Minimal Lepton Flavour Violation and leptogenesis

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We investigate the viability of leptogenesis in models with three heavy right-handed neutrinos, where the charged-lepton and the neutrino Yukawa couplings are the only irreducible sources of lepton-flavour symmetry breaking (Minimal Lepton Flavour Violation hypothesis). We analyze the impact of the CP violating phases responsible for leptogenesis on low-energy flavour changing observables.

1 Introduction

In the Standard Model (SM) the structure of the fermion mass matrices controls the physics of flavour. In the quark sector for instance, up and down quarks have mass matrices which are approximately aligned and the eigenvalues are several orders of magnitude smaller than the electroweak scale. This results in effective GIM and Cabibbo suppressions of flavour changing rates and guarantees a good level of predictivity to the theory. It would be useful to extend the SM connection between flavour and masses in models of new physics which predict new flavoured particles at the TeV scale, that could easily conflict with bounds from flavour experiments. This is the idea behind the Minimal Flavour Violation (MFV) principle [1,2,3]. It is a general and flexible hypothesis that has been implemented in strongly-interacting theories [1], low-energy supersymmetry [2,3], multi Higgs [3,4] and GUT [5] models.

The SM Lagrangian is modified by the presence of high-energy new physics by the appearance of non-renormalizable operators, constructed from SM and Yukawa fields and suppressed by inverse powers of the mass of the new flavoured particles, Λ_{LFV}. The MFV recipe for model building requires that the new physics operators obey those flavour symmetry properties of the SM which are responsible for the suppression of flavour changing effects. In the quark sector there is essentially no ambiguity because the largest group of unitarity field transformations that commutes with the gauge group, $\mathcal{G}_q = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times U(1)^3$, is broken only by the Yukawa couplings. The invariance of the SM Lagrangian under $\mathcal{G}_q$ can be formally recovered elevating the Yukawa matrices to spurion fields with the appropriate transformation properties under $\mathcal{G}_q$. The hypothesis of MFV states that this must be sufficient to make invariant under $\mathcal{G}_q$ also the new physics operators. The quark sector has been extensively tested in flavour experiments and it seems plausible that some kind of MFV principle is really at work in the underlying theory.

Apart form the analogy with quarks, the definition of a Minimal Lepton Flavour Violation (MLFV) principle [6] is demanded by a severe flavour problem in the lepton sector as well. For instance, the charged lepton radiative decays $l_i \to l_j \gamma$ are mediated by the effective operator

$$\frac{\Delta^{RL}_{ij}}{\Lambda^2_{LFV}}\, g\, H^\dagger \bar{e}^i_R \sigma^{\sigma\rho} \mathrm{L}^j_L F_{\sigma\rho} \equiv \frac{\Delta^{RL}_{ij}}{\Lambda^2_{LFV}}\, O_{RL} \tag{1}$$

where the matrix $F_{\sigma\rho}$ is the field strength tensor of the gauge group, g the corresponding gauge coupling and Δ^{RL} is a flavour changing matrix which mediates the transition. If unnaturally small dimensionless parameters do not enter this expression, the experimental limit for $\mu \to e\gamma$ pushes the scale Λ_{LFV} to values $> 10^5$ TeV, well above the electroweak scale.

The implementation of a MFV principle in the lepton sector is not as straightforward as in the quark sector. If neutrinos are Majorana particles, neutrino masses themselves are generated by a non-renormalizable operator suppressed by the scale of lepton number violation (LNV)

$$m_\nu \propto g^{ij}_\nu \, \frac{(HL_L)^t_i (HL_L)_j}{\Lambda_{LNV}}\,, \tag{2}$$

so that (i) we face a two scale problem, probably with the hierarchy $\Lambda_{LNV} \gg \Lambda_{LFV}$; (ii) and we have to make hypotheses about the origin of the effective operator in eq.(2). In this work, we assume that neutrino masses are generated by the standard see-saw mechanism with three heavy right-handed neutrinos and we explore the connection with the generation of the baryon asymmetry of the universe [7]. However different implementations of the MFV idea in the lepton sector are possible [6,8] and do not necessarily involve an extension of the SM field content.

2 MLFV with three right-handed neutrinos

We consider three right-handed Majorana neutrinos ν^i_R in addition to the three SM left-handed doublets L^i_L and right-handed charged lepton singlets e^i_R. The maximal flavour group is

$$\mathcal{G}_l = SU(3)_{L_L} \times SU(3)_{e_R} \times O(3)_{\nu_R}\,, \tag{3}$$

in close analogy with quark sector: the ν_R's are the counterpart of right-handed up quarks and the irreducible sources of flavour breaking are the charged lepton and neutrino Yukawa couplings. We choose not to take the Majorana mass term for right-handed neutrinos as an independent spurion, but we assume that it is proportional to the identity matrix in flavour space,

$$M_R = \Lambda_{LNV} I \tag{4}$$

thus breaking the $SU(3)_{\nu_R}$ symmetry to $O(3)_{\nu_R}$. This assumption increases the predictivity of the model and can be excluded experimentally.

The symmetry braking lagrangian is

$$\begin{aligned} \mathcal{L}_{Sym.Br.} &= -\lambda^{ij}_e\, \bar{e}^i_R (H^\dagger L^j_L) + i\lambda^{ij}_\nu \bar{\nu}^i_R (H^T \tau_2 L^j_L) + \text{h.c.} \\ &\to -v\lambda^{ij}_e\, \bar{e}^i_R e^j_L - v\lambda^{ij}_\nu\, \bar{\nu}^i_R \nu^j_L + \text{h.c.} \end{aligned} \tag{5}$$

The irreducible sources of lepton symmetry breaking are the Yukawa matrices λ_ν and λ_e, transforming as $(\bar{3},1,3)$ and $(\bar{3},3,1)$ under the symmetry group $\mathcal{G}_l$.

3 Connection between low energy observables and leptogenesis

Having identified the irreducible sources of flavour symmetry breaking and their transformation properties under the flavour group $\mathcal{G}_l$, we are now ready to build the non-renormalizable operators suppressed by inverse powers of Λ_{LFV} that contribute to flavour violating processes. According to the MLFV hypothesis, these operators must be invariant combinations of the SM fields and the spurions λ_e and λ_ν. The complete list of the operators contributing at leading order to LFV processes is given in Refs.[6,9].In the case of charged lepton radiative decays there are two six-dimensional operators,

$$\begin{aligned} O_{RL}^{(1)} &= g' H^\dagger \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta^{RL} L_L B_{\mu\nu} \ , \\ O_{RL}^{(2)} &= g H^\dagger \bar{e}_R \sigma^{\mu\nu} \tau^a \lambda_e \Delta_{FCNC} L_L W_{\mu\nu}^a \ , \end{aligned} \tag{6}$$

which enter the effective Lagrangian for $l_i \to l_j \gamma$:

$$\mathcal{L}_{eff} = \frac{1}{\Lambda_{LFV}^2} \left(c_{RL}^{(1)} O_{RL}^{(1)} + c_{RL}^{(2)} O_{RL}^{(2)} \right) \ , \tag{7}$$

where g' (g) and $B_{\mu\nu}$ ($W_{\mu\nu}^a$) are the coupling constant and the field strength tensor of the $U(1)_Y$ ($SU(2)_L$) gauge group.

The matrix Δ^{RL} is a spurion combination transforming as $(\bar{3}, 3)$ under the flavour group $\mathcal{G}_l$. In the basis where λ_e is diagonal and at leading order in $\lambda_e = m_e/v$, it is given by

$$\Delta^{RL} = \lambda_e \left(\lambda_\nu^\dagger \lambda_\nu \right) \tag{8}$$

In the quark sector the analogue quantity $\lambda_u^\dagger \lambda_u$ can be expressed in terms of quark masses and CKM angles, so that the only unknown in the predictions for flavour changing branching ratios is the overall normalization given by Λ_{FV}, coming from the power suppression of new physics operators:

$$BR(quark\, MFV) \sim \frac{f(m_u, m_d, V_{CKM})}{\Lambda_{FV}^4} \tag{9}$$

In the lepton sector instead we can obtain $\lambda_\nu^\dagger \lambda_\nu$ extracting λ_ν from the see-saw relation

$$m_\nu^{eff} \equiv v^2 \lambda_\nu^T M_R^{-1} \lambda_\nu = U_{PMNS}^* m_{diag} U_{PMNS}^\dagger \tag{10}$$

written in the basis where the charged lepton Yukawa matrix λ_e and the heavy Majorana neutrinos mass matrix M_R are diagonal. We find

$$\lambda_\nu = \frac{M_R^{1/2}}{v} O \ H(\phi_1, \phi_2, \phi_3) \ m_{diag}^{1/2} \ U_{PMNS}^\dagger \tag{11}$$

where O is a real orthogonal matrix which depends on three real parameters and H a complex hermitian matrix which also depends on three real parameters $\phi_{1,2,3}$; $H \to I$ in the CP limit [11]. In our definition of MFV, right-handed neutrinos are degenerate in mass so that M_R in (11) is a number that we identify with Λ_{LNV} and the matrix O can be rotated away (the Lagrangian is $O(3)$ invariant). Using eq.(11), the basic unit that takes part into low energy flavour changing rates in the MFV framework can be written as

$$\lambda_\nu^\dagger \lambda_\nu = \frac{M_R}{v^2} U_{PMNS}\, m_{diag}^{1/2} H^2 m_{diag}^{1/2} U_{PMNS}^\dagger . \tag{12}$$

Let us count the parameters in eq.(12): there are 9 parameters in principle measurable at low energy (MNS angles, Dirac and Majorana phases, neutrino masses) and 4 unknown parameters:

the normalization M_R and $\phi_{1,2,3}$ in the matrix H, which disappear in the see-saw relation (10). With non degenerate right-handed neutrinos we would have 5 unknown parameters more (the 2 mass splittings of M_R and the 3 angles in O).

Assuming that all the baryon asymmetry η_B of the universe has been generated through sphaleron effects by the lepton asymmetry produced in the out-of-equilibrium decays of right-handed neutrinos, we can use the observed value of $\eta_B = (6.3 \pm 0.3) \times 10^{-10}$ to get some information on the high energy parameters M_R and $\phi_{1,2,3}$. In fact, in the one-flavour approximation leptogenesis depends on the combination

$$\lambda_\nu \lambda_\nu^\dagger = \frac{M_R}{v^2}\, H\, m_{diag}\, H\ . \tag{13}$$

4 Analysis of leptogenesis

A necessary condition for generating a lepton asymmetry is the non-degeneracy of heavy neutrinos. We in general expect that the tree-level degeneracy of heavy neutrinos is lifted by radiative corrections. In MFV models the most general form of the allowed mass-splittings is

$$\begin{aligned}\frac{\Delta M_R}{M_R} &= c_\nu \left[\lambda_\nu\lambda_\nu^\dagger + (\lambda_\nu\lambda_\nu^\dagger)^T\right] \\ &+ c_{\nu\nu}^{(1)}\left[\lambda_\nu\lambda_\nu^\dagger\lambda_\nu\lambda_\nu^\dagger + (\lambda_\nu\lambda_\nu^\dagger\lambda_\nu\lambda_\nu^\dagger)^T\right] \\ &+ c_{\nu\nu}^{(2)}\left[\lambda_\nu\lambda_\nu^\dagger(\lambda_\nu\lambda_\nu^\dagger)^T\right] + c_{\nu\nu}^{(3)}\left[(\lambda_\nu\lambda_\nu^\dagger)^T\lambda_\nu\lambda_\nu^\dagger\right] \\ &+ c_{\nu l}\left[\lambda_\nu\lambda_e^\dagger\lambda_e\lambda_\nu^\dagger + (\lambda_\nu\lambda_e^\dagger\lambda_e\lambda_\nu^\dagger)^T\right] + \ldots\end{aligned} \tag{14}$$

In a model-independent approach we cannot fix the coefficients in (14) but in a perturbative scenario we expect $c_\nu \sim g_{eff}/4\pi$ and $c_{\nu\nu}^{(i)}$, $c_{\nu l} \sim g_{eff}^2/(4\pi)^2$. From eq.(14) we can derive some general properties of leptogenesis in MFV models: (i) the term proportional to c_ν does not generate an asymmetry by itself, but (ii) sets the order of magnitude of the mass splitting and naturally gives the condition for resonant leptogenesis: the mass splitting of right-handed neutrinos is comparable to the decay width,

$$\Delta M_{R_{ij}} \sim \Gamma_j = M_R \frac{|\,\lambda_\nu\lambda_\nu^\dagger|_{jj}}{8\pi}\ . \tag{15}$$

(iii) The right amount of leptogenesis can be generated even with $\lambda_e = 0$, provided that all the three parameters $\phi_{1,2,3} \neq 0$. Since $\lambda_\nu \sim \sqrt{M_R}$, (iv) for low values of M_R ($\leq 10^{12}$ GeV) the asymmetry generated by the $c_{\nu l}$ term dominates but is typically too small to match the observed value of η_B. In this regime we find the flat dependence on M_R typical of resonant leptogenesis. At $M_R \geq 10^{12}$ GeV the quadratic terms $c_{\nu\nu}^{(i)}$ dominate the generation of the asymmetry, which grows linearly with M_R. These specific features of resonant leptogenesis in MFV can be derived with a general analysis of CP-invariants and reproduced analytically[7]. Properties (i),(iv) explain the characteristic behaviour of η_B as function of M_R, shown in fig. 1.

In deriving this result we used the analytic formulae for leptogenesis without flavour effects of Ref.[12] and assumed a loop hierarchy between the coefficients of the mass splittings. Under these assumptions, we find that the right size for η_B can be reached for $M_R \geq 10^{12}$ GeV. The regime $M_R \gg 10^{12}$ GeV is particularly interesting since in this case the CP-violating parameters ϕ_i are very small and we recover the predictive scheme of Ref.[6].

A MFV model is for instance the Minimal Supersymmetric Standard Model with degenerate right-handed neutrinos at the GUT scale. This scenario has been analyzed in Ref.[10] where also flavour effects were included in the leptogeensis analysis.

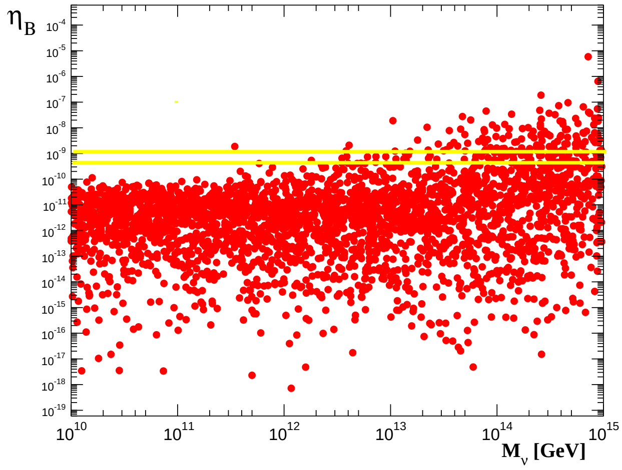

Figure 1: Baryon asymmetry η_B as a function of the right-handed neutrinos mass M_R. The experimental value in log-scale is 9.20 ± 0.02; the larger error band takes into account the theoretical uncertainties of a model-independent analysis, in particular the conversion factor of the leptonic asymmetry into the baryonic one, which depends on the field content of the model.

5 Results for lepton radiative decays

In the MLFV framework, the effective Lagrangian in eq.(2) relevant for lepton radiative decays leads to the branching ratio $l_i \to l_j\gamma$

$$BR_{l_i\to l_j\gamma} \equiv \frac{\Gamma(\ell_i \to \ell_j\gamma)}{\Gamma(\ell_i \to \ell_j\nu_i\bar{\nu}_j)} = 384\,\pi^2 e^2\,\frac{v^4}{\Lambda^4_{LFV}}\left|\left(\lambda_\nu^\dagger\lambda_\nu\right)_{ij}\right|^2 |c^{(2)}_{RL} - c^{(1)}_{RL}|^2$$
$$\sim \left(\frac{\Lambda_{LNV}}{\Lambda^2_{LFV}}\right)^2 f\left(m_\nu^{eff}, m_l, U_{PMNS}, \phi_i\right) \tag{16}$$

to be compared with eq.(9), which represents a generic branching ratio in MFV in the quark sector. An important difference is that in the lepton sector the ϕ_i parameters, non-measurable in low energy experiments, enter the branching ratio. As shown in fig. 2, they typically produce an enhancement, but weaken the predictivity of the model. However for large values of M_R their effect is moderate and MFV predicts $BR(\mu \to e\gamma)/BR(\tau \to \mu\gamma) \leq 1$. The second important difference between eqs.(9) and (16) is the absolute normalization of the branching ratios, that now depends on both the scales of lepton number and lepton flavour violation. For this reason, we cannot directly interpret the experimental bounds on $l_i \to l_j\gamma$ as lower limits on the scale Λ_{FV} without independent information on $\Lambda_{LNV} \equiv M_R$. Leptogenesis provided this information, since the baryon asymmetry is most naturally reproduced for $M_R \geq 10^{12}$ GeV. For Λ_{FV} around the TeV scale, the branching ratio for $\mu \to e\gamma$ is expected to be in the reach of the MEG experiment.

6 Conclusions

In this talk we studied leptogenesis in the MFV scenario with right-handed Majorana neutrinos degenerate in mass. Radiative corrections lift the tree-level degeneracy of right-handed neutrinos and induce mass-splittings proportional to the neutrino and charged lepton Yukawa couplings. We showed that leptogenesis is viable and most efficient at high values of right-handed neutrino

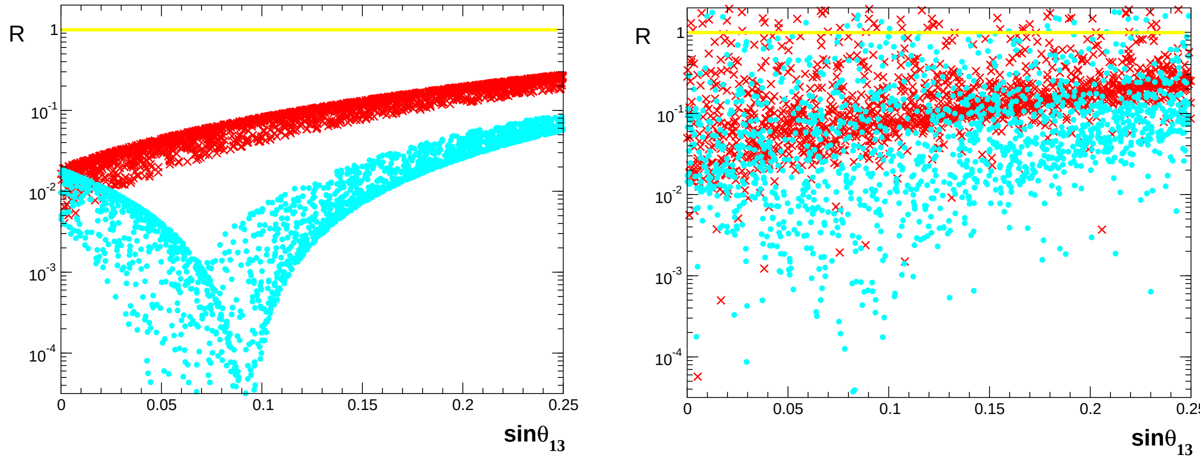

Figure 2: Effect of high energy CP-violating parameters ϕ_i on $R = BR(\mu \to e\gamma)/BR(\tau \to \mu\gamma)$. The light blue circles correspond to the PMNS phase $\delta = 0$, the red crosses to $\delta = \pi$. Left panel: imposing ϕ_i=0. Right panel: general result with the ϕ_i parameters imposing that the leptogenesis constraint is satisfied.

masses ($\geq 10^{12}$ GeV), where it is driven by the mass-splittings quartic in the neutrino Yukawa couplings. As a consequence, the predictions for $\mu \to e\gamma$ are enhanced and should be observable in next experiments, at least for natural values of the scale of new physics. High energy CP-violating parameters, that disappear in the see-saw relation but take part into leptogenesis, have a significative impact on low-energy processes. The expectation $BR(\mu \to e\gamma)/BR(\tau \to \mu\gamma) \ll 1$, valid in the CP limit, is recovered in the regime of very heavy right-handed neutrinos ($M_R \gg 10^{12}$ GeV).

Acknowledgments

I would like to thank my collaborators V. Cirigliano and G. Isidori and the organizers of the 42nd Rencontres de Moriond for an excellent conference and financial support. I also acknowledge Spanish MEC and FEDER (grant FPA2005-01678).

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