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# Radiative corrections to dark matter annihilation: Recent developments

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# Outline

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1. Introduction and Motivation
2. Radiative corrections to neutralino pair annihilation
3. Few numerical examples
4. Conclusion and Outlook

# Introduction

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New physics provides interesting candidates for cold dark matter

Consider Minimal Supersymmetric Standard Model (MSSM) with R-parity conservation

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$$\frac{dn}{dt} = -3Hn - \langle\sigma_{\text{ann}}v\rangle(n^2 - n_{\text{eq}}^2) \rightarrow \Omega_{\text{CDM}}h^2 \propto n_0 \propto \frac{1}{\langle\sigma_{\text{ann}}v\rangle}$$

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$$0.1053 < \Omega_{\text{CDM}}h^2 < 0.1193$$

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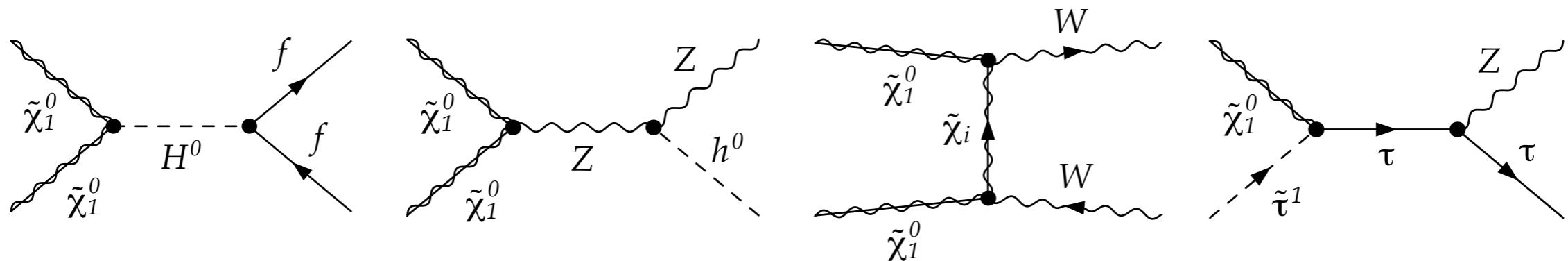
[Komatsu *et al.* (WMAP) 2010]

## Public program packages

DarkSUSY [Gondolo *et al.* 2000-2010], micrOMEGAs [Bélanger *et al.* 2003-2010],  
SuperIso Relic [Arbey and Mahmoudi 2009]

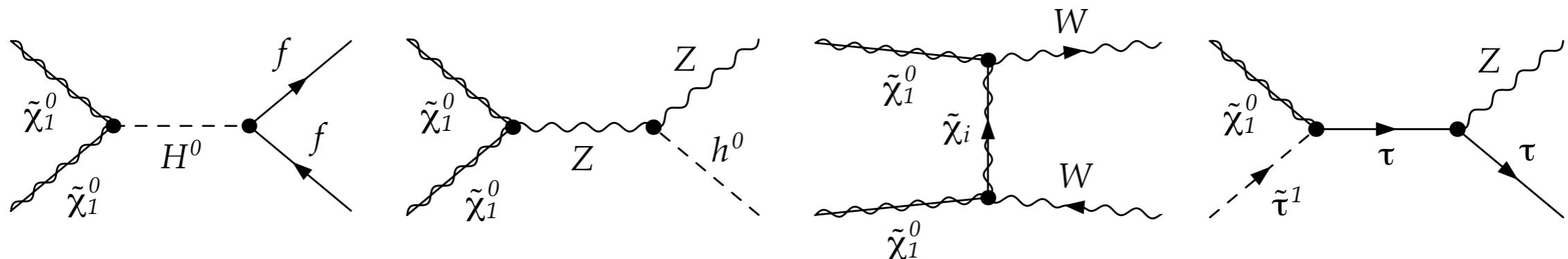
# Motivation: Why radiative corrections...?

All (co)annihilation processes are implemented in public codes at leading order



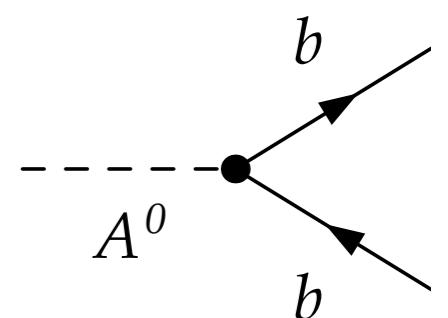
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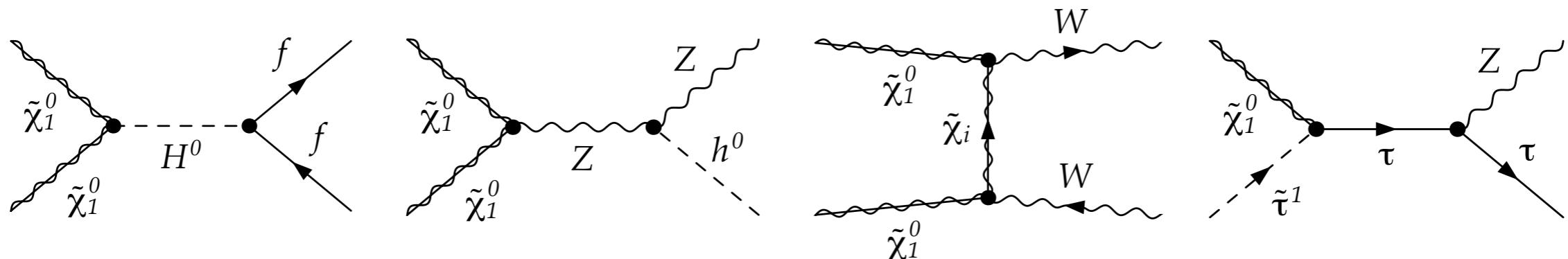
Higher order corrections included only for a few very sensitive quantities,  
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$$h_{Abb} \propto \frac{\overline{m}_b(Q)}{1 + \Delta_b} \tan \beta$$



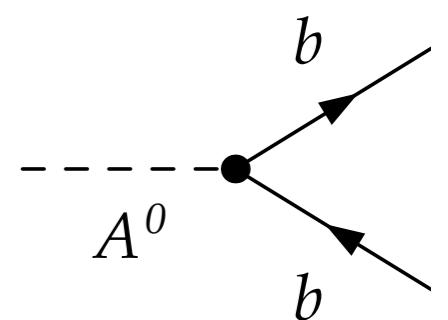
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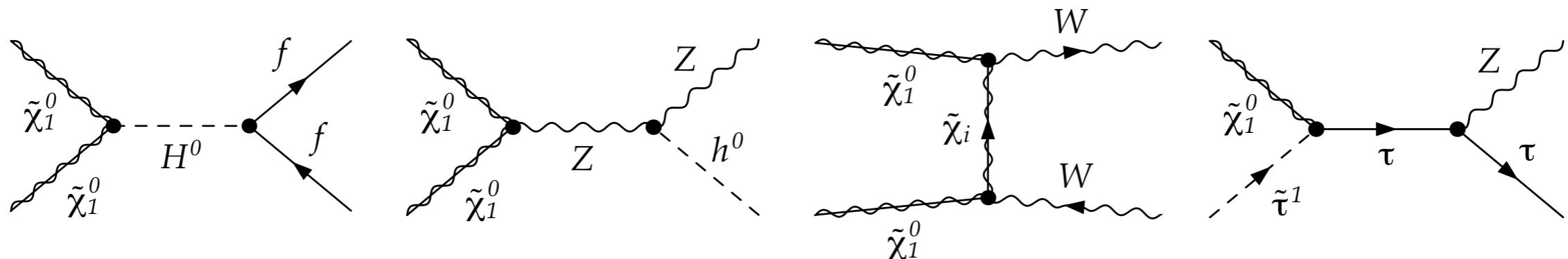
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Better theoretical precision needed to keep up with experimental improvements

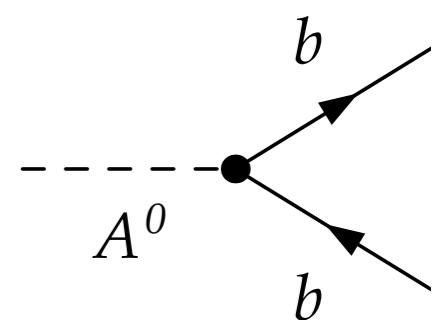
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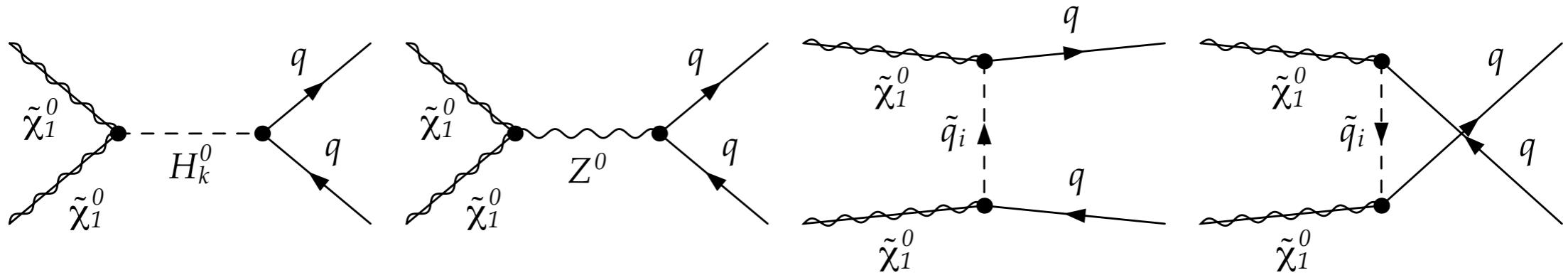


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QCD corrections numerically most important due to strong coupling constant,  
but also electroweak corrections can have a sizeable impact

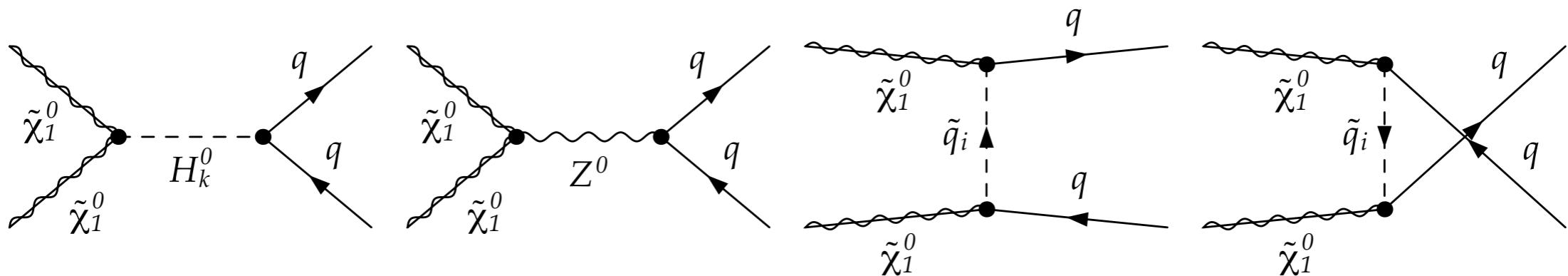
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## Higgs-exchange dominant in mSUGRA

- ▶ Low  $m_{1/2}$  (if not excluded by LEP)

$$\tilde{\chi}\tilde{\chi} \rightarrow h^0 \rightarrow b\bar{b}$$

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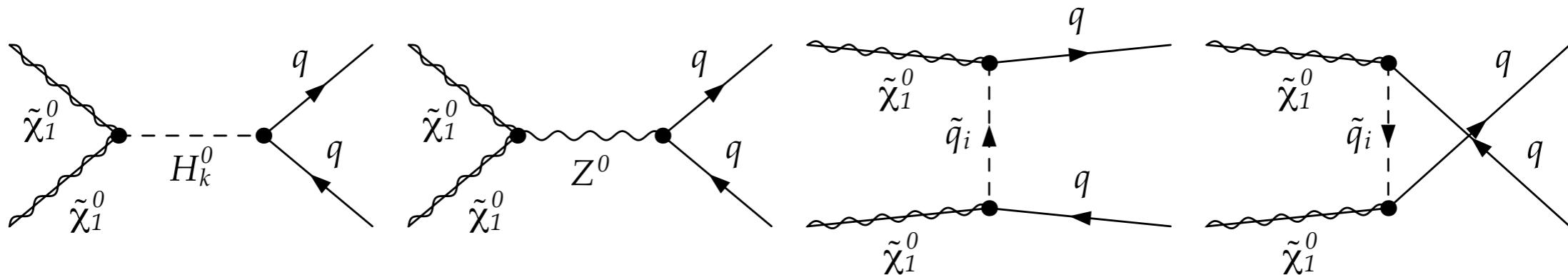
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## Relax scalar or gaugino mass unification

- ▶ Non-universal Higgs masses (NUHM) or “compressed SUSY” (non-univ. gaugino masses)
- ▶ A-Funnel already at low  $\tan\beta$
- ▶ Larger higgsino-comp. favours  $Z^0$ -exchange

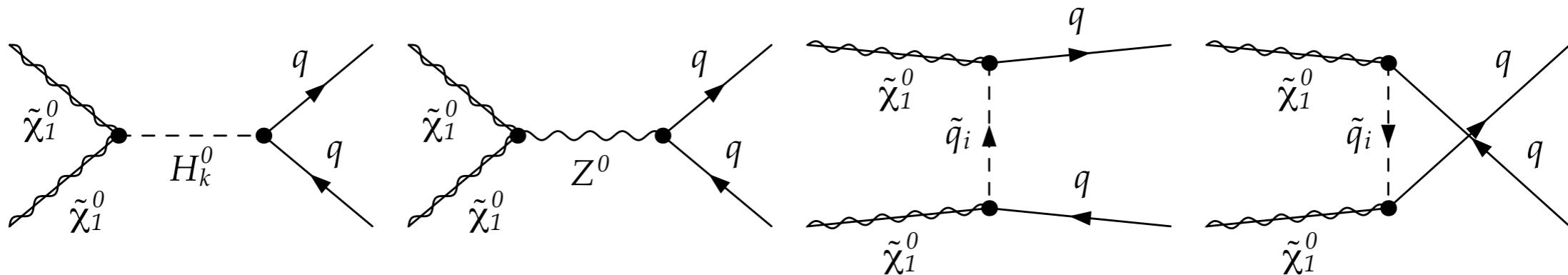
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## Large mass splitting favours squark exchange

- ▶ Large trilinear coupling  $A_0$

$$\tilde{\chi}\tilde{\chi} \rightarrow t\bar{t}$$

[Herrmann, Klasen, Kovařík, PRD 80 (2009)]

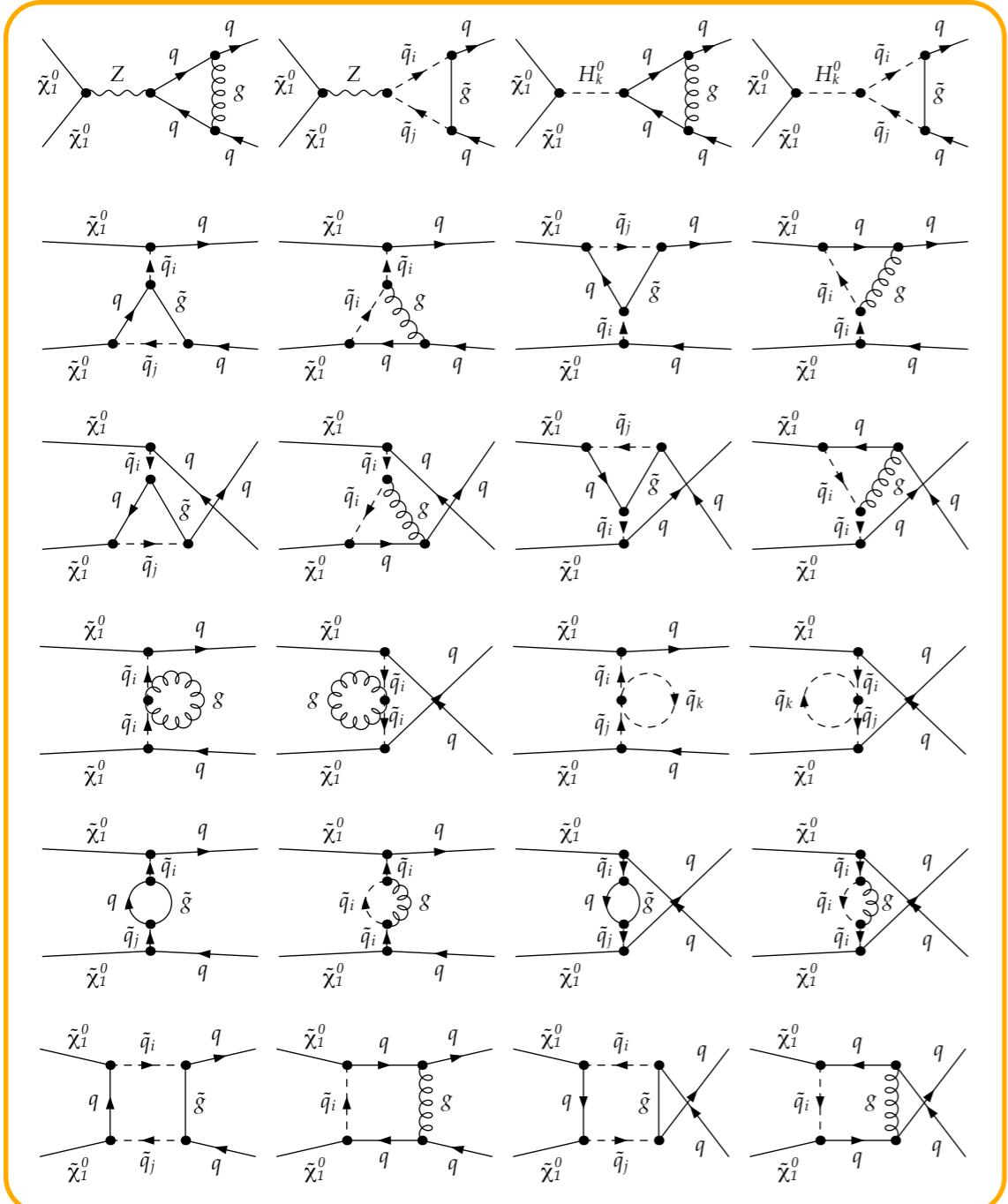
# Corrections of order $\alpha_s$

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One-loop contributions combined with real gluon emission using dipole subtraction method

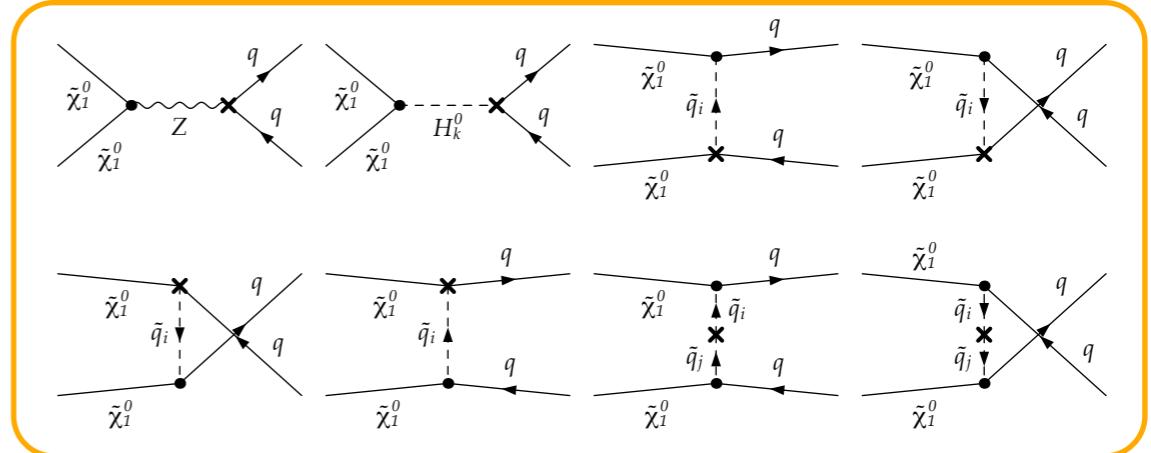
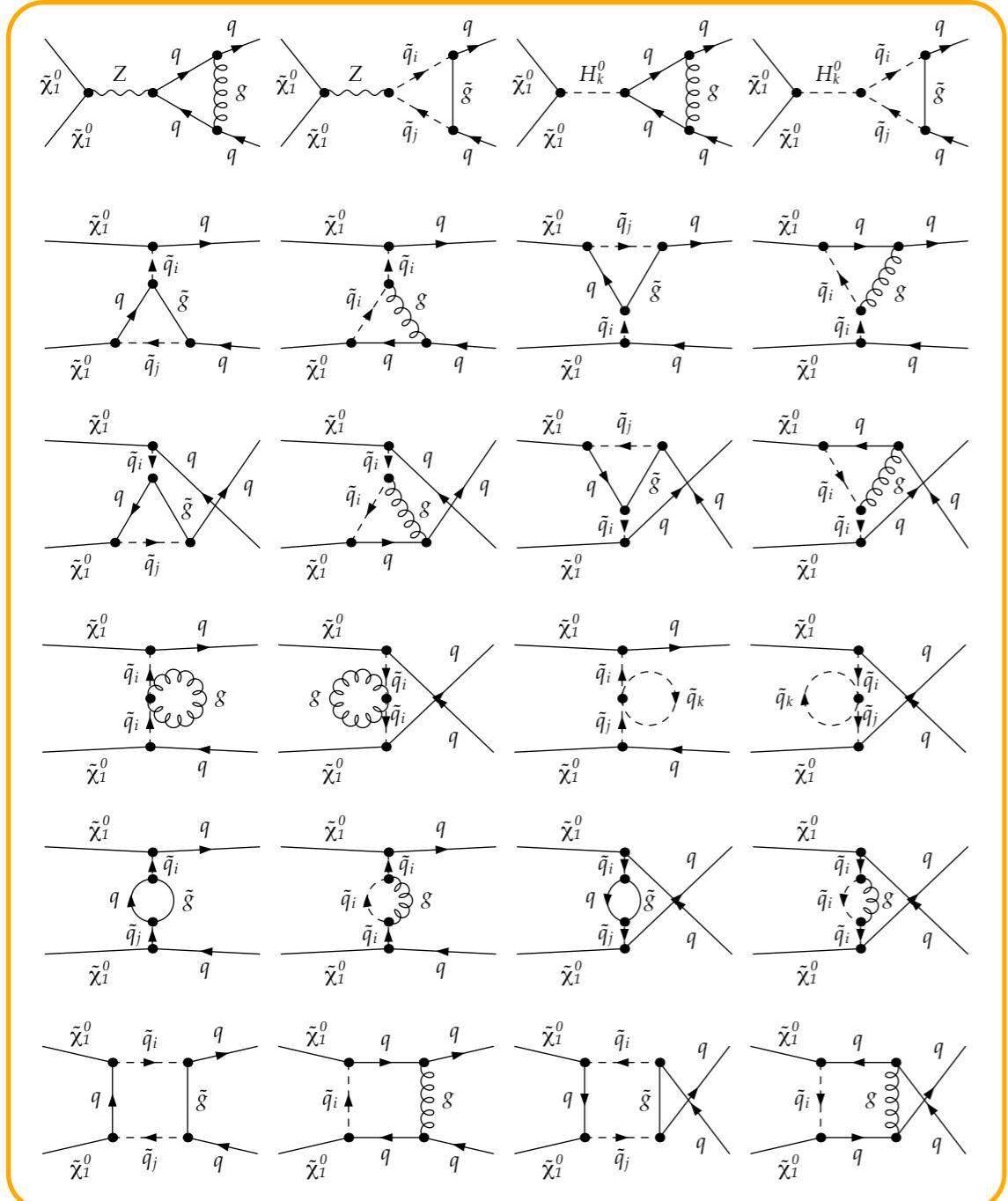
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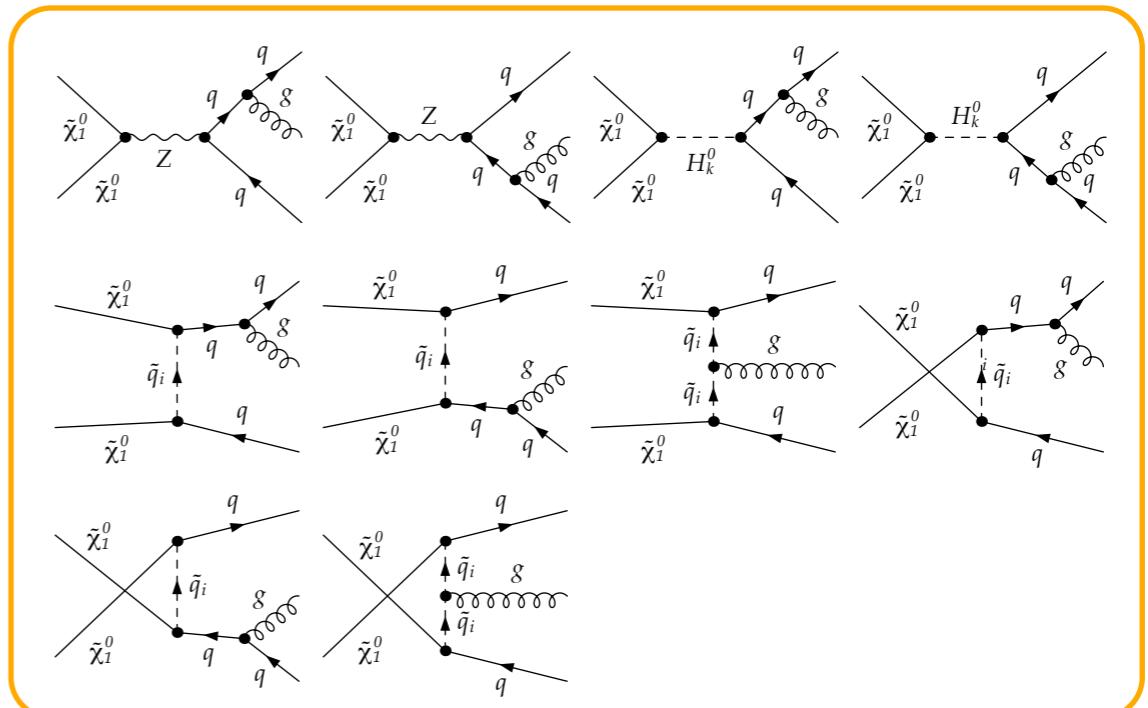
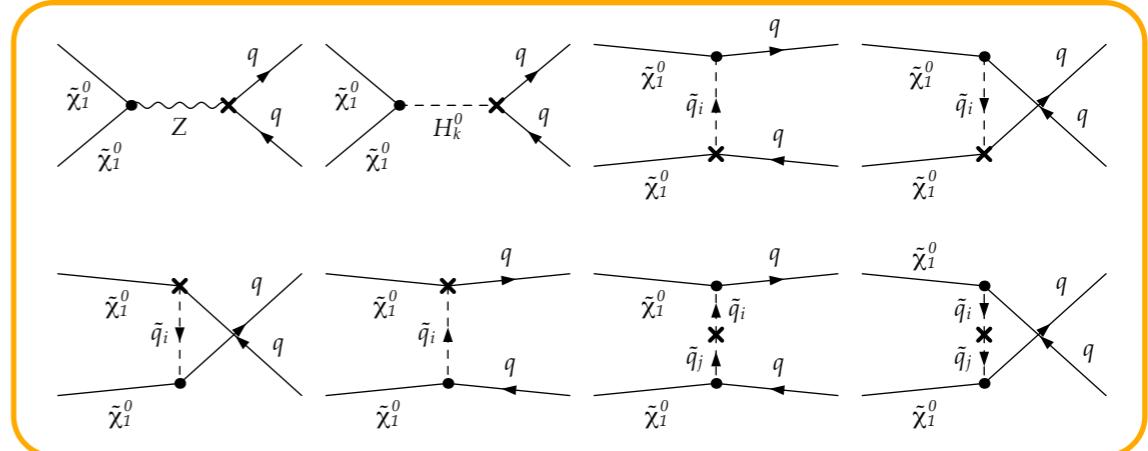
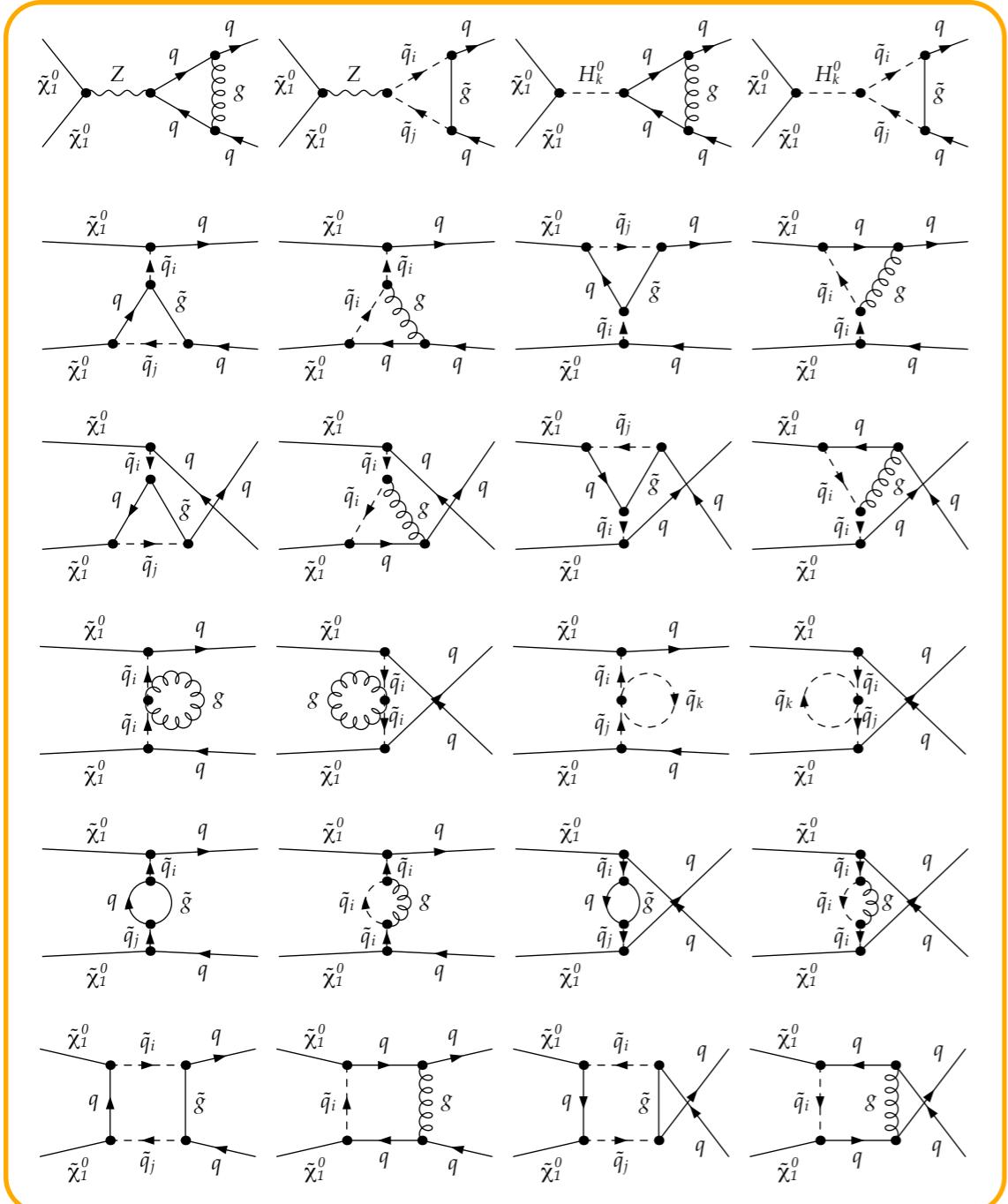
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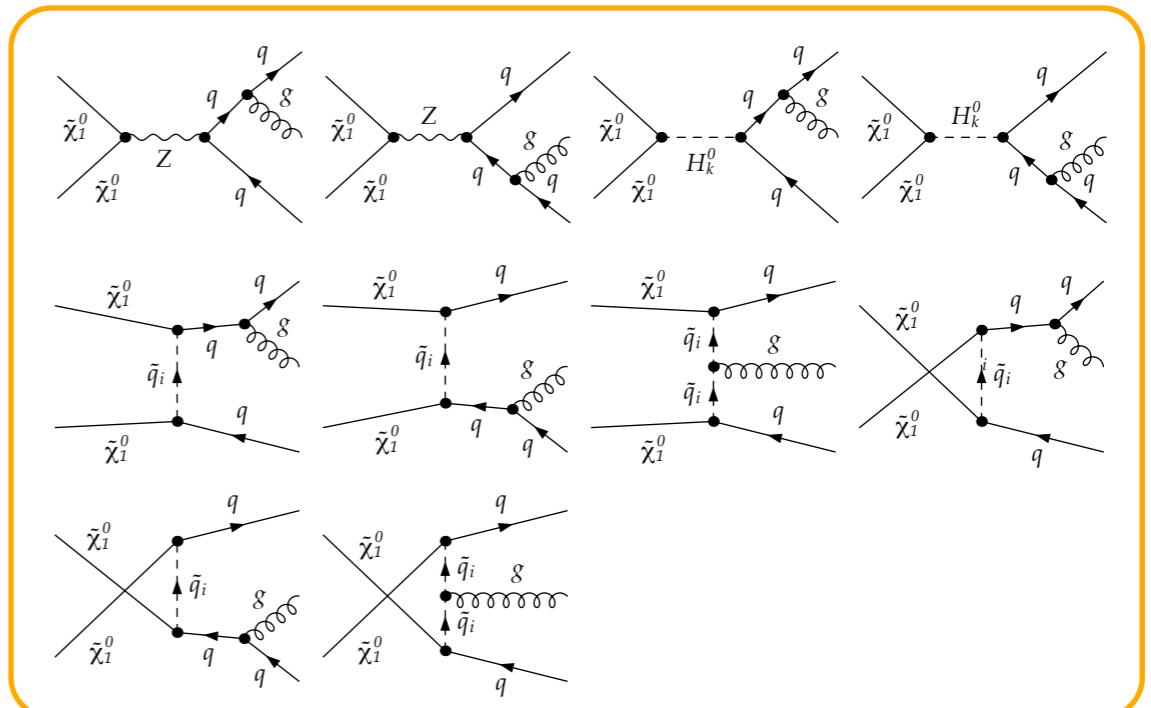
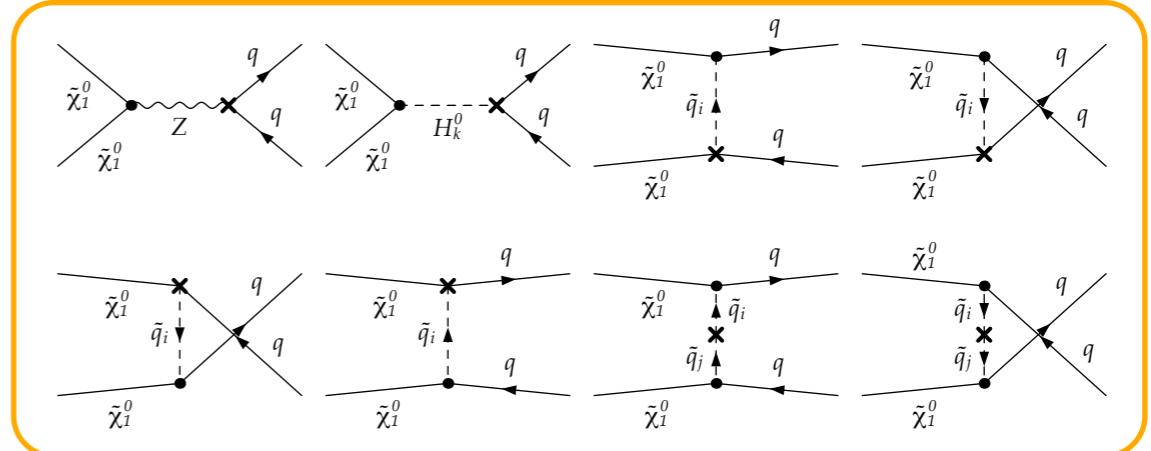
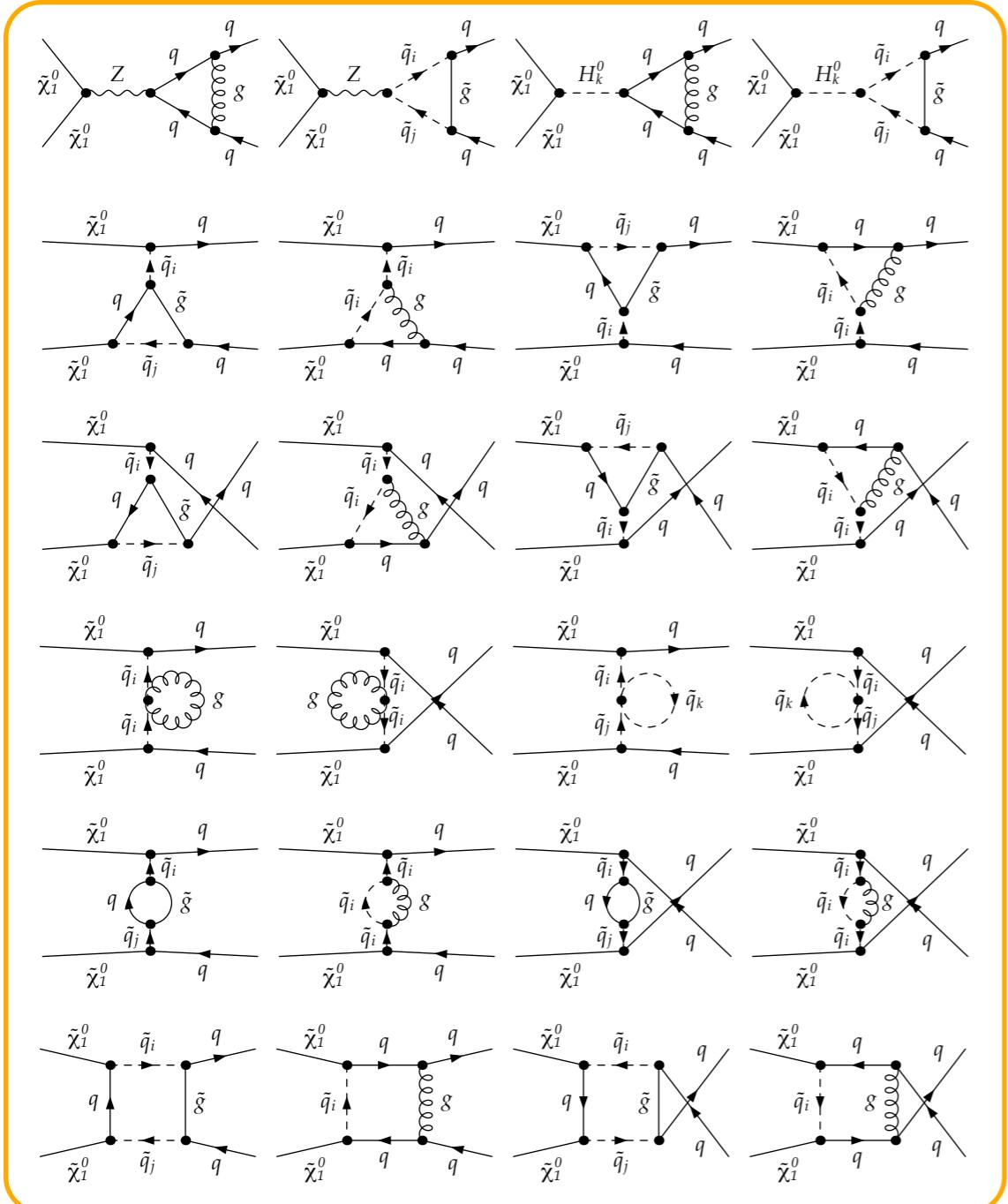
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Numerical implementation can serve as extension for micrOMEGAs or DarkSUSY

# Corrections of order $\alpha_s$

Virtual corrections at the one-loop level

Diagrams calculated in the  $\overline{\text{DR}}$  renormalization scheme (preserving Supersymmetry)

Ultraviolet divergences (showing up as poles) removed by on-shell renormalization

Real gluon emission from final state quarks

Poles in gluon emission diagrams cancel remaining infrared divergences

Dipole subtraction method allows for separate numerical integration

$$\sigma_{\text{NLO}} = \int_2 \left[ d\sigma^V + \int_1 d\sigma^A \right] + \int_3 \left[ d\sigma^R - d\sigma^A \right]$$

[Catani et al. 2000-2002]

Improvements for Yukawa couplings relevant for Higgs-exchanges

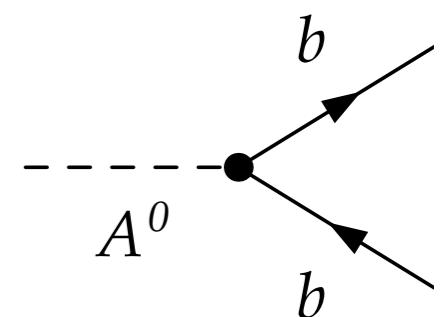
Decays of Higgs-bosons to quarks known up to  $\mathcal{O}(\alpha_s^4)$

SUSY-QCD resummation known to be relevant at large  $\tan\beta$

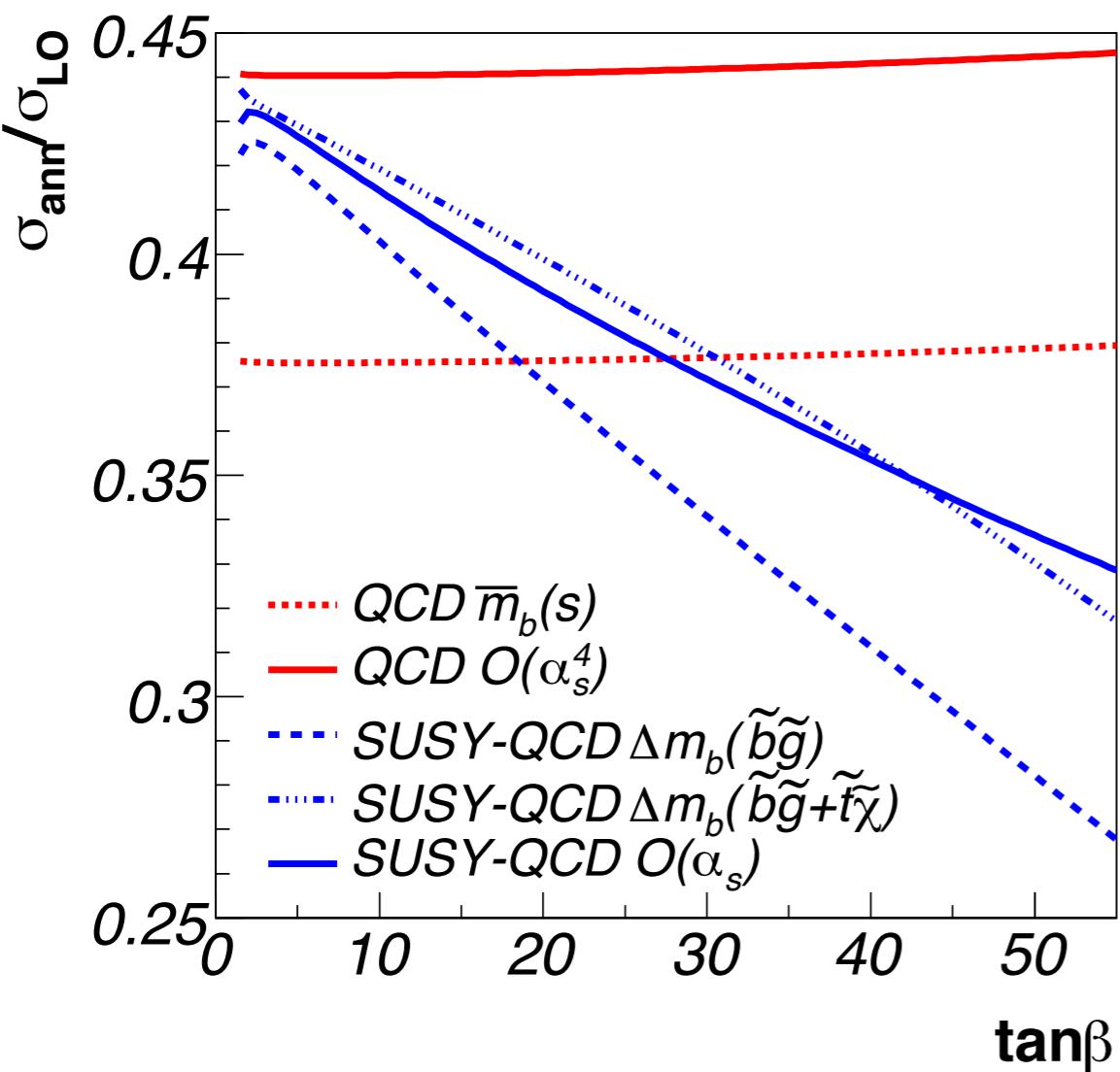
$$h_{Abb} \propto \frac{\bar{m}_b(Q)}{1 + \Delta_b} \tan \beta$$

[Braaten & Leveille 1980, Chetyrkin et al. 1995, Chetyrkin 1997, Chetyrkin et al. 2005]

[Carena et al. 2000, Guasch et al. 2003]

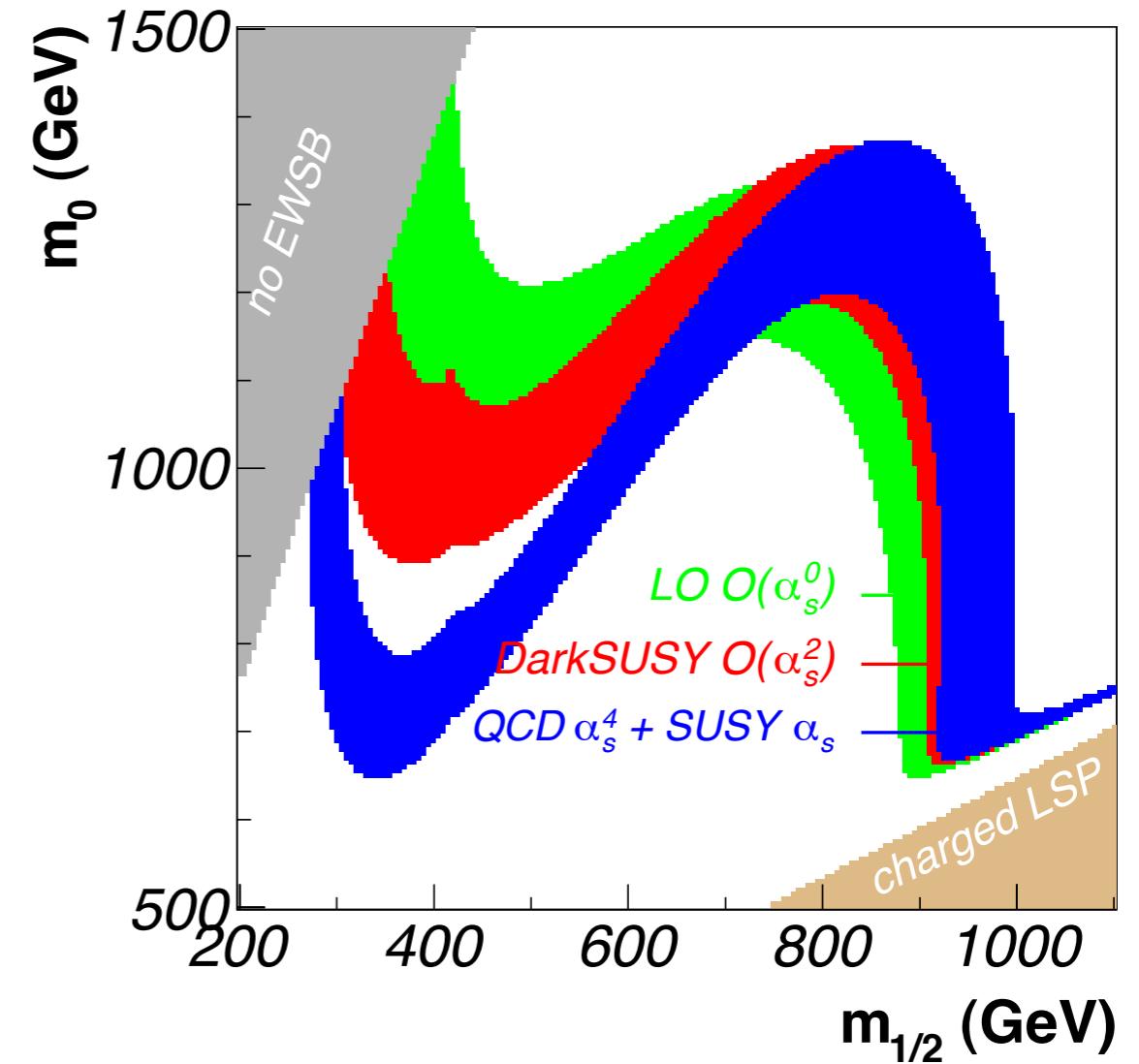
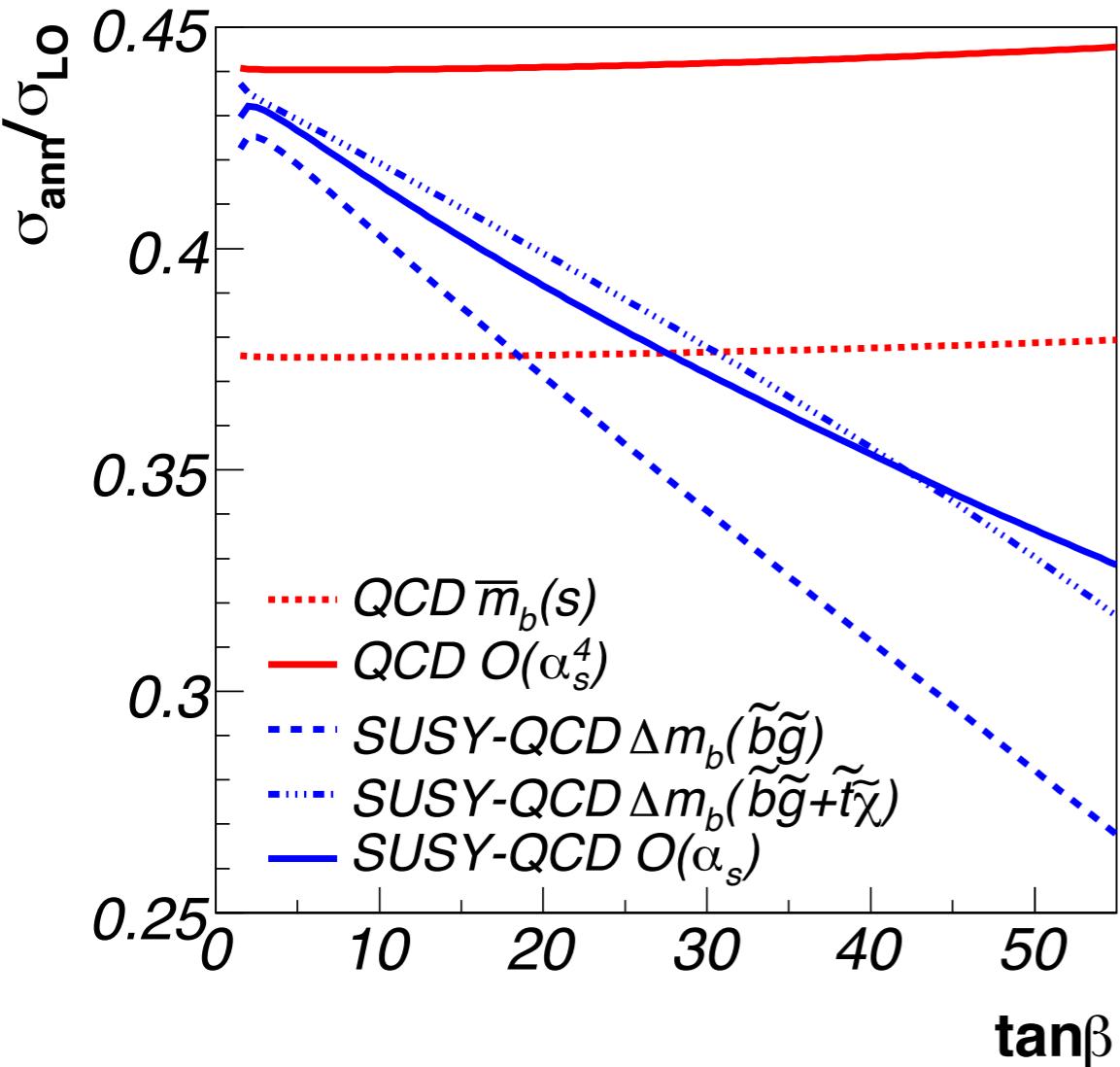


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Main effects on cross-section here due to QCD and SUSY-QCD mass resummation

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Impact of corrections more important than experimental uncertainty

favoured region shifted to smaller masses in order to compensate effect on cross-section

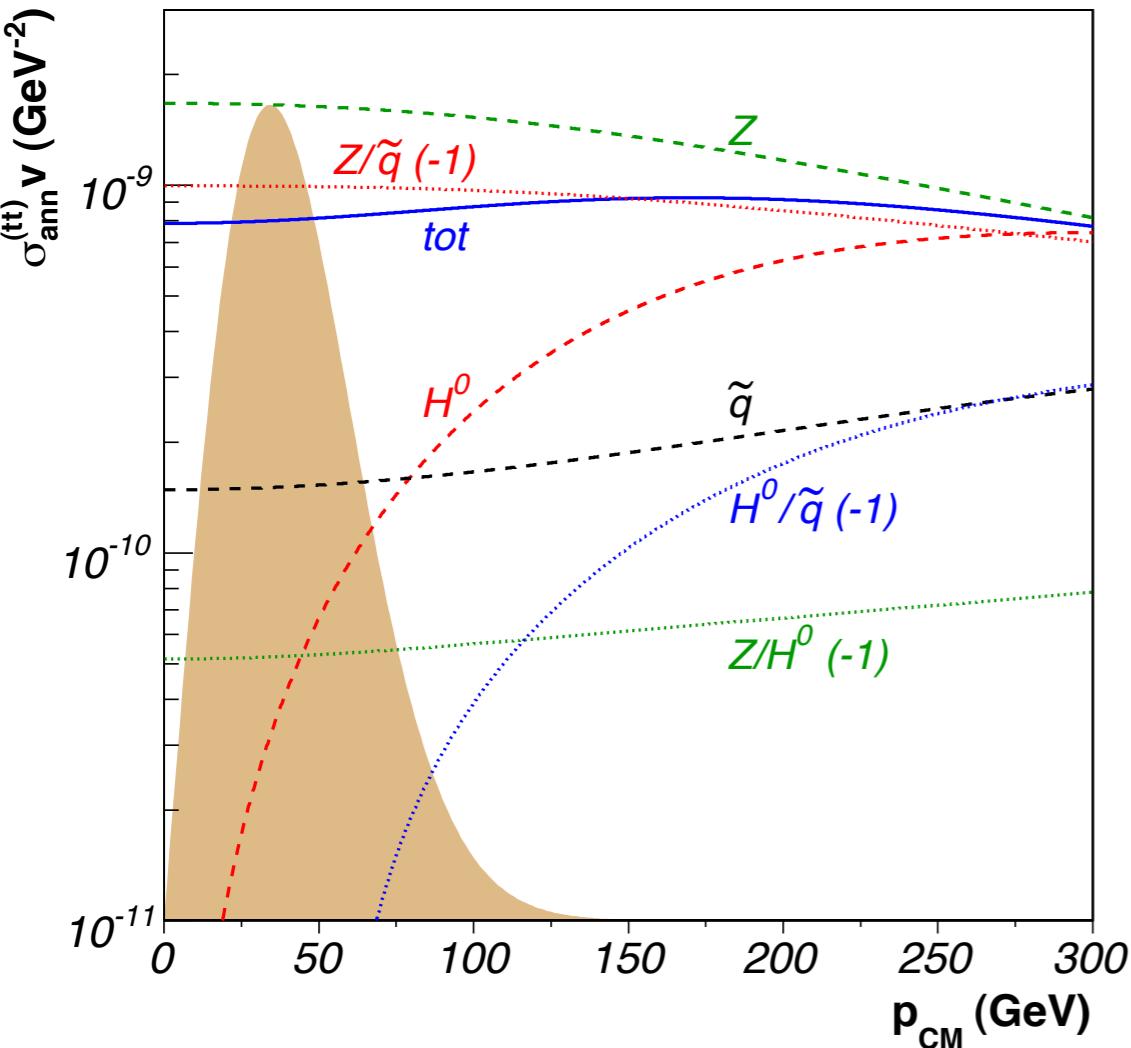
Effect reversed around the Higgs-pole due to corrections to decay width

[Herrmann and Klasen, PRD 76: 117704 (2007)]

# Case of dominant Z- or squark-exchange

$m_0$	1500 GeV	$\Omega_{\tilde{\chi}} h^2$	0.104
$M_{1,2}$	600 GeV	$t\bar{t}$	50.4%
$M_3$	266 GeV	$m_{\tilde{\chi}}$	235.6 GeV
$A_0$	0	$m_{\tilde{t}}$	939.0 GeV
$\tan \beta$	10		
$\text{sgn}(\mu)$	+		

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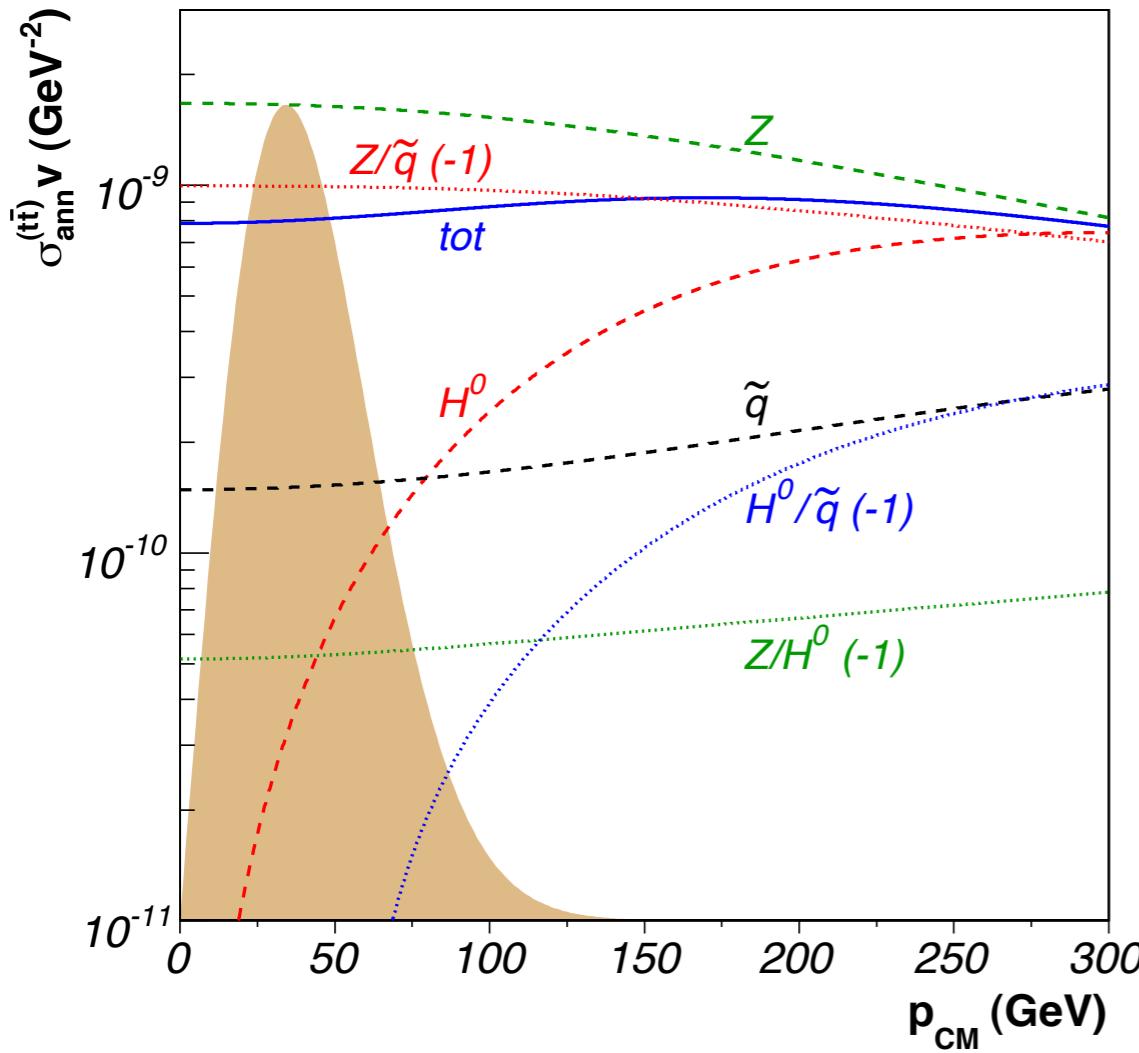


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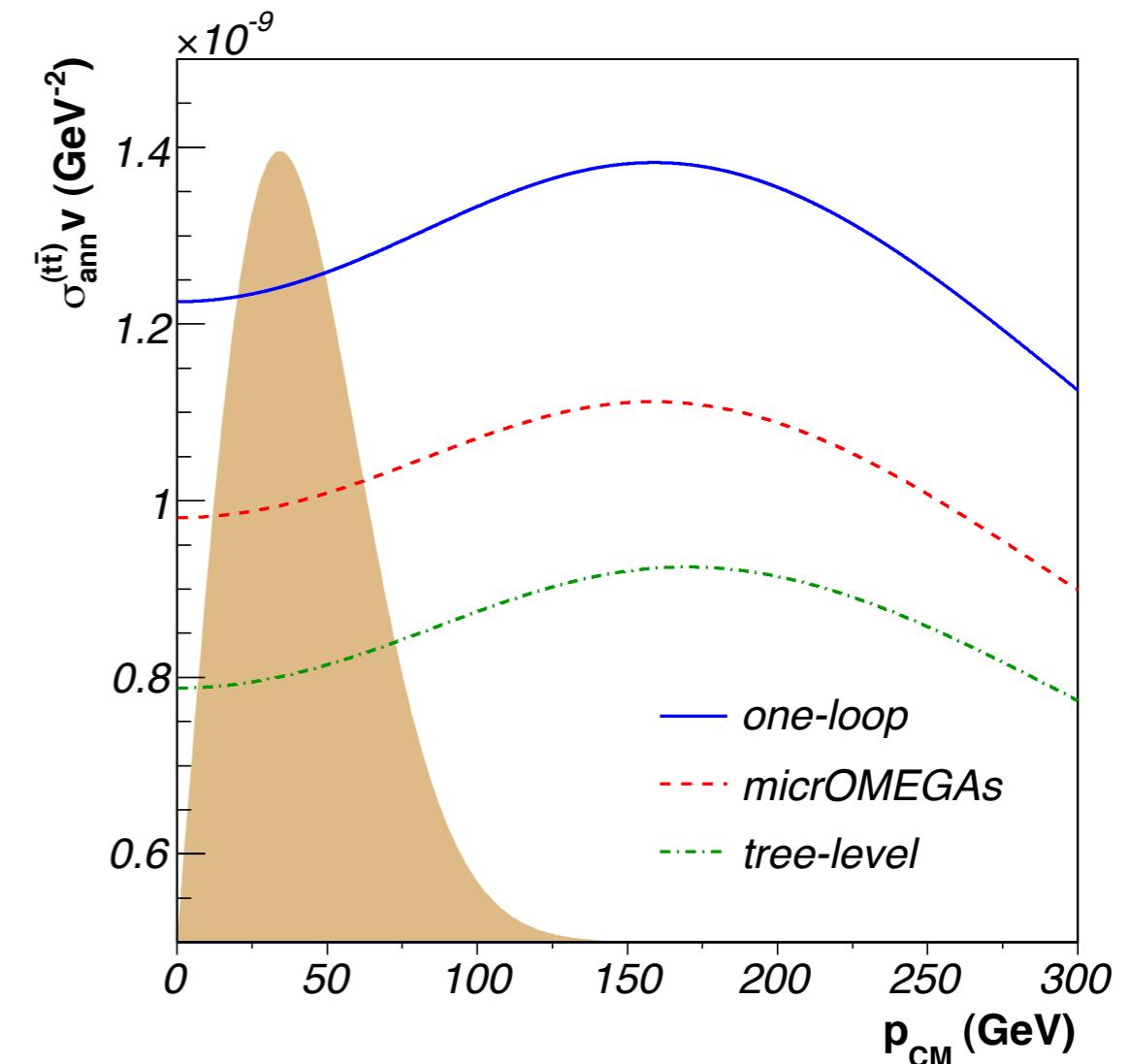
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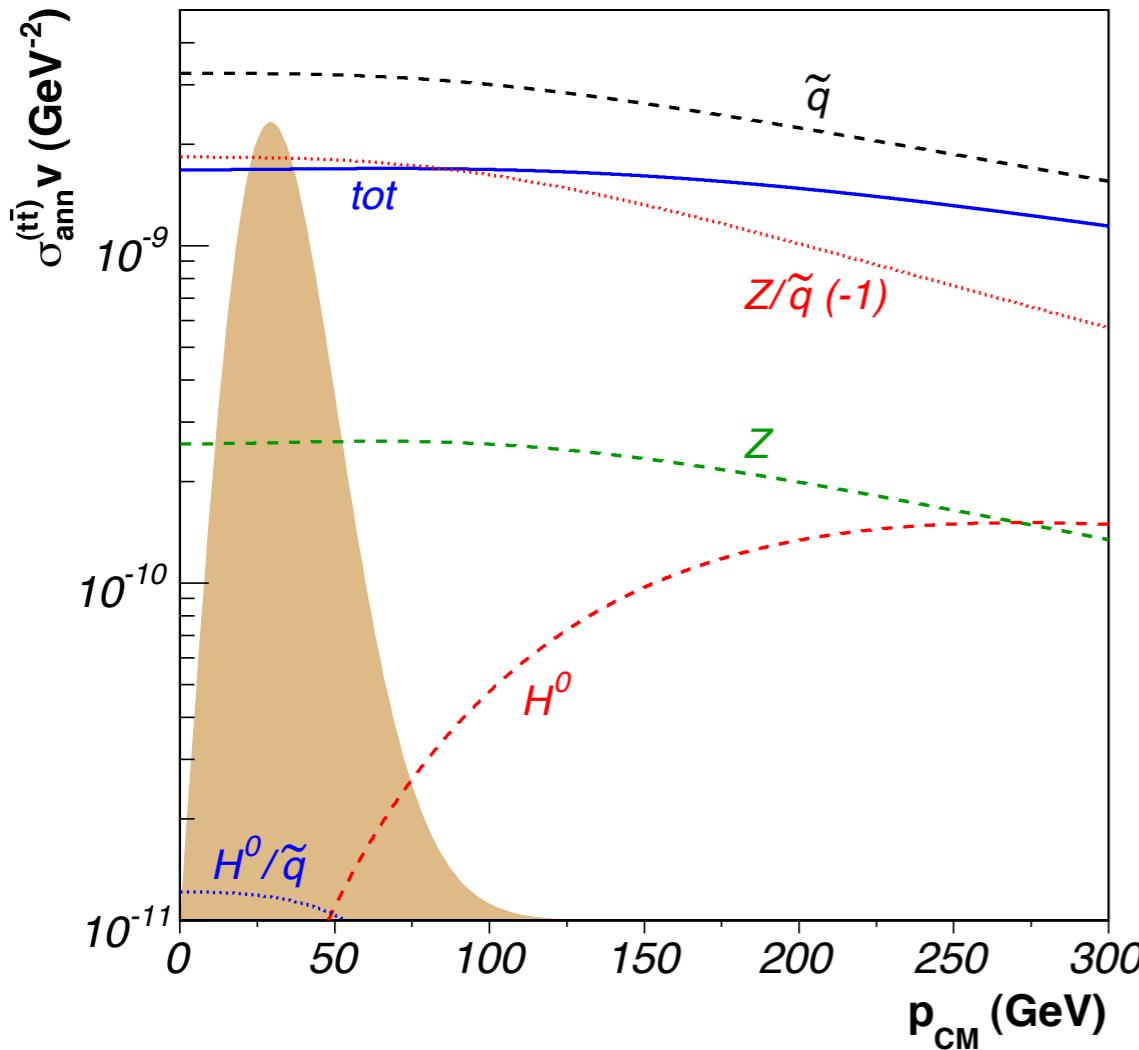
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One-loop corrections increase cross-section by about 50% w.r.t. tree-level approximation

[Herrmann, Klasen, Kovařík, PRD 80: 085025 (2009)]



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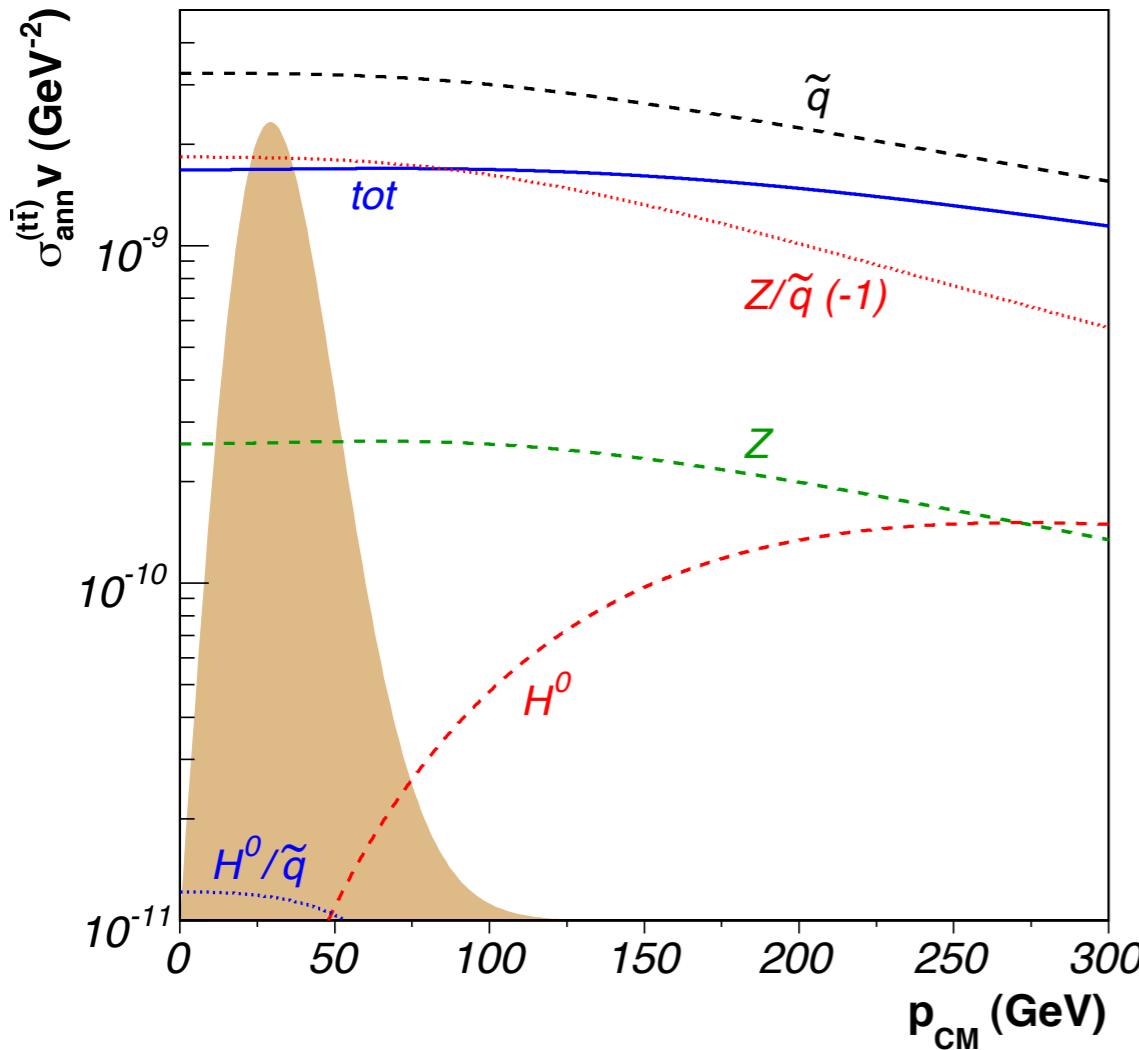


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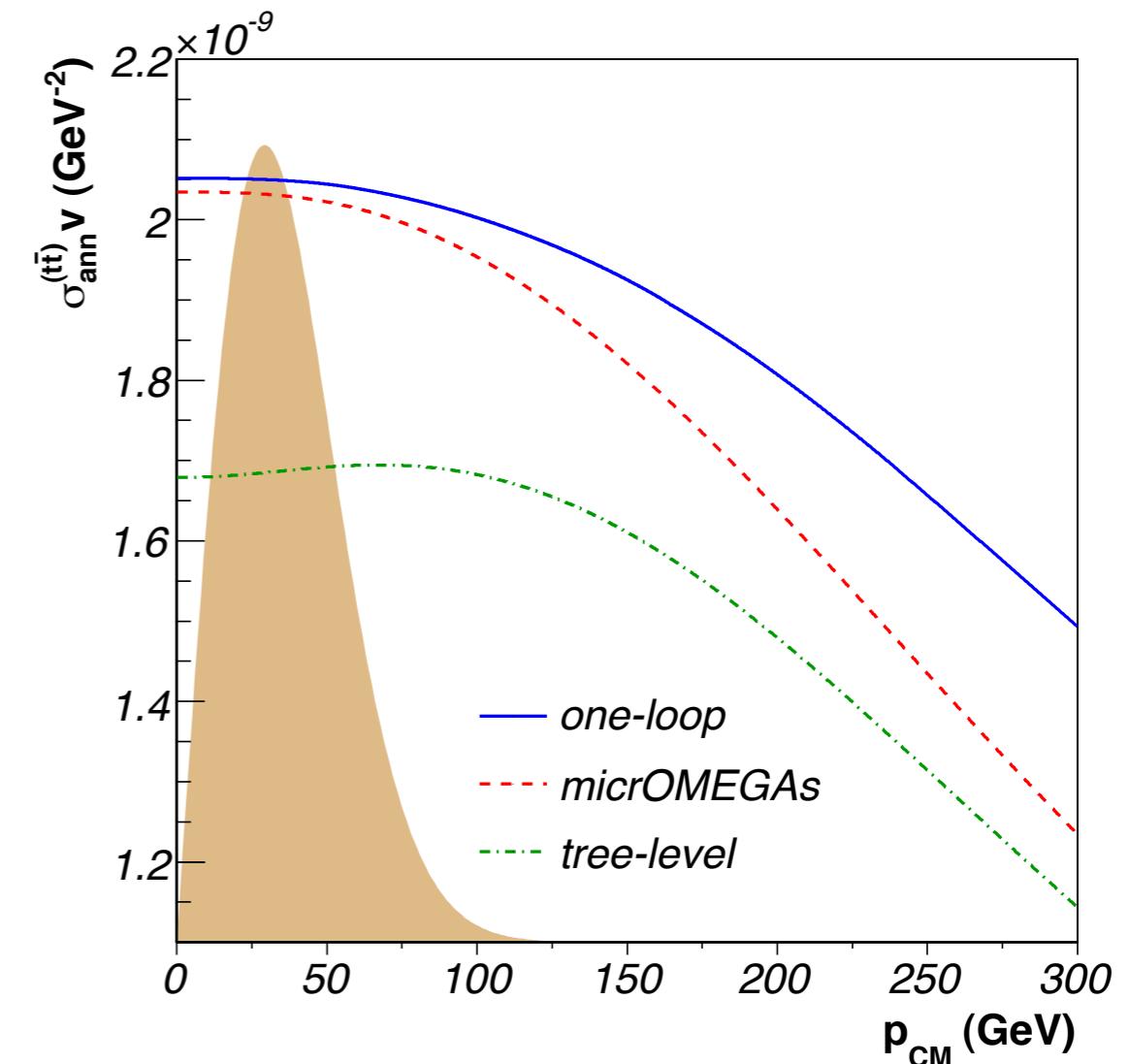
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One-loop corrections increase cross-section by about 25% w.r.t. tree-level approximation

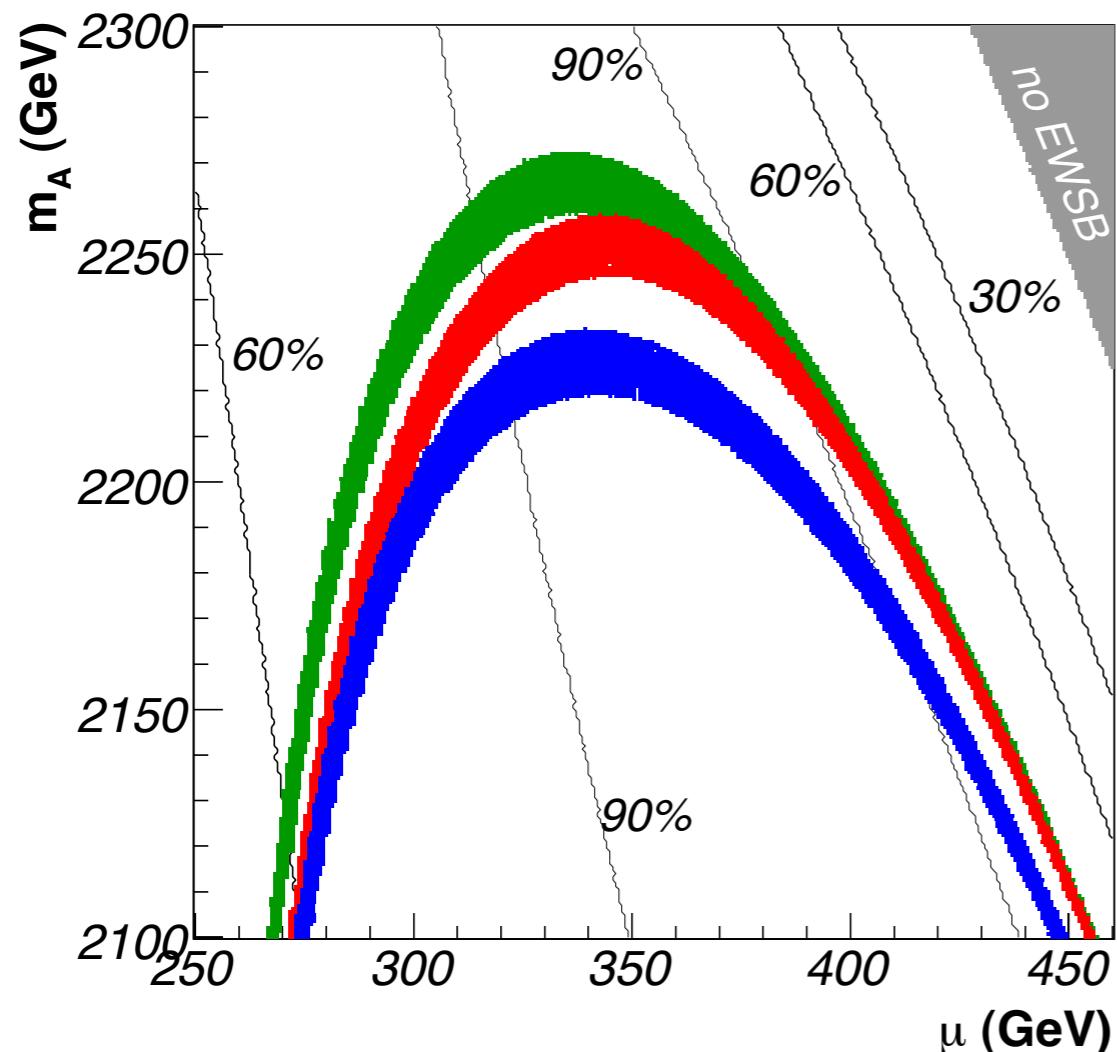
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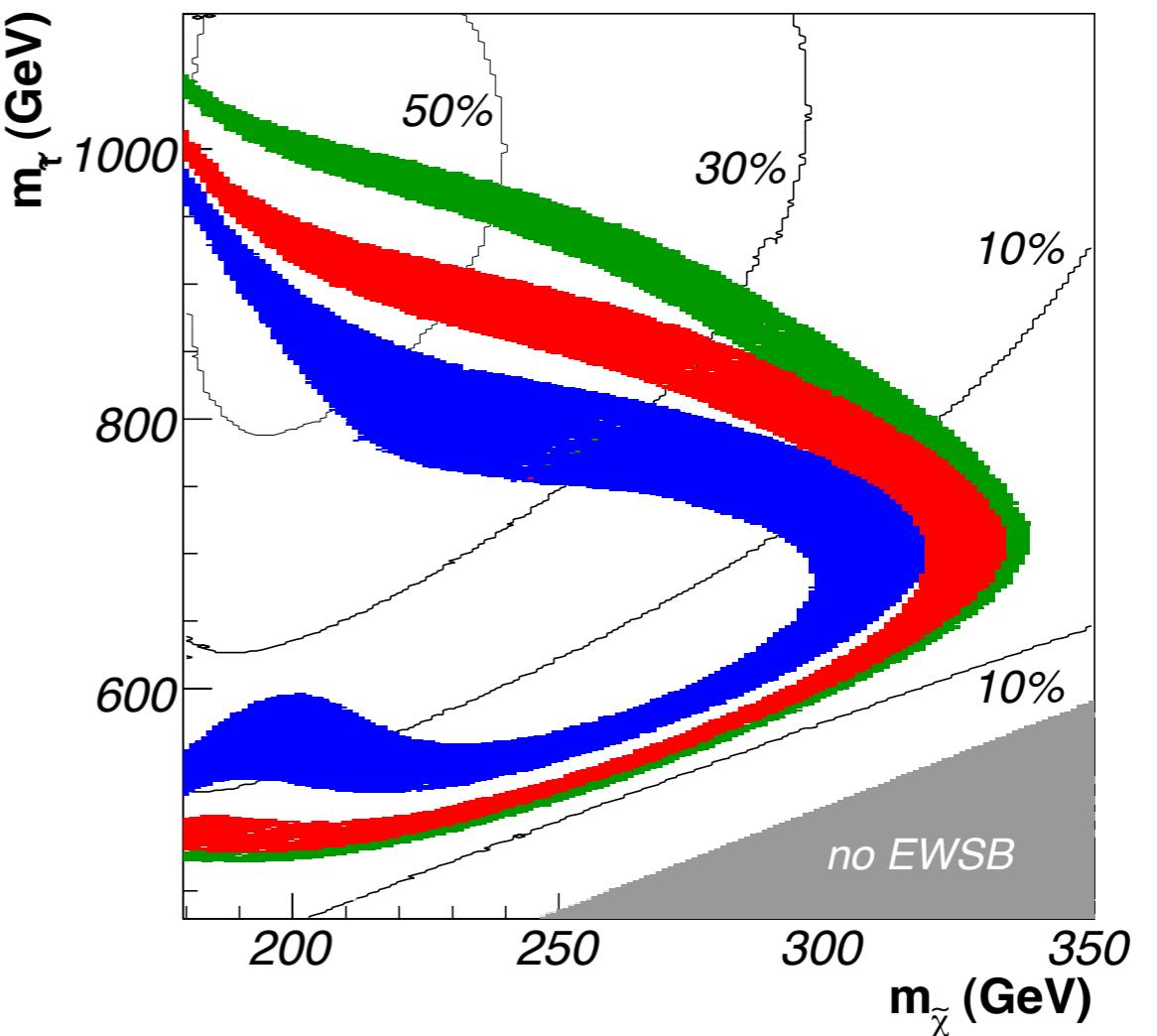
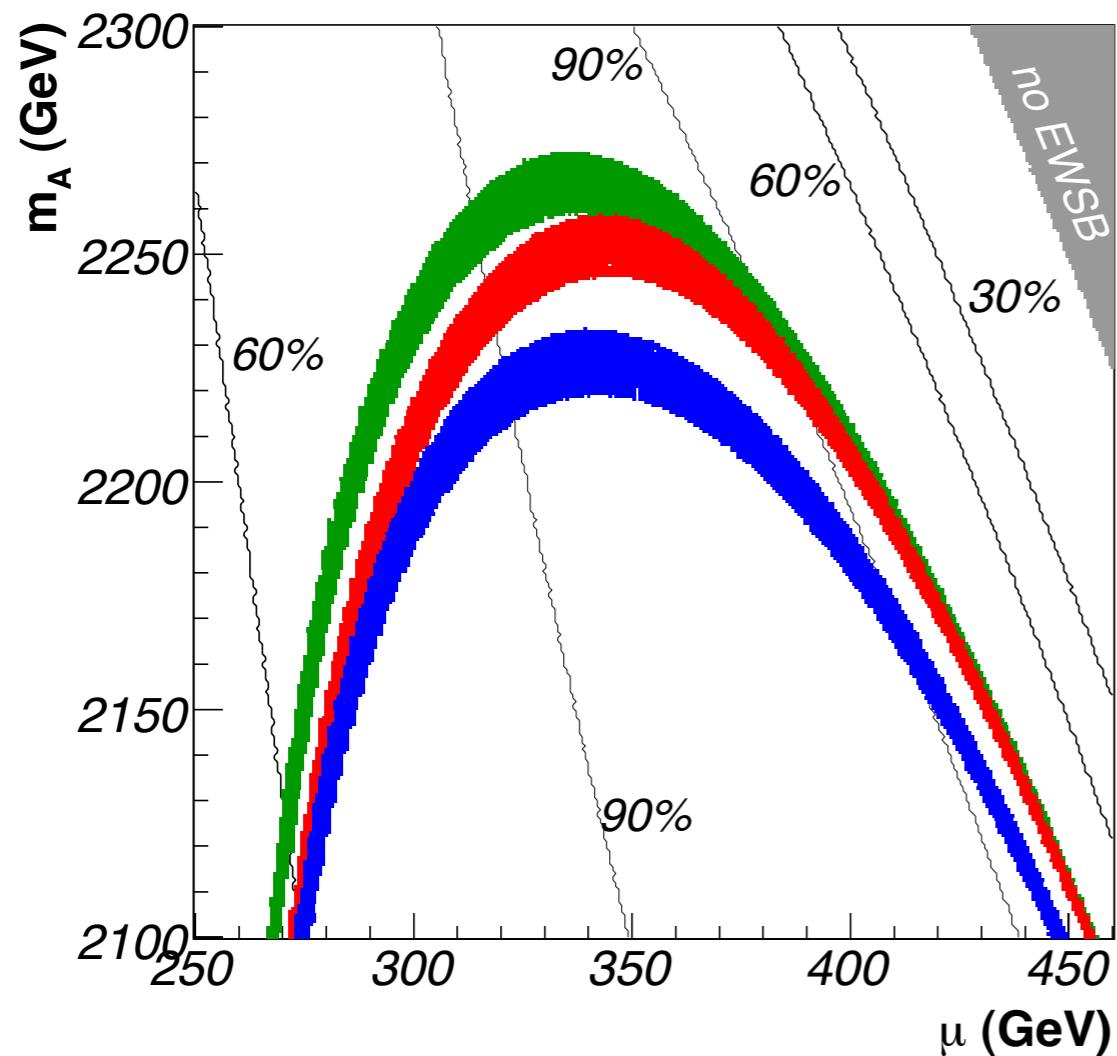
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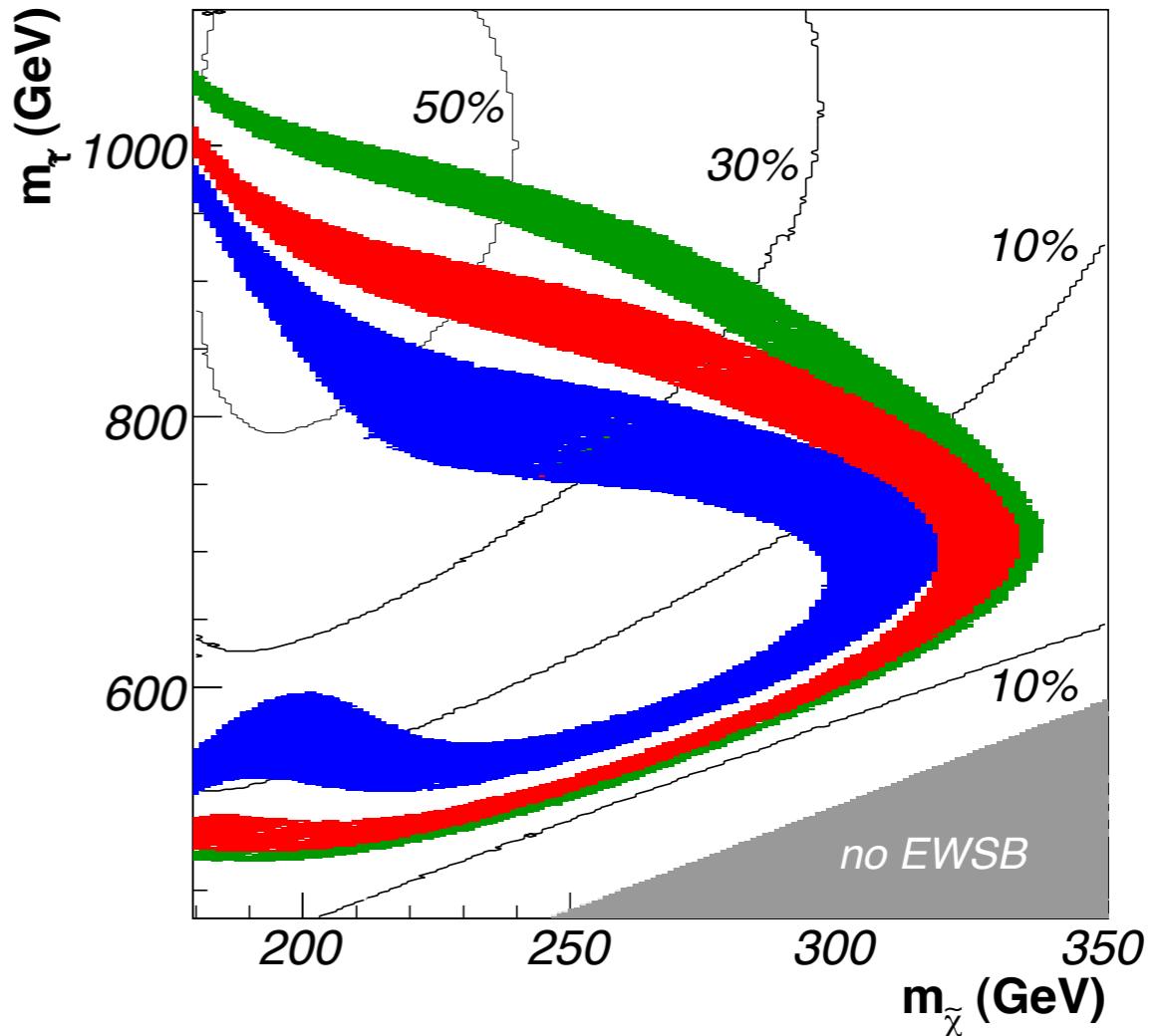
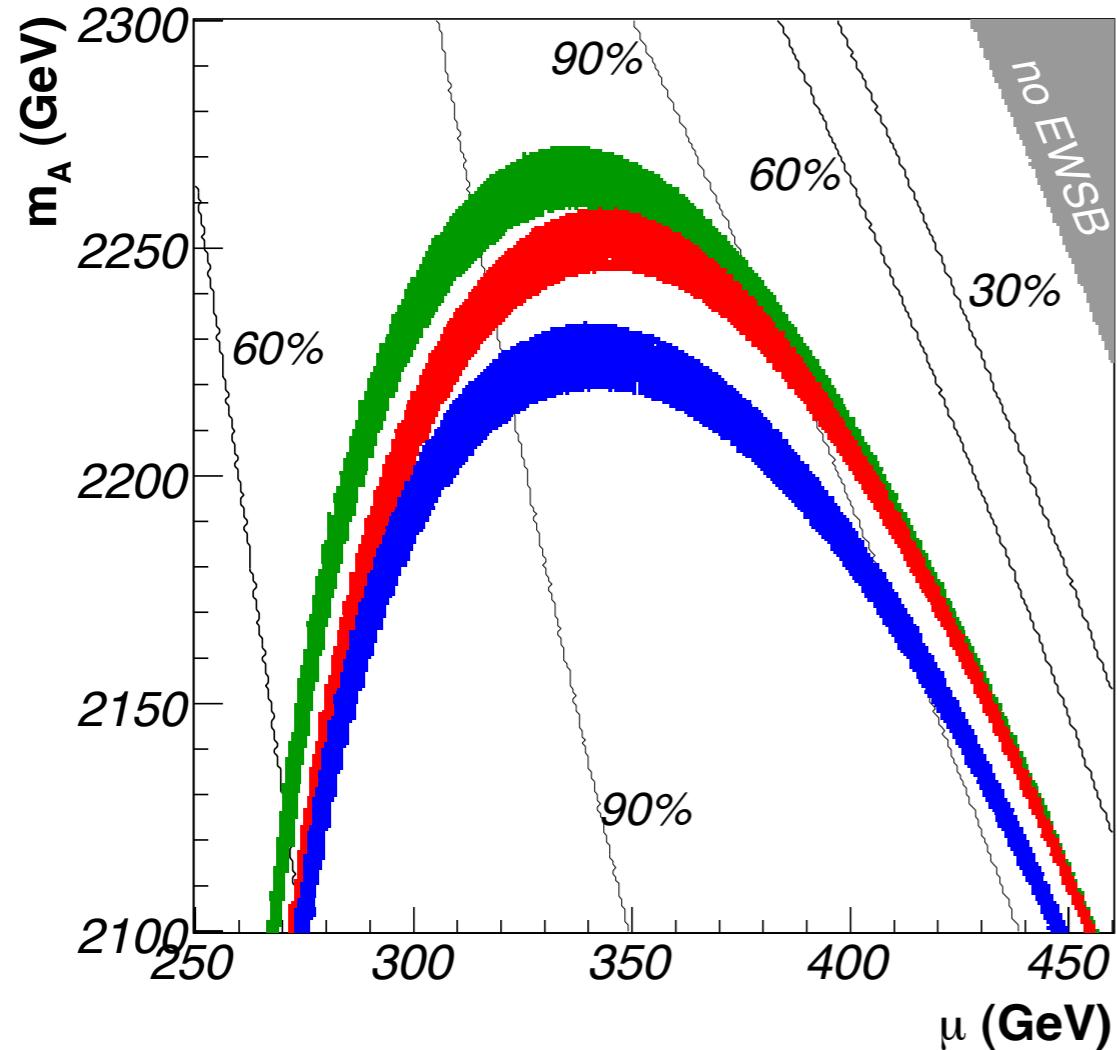
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Important impact of the cosmologically favoured regions of parameter space,  
e.g. shift of about 50 GeV for  $m_A$  or almost 200 GeV for  $m_{\text{stop}}$

Numerical effect larger than experimental uncertainty in relevant regions of parameter space

[Herrmann, Klasen, Kovařík, PRD 80: 085025 (2009)]

# Conclusion and Perspectives

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Relic density calculation allows to obtain constraints on the MSSM parameter space, that are complementary w.r.t. collider data and precision measurements

Impact of QCD corrections to neutralino annihilation into (heavy) quarks  
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Results to be generalized to 1<sup>st</sup> and 2<sup>nd</sup> generation quarks [Herrmann, Klasen, Kovařík (in progress)]

QCD corrections also relevant for co-annihilations

with neutralinos or charginos [Herrmann, Klasen, Kovařík (to be published)]

with lightest squark [Freitas 2007, Herrmann *et al.* (in preparation)]

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Further uncertainties due to renormalization group evolution [Bélanger et al. 2005],  
cosmological model [Arbey et al. 2009], QCD equation of state [Hindmarsh et al. 2005], ...