A ("novel") Symmetry Breaking mechanism in Extra Dimensions

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Brief Introduction: Framework

- A (4+*d*)D SU(N) gauge field is equivalent to
  - 4D vector boson degree of freedom $\longrightarrow$ 1
  - 4D scalars degree of freedom $\longrightarrow$ *d*

$$A_M = \{A_\mu, A_i\} \quad \longleftrightarrow \quad \begin{cases} A_\mu & = \text{Vector Bosons} \\ A_i & = \text{Scalar Bosons} \end{cases}$$

- The Scalar Components can play the role of the Higgs: **Gauge-Higgs Unification** (Fairly-Manton '79)

- (4+*d*) Gauge Symmetry protects the Higgs from quadratic divergences: **Solution of the Hierarchy Problem**

Brief Introduction: ...

Now in general 2 problems have to be solved:

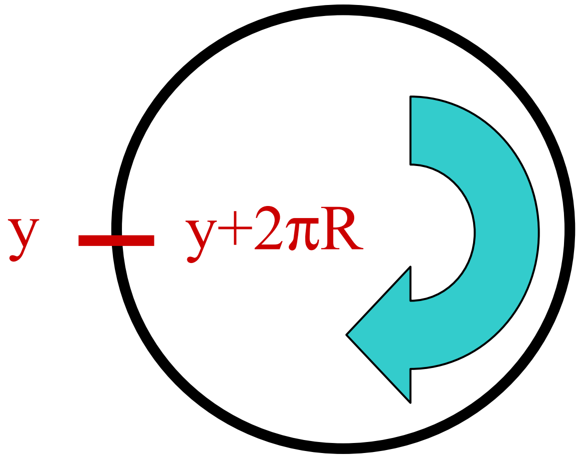
1. Mechanism for HIDING the Extra Dim:
   - No experimental evidence of E.D. at energies presently available: $1/R >> 1$ TeV;

2. Mechanism for BREAKING Gauge Sym:
   - No scalar potential to drive Electro-Weak symmetry breaking;
   - For model building reasons one has to start from larger gauge group.

Basics on 5D Compactification

$$y \xrightarrow{\ \tau\ } \tau[y] = y + 2\pi R \equiv y$$

$$\mathbb{R} \longrightarrow \mathbb{S}^1 = \mathbb{R}/\Lambda$$



- Periodic Boundary Conditions: $A_M(x, \tau[y]) = A_M(x, y)$

$$A_M(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_k e^{i\frac{k\cdot y}{R}} \tilde{A}_M^{(k)}(x) \qquad m_k^2 = \left(\frac{k}{R}\right)^2 \qquad \begin{cases} k = 0 \;\rightarrow\; \text{massless mode} \\ \longrightarrow \tilde{A}_M^{(k)}(x) \text{ 4D KK modes} \\ k \neq 0 \;\rightarrow\; \text{massive modes} \end{cases}$$

- One 4D massless boson (vector/scalar) + tower of massive KK-modes;
- The 4D theory has an unbroken SU(N) symmetry;

- General (Scherk-Schwarz) Boundary Conditions:

[Scherk and Schwarz '79]

$$A_M(x,\tau[y]) = T\ A_M(x,y)\ T^\dagger \qquad \left(T = e^{i\alpha H} \in SU(N)\right)$$

$$A_M(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_k e^{i\frac{k\cdot y}{R}} \left[e^{i\alpha\frac{y}{R}H}\ \tilde{A}_M^{(k)}(x)\ e^{-i\alpha\frac{y}{R}H} \right]$$

$$m_k^2 = \begin{cases} \left(\frac{k}{R}\right)^2 & \to \quad A_M^a = \text{unbroken modes} \ \ (\text{i.e. } [\lambda_a, H] = 0) \\ \left(\frac{k+q_{\hat a}\alpha}{R}\right)^2 & \to \quad A_M^{\hat a} = \text{broken modes} \ \ (\text{i.e. } [\lambda_{\hat a}, H] = q_{\hat a}\alpha) \end{cases}$$

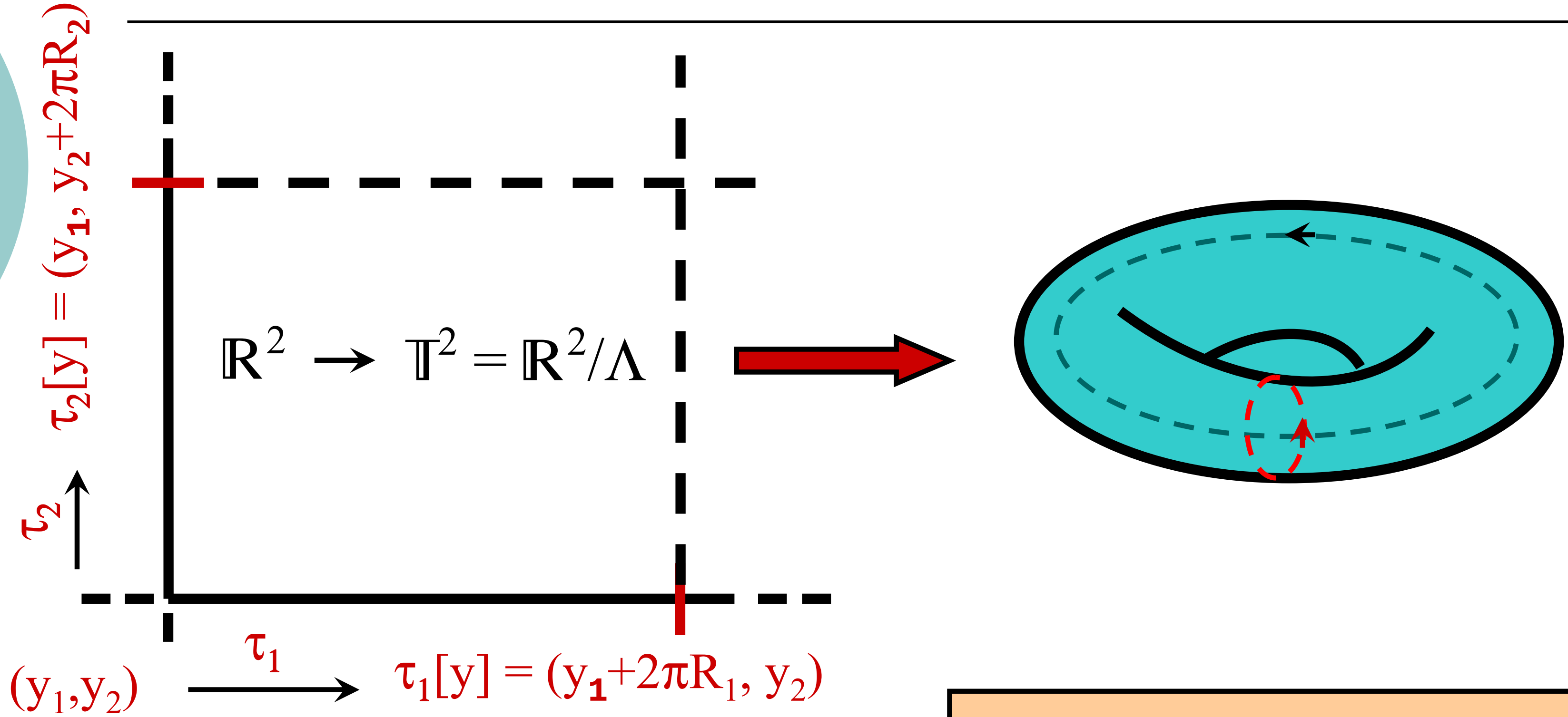
- SS Boundary Conditions can break the Gauge Symmetry
  - If the "phase" $\alpha \neq 0$ then there are no 0-modes associated to the components $A_M^{\hat a}$; the symmetry breaking is rank preserving

$$SU(N) \longrightarrow \mathcal{G} = \{\lambda_a / [\lambda_a, H] = 0\} \supseteq U(1)^{N-1}$$

- The (non-integrable) phase α is associated to the vev of the scalar components $\langle A_i^a \rangle$ (Continuous Wilson Line)
  - α is fixed minimizing the one-loop effective potential. Dynamical (Spontaneous) Symmetry Breaking - **Hosotani mechanism**

[Luscher '83, Hosotani '83]

Basics on 6D Compactification



- Periodic Boundary Conditions:

$$A_M(x, \tau_1[y]) = A_M(x, y)$$

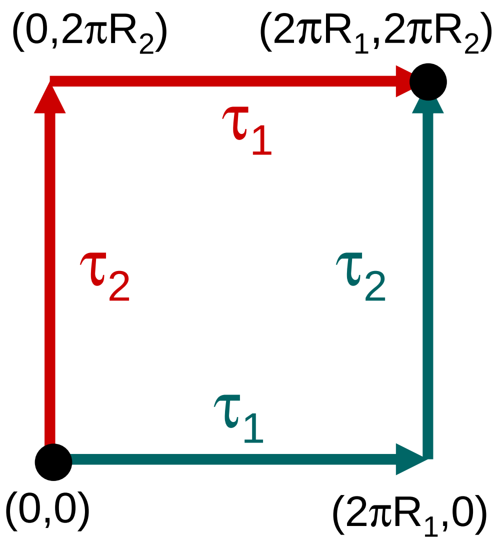
$$A_M(x, \tau_2[y]) = A_M(x, y)$$

- General (Scherk-Schwarz) Boundary Conditions:

$$\mathrm{A}_M(x, \tau_1[y]) = T_1\ A_M(x,y)\ T_1^\dagger \qquad \left(T_1 = e^{i\alpha_1 H_1} \in SU(N)\right)$$

$$\mathrm{A}_M(x, \tau_2[y]) = T_2\ A_M(x,y)\ T_2^\dagger \qquad \left(T_2 = e^{i\alpha_2 H_2} \in SU(N)\right)$$

- Can we choose arbitrary B.C. T_1, T_2 along (y_1, y_2) ?



$$A_M(2\pi R_1, 2\pi R_2) \equiv A_M(\tau_1 \cdot \tau_2[0]) = T_1 \cdot T_2\,[A_M(0,0)]$$

$$A_M(2\pi R_1, 2\pi R_2) \equiv A_M(\tau_2 \cdot \tau_1[0]) = T_2 \cdot T_1\,[A_M(0,0)]$$

['t Hooft '79, 't Hooft '81]

$$T_1 \cdot T_2 = (\qquad) T_2 \cdot T_1$$

't Hooft Consitency Condition

‘t Hooft (magnetic) Flux

- Translations in guage space have to “commute” modulo an element of the center (i.e. identity) of SU(N):

$$T_1 \cdot T_2 = e^{2\pi i \frac{m}{N}} T_2 \cdot T_1$$

- The **‘t Hooft (magnetic) flux *m***:
  - Is an integer number keeping values between 0,…,N-1;
  - Is a topological quantity that identifies equivalence classes of possible vacuum solutions (SU(N)/Z_N “instantons”);

- Boundary Conditions are referred (not unanimously) as:
  - **Untwisted** B.C. if ***m=0*** (mod N)
  - **Twisted** B.C. if ***m≠0*** (mod N)

- **Symmetry Breaking patterns depend on m**

Symmetry Breaking Pattern *m=0*

- The translations T_1 and T_2 commute and they can be chosen in the commuting sub-algebra of SU(N):

$$\mathrm{T}_i = e^{i\alpha_i H_i} \quad \Longleftrightarrow \quad [H_1, H_2] = 0$$

with $\alpha_i \in [0, 2\pi)$

- The parameters α_1 and α_2 are "free" at tree-level and are fixed once the one-loop effective potential is minimized (Hosotani Mechanism). If $\alpha_i \neq 0$ the symmetry is broken;

- The **Symmetry Breaking** is **Rank Preserving** (Hosotani)

$$m_k^2 = \left[\left(\frac{k_1 + q_1\alpha_1}{R_1}\right)^2 + \left(\frac{k_2 + q_2\alpha_2}{R_2}\right)^2\right]$$

$$\begin{cases} q_{1,2} = 0 & \text{unbroken sector } \mathcal{G} \\ q_{1,2} \neq 0 & \text{broken sector } SU(N)/\mathcal{G} \end{cases}$$

$$SU(N) \longrightarrow \mathcal{G} = \{\lambda_a / [\lambda_a, H_{1,2}] = 0\} \supseteq U(1)^{N-1}$$

Dynamical & Spontaneous

Symmetry Breaking pattern $m \neq 0$

- The translations T_1 and T_2 **DO NOT** commute and they cannot be chosen in the commuting sub-algebra of SU(N):

$$\mathrm{T}_i = e^{i\alpha_i \hat{\lambda}_i} \Longleftrightarrow \left[\hat{\lambda}_1, \hat{\lambda}_2\right] \neq 0$$

- For a given m the possible T_i have been classified in terms of 2 constant matrices P,Q and 4 integer coefficients (s_i, t_i):

$$\mathrm{T}_i = \omega_i P^{t_i} Q^{s_i} \qquad \begin{cases} PQ = e^{\frac{2\pi i}{N}} QP \\ \mathrm{s}_1 t_1 - s_2 t_2 = m \end{cases}$$

('t Hooft '81, Van Baal '85)

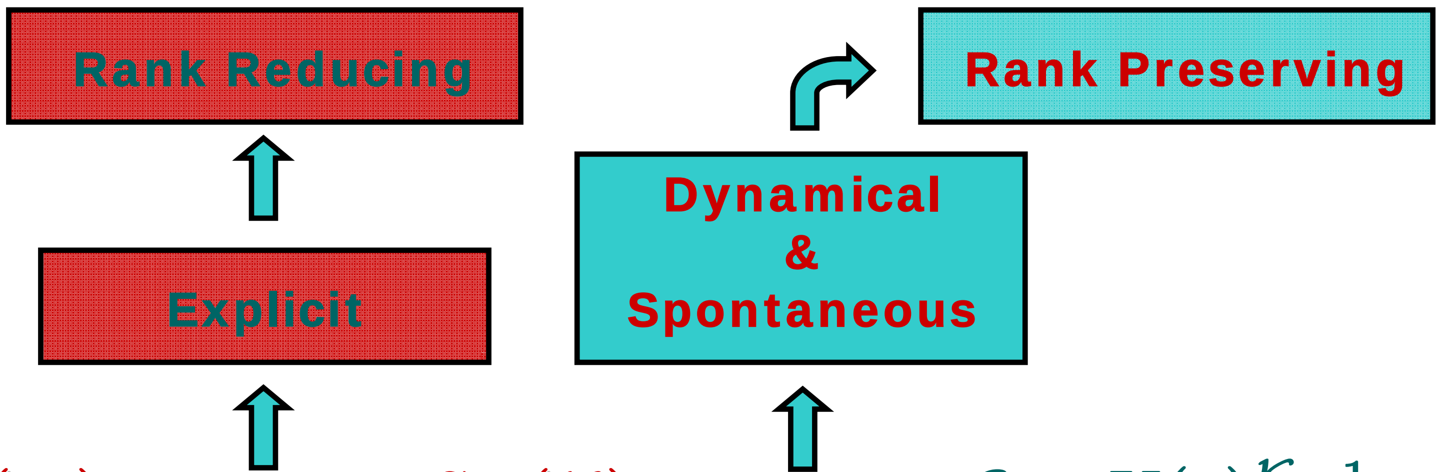
- The parameters α_i are no longer arbitrary (also at tree level) but fixed by previous conditions (Discrete Wilson Lines)

- This induces a **Rank Reducing** **Symmetry Breaking** pattern

$$SU(N) \longrightarrow SU(\mathcal{K}) \qquad \mathcal{K} = \text{g.c.d.}(\text{m,N})$$

- If $\mathcal{K} > 1$ there is a residual gauge invariance:
  - The ω_i are in general non trivial elements of SU($\mathcal{K}$) and we can apply to them the discussion done for m=0 (ω_i commute);
  - A second dynamical (spontaneous) symmetry breaking *a la* Hosotani is possible for SU($\mathcal{K}$);

- The complete Symmetry Breaking pattern now reads:



$$SU(N) \longrightarrow \otimes \longrightarrow SU(\mathcal{K}) \longrightarrow \otimes \longrightarrow \mathcal{G} \supseteq U(1)^{\mathcal{K}-1}$$

Conclusions & Outlook

- Discussed Scherk-Schwarz symmetry breaking patterns in 5D and 6D compactifications;
- Novelty of 6D compactification: Untwisted vs. Twisted case
  - Untwisted sectors: Dynamical SSB a la Hosotani (continuous WL)
  - Twisted sectors: Explicit SB + Dynamical SSB a la Hosotani (discrete + continuous WL);
- New possibilities for Model Building in Extra Dimensions (with no need to introduce orbifolds compactification or complementary to it):
  - Symmetry Breaking from ‘t Hooft Fluxes;
  - Chirality from Magnetic Fluxes (background).