The Physics of Glueballs

Vincent Mathieu

Université de Mons, Belgique

Strasbourg, January 2009

Outline

References

V. Mathieu, N. Kochelev, V. Vento
"The Physics of Glueballs"
invited review for Int. J. Mod. Phys. E
arXiv:0810.4453 [hep-ph]

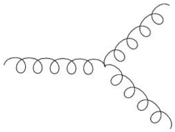
V. Mathieu, F. Buisseret and C. Semay
"Gluons in Glueballs: Spin or Helicity ? "
Phys. Rev. D **77**, 114022 (2008), [arXiv:0802.0088 [hep-ph]]

V. Mathieu, C. Semay and B. Silvestre-Brac
"Semirelativistic Potential Model for Three-gluon Glueballs."
Phys. Rev. D**77** 094009 (2008), [arXiv:0803.0815 [hep-ph]]

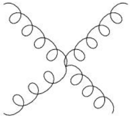
N. Boulanger, F. Buisseret,V. Mathieu, C. Semay
"Constituent Gluon Interpretation of Glueballs and Gluelumps."
arXiv:0806.3174 [hep-ph]
accepted for publication in Eur. Phys. Jour. A

Introduction - QCD

QCD = gauge theory with the color group $\mathcal{SU}(3)$



$$
\begin{aligned}
\mathcal{L}_{QCD} &= -\frac{1}{4}\mathrm{Tr}\, G_{\mu\nu}G^{\nu\mu} + \sum \bar{q}(\gamma^\mu D_\mu - m)q \\
G_{\mu\nu} &= \partial_\mu A_\mu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]
\end{aligned}
$$



Quark = fundamental representation $\mathbf{3}$
Gluon = Adjoint representation $\mathbf{8}$
Observable particles = color singlet $\mathbf{1}$

Mesons: $\mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{1}\oplus\mathbf{8}$

Baryons: $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3} = \mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10}$

Glueballs: $\mathbf{8}\otimes\mathbf{8} = (\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{27})\oplus(\mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}})$

$\mathbf{8}\otimes\cdots\otimes\mathbf{8} = \mathbf{1}\oplus\mathbf{8}\oplus\ldots$

Colored gluons $\rightarrow$ color singlet with only gluons

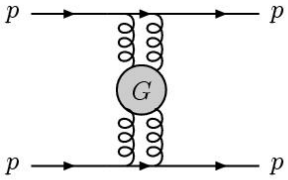
Prediction of the QCD
Production in gluon rich processes (OZI forbidden,...)
Closely linked to the Pomeron:

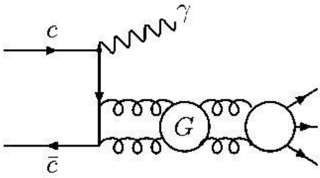
$$J = 0.25M^2 + 1.08$$

Mixing between glueball 0^{++} and light mesons

Candidates: $f_0(1370)$ $f_0(1500)$ $f_0(1710)$

One scalar glueball between those states [Klempt, Phys. Rep. **454**].





Pure states: $|gg\rangle$, $|n\bar{n}\rangle$, $|s\bar{s}\rangle$

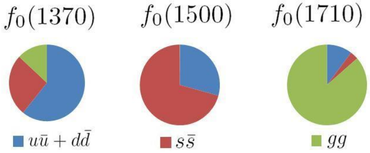
$$|G\rangle = |gg\rangle + \frac{\langle n\bar{n}|gg\rangle}{M_{gg} - M_{n\bar{n}}} |n\bar{n}\rangle + \frac{\langle s\bar{s}|gg\rangle}{M_{gg} - M_{s\bar{s}}} |s\bar{s}\rangle$$

Analysis of

Production: $J/\psi \to \gamma f_0, \omega f_0, \phi f_0$ Decay: $f_0 \to \pi\pi, K\bar{K}, \eta\eta$

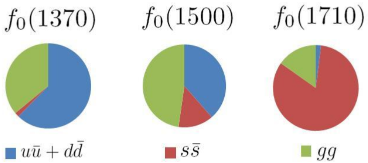
Two mixing schemes

Cheng *et al* [PRD**74**, 094005 (2006)]



$$M_{n\bar{n}} < M_{s\bar{s}} < M_{gg}$$

Close and Kirk [PLB**483**, 345 (2000)]

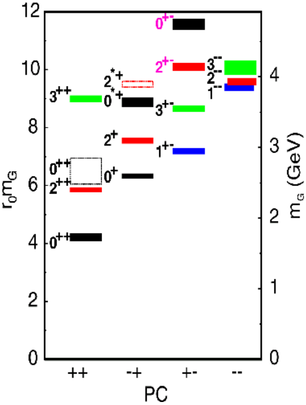


$$M_{n\bar{n}} < M_{gg} < M_{s\bar{s}}$$

Investigation of the glueball spectrum (pure gluonic operators) on a lattice by Morningstar and Peardon [Phys. Rev. D**60**, 034509 (1999)]
Identification of 15 glueballs below 4 GeV

$$M(0^{++}) = 1.730 \pm 0.130 \text{ GeV}$$
$$M(0^{-+}) = 2.590 \pm 0.170 \text{ GeV}$$
$$M(2^{++}) = 2.400 \pm 0.145 \text{ GeV}$$

Quenched approximation (gluodynamics)
→ mixing with quarks is neglected



Lattice studies with $n_f = 2$ exist. The lightest scalar would be sensitive to the inclusion of sea quarks but no definitive conclusion.

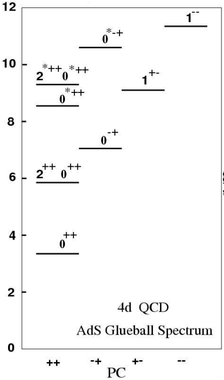
AdS/QCD

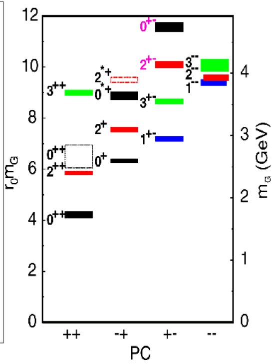
AdS/CFT correspondance:
Correspondance between conformal theories and string theories in AdS spacetime
QCD not conformal → breaking conformal invariance somehow

Introduction of a black hole in AdS to break conformal invariance

Parameter adjusted on 2^{++}

Same hierarchy but some states are missing (spin 3,...)





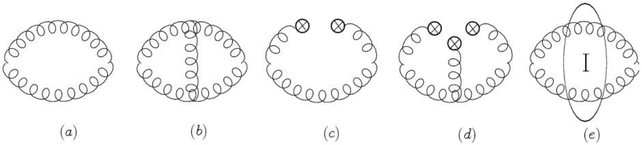
[R. C. Brower *et al.*, Nucl. Phys. B**587**, 249 (2000)]

Gluonic currents: $J_S(x) = \alpha_s G^a_{\mu\nu}(x) G^a_{\mu\nu}(x) \qquad J_P(x) = \alpha_s G^a_{\mu\nu}(x) \widetilde{G}^a_{\mu\nu}(x)$

$$\Pi(Q^2) = i \int d^4x \, e^{iq\cdot x} \langle 0 | T J_G(x) J_G(0) | 0 \rangle = \frac{1}{\pi} \int_0^\infty \frac{\mathrm{Im}\Pi(s)}{s + Q^2} ds$$

Theoretical side (OPE):

$$J_G(x) J_G(0) = C_{(a)+(b)+(e)} \mathbf{1} + C_{(c)} G^a_{\mu\nu} G^{\mu\nu}_a + C_{(d)} f_{abc} G^a_{\alpha\beta} G^b_{\beta\gamma} G^b_{\gamma\alpha} + \cdots$$



Confinement parameterized with condensates $\langle 0 | \alpha_s G^a_{\mu\nu} G^{\mu\nu}_a | 0 \rangle, \ldots$

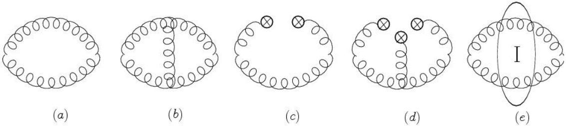
Phenomenological side:

$$\mathrm{Im}\Pi(s) = \sum_i \pi f^2_{G_i} m^4_{G_i} \delta(s - m^2_{G_i}) + \pi \theta(s - s_0) \mathrm{Im}\Pi(s)^{\mathrm{Cont}}$$

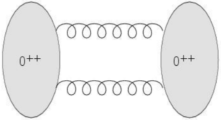
Sum rules calculation by Forkel [Phys. Rev. D**71**, 054008 (2005)]

$$M(0^{++}) = 1.25 \pm 0.20 \text{ GeV} \quad M(0^{-+}) = 2.20 \pm 0.20 \text{ GeV} \tag{1}$$

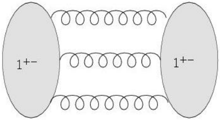
Two gluons in interaction

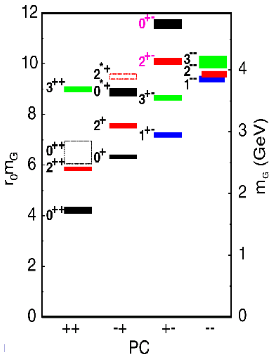


Two gluons → $C = +$

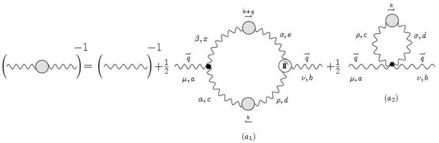


Three gluons → $C = -$

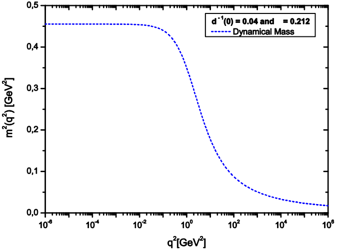




Gluons massless in the Lagrangian
Non-perturbative effects → dynamical mass
Aguilar and Papavassiliou [Eur. Phys. J. A**35**,189 (2008)]



$$m^2(q^2) = m_0^2 \left[\frac{\ln\left(\frac{q^2+\rho m_0^2}{\Lambda^2}\right)}{\ln\left(\frac{\rho m_0^2}{\Lambda^2}\right)} \right]^{\gamma}$$



Gluons ∼ heavy quarks

Gluonium models for the low-lying glueballs with Spinless Salpeter Hamiltonian

$$H_{gg} = 2\sqrt{\boldsymbol{p}^2 + m^2} + V(r)$$

bare mass $m = 0$ and effective mass $\mu = \langle \Psi | \sqrt{\boldsymbol{p}^2} | \Psi \rangle$ state dependent (Simonov 1994)
Gluons spin-1 particles with the usual rules of spin coupling

Brau and Semay [Phys. Rev. D**70**, 014017 (2004)]

Two-gluon glueballs $\rightarrow C = +$ and $P = (-1)^L$

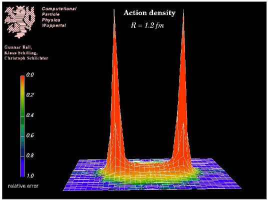
$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ with $S = 0, 1, 2$.

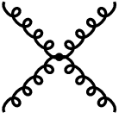
$$H^0 = 2\sqrt{\boldsymbol{p}^2} + \frac{9}{4}\sigma r - 3\frac{\alpha_s}{r}$$

Cornell potential does not lift the degeneracy between states with different S

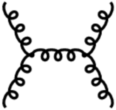
Corrections of order $\mathcal{O}(1/\mu^2)$, with

$$\mu = \langle \Psi | \sqrt{\boldsymbol{p}^2} | \Psi \rangle$$





Structures coming from the OGE



$$V_{\text{oge}} = \lambda \Bigg[\left(\frac{1}{4} + \frac{1}{3}\boldsymbol{S}^2 \right) U(r) - \frac{\pi}{\mu^2}\delta(\boldsymbol{r}) \left(\frac{5}{2}\boldsymbol{S}^2 - 4 \right)$$
$$\frac{3}{2\mu^2}\frac{U'(r)}{r}\boldsymbol{L}\cdot\boldsymbol{S} - \frac{1}{6\mu^2}\left(\frac{U'(r)}{r} - U''(r) \right) T \Bigg]$$

with $\lambda = -3\alpha_S$ and $U(r) = \exp(-\mu r)/r$.

Eigenvalue problem → decomposition in a basis

$$H|\psi\rangle = E|\psi\rangle \quad |\psi\rangle = \sum_i \alpha_i |\phi^i\rangle$$

Matrix elements: $\mathcal{H}^{ij} = \langle\phi^j|H|\phi^i\rangle = \mathcal{T}^{ij} + \mathcal{V}^{ij}, \quad \mathcal{N}^{ij} = \langle\phi^j|\phi^i\rangle = \delta^{ij}$

2-body problem → one radial variable r

$$H = 2\sqrt{\boldsymbol{p}^2} + \frac{9}{4}\sigma r - 3\frac{\alpha_s}{r}$$

Lagrange mesh (x_i) based on Laguerre polynomials
Not a variational procedure ! Matrix elements evaluated on the mesh (x_i)

$$\mathcal{V}^{ij} = \langle\phi^j|V|\phi^i\rangle = V(x_i)\delta^{ij}$$

$\langle\phi^j|\mathbf{p}^2|\phi^i\rangle$ analytical → computation of $\langle\phi^j|\sqrt{\mathbf{p}^2 + m^2}|\phi^i\rangle$

Applications to nuclear physics
D. Baye and P.-H. Heenen, J. Phys. A**19**, 2041 (1986)
C. Semay, D. Baye, M. Hesse and B. Silvestre-Brac, Phys. Rev. E**64**, 016703 (2001)

TWO-GLUON GLUEBALLS $C = +$

Smearing of the attractive $\delta^3(\boldsymbol{r})$ by a gluon size

$$\rho(\boldsymbol{r}, \gamma) = \exp(-r/\gamma)/r$$

Replacement of potentials by convolutions with the size function

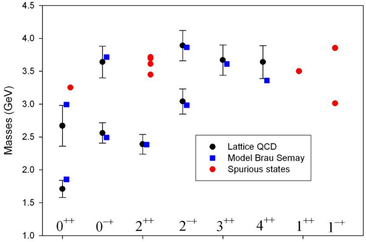
$$V(\boldsymbol{r}) \to \widetilde{V}(\boldsymbol{r}, \gamma) = \int V(\boldsymbol{r} + \boldsymbol{r}^*) \rho(\boldsymbol{r}^*, \gamma) d\boldsymbol{r}^*$$

Parameters

$$\sigma = 0.21 \text{ GeV}^2 \quad \alpha_s = 0.50 \quad \gamma = 0.5 \text{ GeV}^{-1}$$

Good agreement with lattice QCD
Gluons = spin 1 → spurious $J = 1$ states and indetermination of states
For instance, 0^{++} can be $(L, S) = (0, 0)$ or $(2, 2)$

J^{PC}	(L, S)		
0^{++}	(0,0)	(2,2)	$(0,0)^*$
0^{-+}	(1,1)	$(1,1)^*$	
2^{++}	(0,2)	(2,0)	(2,2)
2^{-+}	(1,1)	$(1,1)^*$	
3^{++}	(2,2)		
4^{++}	(2,2)		
1^{++}	(2,2)		
1^{-+}	(1,1)	$(1,1)^*$	

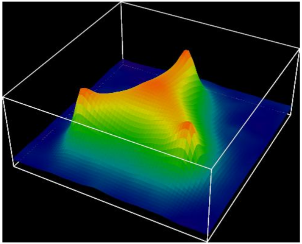


The Model

Extension of the model to three-gluon systems

$$H = \sum_{i=1}^{3} \sqrt{\boldsymbol{p}_i^2} + \frac{9}{4}\sigma \sum_{i=1}^{3} |\boldsymbol{r}_i - \boldsymbol{R}_{\text{cm}}| + V_{\text{OGE}}$$

Same parameters $(\sigma, \alpha_S, \gamma)$ as for two-gluon glueballs



$$V_{\text{OGE}} = -\frac{3}{2}\alpha_S \sum_{i<j=1}^{3} \left[\left(\frac{1}{4} + \frac{1}{3}\boldsymbol{S}_{ij}^2 \right) U(r_{ij}) - \frac{\pi}{\mu^2}\delta(\boldsymbol{r}_{ij}) \left(\beta + \frac{5}{6}\boldsymbol{S}_{ij}^2 \right) \right]$$
$$-\frac{9\,\alpha_S}{4\mu^2} \sum_{i<j=1}^{3} \boldsymbol{L}_{ij} \cdot \boldsymbol{S}_{ij} \frac{1}{r_{ij}} \frac{d}{dr_{ij}} U(r_{ij}) \quad \text{with } U(r) = \frac{e^{-\mu r}}{r}$$

We find the eigenvalue of this operator thanks to a Gaussian basis

Three-gluon glueballs $C = -$

Gaussian functions $\rightarrow$ analytical matrix elements

$$|\phi_i(\boldsymbol{x}, \boldsymbol{y}, A, u)\rangle = \exp\left[-\tilde{\boldsymbol{z}} A_i \boldsymbol{z}\right] \mathcal{Y}_{LM}(\tilde{u}_i \boldsymbol{z}) = \exp\left[-\tilde{\boldsymbol{z}} A_i \boldsymbol{z}\right] |\tilde{u}\boldsymbol{z}|^L Y_{LM}(\widehat{\tilde{u}_i \boldsymbol{z}})$$

$$\langle \phi_i | V(|\alpha \boldsymbol{x} + \beta \boldsymbol{y}|) | \phi_j \rangle \rightarrow \int_0^\infty V(r) e^{-kr^2} dr.$$

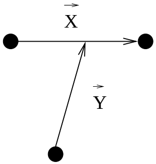
Three-body systems, $\tilde{\boldsymbol{z}} = \begin{pmatrix} \boldsymbol{x} & \boldsymbol{y} \end{pmatrix}$ $\rightarrow$five variational parameters
Fourier transform is also a Gaussian[a]:

$$A_i = \begin{pmatrix} a_i & c_i \\ c_i & b_i \end{pmatrix}$$

$$\mathcal{F}\left[|\phi_i(\boldsymbol{x}, \boldsymbol{y}, A, u)\rangle\right] \propto \phi_i(\boldsymbol{p}, \boldsymbol{q}, A^{-1}/4, A^{-1}u)\rangle$$

$$\tilde{u}_i = \begin{pmatrix} d_i & e_i \end{pmatrix}$$

$\rightarrow$ Computation of $\langle \phi_i | \sqrt{\boldsymbol{p}^2 + m^2} | \phi_j \rangle$
Basis infinite dimensional but we fix the number of Gaussian
$\rightarrow$ the energy is an upper bound of the real energy
Variational problem realized with MINUIT :



$$\frac{\partial E}{\partial A_i} = \frac{\partial E}{\partial u_i} = 0 \quad \forall i$$

[a] Silvestre-Brac and V.M., PRE**76**, 046702 (2007); PRE**77**, 036706 (2008)

Three-gluon glueballs $C = -$

The results

Gluons with spin $\rightarrow \boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$

Good results for 1^{--} and 3^{--} but higher $2^{--} \leftarrow$ symmetry

Disagreement with lattice QCD for $PC = +-$

This model cannot explain the splitting ~ 2 GeV between 1^{+-} and 0^{+-}

$0^{+-}, 1^{+-}, 2^{+-}, 3^{+-}$ are $L = 1$ and degenerate in a model with spin

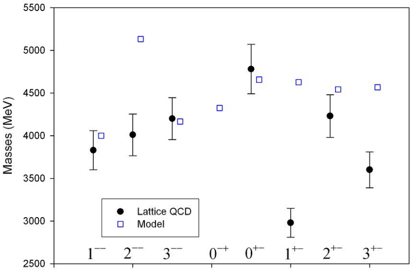
J^{PC}	(L,S)	J^{PC}	(L,S)
1^{--}	(0,1)	0^{+-}	(1,1)
2^{--}	(0,2)	1^{+-}	(1,1)
3^{--}	(0,3)	2^{+-}	(1,1)
0^{-+}	(1,1)	3^{+-}	(1,2)

Solution: Implementation of the helicity formalism for three transverse gluons.

$$d_{abc} A^a_\mu A^b_\nu A^c_\rho \qquad [(\mathbf{88})_{\mathbf{8}_s}\mathbf{8}]^{\mathbf{1}} \qquad C = -$$

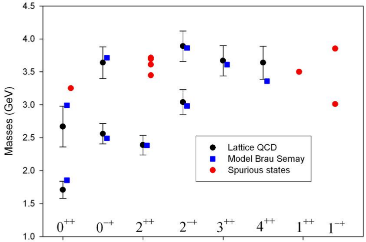
$$f_{abc} A^a_\mu A^b_\nu A^c_\rho \qquad [(\mathbf{88})_{\mathbf{8}_a}\mathbf{8}]^{\mathbf{1}} \qquad C = +$$

S	S_{int}	Symmetry	J^{PC}
0	0	1 A	0^{-+}
1	0, 1, 2	1 S, 2 MS	1^{--}
2	1, 2	2 MS	2^{--}
3	2	1 S	3^{--}



Helicity Formalism for two-gluon glueballs

Yang's theorem: $\rho \nrightarrow \gamma\gamma$
$\gamma\gamma \sim gg \rightarrow J \neq 1$
$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ cannot hold for relativistic system. $\boldsymbol{J}$ is the only relevant quantum number.
Solution: Helicity formalism for transverse gluons with helicity $s_i = 1$. Only two projection, $\lambda_i = \pm 1$.



Formalism to handle with massless particles [Jacob and Wick (1959)]
State J^{PC} in term of usual (L, S) states (with $\Lambda = \lambda_1 - \lambda_2$):

$$|J, M; \lambda_1, \lambda_2\rangle = \sum_{L,S} \left[\frac{2L+1}{2J+1}\right]^{1/2} \langle L0S\Lambda |\, J\Lambda\,\rangle \langle s_1\lambda_1 s_2 - \lambda_2 |\, S\Lambda\,\rangle \left|{}^{2S+1}L_J\right\rangle .$$

Not eigenstates of the parity and of the permutation operator.

$$\begin{aligned} \mathtt{P}\,|J, M; \lambda_1, \lambda_2\rangle &= \eta_1\eta_2(-1)^J\,|J, M; -\lambda_1, -\lambda_2\rangle\,, \\ P_{12}\,|J, M; \lambda_1, \lambda_2\rangle &= (-1)^{J-2s_i}\,|J, M; \lambda_2, \lambda_1\rangle\,, \end{aligned}$$

Construction of states symmetric (bosons) and with a good parity → selection rules.

$$
\begin{aligned}
\left|S_+;(2k)^+\right\rangle &\Rightarrow 0^{++},2^{++},4^{++},\ldots\\
\left|S_-;(2k)^-\right\rangle &\Rightarrow 0^{-+},2^{-+},4^{-+},\ldots\\
\left|D_+;(2k+2)^+\right\rangle &\Rightarrow 2^{++},4^{++},\ldots\\
\left|D_-;(2k+3)^+\right\rangle &\Rightarrow 3^{++},5^{++},\ldots
\end{aligned}
$$

States expressed in term of the usual basis (useful to compute matrix elements!).
Low-lying states:

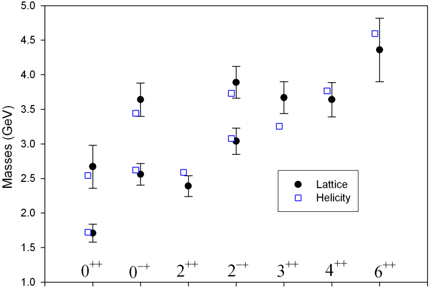
$$
\begin{aligned}
\left|S_+;0^+\right\rangle &= \sqrt{\frac{2}{3}}\left|{}^1S_0\right\rangle + \sqrt{\frac{1}{3}}\left|{}^5D_0\right\rangle,\\
\left|S_-;0^-\right\rangle &= \left|{}^3P_0\right\rangle,\\
\left|D_+;2^+\right\rangle &= \sqrt{\frac{2}{5}}\left|{}^5S_2\right\rangle + \sqrt{\frac{4}{7}}\left|{}^5D_2\right\rangle + \sqrt{\frac{1}{7}}\left|{}^5G_2\right\rangle,\\
\left|D_-;3^+\right\rangle &= \sqrt{\frac{5}{7}}\left|{}^5D_3\right\rangle + \sqrt{\frac{2}{7}}\left|{}^5G_3\right\rangle,\\
\left|S_-;2^-\right\rangle &= \sqrt{\frac{2}{5}}\left|{}^3P_2\right\rangle + \sqrt{\frac{3}{5}}\left|{}^3F_3\right\rangle.
\end{aligned}
$$

Same hierarchy as the lattice QCD
Application with a simple Cornell potential

$$H^0 = 2\sqrt{\boldsymbol{p}^2} + \frac{9}{4}\sigma r - 3\frac{\alpha_s}{r}.$$

Parameters:

$$\sigma = 0.185 \text{ GeV}^2 \qquad \alpha_s = 0.45$$



Addition of an instanton induced interaction to split the degeneracy between the 0^{++} and 0^{-+}

$$\Delta H_I = -P\,\mathcal{I}\,\delta_{J,0} \qquad \text{with } \mathcal{I} = 450 \text{ MeV}.$$

Instanton attractive in the scalar channel and repulsive in the pseudoscalar and equal in magnitude
Very good agreement without spin-dependent potential
Extension for three-body systems ?

Three-gluon glueballs with transverse gluons

$\mathcal{SU}(2) \times \mathcal{SU}(3)$ decomposition for two gluons → to the lowest J:

$$\square^{a} \otimes \square^{b} = \square\square^{(ab)} \oplus \bullet^{(ab)} \oplus \square^{[ab]}$$

Lowest J allowed for 3 gluons with helicity:

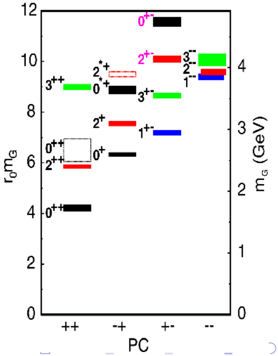
$$\square^{a} \otimes \square^{b} \otimes \square^{c} = \square\square\square^{(abc)} \oplus \square^{(abc)} \oplus \cdots \oplus \bullet^{[abc]}$$

Low-lying states are $J = 1$ and $J = 3$ with symmetric colour function and $J = 0$ with an antisymmetric colour function

Low-lying states are the $1^{\pm -}$, $3^{\pm -}$ and the $0^{\pm +}$

$J = 0^{P-}$ are not allowed for three-gluon glueballs
→ 0^{+-} four transverse gluons

The result should be confirmed by a detailed analysis of the three-body helicity formalism
These developments are under construction

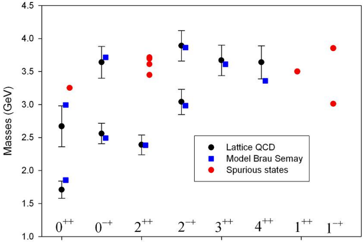


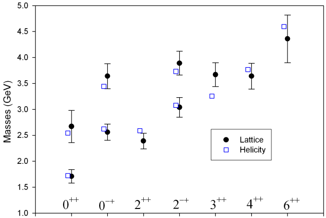
Model with spin-1 gluons reproduces the pure gauge spectrum but spin-dependent potentials should be added and the spectrum is plagued with unwanted states

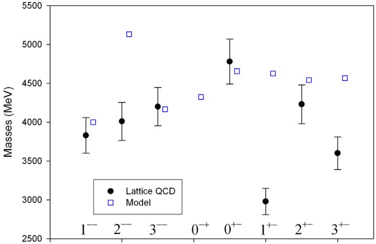
Three-gluon glueballs with massive gluon cannot reproduce the lattice data

With two transverse gluons, a simple linear+Coulomb potential reproduce the lattice QCD results.

Implementation of the helicity formalism for three transverse gluons should solve the hierarchy problem







V. Mathieu, N. Kochelev, V. Vento
"The Physics of Glueballs"
invited review for Int. J. Mod. Phys. E
arXiv:0810.4453 [hep-ph]

V. Mathieu, F. Buisseret and C. Semay
"Gluons in Glueballs: Spin or Helicity ? "
Phys. Rev. D **77**, 114022 (2008), [arXiv:0802.0088 [hep-ph]]

V. Mathieu, C. Semay and B. Silvestre-Brac
"Semirelativistic Potential Model for Three-gluon Glueballs."
Phys. Rev. D**77** 094009 (2008), [arXiv:0803.0815 [hep-ph]]

N. Boulanger, F. Buisseret,V. Mathieu, C. Semay
"Constituent Gluon Interpretation of Glueballs and Gluelumps."
arXiv:0806.3174 [hep-ph]
accepted for publication in Eur. Phys. Jour. A