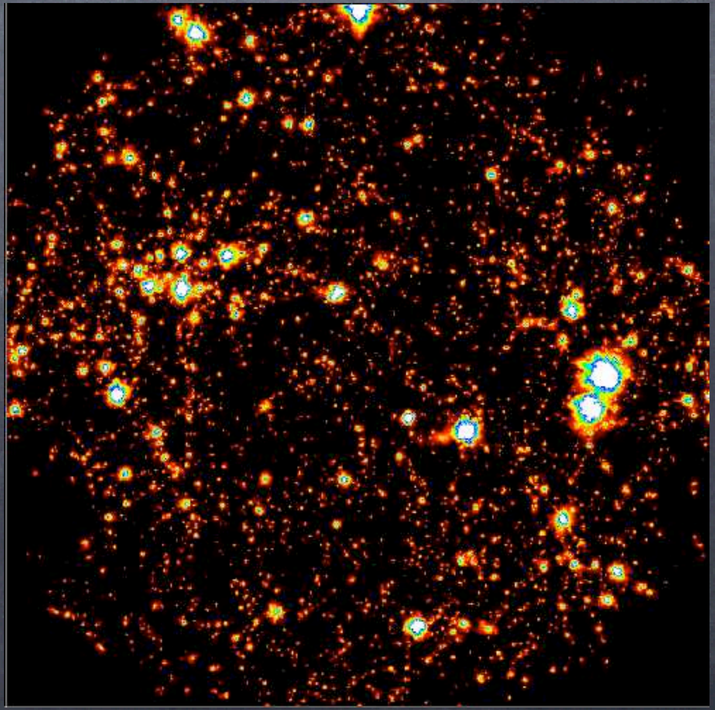


(Vogt and Ensslin 2005)

- Very detailed MHD simulations

(Dubois and Teyssier 2008, Donnert et al 2008...)

$B \simeq 5\mu G$



200 Mpc

High redshift objects:

(Athreya et al 98, Pentericci et al 02, Kronberg et al 07...)

Faraday rotation in radio proto-galaxies and quasars at z>2

$B \simeq \mu\text{G}$ $\ell_{\text{corr}} \simeq \text{kpc}$

OBSERVATIONS - Summary

- Magnetic fields of order microGauss in all observed objects
- Correlated on scales of the order of the object size
- Probably grown in short times (or present previously)

$$\Omega_B = \frac{B^2}{\rho_c} \simeq 0.06\,\Omega_{\text{rad}} \left(\frac{B}{\mu\text{G}}\right)^2$$

Sun: 1 G, Earth: 1/2 G

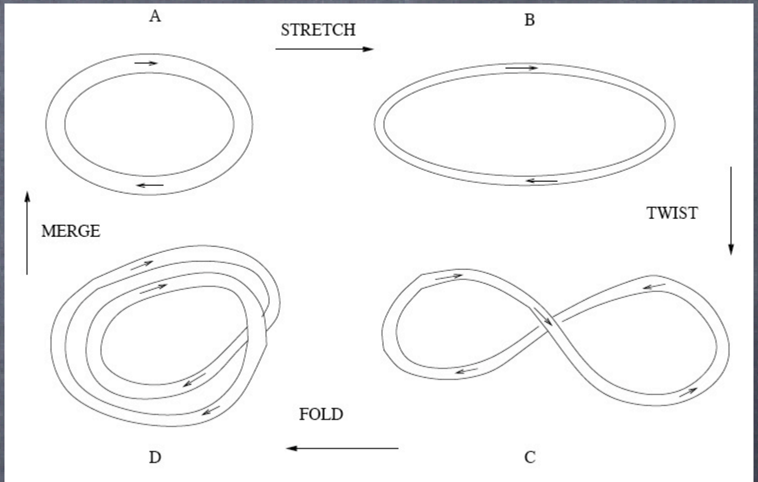
OBSERVATIONS – Amplification

High electrical conductivity ⟶ Conservation of magnetic flux

- Amplification by structure collapse:

$$B_{\rm fin} = B_{\rm in}\left(\frac{L_{\rm in}}{L_{\rm fin}}\right)^2 = B_{\rm in}\left(\frac{\rho_{\rm fin}}{\rho_{\rm in}}\right)^{2/3} \quad \begin{cases} B_{\rm in}^{\rm gal} \simeq 10^{-10}{\rm Gauss} \\ B_{\rm in}^{\rm clu} \simeq 10^{-8}{\rm Gauss} \end{cases}$$

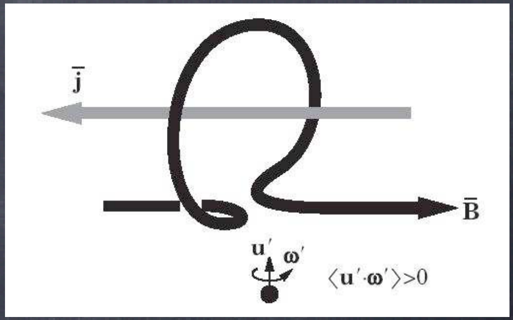
- Amplification by galactic dynamo:



INGREDIENTS high conductivity, turbulence, differential rotation

RESULTS exponential growth to equipartition, axisymmetric

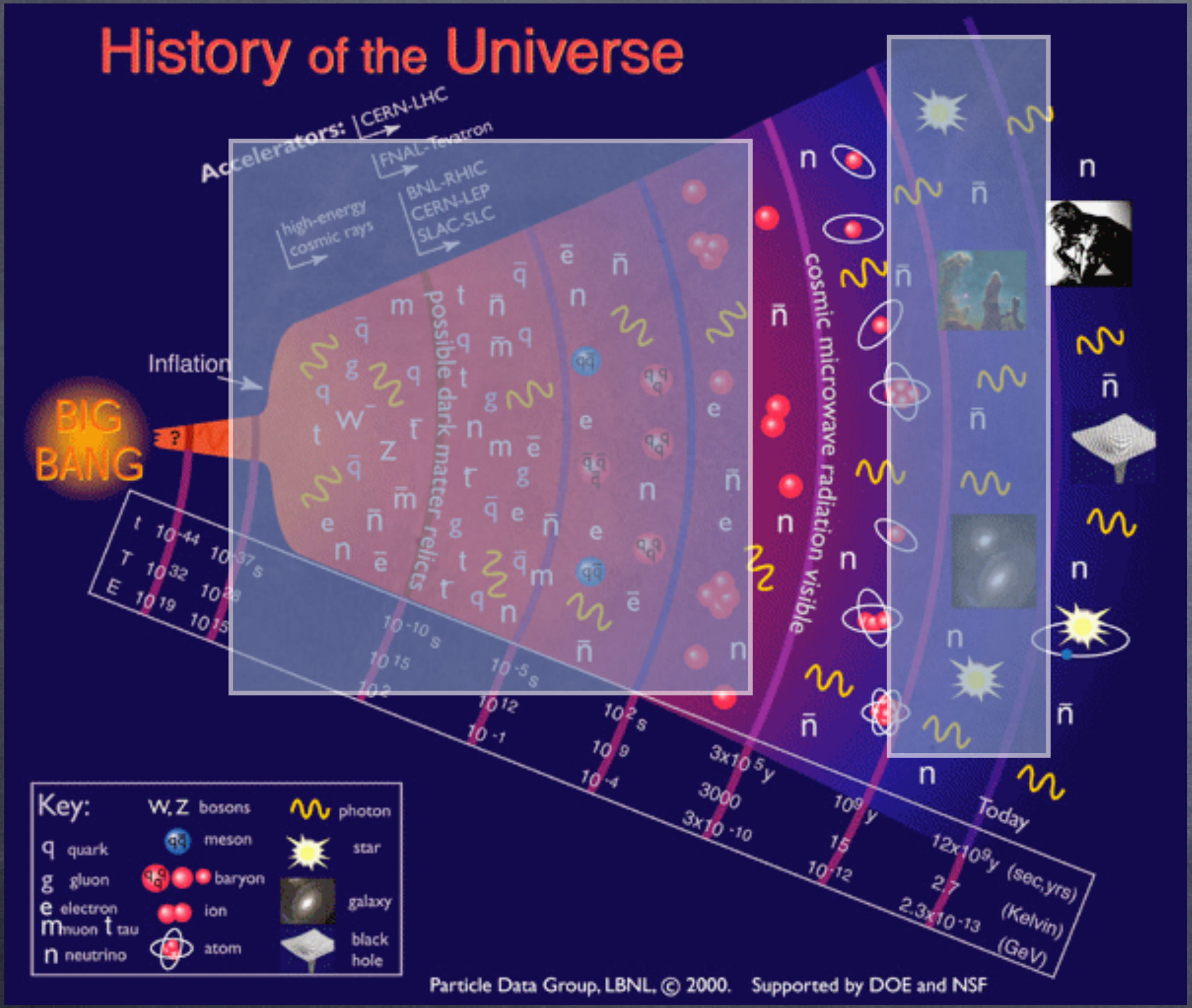
PROBLEMS needs many rotations, reaches saturation?



$$B_{\rm in}^{\rm gal} \simeq 10^{-21}{\rm Gauss} \qquad (t_{\rm gal} \simeq 10{\rm Gy})$$

(Brandenburg and Subramanian 2005)

OUTLINE OF THE SEMINAR:



POSSIBLE GENERATION MECHANISMS

GENERATION MECHANISMS

$B \simeq 10^{-9}\text{Gauss} \quad \text{or} \quad 10^{-21}\text{Gauss}$ at about 100 kpc

- Generation after recombination

  good: known physics

  bad: non-linear physics, correlation scale too small

- Generation prior to recombination

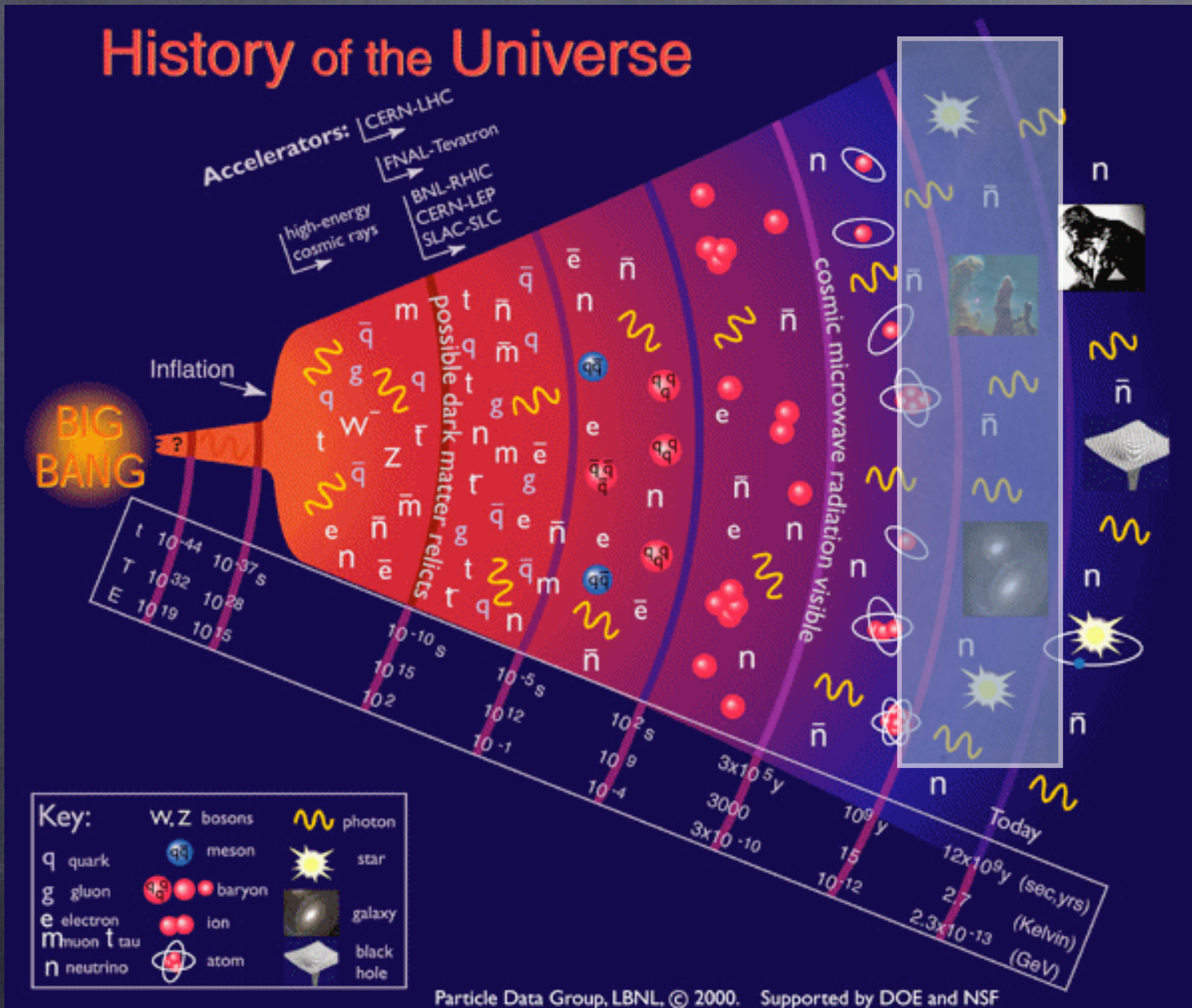
  CAUSAL good: quite easy to get

  bad: correlation scale too small

  A-CAUSAL good: generated at any scale

  bad: not very predictive

more than 100 proposed mechanisms but no preferred one



GENERATION
AFTER
RECOMBINATION

Generation after recombination

- BIERMANN BATTERY $$\frac{\partial B}{\partial t} = \nabla \times v \times B - \frac{\Delta B}{4\pi\sigma} - \frac{\nabla n_e \times \nabla P_e}{e n_e^2}$$

galactic: shocks by SN explosions in proto-galaxies (problem: scale)
both: radiation pressure at reionisation + density fluctuations

- EJECTION galactic: from stars (problem: mean to zero)
  cluster: from galaxies and/or AGN
  (open questions: mix? amplitude? metal enrichment)

- CLUSTERS: SMALL SCALE TURBULENT DYNAMO

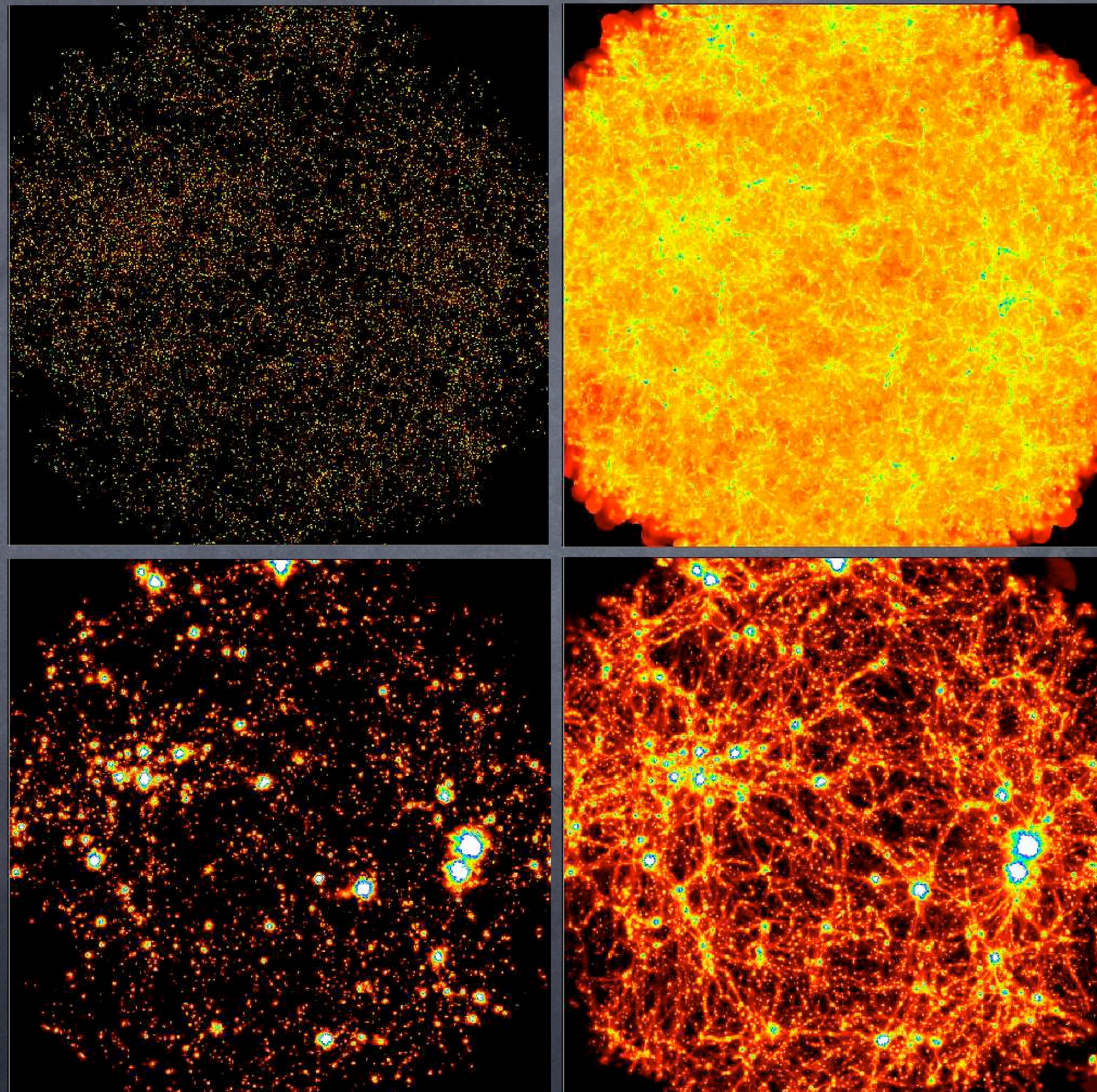
open questions: what generates it (mergers)?
scale (reversal of the field, Faraday rotation)?
saturation?

(Hanaya et al 2005, Biermann and Galea 2003, Langer et al 2005, Colgate and Li 2000, Subramanian 2008, Schekochihin and Cowley 2005...)

MHD simulation (Donnert et al 2008):

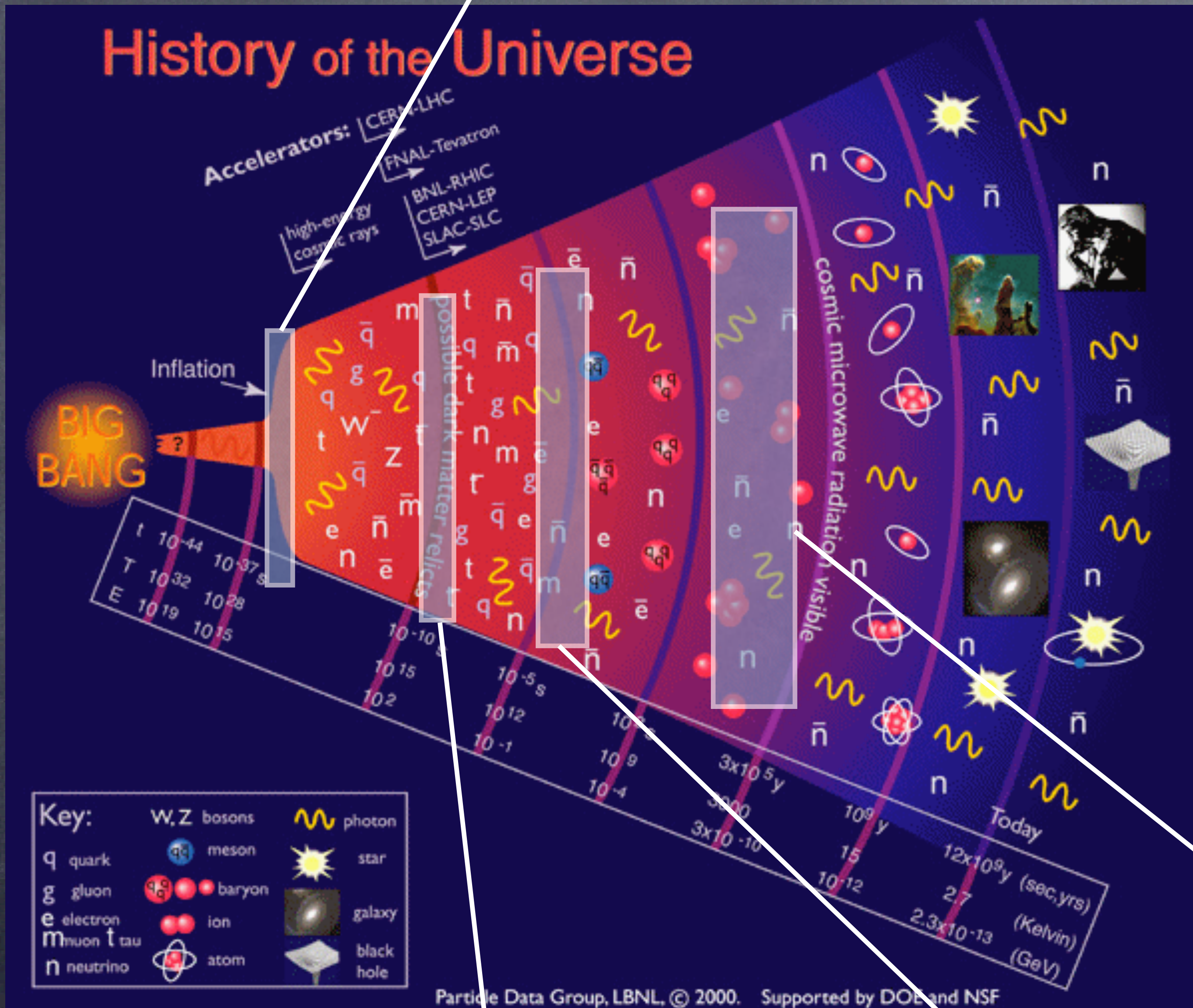
EJECTION PRIMORDIAL

z=4 z=4



z=0 z=0

inflation (a-causal generation)



PRIMORDIAL GENERATION

(causal generation)

plasma dynamics

EW phase transition

QCD phase transition

Generation prior to recombination

GENERATION BY PLASMA DYNAMICS + VORTICITY

$$\partial_t B + \nabla \times E = 0$$
$$\nabla \times B - \partial_t E = 4\pi J$$

$$J = en(v_p - v_e) \qquad \Box B = 4\pi en(\Omega_p - \Omega_e)$$

electrons do Thomson scattering (Harrison 1973)

- vorticity by wiggly strings, by second order perturbations?

- PROBLEM: very small scales?

Vachaspati and Vilenkin 1991, Davis and Dimopoulos 2005, Battefeld et al 2007...
Berezhiani and Dolgov 2003, Gopal and Sethi 2004, Matarrese et al 2004, Takahashi et al 2005...

Generation prior to recombination

PRIMORDIAL PHASE TRANSITIONS

- FIRST ORDER charge separation at bubble walls + amplification by MHD turbulence (both EW and QCD)

Hogan 1983, Quashnock et al 1989, Cheng and Olinto 1994, Baym et al 1996, Sigl et al 1996, Ahonen and Enqvist 1997...

- SECOND ORDER EW generated by the symmetry breaking connected to baryogenesis

Vachaspati 1991, Davidson 1996, Grasso and Riotto 1997, Hindmarsh and Everett 1997, Tornkvist 1998...

- PROBLEM : causal, so very small scales $L \leq \eta_{EW} \simeq 10^{-4}\mathrm{pc}$

Generation prior to recombination

INFLATION

- A-CAUSAL : generated at all scales

- PROBLEM : need to break conformal invariance $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
  otherwise vacuum fluctuations not amplified

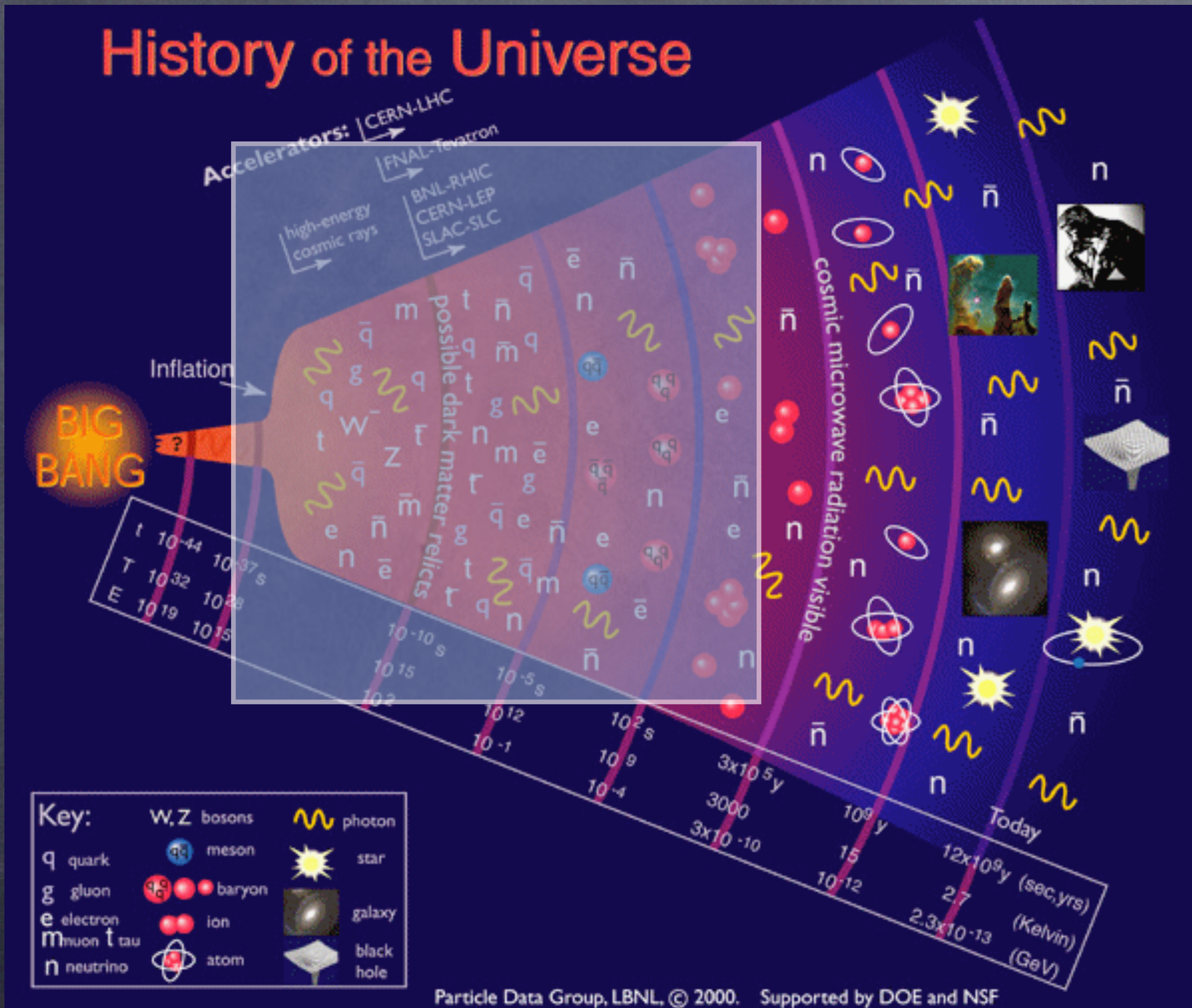
- coupling of em field to the metric, to the inflaton, in string theory context...

$RF_{\mu\nu}F^{\mu\nu}$ $e^{\alpha\phi}F_{\mu\nu}F^{\mu\nu}$ $f(\phi)F_{\mu\nu}F^{\mu\nu}$

Turner and Widrow 1988, Ratra 1992, Martin and Yokoyama 2007...

- PROBLEM : amplitude very model dependent

OUTLINE OF THE SEMINAR:



IF THEY ARE PRIMORDIAL

MODEL AND EVOLUTION

CONSTRAINTS

OBSERVATIONAL EFFECTS

Primordial magnetic field – Model

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

FRW : homogeneity and isotropy

$$ds^2 = a(t)^2(-dt^2 + dx_i dx^i)$$

$$T_{\mu\nu} = \begin{pmatrix} -\rho\, g_{00} & 0 \\ 0 & P g_{ij} \end{pmatrix}$$

- MF breaks FRW symmetries $\quad T^B_{\mu\nu} = \begin{pmatrix} -\frac{B^2}{2}\, g_{00} & 0 \\ 0 & -\frac{B^2}{2}\, g_{ij} - B_i B_j \end{pmatrix}$
- first order perturbation in FRW $\quad G_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G\,(T_{\mu\nu} + T^B_{\mu\nu})$
- Stochastic field, statistically homogeneous, isotropic and gaussian

$$\langle B_i \rangle = 0 \qquad \langle B^2 \rangle \neq 0$$

Primordial magnetic field – Model

Divergence free

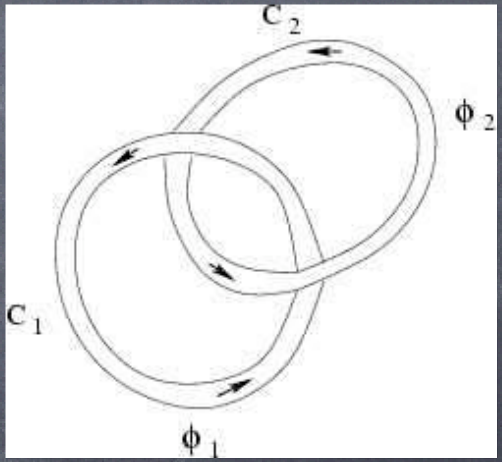
Power spectrum

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{q})\rangle = \delta(\mathbf{k}-\mathbf{q})[\,(\delta_{ij} - \hat{k}_i\hat{k}_j)S(k) + \mathrm{i}\epsilon_{ijm}\hat{k}^m A(k)]$$

energy density $\quad E_B = \int_0^\infty dk\, k^2 S(k)$

helicity density $\quad H = \int_0^\infty dk\, k A(k)$

$$H = \frac{1}{V}\int_V d^3x\, \mathbf{A}\cdot\mathbf{B}$$



Mean amplitude of the MF on a given scale λ

$$B_\lambda^2 = \int dk\, k^2\, S(k)\, e^{-k^2\lambda^2}$$

Primordial magnetic field – Model

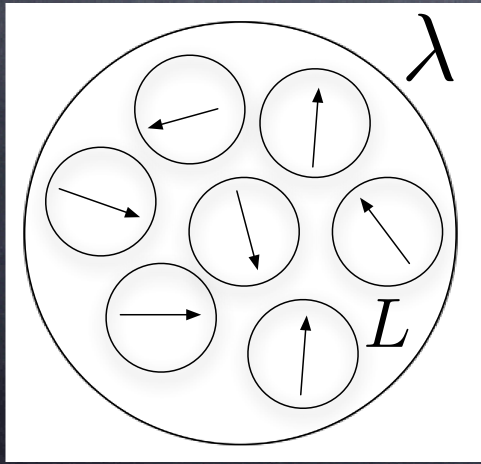
CAUSAL FIELD: $\langle B_i(\mathbf{x}) B_j(\mathbf{x}+\mathbf{r}) \rangle = 0$ for $r > L$ $L \leq$ horizon

correlation function compact support $\longrightarrow$ power spectrum analytic

$$S(k) \propto k^2 \,, k^4 \ldots$$

DIVERGENCE FREE IMPLIES
NO RANDOM WALK: $n \neq 0$

$$B_\lambda^2 = B_L^2 \left(\frac{L}{\lambda} \right)^{n+3}$$



$\longrightarrow$ Cluster scale today $0.1\,\mathrm{Mpc}$

$\longrightarrow$ Horizon scale at generation $10^{-4}\,\mathrm{pc}$

Primordial magnetic field – Evolution

$$ds^2 = a(t)^2(-dt^2 + dx_i dx^i)$$

conformal transformation $\quad g_{\mu\nu} = a(t)^2 \eta_{\mu\nu} \longrightarrow$ flat spacetime

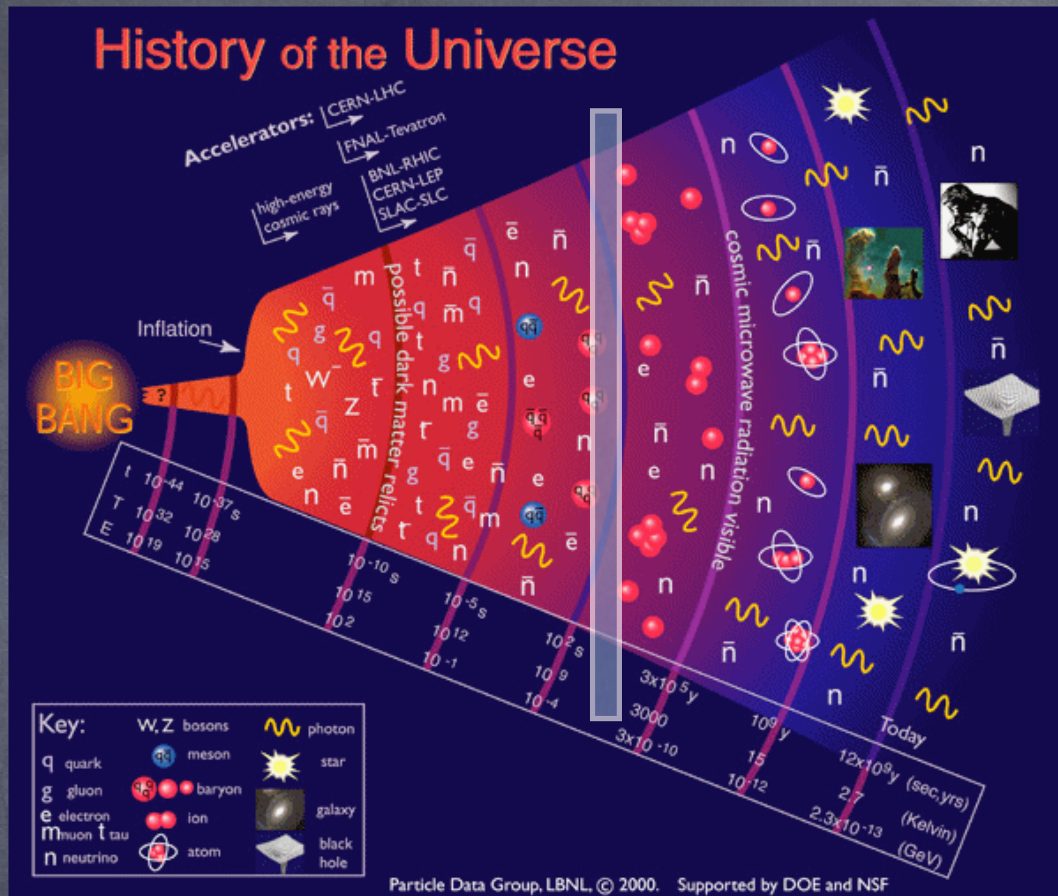
The equation of motions are the same for $B = a(t)^2 \mathcal{B}$

induction equation: $$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\Delta B}{4\pi\sigma}$$

IDEAL MDH limit $\sigma \longrightarrow \infty$: flux and helicity are conserved

$$\mathcal{B} \propto a^{-2}(t)$$

Primordial magnetic field – Evolution



NEUTRINO DECOUPLING

ELECTRONS NON RELATIVISTIC

TURBULENT PHASE

$$\nu \simeq \ell_{\nu e}$$

Turbulent cascade

VISCOUS PHASE

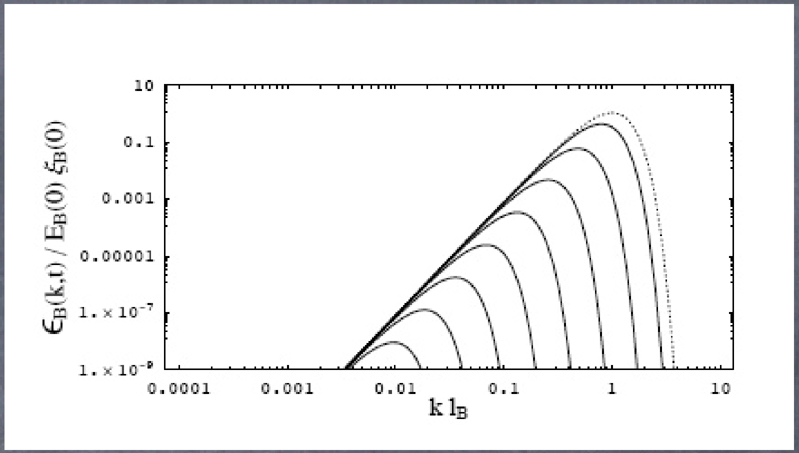
$$\nu \simeq \ell_{\gamma e}$$

Magnetosonic waves in viscous fluid: damping of magnetic energy

Primordial magnetic field – Evolution

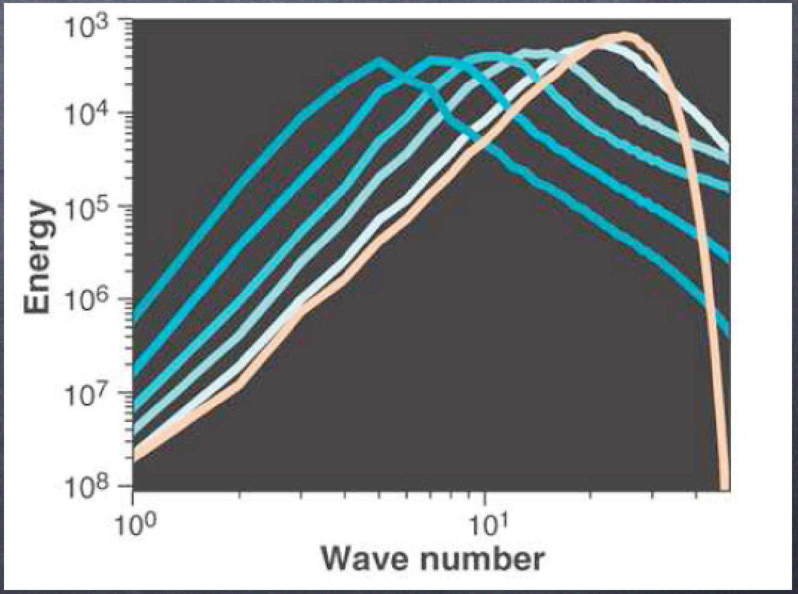
TURBULENT PHASE

- non-helical field: DIRECT CASCADE



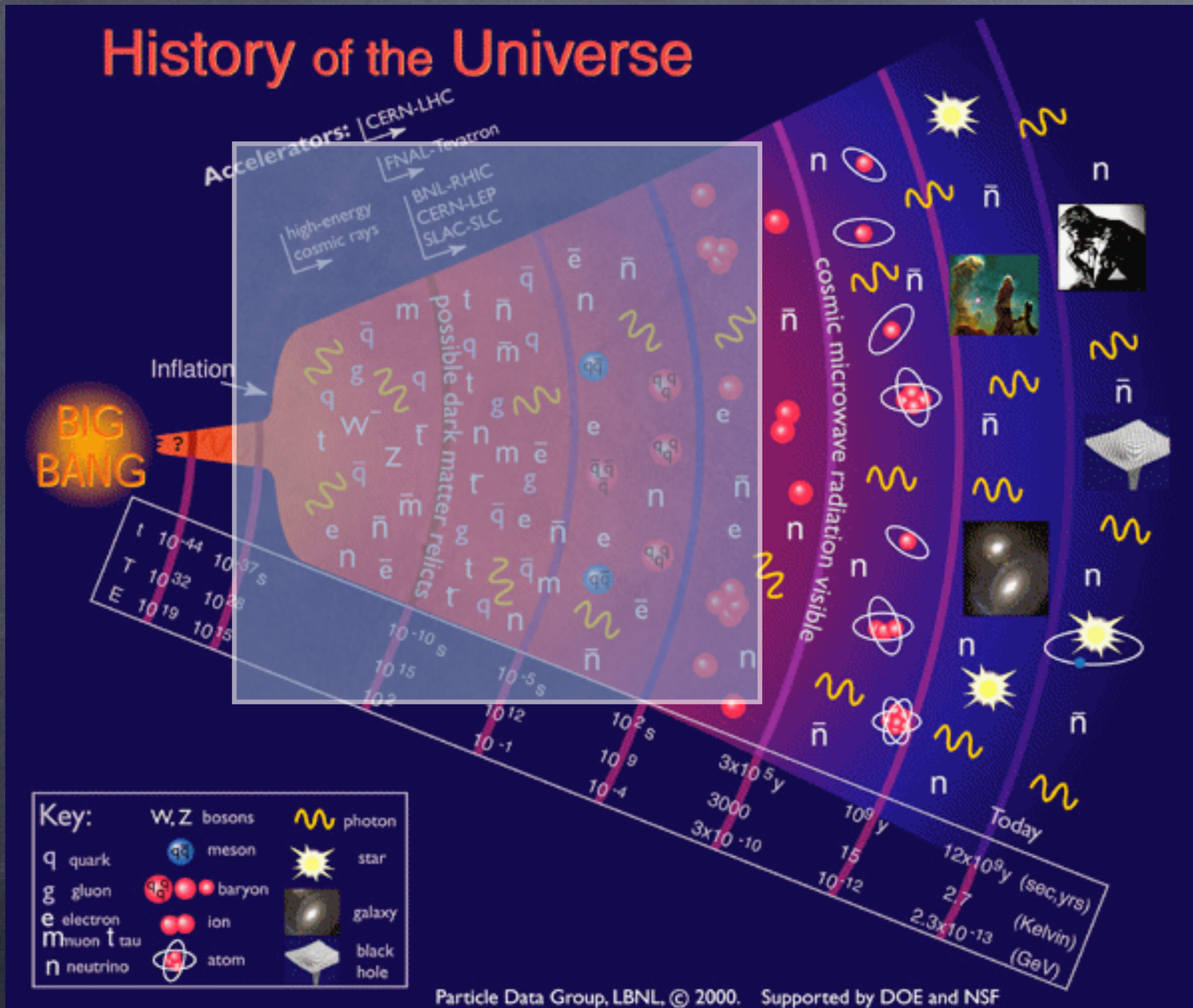
- helical field: INVERSE CASCADE

Magnetic energy is transferred to larger scales



Christensson et al 2002, Banerjee and Jedamzik 2004, Campanelli 2007...

OUTLINE OF THE SEMINAR:



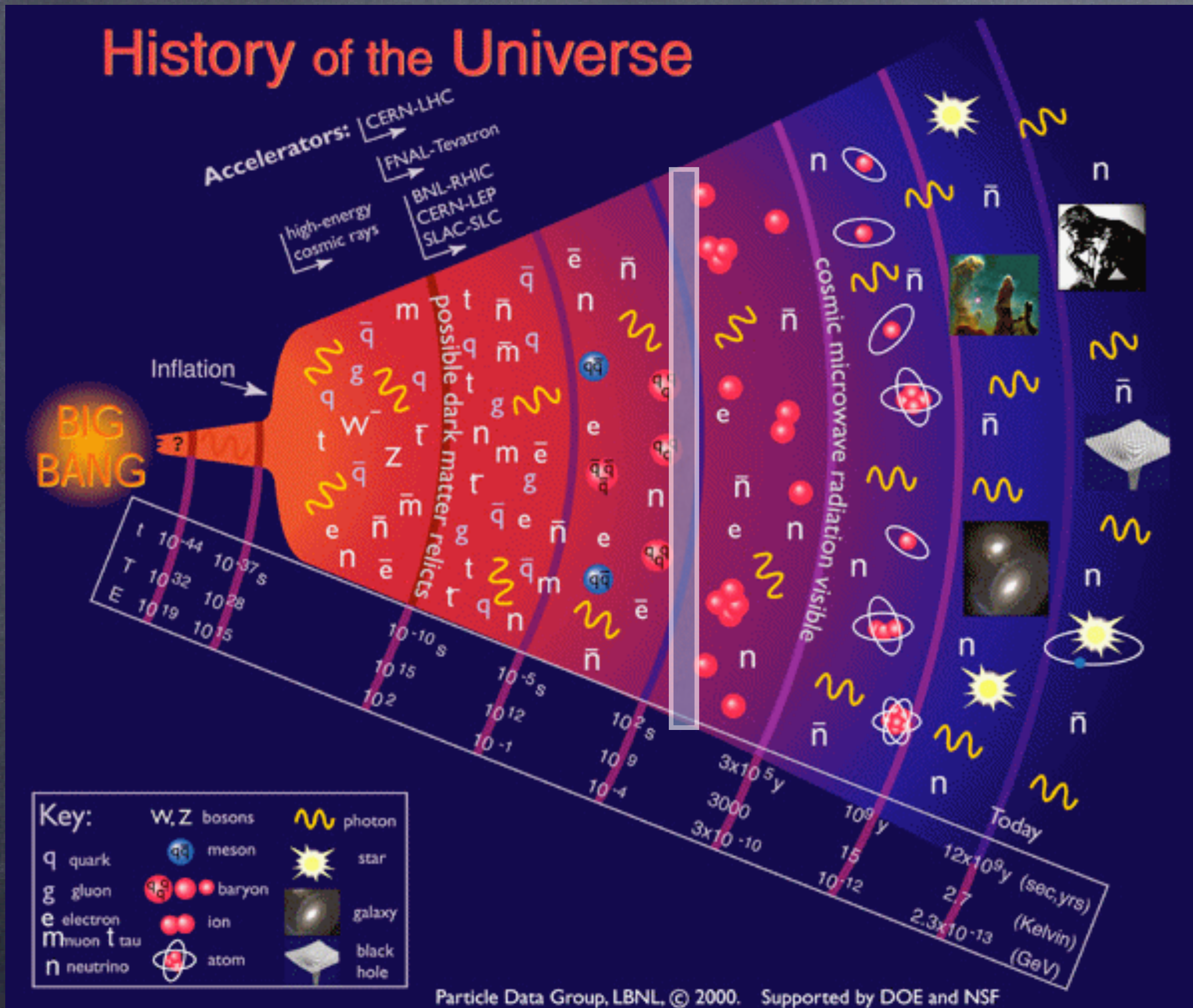
IF THEY ARE PRIMORDIAL

MODEL AND EVOLUTION

CONSTRAINTS

OBSERVATIONAL EFFECTS

NUCLEOSYNTHESIS



- Abundance of neutrons depends on universe expansion
- MF influences universe expansion
- Abundances of elements constrains MF intensity

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \longrightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\rm tot}$$

Primordial magnetic field – Constraints

$$B \simeq 10^{-9}\text{Gauss} \quad \text{or} \quad 10^{-21}\text{Gauss} \qquad \text{at about 100 kpc}$$

- CAUSAL GENERATION AT 100 GeV:

$$n = 2 \qquad B_{0.1\text{Mpc}} \leq 10^{-27}\text{Gauss}$$

- INFLATIONARY GENERATION AT 10^15 GeV:

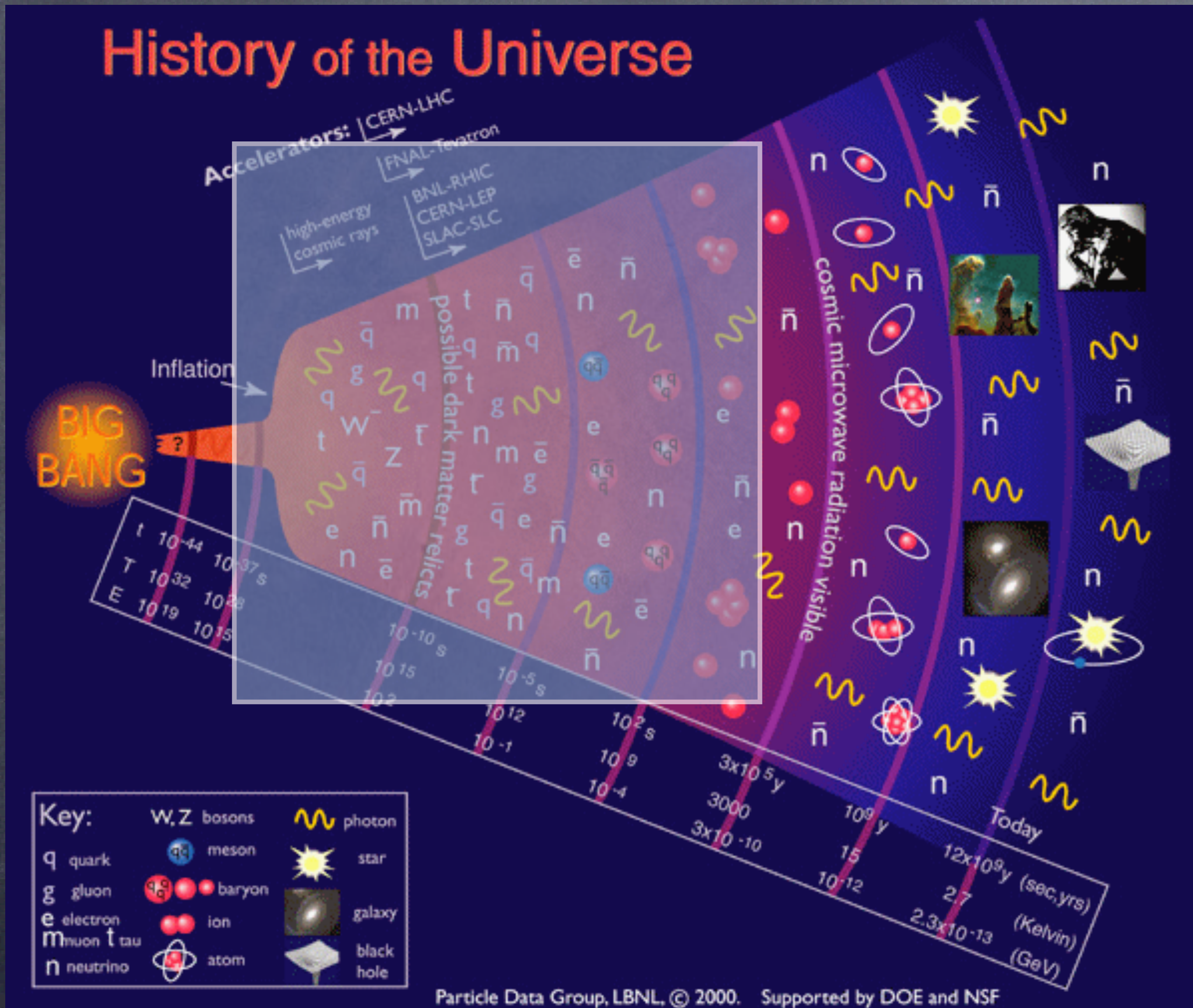
$$n = 0 \qquad B_{0.1\text{Mpc}} \leq 10^{-39}\text{Gauss}$$

$$n \to -3 \qquad B_{0.1\text{Mpc}} \leq 10^{-9}\text{Gauss}$$

APPLIES TO HELICAL FIELD?

CC and Durrer 2001

OUTLINE OF THE SEMINAR:



IF THEY ARE PRIMORDIAL

MODEL AND EVOLUTION

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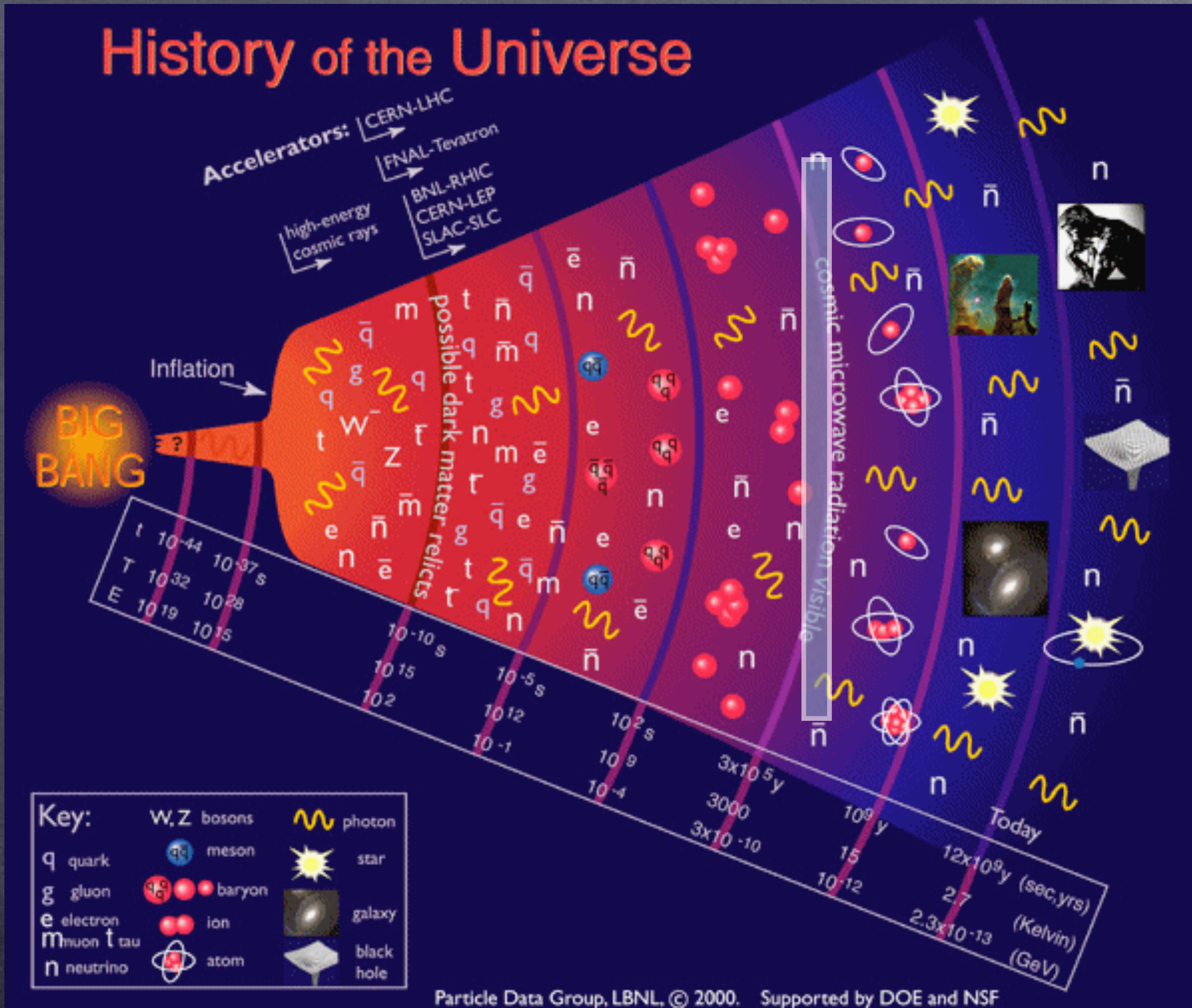
OBSERVATIONAL EFFECTS

Primordial magnetic field – Observational effects



MF breaks
FRW
symmetries

$$T^B_{\mu\nu} = \begin{pmatrix} -\frac{B^2}{2}\, g_{00} & 0 \\ 0 & -\frac{B^2}{2}\, g_{ij} - B_i B_j \end{pmatrix}$$

Primordial magnetic field – Observational effects

- CMB measures the isotropy of the universe $\frac{\delta T}{T} \sim 10^{-5}$
- MF is a source of anisotropy in the CMB

$$\delta G_{\mu\nu} = 8\pi G(\delta T_{\mu\nu} + T^B_{\mu\nu})$$

radiation, baryons, dark matter, neutrinos...

MF energy momentum tensor: scalar, vector, tensor modes

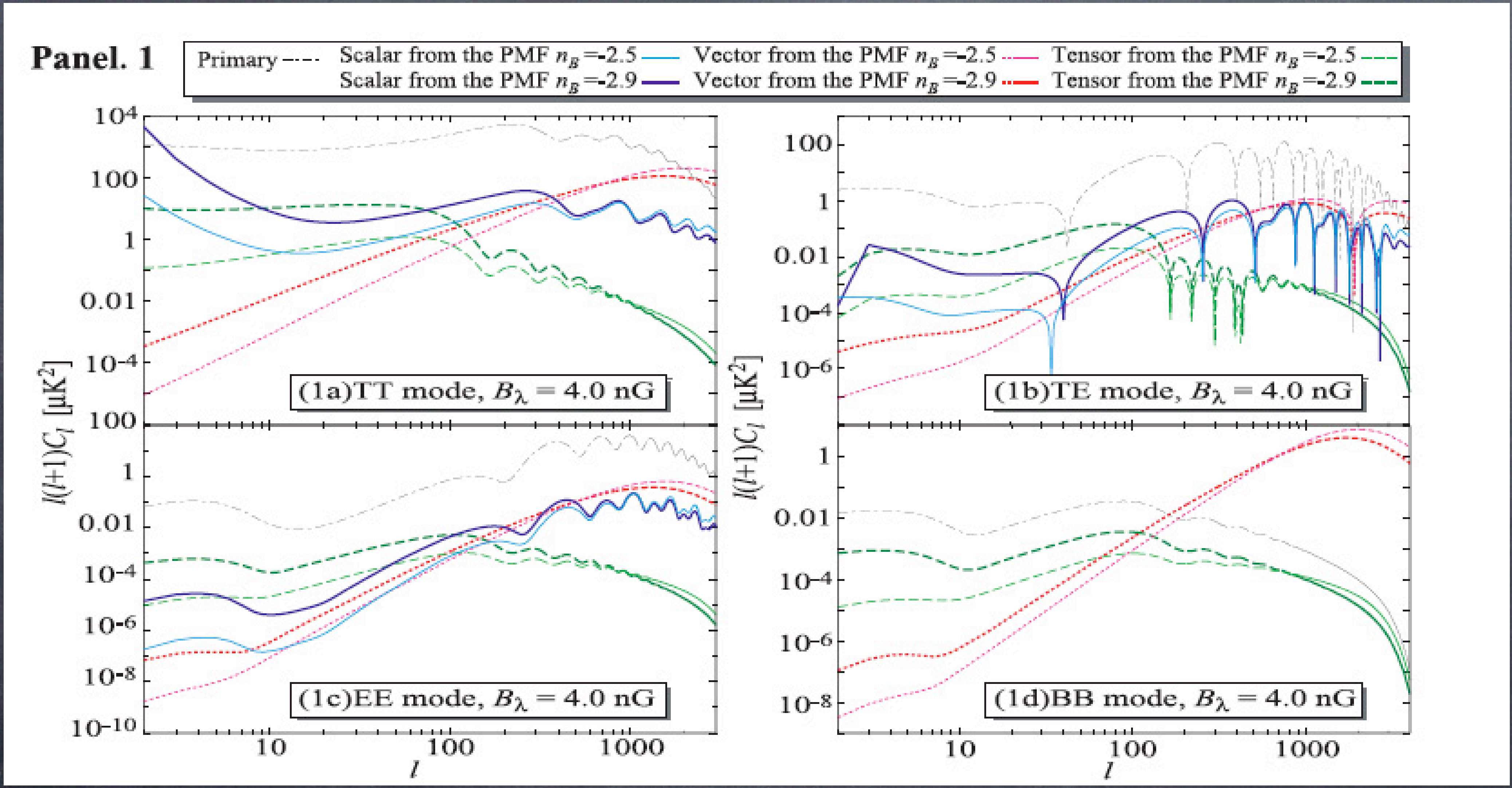
$$\ell^2 C_\ell \sim \left(\frac{\Omega_B}{\Omega_{\rm rad}}\right)^2 \sim 10^{-14} \left(\frac{B}{10^{-9}{\rm Gauss}}\right)^4$$

$(n \to -3)$

$B \simeq 6{\rm nG}$ 1% effect in the temperature anisotropy

$B \simeq 0.3{\rm nG}$ 1% effect in B polarisation (r=0.1)

Primordial magnetic field – CMB effects



Yamakazi et al 2008, Finelli et al 2008, Giovannini and Kunze 2007, Kahniashvili and Ratra 2006, Lewis 2004, Mack et al 2002, Koh and Lee 2002...

Primordial magnetic field - CMB effects

- Parity odd cross-correlations from helical field

$$\ell^2 C_\ell^{TB} \sim \ell^2 C_\ell^{EB} \sim 10^{-11} \quad \text{at } \ell = 50$$

Pogosian et al 2001, CC et al 2004

- Faraday rotation of CMB polarisation

rotation of E in B: $\ell^2 C_\ell^{B} \sim 10^{-14} \quad \text{at } \ell = 10^4 \,, \; \nu = 30\text{GHz}$

Kosowsky and Loeb 1996, Campanelli et al 2004, Kosowsky et al 2005...

- Non-gaussianity: B is gaussian, the source B^2 is not

$$\frac{b_{\ell\ell\ell}}{(C_\ell)^{3/2}} = \mathcal{O}(1)$$

Brown and Crittenden 2005, CC et al in preparation

Conclusions

The origin of MF observed in astrophysical objects is still unclear

Observations:

SKA radiotelescope: all-sky rotation measure survey
evolution of magnetised structure from z>3
detect MF in the IGM?

PLANCK: detect B polarisation and large multipoles
parity-odd cross-correlations?
non-gaussianity?

Theory:

- deeper understanding of generation from ejection and small scale dynamo processes
- are primordial helical fields the solution?

The alpha-omega mean field dynamo

induction equation $\frac{\partial B}{\partial t} = \nabla \times (V \times B)$

mean and random components $B = \langle B \rangle + b$, $V = \langle V \rangle + v$

substituting, averaging, eliminating small scale averaged components

$$\frac{\partial \langle B \rangle}{\partial t} = \nabla \times (\langle V \rangle \times \langle B \rangle) + \nabla \times \langle v \times b \rangle$$

electromotive force

quasi-linear approximation + homogeneity and isotropy

$$\langle v \times b \rangle = \alpha \langle B \rangle - \beta \nabla \times \langle B \rangle$$

$$\beta = \frac{\tau}{3} \langle v^2 \rangle \qquad \alpha = -\frac{\tau}{3} \langle v \cdot \nabla \times v \rangle$$

field with poloidal and toroidal components

$$\frac{\partial \langle B_\phi \rangle}{\partial t} = -\Omega \langle B_r \rangle$$

omega: poloidal into toroidal

$$\frac{\partial \langle B_r \rangle}{\partial t} = -\frac{\partial \alpha \langle B_\phi \rangle}{\partial z}$$

alpha: toroidal into poloidal

$$\alpha = \alpha_0 \frac{z}{h} \qquad B \propto e^{\gamma t} \qquad \gamma = \sqrt{\frac{\alpha_0 \Omega}{h}} \simeq 2\mathrm{Gy}^{-1} \longrightarrow B_{\mathrm{in}}^{\mathrm{gal}} \simeq 10^{-21}\,\mathrm{Gauss}$$

(Brandenburg and Subramanian 2005)