Event shapes for hadron colliders

Gavin P. Salam (in collaboration with Andrea Banfi & Giulia Zanderighi)

LPTHE, Universities of Paris VI and VII and CNRS

LPNHE, Jussieu Paris, July 2005 A wealth of information about QCD lies in its final states. Problem is how to extract it.



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Event shapes for hadron colliders (4/26)



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Event shapes for hadron colliders (5/26) Lacements $\alpha_{\alpha}(Q)$ Event shapes: high information content

Q (GeV)



Much knowledge has been extracted from event-shapes in e^+e^- and DIS:

• α_s fits

- Tuning of Monte Carlos
- Colour factor fits (C_A, C_F, \dots)

 Studies of analytical hadronisation models (1/Q, shape functions, ...)

But mostly neglected so far at hadron colliders

> ('91) CDF broadening ('91) D0 Thrust ('02)



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Various processes:

• $pp \rightarrow W/Z/H$ boson + jet

• $pp \rightarrow 2$ jets

Standard applications (e.g.)

- Measure α_s
- As for 3-jet/2-jet ratio in *pp*, reduce dependence on PDFs
- But for event-shapes → *distribution*
- Far more information than 3-jet/2-jet ratio

Banfi Marchesini Smye Zanderighi '01 Main subject of this talk

New territory

 4-jet (2 + 2) topology → novel perturbative structures

soft colour evln matrices

- 3 & 4-jet topologies (& g-jets)
 → rich environment for analytical non-pert. studies
- Underlying event test models (analytical & MC).

Variety of event-shape observables \rightarrow complementary information \rightarrow disentangle the different physics issues.

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3 jets: colour state of any pair *fixed by third parton* (colour conservation).

4 jets: a given pair can be in various colour states. Soft virtual corrections mix colour states.

Resummation leads to *matrix evolution equation for colour state of amplitudes* ('soft anomalous dimenions') Developed at Stony Brook: Botts, Kidonakis, Oderda & Sterman '89–9



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Fixed order

- Event shapes trivial for Born events (e.g. $p\bar{p} \rightarrow 2$ jets, thrust=1)
- First non-trivial order (LO) is Born + 1 parton, *i.e.* $p\bar{p} \rightarrow 3$ jets
- For NLO, need a program like NLOJET++ ($p\bar{p} \rightarrow 3$ jets @ NLO)

Nagy, '01 & '03

Also:

- Kilgore-Giele code ($p\bar{p} \rightarrow 3$ jets @ NLO),
- MCFM ($p\bar{p} \rightarrow W/Z/H + 2$ jets @ NLO)

Campbell & Ellis '02

Resummation

- In e^+e^- it was always done by hand, one observable at a time.
- Next-to-leading logs (NLL) are tedious, complicated, error-prone.
- Recently automated: Computer-Automated Expert Semi-Analytical Resummer (CAESAR). Banfi, GPS & Zanderighi '01-'04
- For $p\bar{p} \rightarrow 2j$ ets, uses 'Stony Brook' soft-colour evolution matrices.
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$$e^+e^-
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e^+e^- , DY, DIS 3 jets

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Average: 1 observable per paper

Analytical work (done once and for all)

- A1. derive a master formula for a generic observable in terms of simple properties of the observable
- A2. formulate the exact applicability conditions for the master formula

Numerical work (to be repeated for each observable)

- N1. let an "expert system" investigate the applicability conditions
- N2. it also determines the inputs for the master formula
- N3. straightforward evaluation of the master formula, including phase space integration etc.

Note: N1 and N2 are core of automation

- a) they require high precision arithmetic to take asymptotic (soft & collinear) limits
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CAESAR flow chart



Requirement: globalness

Global observable:

e.g. total e^+e^- Broadening, B

making $B \ll 1$ restricts emissions everywhere.

Coherence + globalness:

 emissions can be resummed as if independent (proved)

> Answers guaranteed to NLL accuracy

Non-Global observable:

Right-hemisphere Broadening, B_R

making $B_R \ll 1$ restricts emissions in right-hand hemisphere (\mathcal{H}_R) .

Tempting to *assume* one can:

- ignore left hemisphere $(\mathcal{H}_{\mathcal{L}})$
- use independent emission approximation in H_R.

WRONG AT NLL ACCURACY

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Resummation of NG observables

All-orders:

Forbid coherent radiation from energy-ordered ensembles of large-angle gluons



Difficulties:

- Logarithms resummed so far only in large-*N_c* limit
- In general, boundary between the two regions may have arbitrary shape.
- It may depend on the pattern of emissions (*e.g.* with jet algorithm).

Appleby & Seymour '02, '03

Resummation of a general non-global observable is tricky. For time-being CAESAR deals only with global observables.NB: (most) Monte Carlo's are also best suited to global observables

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Theoretical calculations are for global observables. But experiments only have detectors in limited rapidity range. (Strictly: series of sub-detectors, of worsening quality as rapidity increases)

Model by cut around beam $|\eta| < \eta_{\max}$ \Rightarrow Problems with globalness

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η_{\max}	3.5	5.0

From kinematics, emissions (*k*) near forward detector edges typically have small transverse momentum:

$k_\perp \sim P_\perp e^{-\eta_0} \ll P_\perp$

If event-shape value is always sufficiently large that such an emission contributes negligibly, then: we can ignore rapidity cut & pretend measurement is global

- Calculate distribution without any rapidity cutoff
- Determine smallest 'typical' value of observable
- Check self-consistency: *i.e.* that in comparison, emissions beyond cutoff contribute negligbly.
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Results that follow based on this (illustrative) event selection:

- Run longitudinally invariant inclusive k_t jet algorithm (could also use midpoint cone)
- Require hardest jet to have $P_{\perp,1} > P_{\perp,\min} = 50 \text{ GeV}$
- Require two hardest jets to be central $|\eta_1|, |\eta_2| < \eta_c = 0.7$

Pure resummed results no matching to NLO (or even LO) Shown for Tevatron run II Some observables are naturally defined in terms of all particles in the event, *e.g. Global Transverse Thrust*

$$T_{\perp,g} \equiv \max_{\vec{n}_T} \frac{\sum_i |\vec{q}_{\perp i} \cdot \vec{n}_T|}{\sum_i q_{\perp i}}, \qquad \tau_{\perp,g} = 1 - T_{\perp,g},$$

and Global Thrust Minor

$$T_{m,g} \equiv \frac{\sum_{i} |\vec{q}_{i}.\vec{n}_{m}|}{\sum_{i} q_{\perp i}}, \qquad \vec{n}_{m} \cdot \vec{n}_{T}$$



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3-jet resolution threshold

Use *exclusive* long. inv. k_t algorithm: successive recombination of pair with smallest closeness measure d_{kl} , d_{kB} :

$$d_{kB} = q_{\perp k}^2$$
, $d_{kl} = \min\{q_{\perp k}^2, q_{\perp l}^2\} \left((\eta_k - \eta_l)^2 + (\phi_k - \phi_l)^2 \right)$.

Define $d^{(n)}$ as smallest d_{kl} , d_{kB} when only *n* pseudo-jets left. Examine (normalised) 3-jet resolution threshold



Generalisation of 3-jet cross section

3-jet resolution threshold

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Generalisation of 3-jet cross section

Results

Probability P(v) that event shape is smaller than some value v:

$$P(v) = \exp\left[-G_{12}\frac{\alpha_s L^2}{2\pi} + \cdots\right], \quad L = \ln\frac{1}{v}$$

Ev.Shp.	G ₁₂	
$ au_{\perp,g}$	$2C_B + C_J$	
$T_{m,g}$	$2C_B + 2C_J$	
<i>y</i> 23	$\frac{1}{2}C_B + \frac{1}{2}C_J$	

 C_B = total colour of Beam partons C_J = total colour of Jet partons

Results

-2

-3

 $ln(\tau_{\perp,g)}$

Probability P(v) that event shape is smaller than some value v:



-6

-5

 C_I = total colour of Jet partons

Results

Probability P(v) that event shape is smaller than some value v:



-6

-7

-5

 $ln(\tau_{\perp,g})$

-3

-2

 $C_J =$ total colour of Jet partons

Results

Probability P(v) that event shape is smaller than some value v:



Beam cut: $au_{\perp,g}\gtrsim 0.15 e^{-\eta_{\max}}$

Results

Probability P(v) that event shape is smaller than some value v:



Results

Probability P(v) that event shape is smaller than some value v:



Event shapes for hadron colliders (20/26) Example observables 2. Forward-suppressed observables

Forward-suppressed observables

Divide event into central region (C, say $|\eta| < 1.1$) and rest of event (\overline{C}). [NB: \exists considerable freedom in definition of C: *e.g.* can also be two hardest jets]

Define central \perp mom., and rapidity:

$$Q_{\perp,\mathcal{C}} = \sum_{i\in\mathcal{C}} q_{\perp i}\,, \quad \eta_{\mathcal{C}} = rac{1}{Q_{\perp,\mathcal{C}}}\sum_{i\in\mathcal{C}} \eta_i\,q_{\perp i}$$

and an *exponentially suppressed forward term*,

:-+ /

$$\mathcal{E}_{ar{\mathcal{C}}} = rac{1}{Q_{\perp,\mathcal{C}}} \sum_{i
otin \mathcal{C}} q_{\perp i} \, e^{-|\eta_i - \eta_\mathcal{C}|} \, .$$

Define a non-global event-shape in C. Then add on $\mathcal{E}_{\overline{C}}$. Result is a global event shape, with suppressed sensitivity to forward region. Event shapes for hadron colliders (20/26) Example observables 2. Forward-suppressed observables

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Event shapes for hadron colliders (21/26) Example observables 2. Forward-suppressed observables



- Split C into two pieces: Up, Down
- Define jet masses for each

$$\rho_{X,C} \equiv \frac{1}{Q_{\perp,C}^2} \Big(\sum_{i \in \mathcal{C}_X} q_i \Big)^2, \qquad X = U, D,$$

Define sum and heavy-jet masses

 $\rho_{S,\mathcal{C}} \equiv \rho_{U,\mathcal{C}} + \rho_{D,\mathcal{C}}, \qquad \rho_{H,\mathcal{C}} \equiv \max\{\rho_{U,\mathcal{C}}, \rho_{D,\mathcal{C}}\},\$

Define global extension, with extra forward-suppressed term

$$\rho_{S,\mathcal{E}} \equiv \rho_{S,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}}, \qquad \rho_{H,\mathcal{E}} \equiv \rho_{H,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}}.$$

• Similarly: total and wide jet-broadenings

$$B_{T,\mathcal{E}} \equiv B_{T,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}}, \qquad B_{W,\mathcal{E}} \equiv B_{W,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}}.$$

Event shapes for hadron colliders (22/26) Example observables 2. Forward-suppressed observables

$P(v) = \exp\left[-G_{12}\frac{\alpha_s L^2}{2\pi} + \cdots\right], L = \ln\frac{1}{v}$					
Ev.Shp.	G ₁₂				
$ ho_{\mathcal{S},\mathcal{E}}$	$C_B + C_J$				
$ ho_{H,\mathcal{E}}$	$C_B + C_J$				
$B_{T,\mathcal{E}}$	$C_B + 2C_J$				
$B_{W,\mathcal{E}}$	$C_B + 2C_J$				
	:				

 C_B = total colour of Beam partons C_J = total colour of Jet partons

Event shapes for hadron colliders (22/26) Example observables 2. Forward-suppressed observables



Beam cuts: $B_{X,\mathcal{E}}, \rho_{X,\mathcal{E}} \gtrsim e^{-2\eta_{\max}}$ [because $\mathcal{E}_{\bar{\mathcal{C}}} \sim k_t e^{-|\eta|}$]



Recoil observables

By momentum conservation

$$\sum_{i\in\mathcal{C}}\vec{q}_{\perp i}=-\sum_{i\notin\mathcal{C}}\vec{q}_{\perp i}$$

Use central particles to define *recoil term*, which is *indirectly sensitive* to non-central emissions

$${\cal R}_{\perp,{\cal C}} \equiv rac{1}{Q_{\perp,{\cal C}}} \left| \sum_{m{i} \in {\cal C}} ec{q}_{\perp m{i}}
ight| \, ,$$

Define event shapes exclusively in terms of *central particles*:

 $\rho_{X,\mathcal{R}} \equiv \rho_{X,\mathcal{C}} + \mathcal{R}_{\perp,\mathcal{C}}, \qquad B_{X,\mathcal{R}} \equiv B_{X,\mathcal{C}} + \mathcal{R}_{\perp,\mathcal{C}}, \dots$

These observables are *indirectly global*

First studied at HERA (B_{zE} broadening)

Results

CAESAR resummation works for observables having *direct exponentiation*:

 $P(v) = e^{Lg_1(\alpha_s L) + g_2(\alpha_s L) + \dots}$

For recoil observables, exponentiation holds fully only after Fourier & other integral transforms (generalised *b*-space resummation).

Manifestation: NLLs $(g_2(\alpha_s L))$ diverge at some $\alpha_s L \sim 1$.

Consequently, cannot extend distribution to v = 0 — must cut before divergence.

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recoil transverse thrust



Quite large effect: \sim 15% of X-sct is beyond cutoff

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recoil thrust minor



Moderate effect: few % of X-sct is beyond cutoff
Summary of observables

Event-shape	Impact of η_{\max}	Resummation	Underlying	Jet
		breakdown	Event	hadronisation
$ au_{\perp,g}$	tolerable	none	$\sim \eta_{\sf max}/{\it Q}$	$\sim 1/Q$
$T_{m,g}$	tolerable	none	$\sim \eta_{\sf max}/{\it Q}$	$\sim 1/(\sqrt{lpha_{s}}Q)$
<i>y</i> ₂₃	tolerable	none	$\sim \sqrt{y_{23}}/Q$	$\sim \sqrt{y_{23}}/Q$
$ au_{\perp,\mathcal{E}}$, $ ho_{\mathbf{X},\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim 1/Q$
$B_{X,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim 1/(\sqrt{lpha_{s}}Q)$
$T_{m,\mathcal{E}}$	negligible	serious	$\sim 1/Q$	$\sim 1/(\sqrt{lpha_{s}}Q)$
У23, <i>Е</i>	negligible	none	$\sim 1/Q$	$\sim \sqrt{y_{23}}/Q$
$ au_{\perp,\mathcal{R}}$, $ ho_{\mathbf{X},\mathcal{R}}$	none	serious	$\sim 1/Q$	$\sim 1/Q$
$T_{m,\mathcal{R}}, B_{X,\mathcal{R}}$	none	tolerable	$\sim 1/Q$	$\sim 1/(\sqrt{lpha_{s}}Q)$
Y 23,R	none	intermediate	$\sim \sqrt{y_{23}}/Q$	$\sim \sqrt{y_{23}}/Q$

NB: there may be surprises after more detailed study, *e.g.* matching to NLO... Grey entries are definitely subject to uncertainty

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- Important that multijet event shapes also be studied in DIS and e^+e^- .
 - Measurements recently published by LEP and in progress at HERA.
 - Theoretical comparisons in pipeline automation facilitates this.

Hadron-collider specificities

- New domain for "rigorous" QCD studies:
 - non-perturbative: underlying event
 - perturbative: Stony Brook soft colour resummation
- Tension between theoretical simplicity (globalness) and experimental measurability (limited rapidity) can be resolved

Next step: matching to NLO

- technology exists (NLOJET++) for *poor-man's* matching, all channels (gg → gg, qq → qq, ...) mixed together.
 - To be 'sensible', matching must be done *channel-by-channel*.
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