AN ESTIMATE of A DILATON MASS FROM QUARKONIUM SPECTRA.

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Outline

- Presentation and Motivations.
- Model Description & Derivation
- Dilaton Mass
- Conclusion

- Fundamental scalars with direct coupling to gauge curvature terms in higher dimensional unified theories (string, KK...) offers a challenge with attractive implications in 4d gauge theories.
- Gauge theories with a dilaton present rather an interesting than worrisome prediction of string theory.
- Vacuum topological structure of theories with dilaton field is drastically changed compared to the non dilatonic ones.
- Main interest.....Two relevant problems:
  - How light dilaton affects the Coulomb problem?
  - Can this low energy model estimate the dilaton mass?

• R. Dick, (1997); E. Saidi, MC, (2000)

Suggest a new scenario to generate color confinement......Challenge to monopole condensation.

It provides interesting mechanism which accommodates both the Coulomb and confining phases.

Color confinement can be signaled through the behavior of the interaction potential at large distances.

Description of the Effective Theory

• The impact of dilaton on a 4d effective nonabelian gauge theory is described by a Lagrangian density:

$$\mathcal{L}(\phi, A) = -\frac{1}{4F(\phi)} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) - J^\mu_a A^a_\mu,$$

• ϕ is the dilaton field and $G^{\mu\nu}$ is the standard field strength tensor of the theory. • $V(\phi)$ denotes the dilaton potential and $F(\phi)$ represents the coupling function depending on the dilaton field.

Several forms of $F(\phi)$ have been proposed in literature.

• The most popular one, $F(\phi) = e^{-k\frac{\phi}{f}}$ (string theory, KK theories)

• $\frac{1}{F(\phi)} = \frac{\phi}{f}$: used first by Cornwall-Soni as a low energy contribution to QCD effective models.

• The problem of the Coulomb gauge theory augmented with dilatonic degrees of freedom is analyzed as follows: • First consider a point like static Coulomb source defined in the rest frame by the current:

$$J_a^\mu = g\delta(r) C_a \nu_0^\mu = \rho_a \eta_0^\mu , \tag{1}$$

C_a is the expectation value of $SU(N_c)$ generator.

• Field equations emerging from this static configuration are:

$$\left[D_\mu, F^{-1}(\phi) G^{\mu\nu} \right] = J^\nu , \tag{2}$$

$$\partial_\mu \partial^\mu \phi = -\frac{\partial V(\phi)}{\partial \phi} - \frac{1}{4} \frac{\partial F^{-1}(\phi)}{\partial \phi} G_a^{\mu\nu} G_a^{\nu\mu} , \tag{3}$$

• By setting $G^{0i}_a = E^i \chi_a = -\nabla^i \Phi_a$, after some algebra, we derive the chromoelectric field,

$$E_a = \frac{Q^a_{eff}(r)}{r^2} \tag{4}$$

Where the effective charge is defined by

$$Q^a_{eff}(r) = \left(g \frac{C_a}{4\pi} \right) F(\phi(r))$$

• Note that it is the running of the effective charge that makes the potential stronger than the Coulomb potential. In other words, Coulomb spectrum is recovered if the effective charge did not run.

Thereby the interaction potential U(r) reads as,

$$U(r) = 2\widetilde{\alpha}_s \int \frac{F(\phi(r))}{r^2} dr \tag{5}$$

with $\alpha_s = \frac{g^2}{4\,\pi}$ and $\widetilde{\alpha} = \frac{\alpha_s}{8\pi}\left(\frac{N_c - 1}{2N_c}\right)$

• Remarkable Formula:

Direct relation between $U(r)$ and $F(\phi(r))$.

Existence of a confining phase in this effective theory is subject to condition,

$$\lim_{r\to\infty} rF^{-1}(\phi(r)) = finite, \tag{6}$$

- Main objective: Solve the field equations and determine analytically $\phi(r)$ and $\Phi_a(r)$.
- For this, $F(\phi)$ and $V(\phi)$ have to be fixed:
  $V(\phi) = \frac{1}{2}m^2\phi$
  $\frac{1}{F(\phi)} = \frac{\phi^2}{f^2+\beta\,\phi^2}$

Dick R, 1999

- Dick used the form: $\frac{1}{F(\phi)} = \frac{\phi^2}{f^2+\beta\,\phi^2}$

with f characterizing the strength of the dilaton-gluon coupling. and β is a parameter in the range $0 < \beta < 1$.

- The radial dependence of the dilaton field and the interquark potential (up to a color factor):

$$\phi(r) = \pm\frac{1}{r}\sqrt{\frac{k}{m} + (y_0^2 - \frac{k}{m})exp(-2mr)}$$

$$V(r) = [\frac{\beta\, g^2}{4\,\pi\, r} - gf\sqrt{\frac{N_c}{2\,(N_c-1)}}ln[e^{2mr} - 1 + \frac{m}{k}y_o^2]$$

with $k^2 = \frac{\alpha_s\, f^2}{8\,\pi}(\frac{N_c-1}{N_c})$

At large distances, interquark potential increses linearly with r: $V(r) \sim r$.

- Occurrence of this behavior implies that this effective theory can serve to model quark confinement.

Dilaton Mass

This part is dedicated to the estimate of the dilaton mass through the phenomenological investigation of the heavy quarkonium spectra with Dick potential $V_D(r)$

The semi-relativistic wave equation:

$$[(p^2 + m_1^2\ )^{1/2} + (p^2 + m_2^2\ )^{1/2} + V(r) - M\ ]\ \psi(r) = 0\ . \tag{7}$$

where M is the total binding meson mass
$M_{n\ell} = E_{n\ell} + m_1 + m_2$

This problem will be addressed using the shifted-l expansion (SLET) method, Barakat, (2001); MC (2008)

- Yields very accurate and rapidly converging energy eigenvalue series

- Handles highly excited states

- Relativistic corrections are included in a consistent way

SLET formula for the energy eigenvalues,

$$E_{n\ell} = E_0 + \frac{\alpha_{(1)}}{r_0^2\left(1 + \frac{E_0 - V(r_0)}{\eta}\right)} + \frac{\alpha_{(2)}}{r_0^2\left(1 + \frac{E_0 - V(r_0)}{\eta}\right)\bar{l}} \tag{8}$$

Consider Dick interquark potential,

$$V_D(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \frac{4}{3} g f \sqrt{\frac{N_c}{2(N_c - 1)}} \ln[exp(2mr) - 1] \tag{9}$$

- There are five input parameters to be specified:

$$m_c,\ m_b,\ \alpha_s, m \text{ and } f \text{ and } .$$

Numerical analysis: c and b quark masses are set to:

$$m_c = 1.89 \text{ GeV and } m_b = 5.19 \text{ GeV}.$$

- For α_s: account for the running of α_s.

We fix the renormalization scale to $\lambda = 2\mu$ where μ is the reduced mass,

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \tag{10}$$

...and use the formula,

$$\alpha_s(\lambda) = \frac{\alpha_s(m_z)}{1 - (11 - \frac{2}{3}n_f)[\alpha_s(m_z)/2\pi]ln(m_z/\lambda)} \tag{11}$$

combined to $\alpha_s(m_z) = 0.118$, we obtain:

$$\alpha_s = 0.31, \qquad \alpha_s = 0.20 \tag{12}$$

• On the other hand, the dilaton parameters m and f are considered as free in our analysis.

Best fit to experimental data of $c\bar{c}$ and $b\bar{b}$ spectrum:

$$m = 57\ MeV \qquad gf\sqrt{\frac{N_c}{2(N_c-1)}} = 430\ MeV$$

Numerical Results

Table 1: Calculated mass spectra $M_{n\ell}$ of $c\bar{c}$ boundstates (in GeV)

State, $n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.	State, $n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.
1S	3.073	3.068	1P	3.546	3.525
2S	3.662	3.663	2P	3.871	-
3S	4.027	4.028	1D	3.787	3.788

Table 2: Calculated mass spectra $M_{n\ell}$ of $b\bar{b}$ boundstates (in GeV)

State, $n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.	State, $n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.
1S	9.450	9.446	1P	9.903	9.900
2S	10.014	10.013	2P	10.227	10.260
3S	10.299	10.348	1D	10.129	-

- The obtained results for charmoniun and bottomonium fit well experimental data when the dilaton mass is given a value about 57 MeV.

CONCLUSIONS

- Review of some aspects of recent work on 4d non abelian gauge theories with a dilaton.

- These string-inspired effective theories can well be relevant to probe physics beyond the standard model.

- Study of possible impacts of the hypothetical fundametal scalar on the Coulomb problem: interesting paradigm for the description of confinement.

- As by product, quarkonium spectra is compatible with dilaton mass about $57 MeV$: value in the mass range for which dilaton could make a credible DM candidate.

• This estimate should be understood as an order estimate and not an exact result. Neverthless, it provides a piece of information on the dilaton mass.

In my opinion, dilaton gauge theories deserves more inverstigation efforts.....offer unexpected rewards in low energy physics and cosmology.

Thank you for your attention