Semi-Analytic Reconstruction of mSUGRA Parameters

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Introduction I

- Few particles at LHC : h, LSP, NLSP, $\tilde{g}$, some squarks & sleptons
- Standard fitting procedure (MINUIT etc . . .) (mSUGRA,General MSSM) works well only if the nb of data $\gg$ nb of fitted parameters
- Aim : Reconstruct mSUGRA parameters using inversion procedure (Lagragian parameters from physical EWSB masses)$\longrightarrow$ Allows to consider more pessimistic scenario (if only few sparticles discovered at LHC)

Introduction II

Main idea is a combination of 2 parts:

- Part I:
  Analytical “inversion" in gaugino sector(Moultaka, Kneur ’98)
  Input : charginos, neutralinos, tan β (arbitrary or derived from another sector)
  Output : gauginos
  - Scenario S1: input $M_{\chi_1^+}$, $M_{\chi_2^+}$,M_{N_2},tan β
    Starting with the Chargino mass matrix:

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix}$$

Introduction III

Inverting the eigenvalues of the mass term ($M_C^\dagger M_C$):

$$\mu^2 = \frac{1}{2}(M^2_{\chi_1^+} + M^2_{\chi_2^+} - 2m^2_W \pm[(M^2_{\chi_1^+} + M^2_{\chi_2^+} - 2m^2_W)^2 - 4(m^2_W \sin 2\beta \pm M_{\chi_1^+} M_{\chi_2^+})^2]^{1/2})$$

$$M_2 = [M^2_{\chi_1^+} + M^2_{\chi_2^+} - 2m^2_W - \mu^2]^{1/2}$$

Problems:

1. Needs both $M_{\chi_1^+}, M_{\chi_2^+}$ (may be diffi cult at LHC..)
2. Symmetry $M_2 \leftrightarrow \mu! \rightarrow$ 2 cases:
   - $\mu < M_2$ gaugino like models
   - $M_2 > \mu$ Higgsino like models (mSUGRA)
3. Quadratic μ Eq. $\rightarrow$ 4-fold ambiguities..

Introduction IV

- Scenario S2: input $M_{\chi_1^+}$, M_{N_1},M_{N_2}

Neutralino mass matrix:

$$M_N = \begin{pmatrix} M_1 & 0 & -m_Z s_W \cos\beta & m_Z s_W \sin\beta \\ 0 & M_2 & m_Z c_W \cos\beta & -m_Z c_W \sin\beta \\ -m_Z s_W \cos\beta & m_Z c_W \cos\beta & 0 & -\mu \\ m_Z s_W \sin\beta & -m_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}$$

Considering the 4 invariants (under diagonalization transformation):

$$TrM_N, \quad \frac{(TrM_N)^2}{2} - \frac{Tr(M_N^2)}{2}$$

$$\frac{(TrM_N)^3}{6} - \frac{TrM \;\; Tr(M_N^2)}{2} + \frac{Tr(M_N^3)}{3}, \quad DetM_N$$

Introduction V

This system gives a unique solution for M_1 as:

$$M_1 = -\frac{P_{2i}^2 + P_{2i}(\mu^2 + M_Z^2 + M_2 S_{2i} - S_{2i}^2) + \mu M_Z^2 M_2 s_w^2 \sin 2\beta}{P_{2i}(S_{2i} - M_2) + \mu(c_w^2 M_Z^2 \sin 2\beta - \mu M_2)}$$

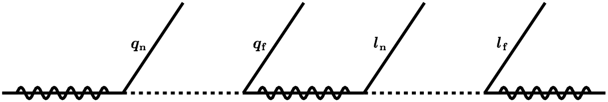
- $S_{2i} \equiv M_{N_2} + M_{N_i}$ $P_{2i} \equiv M_{N_2} M_{N_i}$
- $M_{N_i} \equiv$ ANY of the remaining neutralinos

Problem

Needs $M_2, \mu, \tan\beta$ (given by another sector)

Introduction VI

- Part II :"gluino and other mass measurements via cascade decays: e.g SPS 1a"(Gjelsten, Miller, Osland '2005)



$\sigma = 35.4$ pb	$\tilde{g}$	23.0%	$\tilde{q}_L$	31.4%						309 fb
		17.0%	$\tilde{b}_1$	35.5%	$\tilde{\chi}_2^0$	12.1%	$\tilde{l}_R$	100%	$\tilde{\chi}_1^0$	259 fb
		9.7%	$\tilde{b}_2$	18.0%						75 fb
										643 fb

  - Masses are very well determined using the kinematical endpoints method"

Gauginos Sector I

- Senario S2 :Same basic equations (but different input/output)

$$P_{ij}^2 + (\mu + M_Z^2 - M_1M_2 + (M_1 + M_2)S_{ij} - S_{ij}^2)P_{ij} + \mu M_Z^2(c_w^2M_1 + s_w^2M_2)\sin 2\beta - \mu^2 - M_1M_2 = 0$$

$$(M_1+M_2-_{ij}-S_{ij})P_{ij}^2+(\mu^2(M_1+M_2)+M_Z^2(c_w^2M_1+s_w^2M_2-\mu\sin 2\beta))P_{ij}+\mu(M_Z^2(c_w^2M_1+s_w^2M_2)\sin 2\beta-\mu M_1M_2)S_{ij} = 0$$

  - $S_{2i} \equiv \tilde{M}_{N_2} + \tilde{M}_{N_i}$
  - $P_{2i} \equiv \tilde{M}_{N_2}\tilde{M}_{N_i}$
- New input from the gluino chain decay : $M_{\tilde{g}}$, M_{N_1},M_{N_2}
- mSUGRA case $M_1 = M_2 = M_3$ (GUT scale) determines M_1, M_2 using the one-loop relation (Q scale)

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2}$$

- Solving the above equations with the new conditions $\rightarrow$ linear equation for $\sin 2\beta$ and μ

Gauginos Sector II

Example

- For a given $M_{\tilde{g}}$, M_{N_1},M_{N_2} from the kinematic endpoints method for the gluino decays for SPS 1a (SUSPECT)

$$m_0 = -A_0 = 100 \text{ GeV} \qquad m_{1/2} = 250 \text{ GeV} \qquad \tan\beta = 10, \mu > 0$$
$$M_{\tilde{g}} = 587.2\text{GeV} \qquad M_{\tilde{N}_1} = 98.37\text{Gev} \qquad M_{\tilde{N}_2} = 176.9\text{Gev}$$

$$\tan\beta = 9.73; \qquad \mu = 362.71$$

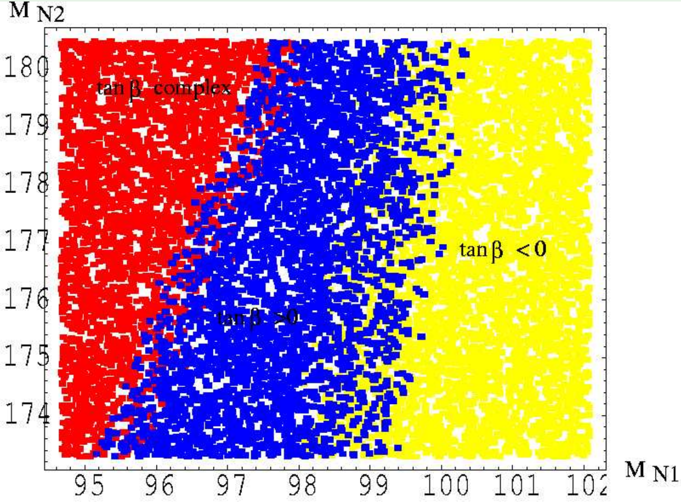
- Take into account the experimental errors :

$$\Delta M_{\tilde{g}} = 7.2\text{GeV} \qquad \Delta M_{\tilde{N}_1} = 3.7\text{Gev} \qquad \Delta M_{\tilde{N}_2} = 3.6\text{Gev}$$

- Scanning the values for $\tan\beta$ with the conditions $\tan\beta$=real & >0 $\rightarrow$ constrains BETTER the errors on $\tilde{M}_{N1}$

Gauginos Sector III

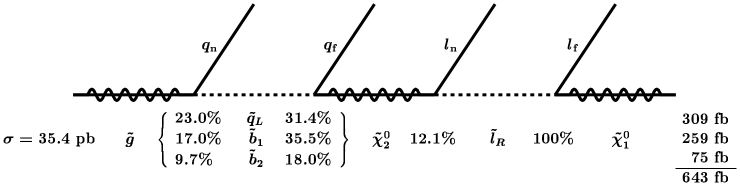
Example



Squarks & Sleptons sector I

Aim : determin m_0 (and $\tan\beta$) & A_0

Strategy for m_0:



- involved $\tilde{q}_L (\neq \tilde{b}, t)$ and $\tilde{l}_R$ obey (at EWSB scale)

$$m^2_{\tilde{u}1} = m^2_{\tilde{u}L} + (\frac{1}{2} - \frac{2}{3} S^2_W) m^2_Z \cos 2\beta$$
$$m^2_{\tilde{e}2} = m^2_{\tilde{e}R} - S^2_W m^2_Z \cos 2\beta$$

Squarks & Sleptons sector II

- Consider linear combination eliminating $\cos 2\beta$ dependence:

$$S_w^2 m_{\tilde{u}1}^2 + (\frac{1}{2} - \frac{2}{3} S_w^2 m_{\tilde{e}2}^2) = S_w^2 m_{\tilde{u}L}^2 + (\frac{1}{2} - \frac{2}{3} S_w^2) m_{\tilde{e}R}^2$$

- Bottom-Up RGE determines $m0$

$$(\frac{1}{2} + \frac{1}{3} S_{w_{GUT}}^2) m_0^2 = \int \frac{d(S_w^2 m_{\tilde{u}L}^2 + (\frac{1}{2} - \frac{2}{3} S_w^2) m_{\tilde{e}R}^2)}{dt}$$

- Top-Down RGE gives back $m_{\tilde{e}R}^2 \longrightarrow \tan\beta$

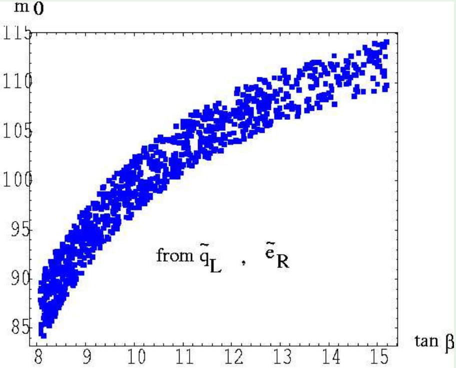
- Experimental mass errors from cascade decay $\longrightarrow$ study error propagation on $m_0 \longrightarrow \tan\beta$

Squarks & Sleptons sector III

Example

$$\Delta m_{\tilde{l}_R} \sim 3.7 \text{ GeV (LHC)} \qquad \Delta m_{\tilde{q}_L} \sim 6 \text{ GeV (LHC)}$$

$$\longrightarrow m_0 \sim 100 \pm 15 \text{ GeV}$$



Squarks & Sleptons sector IV

Strategy for A_0

- Sbottom $(mass)^2$ matrix :

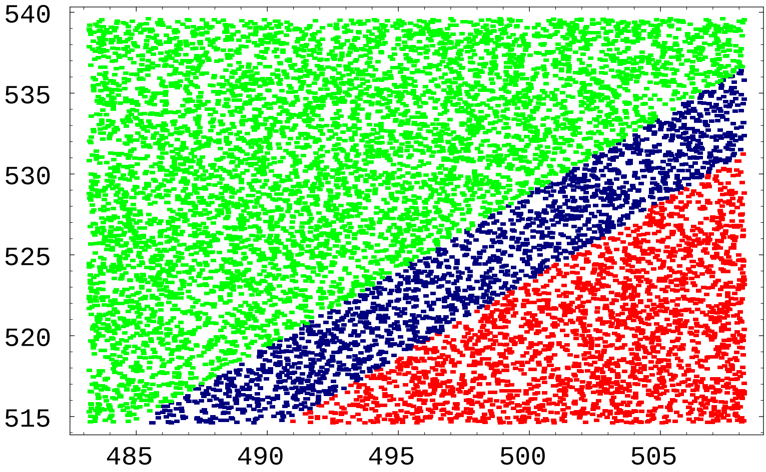
$$M_{\tilde{b}} = \begin{pmatrix} M_Q^2 + m_b^2 - (\frac{1}{3}m_W^2 + \frac{1}{6}m_Z^2)\cos 2\beta & m_b\,(A_b - \mu\tan\beta) \\ m_b(A_b - \mu\tan\beta) & m_{b_R}^2 + m_b^2 + \frac{1}{3}(m_W^2 - m_Z^2)\cos 2\beta \end{pmatrix}$$

- Solving for A_b gives:

$$\begin{aligned} A_b = & \pm \frac{1}{12 m_b}\sqrt{36(m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2)^2 - (-6m_{b_R}^2 + 6m_Q^2 + \cos 2\beta(-4m_W^2 + m_Z^2))^2} \\ & + \mu\tan\beta \end{aligned}$$

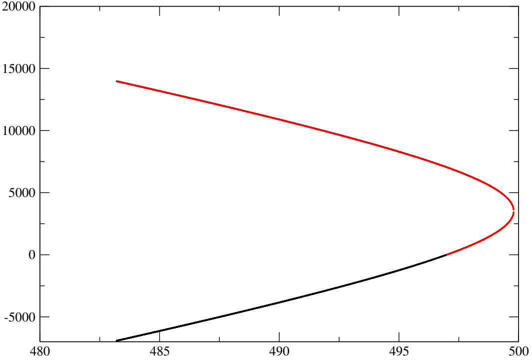
- Experimental $m_{\tilde{b}_{1,2}}$ errors scans $\rightarrow$ variation of A_b
- Bottom-Up RGE $\rightarrow A_0$

Squarks & Sleptons sector V



Squarks & Sleptons sector VI

- Solving A_b from $m^2_{\tilde{b}_1}$



Conclusions I

So Far

- BAD $\tan\beta$ from Gauginos sector, BETTER one from sleptons sector
- m_0 from Squaks & Sleptons sector
- Very BAD A_0 from Sbottoms sector
- Theoretical conditions helps to constrain some experimental error's inputs
- Preliminary work (1-loop RGEs, etc): needs to add some radiative corrections$\longrightarrow$ not fully taken into account yet
- Not (yet) competitive with elaborated fitting tools, but useful theoretical guide: illustrates well sensitivity to parameters, often obscure from “blind" fits.

Conclusions II

What about the Higgs Sector?

- Higgs (*mass*)^2 matrix

$$\begin{pmatrix} m_Z^2 \cos^2\beta + m_A^2 \sin^2\beta & -(m_Z^2 + m_A^2)\sin\beta\cos\beta \\ -(m_Z^2 + m_A^2)\sin\beta\cos\beta & m_Z^2 \sin^2\beta + m_A^2 \cos^2\beta \end{pmatrix}$$

- Inversion, gives

$$\bar{M}_A^2 = \frac{\bar{m}_h^2(m_Z^2 - \bar{m}_h^2)}{m_Z^2 \cos^2 2\beta - \bar{m}_h^2}$$

- Next:

$$m_{H_u}^2 = \frac{\bar{M}_A^2 - (\mu^2 + m_Z^2/2)(\tan^2\beta - 1)}{1 + \tan^2\beta} \quad m_{H_d}^2 = M_A^2 - M_{H_u}^2 - 2\mu^2$$

Conclusions III

- Full 1(2)-loop Higgs R.C. preserve *linear* M_A^2 solution for $\tan\beta$!!

$$\bar{m}_h^2 = M_{h,pole}^2 - 3g^2 \frac{m_t^4}{8\pi^2 m_W^2 \sin^2\beta} (\ln \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t} + ..)$$

Conclusions IV

Example

- $m_h = 111.95$, $\Delta m_h = 2.5, m_A = 403.83$